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THE INFLUENCE OF ASYMMETRIC SPACE FILLING IN THE HUMAN LUNG

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ABSTRACT

This paper will present a comparison in terms of respiratory impedance values, between a symmetric morphologic structure and an asymmetric structure of the airways. The model of the respiratory tree is introduced as a recurrent, fractal structure. The intrinsic properties posed by such a system affect directly the respiratory impedance, in a frequency dependent manner. It is therefore important to determine the range of frequencies where these influences are minimal, thus to reduce the effects of inter-patient variability. The results are analyzed by means of complex impedance and Bode plots, on a wide range of frequencies.

Keywords: *fractals, respiratory tree, non-integer order systems, impedance, self-organizing*

2000 *Mathematics Subject Classification*: 92B05, 93A30, 26A33.

1. Introduction

The structure of the human airways in the lungs is not a perfectly symmetric fractal tree; however, with a certain degree of robustness, approximations can be made to ease the computational efforts in simulation studies [1],[2],[3]. Many authors have addressed the intrinsic fractal structures in the human body [1],[4]. In his recent publication, Weibel discusses the reduction of diameter and length by a constant factor with respect to both blood vessels and airways [1]. He recognizes the theoretical contributions of Murray [2], i.e. that the dissipation of energy due to flow of blood or air in a branched tube system can be minimized if the diameter of the two daughter-branches are related to the diameter of the parent as in $d^3_{parent} = d^3_1 + d^3_2$ [3].

In this contribution, we investigate the frequency response of the respiratory system in terms of its equivalent electrical impedance, comparing the influence of an asymmetric and a symmetric tree structure. The comparison between the two responses will indicate whether or not the asymmetry is significant or not in modelling the respiratory impedance and indicate the frequency range where these effects are minimal.

2.Methods and Results

It has been demonstrated by a systematic analysis that the airway tree in different species shows a common fractal structure as in table 1, despite of some gross differences in airway morphology [1],[5]. Nevertheless, we propose to investigate also the case of asymmetric branching in the human lungs. The *asymmetric* case is when the bifurcations are still dichotomous, but they occur in non-sequent levels, as given in table 2. The parameter Δ denotes the asymmetry index. In this case, a parent airway will split into two daughters: one of subsequent level $m + 1$ and one of level $m + 1 + \Delta$. This latter anatomical context is agreed by another group of authors: [6].

An equivalent representation of the respiratory tree as an asymmetric electrical network is given in figure 2. Figure 3 shows the number of branches that are in one generation, for the symmetric and asymmetric lung structure. Notice the different slope which characterizes the space-filling distribution; the top-figure shows that the slope is lower in the asymmetric tree section than in the symmetric tree section.

Since the symmetry is lost, one cannot simplify the electrical network to a lumped ladder network equivalent [7]. Therefore, one must calculate explicitly

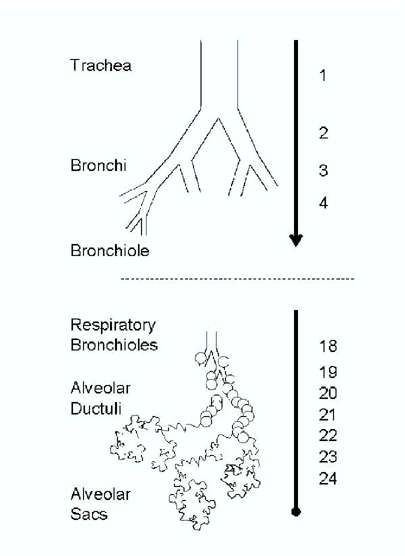

Figure 1: Schematic representation of the bronchial tree: generations 1-16 transport gas and 17-24 provide gas exchange.

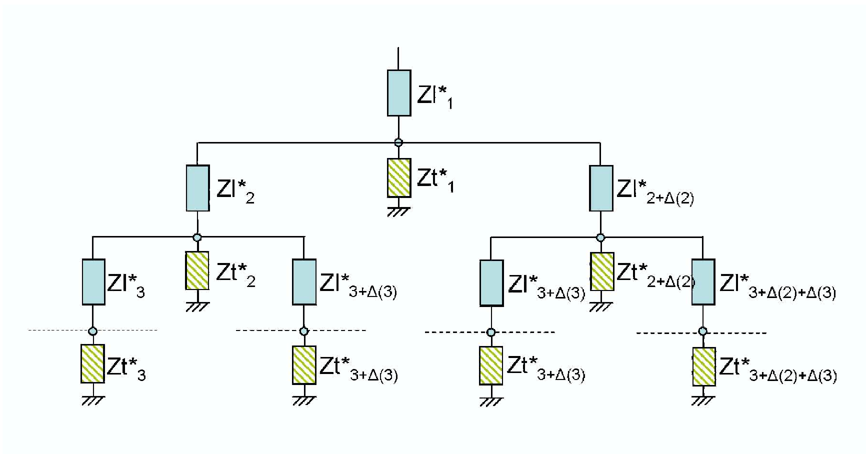

Figure 2: Asymmetric representation for the first four generations, in its electrical equivalent.

Depth m	Length ℓ (cm)	Radius R (cm)	Wall thickness h (cm)	Cartilage fraction κ
1	10.0	0.80	0.3724	0.67
2	5.0	0.6	0.1735	0.5000
3	2.2	0.55	0.1348	0.5000
4	1.1	0.40	0.0528	0.3300
5	1.05	0.365	0.0409	0.2500
6	1.13	0.295	0.0182	0.2000
7	1.13	0.295	0.0182	0.0922
8	0.97	0.270	0.0168	0.0848
9	1.08	0.215	0.0137	0.0669
10	0.950	0.175	0.0114	0.0525
11	0.860	0.175	0.0114	0.0525
12	0.990	0.155	0.0103	0.0449
13	0.800	0.145	0.0097	0.0409
14	0.920	0.140	0.0094	0.0389
15	0.820	0.135	0.0091	0.0369
16	0.810	0.125	0.0086	0.0329
17	0.770	0.120	0.0083	0.0308
18	0.640	0.109	0.0077	0.0262
19	0.630	0.100	0.0072	0.0224
20	0.517	0.090	0.0066	0.0000
21	0.480	0.080	0.0060	0.0000
22	0.420	0.070	0.0055	0.0000
23	0.360	0.055	0.0047	0.0000
24	0.310	0.048	0.0043	0.0000

Table 1: The tube parameters for the sub-glottal airways depths, whereas depth 1 denotes the trachea and depth 24 the alveoli, as used in [1]

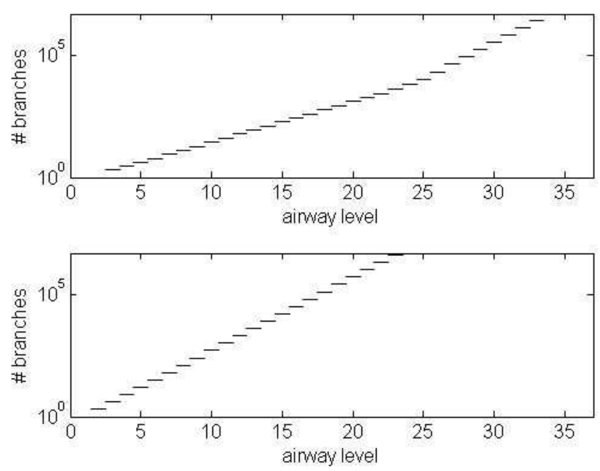

Figure 3: Number of branches for each generation, in the asymmetric (top) and symmetric (bottom) generation. Notice that the Y-axis is logarithmic.

Depth m	Length ℓ (cm)	Radius R (cm)	Wall thickness h (cm)	Cartilage fraction κ	bifurcation Δ
1	10.0	0.80	0.3724	0.67	1
2	5.0	0.6	0.1735	0.5000	2
3	2.2	0.55	0.1348	0.5000	3
4	1.1	0.40	0.0528	0.3300	3
5	1.05	0.365	0.0409	0.2500	3
6	1.13	0.295	0.0244	0.2000	3
7	1.13	0.295	0.0244	0.0926	3
8	0.97	0.270	0.0205	0.0851	3
9	1.08	0.215	0.0149	0.0671	3
10	0.860	0.175	0.0126	0.0526	3
11	0.950	0.175	0.0126	0.0525	3
12	0.990	0.155	0.0118	0.0450	3
13	0.800	0.145	0.0114	0.0410	3
14	0.920	0.140	0.0112	0.0389	3
15	0.820	0.135	0.0111	0.0370	3
16	0.810	0.125	0.0107	0.0329	3
17	0.770	0.120	0.0105	0.0309	3
18	0.640	0.109	0.01	0.0262	3
19	0.630	0.100	0.0096	0.0224	3
20	0.517	0.090	0.0091	0.0000	3
21	0.480	0.080	0.0085	0.0000	3
22	0.420	0.070	0.0079	0.0000	3
23	0.360	0.055	0.0067	0.0000	2
24	0.310	0.048	0.0060	0.0000	2
25	0.250	0.038	0.0050	0.000	1
26	0.11	0.0315	0.0042	0.000	0
27	0.131	0.0265	0.0036	0.000	0
28	0.105	0.024	0.0032	0.000	0
29	0.075	0.0215	0.0029	0.000	0
30	0.059	0.04	0.0052	0.000	0
31	0.048	0.04	0.0052	0.000	0
32	0.048	0.04	0.0052	0.000	0
33	0.048	0.04	0.0052	0.000	0
34	0.048	0.04	0.0052	0.000	0
35	0.048	0.04	0.0052	0.000	0

Table 2: The tube parameters for the sub-glottal airways depths, whereas depth 1 denotes the trachea and depth 35 the alveoli, as used in [6]

the impedance from level 36 to level 1. To avoid complex numerical formulations, the impedance along the longest path was calculated. One should notice that from level 26 onward, the asymmetry index is zero, therefore symmetric bifurcation occurs [6]. The effect of this change in the asymmetry index is visible in figure 3, i.e. a change in the slope. The initial values in the trachea are imposed similarly as in the symmetric case [6]. Figure 4 shows the total impedance by means of its complex representation (left) and its Bode plot (right), for the symmetric and the asymmetric tree, whereas each airway tube is modelled by a gamma $R-L-C$ circuit in both representations [8].

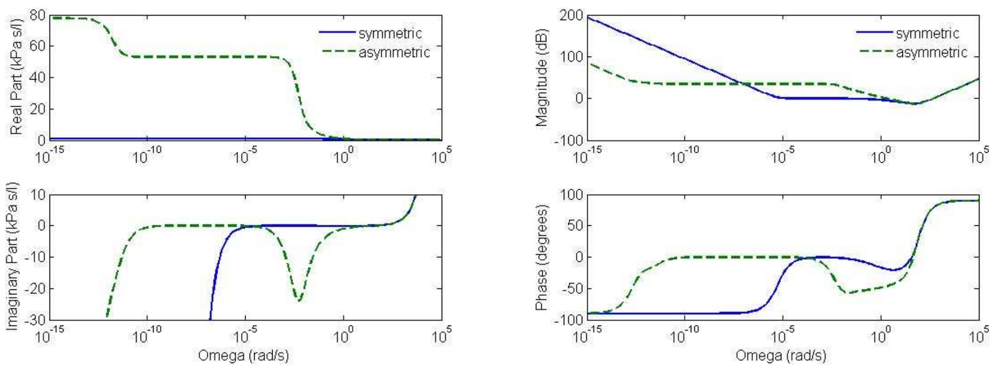

Figure 4: Impedance by means of complex (left) and Bode-plot (right) representation, symmetric (continuous line) and the asymmetric (dashed line) tree.

It is significant to observe that in the frequency interval of clinical interest, $\omega \in [25, 300]$ rad/s, the two impedances tend to behave similarly. A detail of figure 4 can be viewed in figure 5. For the asymmetric case, we have a decrease of about -10dB/dec and a phase of approximately -50^o. In the standard clinical range of frequencies for the forced oscillation lung function test, namely 4-48 Hz, both models give similar results, as depicted in figure 6 below.

3.Conclusions

In this paper, it is observed that the asymmetric structure of the respiratory tree affects the impedance of the respiratory system; however, the frequency range of clinical interest in pre-diagnosis is not significantly affected (4-48 Hz). We conclude that asymmetry plays an important role when viscoelastic effects are envisaged (below 4 Hz) and that simulation studies should take into account

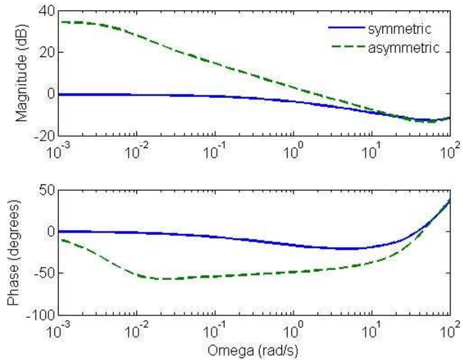

Figure 5: Detailed view of the impedance by means of the Bode-plot representation, for the symmetric (continuous line) and the asymmetric (dashed line) tree.

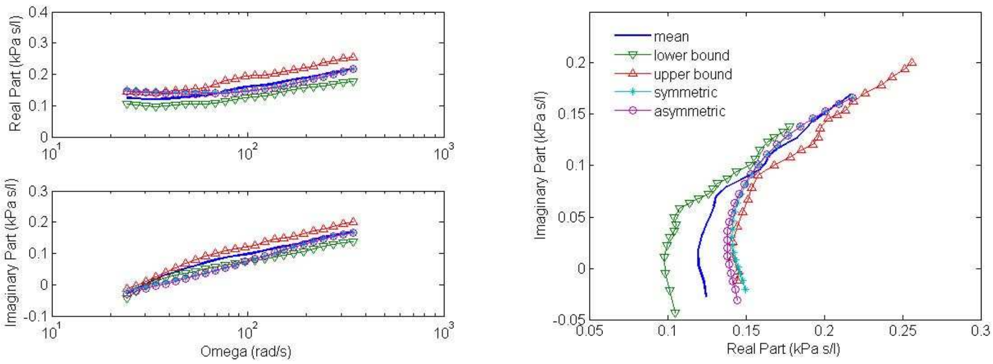

Figure 6: The estimated impedance within the measured frequency range for the symmetric (*) and the asymmetric (o) case, against averaged data from healthy subjects (left) and equivalent polar plot representation (right).

these effects to avoid biased results. Moreover, from a pathologic standpoint, asymmetry becomes important when respiratory diseases are investigated in terms of their effects on morphology, structure and air flow.

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