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CHARACTERIZATION OF GENERALIZED γ-CLOSED SETS

C. Carpintero, E. Rosas and M. Salas Brown

Abstract. A new classes of sets called generalized γ-semi closed, semi generalized γ-semi closed and semi generalized γ-closed are introduced. Some of its basic properties are studied and characterized.

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1. Introduction

In [14], Kasahara intoduced the concept of operator associated with a topology τ of a space X as a map α from τ to $\wp(X)$ such that $U \subseteq \alpha(U)$ for all $U \in \tau$. Ogata [20], studies and extends these notions and introduced the concept of τ_α open sets as a generalization of open sets. E. Rosas et al. [21], modified the domain of the operator, taking this from $\wp(X)$ into $\wp(X)$, and find new forms of generalization open sets. In several articles, see [5],[6],[7],[10] [21] are given characterizations of these classes of sets, also have been found new properties and further characterizations of these sets. Others mathematicians taking specific operators as $\alpha(A) = Cl(Int(A))$ [16] (respectively $\alpha(A) = Int(Cl(A))$, $\alpha(A) = Cl(Int(Cl(A)))$[1], $\alpha(A) = Int(Cl(Int(A)))$[19]) defined the notions of semi open (respectively preopen, α-open, β-open) as $A \subseteq X$ is said semi open, (respectively preopen, α-open, β-open) if $A \subseteq Cl(Int(A))$, (respectively $A \subseteq Int(Cl(A))$, $A \subseteq Int(Cl(Int(A)))$, $A \subseteq Cl(Int(Cl(A)))$. In the same form $A \subseteq X$ is semiclosed, (respectively preclosed, α-closed, β-closed) if its complement is semi open, (respectively preopen, α-open, β-open). In [9], Csaszar introduced on a topological space X and $\gamma : exp(X) \mapsto exp(X)$ a mapping that satisfies the following conditions: (1) if $A \subset B$, then $\gamma(A) \subset \gamma(B)$,(2) $\gamma(\emptyset) = \emptyset$, $\gamma(X) = X$ and (3) for $A \subset X$ and an open set $G \subset X$, $G \cap \gamma(A) \subset \gamma(G \cap A)$. In this article, using monotone operators associated with a topology, we characterize the generalized open sets, as well as, we give some characterizations of generalized γ-semi closed, semi generalized γ-semi closed and semi generalized γ-closed sets.

2. Preliminaries

Throughout this paper, (X, τ) always mean topological space in which no separation axioms are assumed unless explicitly stated. Let $A \subseteq X$, $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$ denote the closure of A and the interior of A with respect to τ, respectively.

Definition 1. Let A be a subset of X. Then:

1. $scl(A) = \cap\{B : A \subseteq B, B$ is a semi closed in $X\}$ is called a semiclosure of A [8];
2. $\alpha - cl(A) = \cap\{B : A \subseteq B, B$ is α-closed in $X\}$ is called the α-closure of A [18];
3. $\beta - cl(A) = \cap\{B : A \subseteq B, B$ is β-closed in $X\}$ is called the β-closure of A [2].

Definition 2. Let A be a subset of X. Then A is called:

1. a generalized closed (g-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an open set [8];
2. a semigeneralized semi closed (sg-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is a semi open set [18];
3. a generalized semi closed (gs-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an open set [3];
4. a generalized α-closed (gα-closed) if $\alpha - cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an open set [17];
5. a generalized β-closed (gβ-closed) if $\beta - cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an open set [11];

Theorem 1. *[13] Let x be a point in X, then $\{x\}$ is either nowhere dense or preopen.*

From the above theorem, we obtain a decomposition of X, namely $X = X_1 \cup X_2$, where

$$X_1 = \{x \in X : \{x\} \text{ is nowhere dense in } X\}$$

and

$$X_2 = \{x \in X : \{x\} \text{ is preopen in } X\}.$$

3. OPERATORS ASSOCIATED TO A TOPOLOGY

Definition 3. *Let A be a subset of X and γ an operator on τ. then A is called γ-open if $A \subseteq \gamma(A)$.*

Observe that all open sets are γ-open sets. But, in general, γ-open sets does not implies open sets.

Example 1. Let $X = \{a, b, c\}$ with the topology $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}$. Define $\gamma : \wp(X) \to \wp(X)$ as follows:

$$\gamma(A) = \begin{cases} Cl(A) & \text{if } b \in A \\ \{a\} & \text{if } b \notin A. \end{cases}$$

Then $\{a, c\}$ is γ-open but is not open.

If γ satisfies the conditions (1), (2) and (3) of Csaszar [9], then γ is an associated operator on τ. But, there exists a monotone operator $\gamma : \wp(X) \to \wp(X)$ for which the conditions (1), (2) and (3) does not necessarily holds.

Example 2. *Let $(\mathbb{R}, \tau)$ be a topological space where τ is the usual topology. Consider $f : \mathbb{R} \to \mathbb{R}$, defined as $f(x) = 0$ for all $x \in \mathbb{R}$. Define γ as follows:*

$$\gamma(A) = f^{-1}(f(A))$$

for all $A \subseteq X$, then $A \subseteq f^{-1}(f(A)) = \gamma(A)$. Hence, γ is a monotone operator on τ. Observe that γ satisfied the conditions (1) and (2) but not (3) of Csaszar [9], because if we consider $U = (0, 2)$ and $A = [3, 4]$, then $U \cap f^{-1}(f(A)) = (0, 2)$ and $f^{-1}(f(U \cap A)) = \emptyset$.

Remark 1. *Let A be a subset of X, then A is called γ-closed if $X - A$ is γ-open.*

Definition 4. *We said that γ is a monotone operator if A, B are subsets of X with $A \subseteq B$ then $\gamma(A) \subseteq \gamma(B)$.*

Remark 2. *Denote by $\Gamma(X)$ the collection of all monotone operators on X. Note that if we take $\gamma(A) = Cl(Int(A))$, (respectively $\gamma(A) = Cl(Int(A))$, $\gamma(A) = Cl(Int(Cl(A)))$, $\gamma(A) = Int(Cl(Int(A)))$), then, we obtain the notions of semi open, (respectively preopen, α-open, β-open) sets.*

Theorem 2. *Let $\gamma \in \Gamma$, if $\{A_i : i \in I\}$ is a collection of γ-open sets and then $\cup_{i \in I} A_i$ is a γ-open set.*

Proof. It is enough to proof that if A_1, A_2 are γ-open set, then $A_1 \cup A_2$ is γ-open set. $A_1 \cup A_2 \subseteq \gamma(A_1) \cup \gamma(A_2)) \subseteq \gamma(A_1 \cup A_2)$.

The following example shows that if , $\gamma \notin \Gamma$, then the union of γ-open set may be not γ-open.

Example 3. Let $X = \{a, b, c\}$ with the topology $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Define $\gamma : \wp(X) \to \wp(X)$ as follows: $\gamma(\{a\}) = \{a\}$, $\gamma(\{b\}) = \{a, b\}$, $\gamma(\{c\}) = \{c, b\}$, $\gamma(\{a, b\}) = \{a, b\}$, $\gamma(\{a, c\}) = \{a, b\}$, $\gamma(\{b, c\}) = \{b, c\}$, $\gamma(X) = X$ and $\gamma(\emptyset) = \emptyset$. Then $\{a\}$ and $\{c\}$ are γ-open but $\{a, c\}$ is not γ-open.

Definition 5. Let A be a subset of X and $\gamma \in \Gamma$, we define the γ-interior of A and the γ-closure of A as follows:

1. $\gamma - Int(A) = \bigcup_{W \subseteq A}\{W : W\ \gamma \text{ open set}\}$;
2. $\gamma - Cl(A) = \bigcap_{A \subseteq F}\{F : F\ \gamma \text{ closed set}\}$.

Theorem 3. *Let $A \subseteq X$ and $\gamma \in \Gamma$, a point $x \in X$ belongs to $\gamma - Cl(A)$ if and only if $U \cap A \neq \emptyset$ for all γ-open set U containing x.*

Proof. Denote by $E = \{y \in X : U \cap A \neq \emptyset, \text{with } U\gamma-\text{open set and } y \in U\}$. We shall prove that $\gamma - Cl(A) = E$. Let $x \notin E$. Then there exists a γ-open set U containing x such that $U \cap A = \emptyset$. This implies that $X - U$ is γ-closed and $\gamma - Cl(A) \subseteq X - U$. It follows that $x \notin \gamma - Cl(A)$. Conversely, let $x \notin \gamma - Cl(A)$. Then there exist a γ-closed F such that $A \subseteq F$ and $x \notin F$. Then we have that $x \in X - F$, $X - F$ and $(X - F) \cap A = \emptyset$. This implies that $x \notin E$. Hence $E \subseteq \gamma - Cl(A)$. Therefore $\gamma - Cl(A) = E$.

Definition 6. Let A be a subset of X, then A is called γ-semi open if there exists $U \in \tau$ such that $U \subseteq A \subseteq \gamma(U)$.

Remark 3. *If $\gamma \in \Gamma$, then A is γ-semi open if and only if $A \subseteq \gamma(Int(U))$.*

Theorem 4. *If $\{A_i : i \in I\}$ is a collection of γ-semi open sets and $\gamma \in \Gamma$, then $\cup_{i \in I} A_i$ is a γ-semi open set.*

Proof. It is enough to proof that if A_1, A_2 are γ-semi open sets, then $A_1 \cup A_2$ is a γ-semi open set. Since A_1, A_2 are γ-semi open, there exist O_1, O_2 open sets such that $O_1 \subseteq A_1 \subseteq \gamma(O_1)$ and $O_2 \subseteq A_2 \subseteq \gamma(O_2)$, then $O_1 \cup O_2 \subseteq A_1 \cup A_2 \subseteq \gamma(O_1) \cup \gamma(O_2)) \subseteq \gamma(O_1 \cup O_2)$.

Definition 7. Let A be a subset of X and $\gamma \in \Gamma$, we define the γ-semi interior of A and the γ-semi closure of A as follows:

1. $\gamma - sInt(A) = \bigcup_{W \subseteq A}\{W : W\ \gamma$ semi open set$\}$;
2. $\gamma - sCl(A) = \bigcap_{A \subseteq F}\{F : F\ \gamma$ semi closed set$\}$.

Theorem 5. *Let $A \subseteq X$ and $\gamma \in \Gamma$, then a point $x \in X$ belongs to $\gamma - sCl(A)$ if and only if $U \cap A \neq \emptyset$ for all γ-semi open set U containing x.*

Proof. The proof is similar to the Theorem 3, doing the necessary changes.

Theorem 6. *Let $\gamma \in \Gamma$, then for each $x \in X$, $\gamma - sInt(\gamma - sCl(\{x\})) = \emptyset$ or $x \in \gamma - sInt(\gamma - sCl(\{x\}))$.*

Proof. If $\gamma - sInt(\gamma - sCl(\{x\})) = \emptyset$, there is nothing to prove. Now, suppose that $\gamma - sInt(\gamma - sCl(\{x\})) \neq \emptyset$, then there exists $y \in \gamma - sInt(\gamma - sCl(\{x\}))$, hence there exists a γ-semi open set S such that $y \in S \subseteq \gamma - sCl(\{x\})$, in consequence, $S \cap \{x\} \neq \emptyset$. Since $x \in S \subseteq \gamma - sCl(\{x\})$. It follows that, $x \in \gamma - sInt(\gamma - sCl(\{x\}))$.

In the notation of the above theorem, we obtain a decomposition of X, namely $X = X_1^* \cup X_2^*$, where

$$X_1^* = \{x \in X : \{x\} \text{ is } \gamma - \text{semi-nowhere dense in } X\}$$

and

$$X_2^* = \{x \in X : \{x\} \text{ is } \gamma - \text{semi-preopen in } X\}.$$

Theorem 7. *Let $\gamma \in \Gamma$, then for each $x \in X$, $\gamma - Int(\gamma - Cl(\{x\})) = \emptyset$ or $x \in \gamma - Int(\gamma - Cl(\{x\}))$. '*

Proof. If $\gamma - Int(\gamma - Cl(\{x\})) = \emptyset$, there is nothing to prove. Now, suppose that $\gamma - Int(\gamma - Cl(\{x\})) \neq \emptyset$, then there exists $y \in \gamma - Int(\gamma - Cl(\{x\}))$, hence there exists a γ- open set S such that $y \in S \subseteq \gamma - Cl(\{x\})$, in consequence, $S \cap \{x\} \neq \emptyset$. Since $x \in S \subseteq \gamma - Cl(\{x\})$. It follows that, $x \in \gamma - Int(\gamma - Cl(\{x\}))$.

In the notation of the above theorem, we obtain a decomposition of X, namely $X = X_1^\gamma \cup X_2^\gamma$, where

$$X_1^\gamma = \{x \in X : \{x\} \text{ is } \gamma - \text{nowhere dense in } X\}$$

and

$$X_2^\gamma = \{x \in X : \{x\} \text{ is } \gamma - \text{preopen in } X\}.$$

4. GENERALIZED γ-SEMI CLOSED SETS

In this section we define the generalized γ-semi closed set and obtain some generalization of them.

Definition 8. Let A be a subset of X and $\gamma \in \Gamma$, A is called generalized γ-semi closed set (gγ-sclosed) if $\gamma - scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an open set.

Theorem 8. *Let $\gamma \in \Gamma$, every γ-closed set is gγ-sclosed in X.*

Definition 9. Let A be a subset of X, the intersection of all open subsets of X containing A is called the kernel of A and is denoted by $ker(A)$.

Lemma 9. *Let $\gamma \in \Gamma$ and $A \subseteq X$. A is gγ-sclosed if and only if $\gamma - sCl(A) \subseteq ker(A)$*

Proof. Let $D = \{W : A \subseteq W, W \text{ open}\}$. Then $ker(A) = \bigcap_{W \in D} W$. If $W \in D$, $A \subseteq W$ then $\gamma - scl(A) \subseteq W$. Hence $\gamma - scl(A) \subseteq ker(A)$.
Conversely, suppose that $\gamma - scl(A) \subseteq ker(A) = \bigcap_{W \in D} W$. Let W be an open set such that $A \subseteq W$, then $W \in D$, hence $\bigcap_{W \in D} W \subseteq W$, in consequence, $\gamma - scl(A) \subseteq W$ and hence A is gγ-sclosed.

Lemma 10. *Let $\gamma \in \Gamma$, $X_2^* \cap \gamma - sCl(A) \subseteq \gamma - sker(A)$ for any subset A of X.*

Proof. Let $x \in X_2^* \cap \gamma - sCl(A)$ and $x \notin ker(A)$, then, there exists an open $W \supset A$, such that $x \notin W$. A subset $F = X - W$ is closed. Since $x \in F$, the $\gamma - sCl(\{x\}) \subseteq \gamma - sCl(\{F\}) \subseteq Cl(\{F\})$. Since $\{x\} \subseteq \gamma - sCl(A)$, follows $\gamma - sInt(\gamma - sCl(\{x\})) \subseteq \gamma - sInt(\gamma - sCl(A)) \subset \gamma - sCl(A)$. Since $x \notin X_1^*$, $\gamma - sInt(\gamma - sCl(\{x\})) \neq \emptyset$. Let $y \in A \cap \gamma - sInt(\gamma - sCl(\{x\}))$, then $y \in A \cap F$, this is a contradiction.

Lemma 11. *Let $\gamma \in \Gamma$ and $A \subseteq X$ if $X_1^* \cap \gamma - sCl(A) \subseteq A$ then A is gγ-sclosed in X.*

Proof. Suppose that $X_1^* \cap \gamma - sCl(A) \subseteq A$. Since $A \subseteq ker(A)$, by the above theorem, $X_2^* \cap \gamma - sCl(A) \subseteq ker(A)$. follows $\gamma - sCl(A) \subseteq (X_1^* \cup X_2^*) \cap \gamma - sCl(A) \subseteq A \cup X_2^* \cap \gamma - sCl(A) \subseteq ker(A)$. Hence A is sgγ-sclosed.

The converse of the above lemma may be not true.

Example 4. Let $X = \{a, b, c, d\}$ with the topology $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Define $\gamma : \wp(X) \to \wp(X)$ as follows:

$$\gamma(A) = \begin{cases} A \cup \{b\} & \text{if } A \neq \{a\} \\ \{a\} & \text{if } A = \{a\}. \end{cases}$$

Take $A = \{c\}$ then $X_1^* = \{c, d\}$, $\gamma - sCl(A) = \{c, d\}$, then $X_1^* \cap \gamma - sCl(A) = \{c, d\}$ is not contained in A, but A is gγ-sclosed in X.

Remark 4. *If $\gamma \in \Gamma$ and $A_{i\in I}$ is a collection of $g\gamma$-sclosed sets in X, then $\bigcap_{i\in I} A_i$ not necessarily is $g\gamma$-sclosed in X. Take in the above example, $A = \{a, d\}$ and $B = \{a, c\}$ are $g\gamma$-sclosed but $A \cap B = \{a\}$ is not $g\gamma$-sclosed.*

Remark 5. *If $\gamma \in \Gamma$ and $A_{i\in I}$ is a collection of $g\gamma$-sclosed sets in X, then $\bigcup_{i\in I} A_i$ not necessarily is $g\gamma$-sclosed in X. If we Take $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\gamma(A) = Int(Cl(Int(A)))$, then $A = \{a\}$ and $B = \{b\}$ are $g\gamma$-sclosed but $A \cup B = \{a, b\}$ is not $g\gamma$-sclosed.*

Theorem 12. *Let $\gamma \in \Gamma$. If A is $g\gamma$-sclosed in X then $\gamma - sCl(A) \setminus A$ contains no nonempty closed sets.*

Proof. Suppose that A is gγ-sclosed, and F a closed set containing in $\gamma - sCl(A) \setminus A$. It follows that $F \subseteq \gamma - sCl(A)$, in consequence, $A \subseteq F^c$ and hence, $\gamma - sCl(A) \subseteq F^c$, this implies that $F \subseteq (\gamma - sCl(A))^c$, therefore $F = \emptyset$.

Theorem 13. *Let $\gamma \in \Gamma$. If A is $g\gamma$-sclosed in X and $A \subseteq B \subseteq \gamma - sCl(A)$, then B is $g\gamma$-sclosed in X.*

Proof. Let U be an open set such that $B \subseteq U$, then $\gamma - sCl(A) \subseteq \gamma - sCl(B) \subseteq \gamma - sCl(\gamma - sCl(A)) = \gamma - sCl(A) \subseteq U$. In consequence B is gγ-sclosed in X.

5. SEMI GENERALIZED γ-SEMI CLOSED SETS

In this section we define the generalized sgγ-sclosed set and obtain some generalization of them.

Definition 10. Let A be a subset of X and $\gamma \in \Gamma$, A is called semi generalized γ-semi closed set (sgγ-sclosed) if $\gamma - scl(A) \subseteq U$ whenever $A \subseteq U$ and U is a γ-semi open set.

Theorem 14. *Let $\gamma \in \Gamma$, every γ-closed set is $sg\gamma$-closed in X.*

Definition 11. Let A be a subset of X, the intersection of all γ-semi open subsets of X containing A is called the γ-semikernel of A and is denoted by $\gamma - sker(A)$.

Lemma 15. *Let $\gamma \in \Gamma$ and $A \subseteq X$. A is $sg\gamma$-sclosed if and only if $\gamma - sCl(A) \subseteq \gamma - sker(A)$*

Proof. Let $D = \{W : A \subseteq W, W\gamma\text{-semi open}\}$. Then $\gamma - sker(A) = \bigcap_{W\in D} W$. If $W \in D$, $A \subseteq W$ then $\gamma - scl(A) \subseteq W$. Hence $\gamma - scl(A) \subseteq \gamma - sker(A)$.
Conversely, suppose that $\gamma - scl(A) \subseteq \gamma - sker(A) = \bigcap_{W\in D} W$. Let W be a γ-semi open set such that $A \subseteq W$, then $W \in D$, hence $\bigcap_{W\in D} W \subseteq W$, in consequence, $\gamma - scl(A) \subseteq W$ and hence A is sgγ-sclosed.

Lemma 16. *Let $\gamma \in \Gamma$, $X_2^* \cap \gamma - sCl(A) \subseteq \gamma - sker(A)$ for any subset A of X.*

Proof. Let $x \in X_2^* \cap \gamma - sCl(A)$ and $x \notin \gamma - sker(A)$, then, there exists a γ-semi open $W \supset A$, such that $x \notin W$. A subset $F = X - W$ is γ-semi closed. Since $x \in F$, the $\gamma - sCl(\{x\}) \subseteq F$. Since $\{x\} \subseteq \gamma - sCl(A)$, follows $\gamma - sInt(\gamma - sCl(\{x\})) \subseteq \gamma - sInt(\gamma - sCl(A)) \subset \gamma - sCl(A)$. Since $x \notin X_1^*$, $\gamma - sInt(\gamma - sCl(\{x\})) \neq \emptyset$. Let $y \in A \cap \gamma - sInt(\gamma - sCl(\{x\}))$, then $y \in A \cap F$, this is a contradiction.

Lemma 17. *Let $\gamma \in \Gamma$ and $A \subseteq X$ if $X_1^* \cap \gamma - sCl(A) \subseteq A$ then A is sgγ-sclosed in X.*

Proof. Suppose that $X_1^* \cap \gamma - sCl(A) \subseteq A$. Since $A \subseteq \gamma - sker(A)$, by the above theorem, $X_2^* \cap \gamma - sCl(A) \subseteq \gamma - sker(A)$. follows $\gamma - sCl(A) \subseteq (X_1^* \cup X_2^*) \cap \gamma - sCl(A) \subseteq A \cup X_2^* \cap \gamma - sCl(A) \subseteq \gamma - sker(A)$. Hence A is sgγ-sclosed.

The converse of the above lemma may be not true.

Example 5. Let $X = \{a, b, c, d\}$ with topology $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}$. Define $\gamma : \wp(X) \to \wp(X)$ as follows:

$$\gamma(A) = \begin{cases} A \cup \{b\} & \text{if } A \neq \{a\} \\ \{a\} & \text{if } A = \{a\}. \end{cases}$$

Take $A = \{c\}$ then $X_1^* = \{c, d\}$, $\gamma - sCl(A) = \{c, d\}$, then $X_1^* \cap \gamma - sCl(A) = \{c, d\}$ is not contained in A, but A is sgγ-sclosed in X.

Remark 6. *If $\gamma \in \Gamma$ and $A_{i \in I}$ is a collection of sgγ-sclosed sets in X, then $\bigcap_{i \in I} A_i$ not necessarily is sgγ-sclosed in X. Take in the above example, $A = \{a, d\}$ and $B = \{a, c\}$ are sgγ-sclosed but $A \cap B = \{a\}$ is not sgγ-sclosed.*

Remark 7. *If $\gamma \in \Gamma$ and $A_{i \in I}$ is a collection of sgγ-sclosed sets in X, then $\bigcup_{i \in I} A_i$ not necessarily is sgγ-sclosed in X. If we Take $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\gamma(A) = Int(Cl(Int(A)))$, then $A = \{a\}$ and $B = \{b\}$ are sgγ-sclosed but $A \cup B = \{a, b\}$ is not sgγ-sclosed.*

Theorem 18. *Let $\gamma \in \Gamma$. A is sgγ-sclosed in X if and only if $\gamma - sCl(A) \setminus A$ contains no nonempty γ-semi closed sets.*

Proof. Suppose that A is sgγ-sclosed, and F a γ-semi closed set containing in $\gamma - sCl(A) \setminus A$, it follows that $F \subseteq \gamma - sCl(A)$, in consequence, $A \subseteq F^c$ and hence, $\gamma - sCl(A) \subseteq F^c$, this implies that $F \subseteq (\gamma - sCl(A))^c$, therefore $F = \emptyset$.
Conversely, suppose that $\gamma - sCl(A) \setminus A$ contains no nonempty γ-semi closed sets, Let $A \subseteq G$, G γ-semi open set. If $\gamma - sCl(A) \subsetneq G$, implies that $\gamma - sCl(A) \cap G^c \neq \emptyset$, since $\gamma - sCl(A)$ and G^c are γ-semi closed sets in X, then, $\emptyset \subsetneq \gamma - sCl(A) \cap G^c \subseteq \gamma - sCl(A) \setminus A$. Therefore, $\gamma - sCl(A) \setminus A$ contains a nonempty γ-semi closed set. This is a contradiction.

Theorem 19. *Let $\gamma \in \Gamma$. If A is sgγ-sclosed in X and $A \subseteq B \subseteq \gamma - sCl(A)$, then B is sgγ-sclosed in X.*

Proof. Let U be an γ-semi open set such that $B \subseteq U$, then $\gamma - sCl(A) \subseteq \gamma - sCl(B) \subseteq \gamma - sCl(\gamma - sCl(A)) = \gamma - sCl(A) \subseteq U$. In consequence, B is sgγ-sclosed in X.

6. SEMI GENERALIZED γ-CLOSED SETS

In this section, we define the generalized sg γ-closed set and obtain some generalization of them.

Definition 12. Let A be a subset of X and $\gamma \in \Gamma$, A is called semi generalized γ-closed set (sgγ-closed) if $\gamma - cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an semi open set.

Theorem 20. *Let $\gamma \in \Gamma$, every γ-closed set is sgγ-closed in X.*

Remark 8. *The sgγ-closed set need not be γ-closed.*

Definition 13. Let A be a subset of X, the intersection of all semi open subsets of X containing A is called the semikernel of A and is denoted by $sker(A)$.

Lemma 21. *Let $\gamma \in \Gamma$ and $A \subseteq X$. A is sgγ-closed if and only if $\gamma - Cl(A) \subseteq sker(A)$*

Proof. Let $D = \{W : A \subseteq W, W \text{ semi open}\}$. Then $sker(A) = \bigcap_{W \in D} W$. If $W \in D$, $A \subseteq W$ then $\gamma - cl(A) \subseteq W$. Hence $\gamma - cl(A) \subseteq sker(A)$.
Conversely, suppose that $\gamma - cl(A) \subseteq sker(A) = \bigcap_{W \in D} W$. Let W be a semi open set such that $A \subseteq W$, then $W \in D$, hence $\bigcap_{W \in D} W \subseteq W$, in consequence, $\gamma - cl(A) \subseteq W$ and hence A is sgγ-closed.

Lemma 22. *Let $\gamma \in \Gamma$, $X_2 \cap \gamma - Cl(A) \subseteq sker(A)$ for any subset A of X.*

Proof. Let $x \in X_2 \cap \gamma - Cl(A)$ and $x \notin sker(A)$, then, there exists a semi open set $W \supset A$, such that $x \notin W$. A subset $F = X - W$ is semi closed. Since the $sCl(\{x\}) = \{x\} \cup Int(Cl(\{x\}))$. Follows that $sCl(\{x\}) \subseteq F$, because $x \in F$. Since $Cl(\{x\}) \subseteq Cl(A)$, follows $Int(Cl(\{x\})) \subseteq Int(Cl(A)) \subset A \cup Int(Cl(A)) = sCl(A)$. Since $x \notin X_1$, $Int(Cl(\{x\})) \neq \emptyset$. Let $y \in A \cap Int(Cl(\{x\}))$, then $y \in A \cap F$. This is a contradiction.

Lemma 23. *Let $\gamma \in \Gamma$. A subset A of X is sgγ-closed in X if and only if $X_1 \cap \gamma - Cl(A) \subseteq A$.*

Proof. Suppose that A is sgγ-closed in X. Let $x \in X_1 \cap \gamma - Cl(A)$ and $x \notin A$, then $x \in X_1$ and $x \in \gamma - Cl(A)$. Since $x \in X_1$, $Int(Cl(\{x\})) = \emptyset$, therefore, $sCl(\{x\}) = \{x\}$, in consequence, $\{x\}$ is semi closed. Let $W = X - \{x\}$, W is semi open and $A \subset W$, Hence $\gamma - Cl(A) \subset W$, this is a contradiction.
Conversely, suppose that $X_1 \cap \gamma - Cl(A) \subseteq A$. Since $A \subseteq sker(A)$, by the above theorem, $X_2 \cap \gamma - Cl(A) \subseteq sker(A)$. follows $\gamma - Cl(A) \subseteq (X_1 \cup X_2) \cap \gamma - Cl(A) \subseteq A \cup X_2 \cap \gamma - Cl(A) \subseteq sker(A)$. Hence A is sgγ-closed.

Theorem 24. *Let $\gamma \in \Gamma$. If $A_{i \in I}$ is a collection of sgγ-closed sets in X, then $\bigcap_{i \in I} A_i$ is sgγ-closed in X.*

Proof. Let $A = \bigcap_{i \in I} A_i$, then for each $i \in I$, A_i is sgγ-closed, follows that, $X_1 \cap \gamma - Cl(A) \subseteq A_i$ for each $i \in I$. Hence, $X_1 \cap \gamma - Cl(A) \subseteq A$.

Remark 9. *If $\gamma \in \Gamma$ and $A_{i \in I}$ is a collection of sgγ-closed sets in X, then $\bigcup_{i \in I} A_i$ not necessarily is sgγ-closed in X. If we Take $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\gamma(A) = Int(Cl(Int(A)))$, then $A = \{a\}$ and $B = \{b\}$ are sgγ-closed but $A \cup B = \{a, b\}$ is not sgγ-closed.*

Theorem 25. *Let $\gamma \in \Gamma$. A is sgγ-closed in X if and only if $\gamma - Cl(A) \setminus A$ contain no nonempty γ-closed sets.*

Proof. Suppose that A is sgγ-closed, and F a γ- closed set containing in $\gamma - Cl(A) \setminus A$, it follows that $F \subseteq \gamma - Cl(A)$, in consequence, $A \subseteq F^c$ and hence, $\gamma - Cl(A) \subseteq F^c$, this implies that $F \subseteq (\gamma - Cl(A))^c$, therefore $F = \emptyset$.
Conversely, suppose that $\gamma - Cl(A) \setminus A$ contains no nonempty γ- closed sets, Let $A \subseteq G$, G γ- open set. If $\gamma - Cl(A) \subsetneq G$, implies that $\gamma - Cl(A) \cap G^c \neq \emptyset$, since $\gamma - Cl(A)$ and G^c are γ- closed sets in X, then, $\emptyset \subsetneq \gamma - Cl(A) \cap G^c \subseteq \gamma - Cl(A) \setminus A$. Therefore, $\gamma - Cl(A) \setminus A$ contains a nonempty γ- closed set. This is a contradiction.

Theorem 26. *Let $\gamma \in \Gamma$. If A is sgγ-closed in X and $A \subseteq B \subseteq \gamma - Cl(A)$, then B is sgγ-closed in X.*

Proof. Let U be an γ-open set such that $B \subseteq U$, then $\gamma - Cl(A) \subseteq \gamma - Cl(B) \subseteq \gamma - Cl(\gamma - Cl(A)) = \gamma - Cl(A) \subseteq U$. In consequence, B is sgγ-closed in X.

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Carlos R. Carpintero F
Department of Mathematics, Universidad De Oriente,
Nucleo De Sucre Cumana, Venezuela
Facultad de Ciencias Básicas, Universidad del Atlántico
Barranquilla, Colombia
email:*carpintero.carlos@gmail.com*

Ennis R. Rosas R
Department of Mathematics, Universidad De Oriente,
Nucleo De Sucre Cumana, Venezuela
Facultad de Ciencias Básicas, Universidad del Atlántico
Barranquilla, Colombia
email:*ennisrafael@gmail.com*

Margot del V. Salas Brown
Department of Mathematics, Universidad De Oriente,
Nucleo De Sucre Cumana, Venezuela
email:*salasbrown@gmail.com*