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SOME PROPERTIES OF AN INTEGRAL OPERATORS DEFINED BY A NEW LINEAR OPERATOR

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Abstract. In this paper, we define a new linear operator and two new general p-valent integral operators for certain analytic functions in the unit disk Δ. It is also shown that the first of these operators maps Ma-Minda type starlike functions into Ma-Minda type convex functions, while the convex mapping are shown to be closed under the second integral operator.

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1. Introduction

Let $\mathcal{A}(p)$ denote the class of functions f of the form:

$$f(z) = z^p + \sum_{k=1}^{\infty} a_{p+k} z^{p+k}, \quad (p \in \mathbb{N} = \{1, 2, ...\}), \tag{1}$$

which are analytic in the open unit disc $\Delta = \{z \in \mathbb{C} : |z| < 1\}$. We write $\mathcal{A}(1) = \mathcal{A}$. A function $f \in \mathcal{A}(p)$ is said to be Ma-Minda type starlike of order γ ($\gamma > 1$) in Δ if

$$\frac{1}{p}\mathfrak{Re}\left(\frac{zf'(z)}{f(z)}\right) < \gamma, \quad z \in \Delta,$$

we denote by $\mathcal{M}_p(\gamma)$, the class of all such functions. A function $f \in \mathcal{A}(p)$ is said to be Ma-Minda type convex of order γ ($\gamma > 1$) in Δ if

$$\frac{1}{p}\mathfrak{Re}\left(1 + \frac{zf''(z)}{f'(z)}\right) < \gamma, \quad z \in \Delta,$$

we denote by $\mathcal{N}_p(\gamma)$, the class of all such functions. If f and g are analytic in Δ, we say that f is subordinate to g in Δ, written $f \prec g$, if there exists Schwarz function

ω, analytic in Δ with $\omega(0) = 0$ and $|\omega(z)| < 1$ in Δ such that $f(z) = g(\omega(z))$, $z \in \Delta$.

For two functions f given by (1) and g given by

$$g(z) = z^p + \sum_{k=1}^{\infty} b_{p+k} z^{p+k}, \quad (p \in \mathbb{N}),$$

their Hadamard product (or convolution) is defined by

$$(f * g)(z) = z^p + \sum_{k=1}^{\infty} a_{p+k} b_{p+k} z^k, \quad (p \in \mathbb{N}).$$

Let $\mathcal{P}$ denote the class of functions of the form:

$$p(z) = 1 + \sum_{k=1}^{\infty} p_n z^n, \tag{2}$$

which are analytic and convex in Δ and satisfy the condition:

$$\mathfrak{Re}(p(z)) > 0, \quad (z \in \Delta).$$

For $a \in \mathbb{R}, c \in \mathbb{R} \setminus \mathbb{Z}_0^-$, where $\mathbb{Z}_0^- := \{..., -2, -1, 0\}$, we introduce a linear operator

$$\mathcal{J}_{\mu,\nu}^{\lambda,p}(a,c) : \mathcal{A}(p) \longrightarrow \mathcal{A}(p)$$

defined by

$$\mathcal{J}_{\mu,\nu}^{\lambda,p}(a,c) f(z) = \phi_{\mu,\nu}^{\lambda,p}(a,c;z) * f(z), \quad (z \in \Delta, f \in \mathcal{A}(p)), \tag{3}$$

where

$$\phi_{\mu,\nu}^{\lambda,p}(a,c;z) := z^p + \sum_{k=1}^{\infty} \frac{(a)_k (p+1)_k (p+1-\mu+\nu)_k}{(c)_k (p+1-\mu)_k (p+1-\lambda+\nu)_k} z^{p+k}, \tag{4}$$

and where $(d)_k$ is the Pochhammer symbol defined by

$$(d)_k := \begin{cases} 1, & k = 0, \\ d(d+1)(d+2)\ldots(d+k-1), & k \in N. \end{cases}$$

Also $0 \le \lambda < 1, \mu, \nu \in \mathbb{R}$ and $\mu - \nu - p < 1$. We note that:
(i) If $\lambda = \mu = 0$ in (3), then we have a linear operator was introduced by Saitoh [21].
(ii) If $a = c = 1$ in (3), then $\mathcal{J}_{\mu,\nu}^{\lambda,p}(a,c) f(z) \equiv \Delta_{z,p}^{\lambda,\mu,\nu} f(z)$, where $\Delta_{z,p}^{\lambda,\mu,\nu} f(z)$ is the fractional operator introduced by Choi [6]. We now introduce the following family of linear operators $\mathcal{I}_{\mu,\nu}^{\lambda,p,\alpha}(a,c)$ analogous to $\mathcal{J}_{\mu,\nu}^{\lambda,p}(a,c)$:

$$\mathcal{I}_{\mu,\nu}^{\lambda,p,\alpha}(a,c) : \mathcal{A}(p) \longrightarrow \mathcal{A}(p)$$

which defined as

$$\mathcal{I}_{\mu,\nu}^{\lambda,p,\alpha}(a,c)f(z) := \psi_{\mu,\nu}^{\lambda,p,\alpha}(a,c;z) * f(z), \tag{5}$$

$$(a \in \mathbb{R}, c \in \mathbb{R} \setminus \mathbb{Z}_0^-, \alpha > -p, 0 \leq \lambda < 1, \mu, \nu \in \mathbb{R}, \mu - \nu - p < 1, z \in \Delta, f \in \mathcal{A}(p)).$$

Where $\psi_{\mu,\nu}^{\lambda,p,\alpha}(a,c;z)$ is the function defined in terms of the Hadamard product by the following condition:

$$\phi_{\mu,\nu}^{\lambda,p}(a,c;z) * \psi_{\mu,\nu}^{\lambda,p,\alpha}(a,c;z) = \frac{z^p}{(1-z)^{\alpha+p}}, \qquad (\alpha > -p). \tag{6}$$

We can easily find from (3)-(5) that

$$\mathcal{I}_{\mu,\nu}^{\lambda,p,\alpha}(a,c)f(z) = z^p + \sum_{k=1}^{\infty} \frac{(c)_k(p+1-\mu)_k(p+1-\lambda+\nu)_k(\alpha+p)_k}{(a)_k(p+1)_k(p+1-\mu+\nu)_k k!} a_{p+k} z^{p+k}. \tag{7}$$

By definition and specializing the parameters λ, μ, p, a, c and α, we obtain: $\mathcal{I}_{0,\nu}^{0,p,1}(p+1,1)f(z) = f(z)$ and $\mathcal{I}_{0,\nu}^{0,p,1}(p,1)f(z) = \mathcal{I}_{0,0}^{1,p,1}(p,1)f(z) = \frac{1}{p}zf'(z)$.
It should be remarked that the linear operator $\mathcal{I}_{\mu,\nu}^{\lambda,p,\alpha}(a,c)f(z)$ is a generalization of many other linear operators considered earlier. In particular, for $f \in \mathcal{A}(p)$, we have the following observations:

- $\mathcal{I}_{0,\nu}^{0,p,\alpha}(a,c)f(z) \equiv \mathcal{I}_p^{\alpha}(a,c)f(z)$, the Cho-Kwon-Srivastava operator [5].
- $\mathcal{I}_{0,\nu}^{0,p,\alpha}(a,a)f(z) \equiv \mathcal{D}^{\alpha+p-1}f(z)$, where $\mathcal{D}^{\alpha+p-1}$ is the Well-known Ruscheweyh derivative of $(\alpha+p-1)$-th order was studied by Goel and Sohi [9].
- $\mathcal{I}_{o,\nu}^{0,p,1}(p+1-\lambda,1)f(z) \equiv \Omega_z^{(\lambda,p)} = \frac{\Gamma(p+1-\lambda)}{\Gamma(p+1)} z^{\lambda}\mathcal{D}_z^{\lambda}f(z), (0 \leq \lambda < 1)$, where $\Omega_z^{(\lambda,p)}$ is the fractional derivative operator defined by Srivastava and Aouf [22] and $\mathcal{D}_z^{\lambda}f(z)$ is the fractional derivative of $f(z)$ of order λ [11], [15], [19].
- $\mathcal{I}_{0,\nu}^{0,1,\alpha-1}(a,c)f(z) \equiv \mathcal{I}_c^{a,\alpha}f(z)$, the linear operator investigated by Hohlov [10].
- $\mathcal{I}_{0,\nu}^{0,1-\alpha,\alpha}(a,c)f(z) \equiv \mathcal{L}_p(a,c)f(z)$, the linear operator studied by Saitoh [21] which yields the operator $\mathcal{L}(a,c)f(z)$ introduced by Carleson and Shaffer for $p=1$ [4].
- $\mathcal{I}_{0,\nu}^{0,p,\alpha}(\alpha+p+1,1)f(z) \equiv \mathcal{F}_{\alpha,p}(f)(z) = \frac{\alpha+p}{z^{\alpha}}\int_0^z t^{\alpha-1}f(t)dt, (\alpha > -p)$, the generalized Bernardi-Libera-Livingston integral operator [7].
- $\mathcal{I}_{0,\nu}^{0,1-\alpha,\alpha}(\lambda+1,\mu)f(z) \equiv \mathcal{I}_{\lambda,\mu}f(z)(\lambda > -1, \mu > 0)$, the Choi-Saigo-Srivastava operator which is closely related to the Carleson-Shaffer operator $\mathcal{L}(\mu,\lambda+1)f(z)$ [4].
- $\mathcal{I}_{0,\nu}^{0,p,1}(p+\alpha,1)f(z) \equiv \mathcal{I}_{\alpha,p}f(z)(\alpha \in \mathbb{Z}, \alpha > -p)$, the operator considered by Liu and Noor [12].

Now by using the linear operator $\mathcal{I}_{\mu,\nu}^{\lambda,p,\alpha}(a,c)f(z)$, defined by (7), we introduce the new classes $\mathcal{S}_{\mu,\nu,\gamma}^{\lambda,p,\alpha}(a,c;h)$ and $\mathcal{K}_{\mu,\nu,\gamma}^{\lambda,p,\alpha}(a,c;h)$ as follows:

Definition 1. *A function $f \in \mathcal{A}(p)$ is said to be in the class $\mathcal{S}^{\lambda,p,\alpha}_{\mu,\nu,\gamma}(a,c;h)$ if it satisfies the following subordination condition:*

$$\frac{1}{\gamma - p}\left(\gamma - \frac{z\left(\mathcal{I}^{\lambda,p,\alpha}_{\mu,\nu}(a,c)f(z)\right)'}{\mathcal{I}^{\lambda,p,\alpha}_{\mu,\nu}(a,c)f(z)}\right) \prec h(z), \quad (z \in \Delta). \tag{8}$$

where $h \in \mathcal{P}, \gamma > p$ and $a, c, \lambda, \mu, \nu, \alpha, f, z$ are same (5).

Definition 2. *A function $f \in \mathcal{A}(p)$ is said to be in the class $\mathcal{K}^{\lambda,p,\alpha}_{\mu,\nu,\gamma}(a,c;h)$ if it satisfies the following subordination condition:*

$$\frac{1}{\gamma - p}\left(\gamma - 1 - \frac{z\left(\mathcal{I}^{\lambda,p,\alpha}_{\mu,\nu}(a,c)f(z)\right)''}{\left(\mathcal{I}^{\lambda,p,\alpha}_{\mu,\nu}(a,c)f(z)\right)'}\right) \prec h(z), \quad (z \in \Delta). \tag{9}$$

It follows from (8) and (9) that

$$f(z) \in \mathcal{K}^{\lambda,p,\alpha}_{\mu,\nu,\gamma}(a,c;h) \Longleftrightarrow zf'(z) \in \mathcal{S}^{\lambda,p,\alpha}_{\mu,\nu,\gamma}(a,c;h).$$

Remark 1. *It is easy to see that, if we choose*

$$\lambda = \mu = 0, \alpha = c = 1, a = p+1, h(z) = \frac{1+z}{1-z} \quad \text{and} \quad z \in \Delta,$$

in the classes $\mathcal{S}^{\lambda,p,\alpha}_{\mu,\nu,\gamma}(a,c;h)$ and $\mathcal{K}^{\lambda,p,\alpha}_{\mu,\nu,\gamma}(a,c;h)$, then it reduces to the classes $\mathcal{M}_p(\gamma)$ was introduced by Polatoğlu et al [20] and $\mathcal{N}_p(\gamma)$, respectively. Moreover, the classes $\mathcal{M}_1(\gamma) =: \mathcal{M}(\gamma)$ and $\mathcal{N}_1(\gamma) =: \mathcal{N}(\gamma)$ was studied by Nishiwaki and Owa [14], Owa and Nishiwaki [17], Owa and Srivastava [18], Srivastava and Attiya [23], Uralegaddi and Desai [24].

Definition 3. *For $r_i \geq 0$, $f_i \in \mathcal{A}(p)$ and $i = 1, 2, ..., n$, by using linear operator (7), define the following respective integral operators:*

$$\mathcal{L}_p(z) = \int_0^z pt^{p-1}\prod_{i=1}^{n}\left(\frac{\mathcal{I}^{\lambda,p,\alpha}_{\mu,\nu}(a,c)f_i(t)}{t^p}\right)^{r_i} dt, \tag{10}$$

$$\mathcal{V}_p(z) = \int_0^z pt^{p-1}\prod_{i=1}^{n}\left(\frac{\left(\mathcal{I}^{\lambda,p,\alpha}_{\mu,\nu}(a,c)f_i(t)\right)'}{pt^{p-1}}\right)^{r_i} dt. \tag{11}$$

The operators $\mathcal{L}_p(z)$ and $\mathcal{V}_p(z)$ reduces to many well-known integral operators by varying the parameters $r_i, \lambda, \mu, \alpha, a, c,$ and p. For example:

Example 1. *If we take $\lambda = \mu = 0, \alpha = c = 1$ and $a = p + 1$, then the integral operators $\mathcal{L}_p(z)$ and $\mathcal{V}_p(z)$ reduces to the operators*

$$F_p(z) = \int_0^z pt^{p-1} \prod_{i=1}^{n} \left(\frac{f_i(t)}{t^p} \right)^{r_i} dt, \tag{12}$$

and

$$G_p(z) = \int_0^z pt^{p-1} \prod_{i=1}^{n} \left(\frac{f_i'(t)}{pt^{p-1}} \right)^{r_i} dt, \tag{13}$$

respectively, which were studied by Frasin [8].

Example 2. *If we take $\lambda = \mu = 0, \alpha = c = p = 1$ and $a = 2$, then the integral operators $\mathcal{L}_p(z)$, reduces to the operator*

$$I_n(f_i)(z) = \int_0^z \prod_{i=1}^{n} \left(\frac{f_i(t)}{t} \right)^{r_i} dt, \tag{14}$$

introduced and studied by Breaz and Breaz [2].

Example 3. *If we take $\lambda = \mu = 0, \alpha = c = p = n = r_1 = 1$ and $a = 2$ in relation (10), we obtain the Alexander integral operator*

$$I_n(f_1)(z) = \int_0^z \left(\frac{f_1(t)}{t} \right) dt, \tag{15}$$

introduced in [1].

Example 4. *If we take $\lambda = \mu = 0, \alpha = c = p = n = 1, r_1 = \beta$ and $a = 2$ in relation (10), we obtain the integral operator*

$$I_n(f_1)(z) = \int_0^z \left(\frac{f_1(t)}{t} \right)^{\beta} dt, \tag{16}$$

studied in [13].

Example 5. *If we take $\lambda = \mu = 0, \alpha = c = p = 1$ and $a = 2$, then the integral operators $\mathcal{V}_p(z)$, reduces to the operator*

$$I_n(f_i)(z) = \int_0^z \prod_{i=1}^{n} \left(f_i'(t) \right)^{r_i} dt, \tag{17}$$

introduced and studied in Breaz et al [3].

2. Closure Property of the Operators $\mathcal{L}_p(z)$ and $\mathcal{V}_p(z)$

Theorem 1. *For $i = 1, 2, ..., n$, let $r_i \geq 0$, $\sum_{i=1}^{n} r_i \leq 1$ and $h \in \mathcal{P}$. If $f_i \in \mathcal{S}_{\mu,\nu,\gamma}^{\lambda,p,\alpha}(a, c; h)$, then $\mathcal{L}_p(z) \in \mathcal{K}_{\mu,\nu,\gamma}^{\lambda,p,\alpha}(a, c; h)$.*

Proof. Since

$$\mathcal{L}'_p(z) = pz^{p-1} \prod_{i=1}^{n} \left(\frac{\mathcal{I}_{\mu,\nu}^{\lambda,p,\alpha}(a, c) f_i(z)}{z^p} \right)^{r_i},$$

it follows that

$$\frac{z\mathcal{L}''_p(z)}{\mathcal{L}'_p(z)} = p - 1 + \sum_{i=1}^{n} r_i \left(\frac{z(\mathcal{I}_{\mu,\nu}^{\lambda,p,\alpha}(a, c) f_i(t))'}{\mathcal{I}_{\mu,\nu}^{\lambda,p,\alpha}(a, c) f_i(t)} - p \right).$$

The relation above is equivalent to

$$\frac{1}{\gamma - p} \left(\gamma - 1 - \frac{z\mathcal{L}''_p(z)}{\mathcal{L}'_p(z)} \right) = \sum_{i=1}^{n} r_i \frac{1}{\gamma - p} \left(\gamma - \frac{z(\mathcal{I}_{\mu,\nu}^{\lambda,p,\alpha}(a, c) f_i(t))'}{\mathcal{I}_{\mu,\nu}^{\lambda,p,\alpha}(a, c) f_i(t)} \right) + (1 - \sum_{i=1}^{n} r_i).$$

The assumption that $f_i \in \mathcal{S}_{\mu,\nu,\gamma}^{\lambda,p,\alpha}(a, c; h)$, yields

$$\frac{1}{\gamma - p} \left(\gamma - \frac{z(\mathcal{I}_{\mu,\nu}^{\lambda,p,\alpha}(a, c) f_i(t))'}{\mathcal{I}_{\mu,\nu}^{\lambda,p,\alpha}(a, c) f_i(t)} \right) \in h(\Delta),$$

for every $z \in \Delta$. Since h is convex, the convex combination of 1 and

$$\frac{1}{\gamma - p} \left(\gamma - \frac{z(\mathcal{I}_{\mu,\nu}^{\lambda,p,\alpha}(a, c) f_i(t))'}{\mathcal{I}_{\mu,\nu}^{\lambda,p,\alpha}(a, c) f_i(t)} \right), \quad (i = 1, 2, ..., n),$$

is again in $h(\Delta)$. This shows that

$$\sum_{i=1}^{n} r_i \frac{1}{\gamma - p} \left(\gamma - \frac{z(\mathcal{I}_{\mu,\nu}^{\lambda,p,\alpha}(a, c) f_i(t))'}{\mathcal{I}_{\mu,\nu}^{\lambda,p,\alpha}(a, c) f_i(t)} \right) + (1 - \sum_{i=1}^{n} r_i)(1) \in h(\Delta),$$

or

$$\frac{1}{\gamma - p} \left(\gamma - 1 - \frac{z\mathcal{L}''_p(z)}{\mathcal{L}'_p(z)} \right) \prec h(z).$$

This completes the proof.

Corollary 2. *If $f \in \mathcal{M}_p(\gamma)$, then the integral operators defined by (12), (14), (15) and (16) are to be in $\mathcal{N}_p(\gamma)$.*

Corollary 3. *Since* $\mathcal{L}_p(z) \in \mathcal{K}_{\mu,\nu,\gamma}^{\lambda,p,\alpha}(a,c;h)$*, it follows that*

$$pz^p \prod_{i=1}^{n} \left(\frac{\mathcal{I}_{\mu,\nu}^{\lambda,p,\alpha}(a,c)f_i(z)}{z^p} \right)^{r_i} \in \mathcal{S}_{\mu,\nu,\gamma}^{\lambda,p,\alpha}(a,c;h).$$

Theorem 4. *For* $i = 1,2,...,n$*, let* $r_i \geq 0$*,* $\sum_{i=1}^{n} r_i \leq 1$ *and* $h \in \mathcal{P}$*. If* $f_i \in \mathcal{K}_{\mu,\nu,\gamma}^{\lambda,p,\alpha}(a,c;h)$*, then* $\mathcal{V}_p(z) \in \mathcal{K}_{\mu,\nu,\gamma}^{\lambda,p,\alpha}(a,c;h)$*.*

Proof. The proof is similar to theorem 1, and is therefore omitted.

Corollary 5. *If* $f_i \in \mathcal{N}_p(\gamma), i = 1,2,...,n,$ $r_i \geq 0$ *and* $\sum_{i=1}^{n} r_i \leq 1$*, then the integral operator defined by (13) and (17) are to be in* $\mathcal{N}_p(\gamma)$*.*

Corollary 6. *Let* $i = 1,2,...,n,$ $r_i \geq 0$*,* $\sum_{i=1}^{n} r_i \leq 1$ *and* $h \in \mathcal{P}$*. If* $f_i \in \mathcal{K}_{\mu,\nu,\gamma}^{\lambda,p,\alpha}(a,c;h)$*, then it follows from theorem 2 that*

$$z^p \prod_{i=1}^{n} \left(\frac{\left(\mathcal{I}_{\mu,\nu}^{\lambda,p,\alpha}(a,c)f_i(t)\right)'}{pt^{p-1}} \right)^{r_i} \in \mathcal{S}_{\mu,\nu,\gamma}^{\lambda,p,\alpha}(a,c;h).$$

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