Learning Indian Arithmetic in the Early Thirteenth Century

Charles Burnett

The most momentous development in the history of pre-modern mathematics is the shift from using roman numerals to using Indian numerals and the 'Indian way' of doing arithmetic that the use of these numerals entailed. Indian numerals were originally Sanskrit symbols that had been introduced into the Islamic world by the early ninth century, when their use was described by the mathematician and astronomer, al-Khwarizmi (ca. 825 A.D.). Al-Khwarizmi's 'On the calculation of the Indians' was, in turn, introduced to a Latin-reading public through a series of translations and adaptations produced from the early twelfth century onwards. This new kind of arithmetic became known as the algorism ('algorismus'), after the Arabic author, and the numerals were described as being either Indian or Arabic. At first there was considerable variety in the forms of numerals used, but by the early thirteenth century, they had become standardised and, with small exceptions (in particular, in the shapes of the '4' and the '5'), became the 'Arabic numerals' that are used universally today. The acceptance of the algorism within the canon of European mathematics was ensured by the magisterial *Liber abbaci* of Leonardo of Pisa (Fibonacci) in two editions (1202 and 1220), and the more popular manuals of Alexander de Villa Dei (the *Carmen de algorismo*) and of John of Sacrobosco (*Algorismus vulgaris*), both slightly later in the thirteenth century.[1]

Nevertheless, such was the novelty of Indian calculation that it took some time for it to be understood and accepted. Particularly difficult was the idea

[1] A recent account of the spread of the algorismus can be found in *Die älteste lateinische Schrift über das indische Rechnen nach al-Hwarizmi*, ed., trans. and comm. by M. Folkerts, with the collaboration of P. Kunitzsch, Abhandlungen der Bayerischen Akademie der Wissenschaften, phil.-hist. Klasse, n.F., 113, Munich, 1997 (English summary on pp. 163-83). The *Liber abbaci* has been edited by B. Boncompagni (Rome, 1857); a new edition by A. Allard is nearing completion. The Latin texts of Alexander de Villa Dei and John of Sacrobosco are available only in J. O. Halliwell, *Rara mathematica*, London 1841, pp. 1-26 (John of Sacrobosco) and 73-83 (Alexander de Villa Dei); modern critical editions are needed. A convenient annotated translation of the major part of Sacrobosco's text can be found in *A Source Book in Medieval Science*, ed. E. Grant, Cambridge, Mass., 1974, pp. 94-102. For arithmetical procedures in the abacus and algorism in the context of the history of numerals, see Georges Ifrah, *The Universal History of Numbers*, English version, London, 1998, pp. 556-66.

that a single symbol could be used to express an infinite range of numbers. In roman numerals, each power of ten is expressed by a different symbol (i, x, c, m etc.); in Indian calculation the symbol remains the same, but the power of ten is indicated by a new symbol for which roman numerals had no equivalent at all: the zero. An idea of the puzzlement that the zero caused can be gained from a note in a late twelfth-century manuscript now in Cambridge, in which, among the several names given to the new symbol, is the 'chimaera', the imaginary beast of mythology.[2]

A half-way stage towards assimilating Indian arithmetic is represented by the 'Gerbertian abacus', which was used for teaching arithmetic from the late tenth century onwards, and in which the Indian numerals were employed only to mark counters which were placed on an abacus board (a piece of wood or thick parchment ruled with lines). In this case, the columns of the abacus themselves indicated the decimal place and a counter for zero was not necessary. In the algorism the numerals were written directly onto the writing surface (whether this was a wax tablet, a tray sprinkled with a thin covering of sand, or a sheet of parchment—later paper), and no lines were drawn to demarcate the decimal places. Some instructions could apply equally to the abacus and the algorism, and sometimes the terminology appropriate to the abacus was carried over to the algorism.

In this article, I would like to show how the essentials of the new arithmetic were taught on the borders between England and Wales sometime in the early thirteenth century, but before the works of Fibonacci, Alexander of Villa Dei and Sacrobosco had become standard.

The evidence comes from a manuscript written, possibly, by a monk in the abbey of Tewkesbury (or a closely-related house),[3] who gathered under a single cover a collection of texts and notes which all more or less concern number. This collection is now the first 120 pages of a manuscript in Cashel (Tipperary, Ireland): G. P. A. Bolton Library (formerly Cathedral Library), Medieval MS 1. Its mathematical contents can be classified as follows:

1. Calendrical material: tables (for the years 1168-1223, and 1140-1642), short texts, some in verse, on how to calculate the various church feasts, elementary astronomy and meteorology.

2. Predictions: on the weather (p. 5); on the recovery of a sick person (the 'sphere of Pythagoras', pp. 17-18); the *Epistola Petosiris* (p. 19); on prognosticating life and death (p. 20).

[2]Cambridge, Trinity College, R.15.16, fol. Av, the different names of zero: 'cifra vel solfra vel nichil t. 0. cimera sipos'.

[3]This provenance is indicated by the presence of a calendar on pp. 71-6 which is derived from the calendar of Tewkesbury Abbey.

3. The algorism. The manuscript includes four works on the algorism. One occurs (pp. 111-7) in an older pamphlet (probably of the twelfth century) which has been attached to the end of the manuscript. This is the *Helcep Sarracenicum* ('Sarracen Calculation'[4]) of Ocreatus, which is distinctive for its use of the first nine roman numerals for the Indian numerals.[5] Another is a complete text on the algorism (pp. 41-58).[6] The third and the fourth are shorter introductions to the algorism (pp. 16-17 and 20-21); these are reproduced and analysed here.

1 The Table

On pp. 20-1 the function of place value is demonstrated through the use of a table (see Plates I-II). Here, each Indian numeral is copied nine times to show how it can stand for the first nine powers of ten; above each repeated Indian numeral the equivalent in roman numerals is given. The lowest digits are on the right. This may originate from the fact that in Arabic one reads from the right. But the result is that it became normal for higher digits to be written on the left, lower on the right.[7] No zeros are included in this table, which could equally show how numerals were disposed on the 'Gerbertian abacus' that was still being used at the time. One may note the cumbersome way in which the powers of ten are expressed in roman numerals; I give their equivalent in Indian numerals:[8]

i	1
x	10
c	100
$\overline{M}$	1000
x$\overline{M}$	10,000
c$\overline{M}$	100,000
$\overline{M}\,\overline{M}$	1,000,000
decies$\overline{M}\,\overline{M}$	10,000,000
centies$\overline{M}\,\overline{M}$	100,000,000

[4]'Helcep' is a transliteration of the Arabic word 'al-ais&b', meaning 'calculation'.

[5]This text has been analysed and edited in Burnett, '*Algorismi vel helcep decentior est diligentia*: the Arithmetic of Adelard of Bath and his Circle', in *Mathematische Probleme im Mittelalter: der lateinische und arabische Sprachbereich*, ed. M. Folkerts, Wiesbaden, 1996, pp. 221-331.

[6]An edition of this text by the present author is to be published in the periodical *Sciamus*.

[7]For the establishment of this norm in the West see C. Burnett, 'Why We Read Arabic Numerals Backwards', in *Ancient and Medieval Traditions in the Exact Sciences, Essays in Memory of Wilbur Knorr*, Stanford, Ca., 2000, pp. 197-202.

[8]It is customary in works on the abacus to put a tilde on the 'm', although the same sign also indicates '1000 times' the numeral: e.g. 'i' is used for '1000', 'x' for '10,000' etc.

2 Some Basic Rules of Arithmetic

Indian arithmetic provides no help for the multiplication of simple digits. For these, school children nowadays learn multiplication tables (usually from $2\times 2 = 4$, $2\times 3 = 6$, $2\times 4 = 8$ to $12\times 12 = 144$). In the Middle Ages they may have done the same, but multiplication tables are also written out, the most detailed of which was composed by Victorius in Late Antiquity.[9] In texts on the algorism some convenient short cuts for multiplication are often found. This is what occurs immediately after the tables of numerals in the Cashel manuscript (p. 21). Comparison with other algorismic texts reveals that the Cashel student had some problems with understanding these short cuts.

(1) Si vis ducere digitum in se, scribe eius decuplum et aufer eum a suo decuplo per differentiam suam ad .x.

'If you wish to multiply a digit (a) by itself, write the product of a x 10 and take it away from the product of 10 times the difference between a and 10.'

What the student should have written was: 'Si vis ducere digitum in se, scribe eius decuplum et aufer *ab eo suam ductionem* per differentiam suam ad .x.': '...take away from the product (of $a \times 10$) the product of $a \times (10 - a)$'; e.g. $7 \times 7 = (7 \times 10) - (7 \times 3)$.[10] A similar mistake is made in the second rule:

(2) Si vis ducere digitum in alium, scribe decuplum minoris et aufer ipsum a decuplo suo per differentiam maioris ad .x.

'If you wish to multiply one digit by another, write the product of the lower number times 10, and take it away from the product of 10 times the difference between the higher number and 10.'

What the student should have written was: 'Si vis ducere digitum in alium, scribe decuplum minoris et aufer *ab eo ductionem maioris* per differentiam suam ad .x.': '...take away from the product (of the lesser number times 10) the product of the greater number times its difference from 10'; e.g. $7 \times 9 = (7 \times 10) - (1 \times 9)$.

Another innovation in Indian arithmetic is that of substituting numerals in the process of calculation. In the case of the 'Gerbertian abacus' one simply had to exchange one counter with another: the usual verbs in the Latin abacus treatises are 'removere' ('take off') and 'ponere' ('put down'). When calculating on a wax-tablet, a board covered with sand, or parchment, one had to erase one number and replace it by another: the usual Latin verbs are 'delere' ('destroy')

[9] See now Abbo of Fleury and Ramsey, *Commentary on Victorius*, ed. A. Peden, London, 2002.

[10] For the correct formulation, see Sacrobosco, *Algorismus vulgaris*, ed Halliwell, p. 12, trans. Grant, p. 98. Other versions can be found in MSS Cambridge, Trinity College, R.15.16, fol. 61r and British Library, Egerton 2261, fol. 226rb: see Burnett, 'Algorismi vel helcep', p. 305. In all these cases a single rule ('if you wish to multiply one digit by another') replaces rules (1) and (2) of the Cashel manuscript.

and 'scribere' ('write'). In this text the terminology of the abacus is still used, though this does not inevitably imply that the student was using the abacus.[11]

(3) Quotiens aliquis multiplicator multiplicat id quod supra ipsum erit, non est addendum id quod ex multiplicatione provenit ei quod supra ipsum erit sed, remoto eo quod supra ipsum erit, simpliciter ponendum est loco illius quod multiplicatione provenit.

'Whenever any multiplier (a) multiplies that (digit) which is above it (b), the product (c) should not be added to b, but rather b should be taken away and c should simply replace b.'

Next comes a rule about a number series:

(4) Si fiat ascensio per impares numeros de proximo ad proximum, si ab unitate incipiatur, ut .i. iii. v. 7., maioris maior medietas in se multiplicata omnium propositorum summam reddit.

'If there is a series of consecutive odd numbers beginning from one—e.g. 1, 3, 5, 7—the product of the larger half of the highest number multiplied by itself is the same as the sum of all the numbers in the series.' The term 'greater middle' refers to the larger of the two unequal parts that an odd number is divided into: in this case 4, since $7 = 3 + 4$; $4 \times 4 = 16 = 1 + 3 + 5 + 7$. This, too, has equivalents in Sacrobosco and the Egerton and Trinity manuscripts.[12] Note the mixture of roman and Indian numerals in this rule.

The last two rules are parallel to the first two, but this time deal with the multiplication of 'articles', i.e. numbers followed by one or more zeros:[13]

(5) Si vis ducere articulum in se, vide a quo digito de(s)cendat et quot unitates proveniunt ex multiplicatione illius digiti in se, tot cente (sic) provenient ex multiplicatione illius articuli in se.

'If you wish to multiply an article by itself, see how many decimal places it is from a unit, and the number that arises from the multiplication of the simple unit in itself will be the number of hundreds that will arise from the multiplication of the article in itself.' E.g. in the case of 700×700, '7' is two decimal places away from the units; one multiplies the digits ($7 \times 7 = 49$), and one makes them into hundreds. The word 'cente' is not attested elsewhere. 'Cente(ni)' ('hundreds') is not strictly accurate here, since the 'article' could be any power of ten higher than the unit. Thus it is worth considering whether 'cente' is not

[11] The verb 'ponere' is used, as well as 'scribere', in the *Helcep Sarracenicum* in the Cashel manuscript: see Burnett, 'Algorismi vel Helcep', p. 241.

[12] See Sacrobosco, *Algorismus vulgaris*, ed Halliwell, p. 19, trans. Grant, p. 100 ('progressio'), and Burnett, 'Algorismi vel helcep', p. 309.

[13] The terminology 'digit' (literally 'finger') and 'article' (literally 'knuckle') derives from the representation of numbers in finger-calculation. Note that, in my translations, 'digit' means any single number used in a calculation, whereas 'number' means the whole number (which could consist of several digits).

a truncation of 'centeni', but rather the student's deliberate attempt to find a way of referring to 'powers of ten'.

(6) Si vis ducere articulum in alium, vide a quo digito uterque descendat, et quot unitates provenient a multiplicatione unius digiti in alium, tot .c. etc.

'If you wish to multiply one article by another, see how many places each one of them is from a digit, and the number of units that arises from the multiplication of the simple digits in themselves, will be the number of hundreds etc.'

These rules are relevant both to the 'Gerbertian abacus' and to the algorism. They have parallels to the 'six rules of multiplication' of Sacrobosco[14] which in turn are similar to some rules in the Egerton and Trinity manuscripts. The Cashel rules differ from all three of these other sources by separating 'multiplication of a number by itself' (i.e. squaring) from 'multiplication of a number by a different number'; such a separation is not arithmetically necessary. This feature, however, is also found in the *Helcep Sarracenicum* in the same Cashel manuscript.[15]

3 The Algorism

The text on pp. 16-17 deals with the algorism itself. It appears to be independent of that on pp. 20-21. For it uses the zero and is more consistent in employing the terminology of the algorism ('delere' and 'scribere') rather than that of the abacus. It gives succinct rules for adding, subtraction, multiplication and division, which are common to all algorisms, but adds a paragraph on how to calculate the highest common factor and the lowest common multiple of several numbers, which is not found in other early algorisms.

(1) Cum numero numerum addere volueris, cui alium addere volueris, prescribas, addendum autem ei supponas, et sic ut numerus prime differentie sub numero prime, numerus secunde sub numero secunde, numerus tertie sub numero tertie sit differentie, et sic secundum ordinem, si plures fuerint differentie, ut semper differentie sibi respondeant. Ex coniunctione ergo suppositi et suprapositi minor numerus quam 10 vel maior vel tantum .x. proveniet. Si minor quam .x. excrescat, idem in loco suprascripti numeri ponatur, deleto suprascripto. Si tantum .x., cifra in loco suprascripti posita, unitas pro .x. in sinistriori proxima transferatur. Si autem maior quam .x., numero superexcreto denario in loco suprascripti posito, unitas similiter in sinistriori loco proximo pro denario transferatur; pro singulis etiam denariis singule unitates in sinistram transferantur partem.

[14]See Sacrobosco, *Algorismus vulgaris*, ed Halliwell, p. 12-13, trans. Grant, p. 98.

[15]This peculiarity is noted in Burnett, 'Algorismi vel Helcep', pp. 241 and 242.

'When you wish to add a number to a number, you put the 'adder' on top, and the 'addend' underneath, in such a way that the digit (of the 'addend') in the first decimal place is under the digit (of the 'adder') in the first decimal place, the digit in the second under the digit in the second, the digit in the third under the digit in the third, and so on, if there are more decimal places; the decimal places (of 'adder' and 'addend') will always match each other. The sum of the higher and lower digits will be less than ten, exactly 10, or more than 10. If the sum is less than ten, that digit should be put in the place of the higher digit, which is erased. If exactly 10, a zero should be put in the place of the higher digit, and a one standing for 10 should be moved to the next decimal place to the left. If more than then, the number in excess of 10 should be put in the place of the higher digit, and again a one standing for 10 should be moved to the next decimal place to the left. (As a general rule) every one that is moved to the left stands for a 10.'

(2) Cum numerum a numero subtrahere volueris, ordine predicto numeros scribas, et deinceps numerum differentie a numero differentie sibi paris tollas, si fieri potest. Quod si fieri nequit, solam unitatem a sinistra differentia demas, que unitas erit tibi pro denario, et sic ab alio denario que oportuerit tollas. Residuum autem in loco suprascripti ponas, et sic habebis propositum. Cave tamen, si a centenario vel a millenario vel deinceps unitas demenda fuerit vel consimilis, ab ultima differentia unitas dematur, et pro singulis cifris novenarius ponatur. Ad ultimum autem pervento, ab eo numerus propositus quasi a denario subtrahatur, residuum autem in loco suprascripti ponatur.

'When you wish to subtract one number from another, write the numbers in the way described before (for addition), and then take the digit in a particular decimal place from the digit in the same decimal place, if that can be done. If that cannot be done, take one from the decimal place on the left and treat it as a 10, and thus take what you need from another[16] 10. Put the remainder in the place of the higher digit, and thus you will have the answer. Make sure, however, in the cases when ones are to be taken from 100s and 1000s and so on, that the one is taken from the last decimal place (to the left), and for each zero a nine is substituted. But when one comes to the last decimal place (to the right), the relevant digit is subtracted from it as if from 10, and the remainder is put in the place of the higher digit.' E.g. if 7 is subtracted from 1000, 9s are substituted for two of the zeros and the 7 is taken from the remaining 10, to produce 993. The text is made less clear by the use of 'ultimus' to describe both the last decimal place on the left, and the last one on the right.

(3) Cum autem numerus per numerum multiplicandus fuerit, multiplicandus prescribas, multiplicatorem autem subscribas, non tamen ut in modis predictis, sed sic ut primus multiplicatoris numerus sit sub ultimo multiplicandi. Deinde

[16] I.e. 10 plus the unit in the same decimal place.

vero ab altiori numero tam multiplicatoris quam multiplicandi incipiatur secundum ordinem suum; numerus ex multiplicatione proveniens ponatur. Multiplicato ergo ultimo multiplicandi per quamlibet multiplicatoris, retrahantur figure in dextram partem, prima in dextram differentiam proximam et quelibet succedat loco alterius, ne tamen differentiam proximam aliqua transiliat.

'When one number is to be multiplied by another, write the multiplicand on top, the multiplier underneath, but not in the same way as before (for addition and subtraction), but in such a way that the first digit (i.e. lowest) of the multiplier is under the last digit (i.e. highest) of the multiplicand. Then one should start from the highest digit, both of the multiplier and the multiplicand, in order; the product should be substituted. Then, when the last digit of the multiplicand has been multiplied by each[17] of the digits of the multiplier, the digits (of the multiplier) are moved to the right, the first digit into the next decimal place to the right and each subsequent one into the place of its neighbour, making sure that none jumps over a decimal place.'[18]

(4) Cum autem numerus per numerum dividendus fuerit, subscribatur dividendus et subscribatur divisor, nullo modorum predictorum, sed sic ut ultimus divisoris sit sub ultimo dividendi, si fieri potest. Quod si fieri non poterit, in dexteram protrahantur partem. Et postea fiat divisio a sinistra incipiendo differentia. Notandum tamen numerum ultimum totiens a sibi superscripto esse demendum quotiens quilibet sequencium a residuo suprascripto sibi demi possit. Ex directo ergo ultimi divisoris denominatio ponenda est iuxta quam divisio fiat, et deinceps protrahantur differentie dextrorsum et alia tunc ponatur denominatio super ultimum numerum divisoris tracta (?) et sic deinceps ut compleatur propositum.

'When one number is to be divided by another, one should write the dividend on top and the divider underneath, but not in any of the previous ways, but rather in such a way that the last (i.e. highest) digit of the divider is under the last (i.e. highest) digit of the dividend, if possible (if that is not possible, the digits are dragged to the right). Then the division should made starting from the decimal place on the left. Note that the last digit should be taken away from the digit above it as many times as each of the digits following it can be taken away from the remainder (?) written above them. The quotient that results from the division should be placed directly above the last digit of the divider; then the digits should be dragged into the decimal places on their right, and another quotient should then be placed and drawn (?) above the last[19] digit of the divider, and so on until the answer is completed.'

[17]'Quamlibet' should mean 'whichever you like' or 'any'; 'quamque' would be expected. The feminine form implies 'figura' which first appears in the next phrase.

[18]This injunction is necessary, presumably, because there might be a temptation to miss out decimal places with zeros in them.

[19]'Proximum' ('next') would be expected.

(5) 1 2 3 4 5 6 7 8 9
10. 100. 1000. 10000. 100000.
20. 200. 2000. 20000. 200000. 2000000.
30. 300. 3000. 30000. 300000.
40. 400. 4000. 40000. 400000. et sic deinceps secundum quamlibet figurarum quemlibet numerum scribe; attendens idoneus eris.

These are examples of writing Indian numerals, which one would have expected to have come first in this text. The student is told: 'Write each number using these numerals (figure). If you pay attention, you will be competent!'

(6) Si propositi fuerint tres[20] numeri et scire volueris maximum numerantem, illos vide si primi duo sint primi ad se invicem aut compositi. Si primi sunt ad se invicem, multiplica unum in alium et qui inde producetur erit minimus numeratus ab illis. Postea vide si ille scilicet qui educitur ex ductu primi in secundum et tertius sint primi ad se invicem aut compositi. Si compositi, quere maximum numerantem illos, divide scilicet unum per alium et minimus divisor erit maximus numerans utrumque. Quere postea minimos in proportione, divide scilicet primo unum postea alium proximum numerantem illos, et illi cum denominatione exeunte erunt .iiii. numeri proportionales. Multiplica ergo primum in quartum, et qui inde producetur erit minimus numeratus a tribus pro(p)ositis numeris. Vide ergo si ille et quartus si primi ad se invicem an compositi et tunc operandum ut supradiximus.

'If you have three numbers and want to know the highest common factor, look at them and see if the first two numbers are prime in respect to each other or composite. If they are prime to each other, multiply one by the other and the produce of this is the lowest common multiple. Then see if the product of the multiplication of the first by the second is prime in respect to the third, or composite. If composite, find the highest common factor, i.e. divide one by the other and the lowest divider will be the highest common factor. Then find the numbers smallest in proportion, i.e. divide first one, then the next one which is a factor of them, and those, added to the denomination (?) that results, will be four numbers in proportion. Then multiply the first and the fourth, and the result will be the lowest common multiple of the three numbers. See, then, if that number and the fourth one are prime or composite to each other, and then proceed as before.'

Here we have something unusual for early algorisms. Unfortunately the abbreviated nature of the text, and possibly some errors on the part of the Cashel student, make it difficult to work out exactly what arithmetical procedures are being described. The subject-matter is the highest common factor ('maximus numerans') and the lowest common multiple ('minimus numeratus'), both of

[20]The abbreviation in the manuscript would normally be read as 'tibi'; however, 'tres' is necessary for the context.

which have to be known when adding fractions with different denominators. If the denominators have common factors (i.e. are 'composite' in respect to one another), then it is necessary to find the highest common factor. If they do not have common factors (i.e. if they are 'prime' in respect to each other), then one must find the lowest common multiple by multiplying them together. What is unclear is why it is necessary to arrange the numbers concerned in a 'proportion' of four terms (which should mean $a : b = c : d$).

4 Conclusions

These texts give summaries of procedures, as an *aide-mémoire* written by a student of arithmetic, rather than fully-explained instructions. No examples are included. It is possible that texts **1** and **2** come from a different source from text **3**, since the former can apply equally to the abacus and the algorism, whereas the latter is specific to the algorism. The procedures described in **2** and **3** are brought together in John of Sacrobosco's *Algorismus vulgaris*, which may therefore represent a later stage in the teaching of the algorism. Both the Cashel student and Sacrobosco use 'cifra' (as a noun in the first declension) for 'zero' rather than the indeclinable 'cif(f)re' (a transliteration of the Arabic αifr) distinctive of earlier English writers,[21] and much of the terminology is shared by both authors: e.g. ordo, differentia, excrescere, transferre, delere, figura, etc. Nevertheless, the phraseology is sufficiently different between the Cashel and Sacrobosco text to suggest that the one is not dependent on the other. Rather, they both represent a common English tradition of the twelfth and early thirteenth century, whose richness and diversity has only recently begun to be appreciated.

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[21]For the use of 'cif(f)re' by Adelard of Bath, 'Ocreatus' and in the *Liber ysagogarum Alchorismi*, see Burnett, 'Algorismi vel helcep', pp. 236-7.

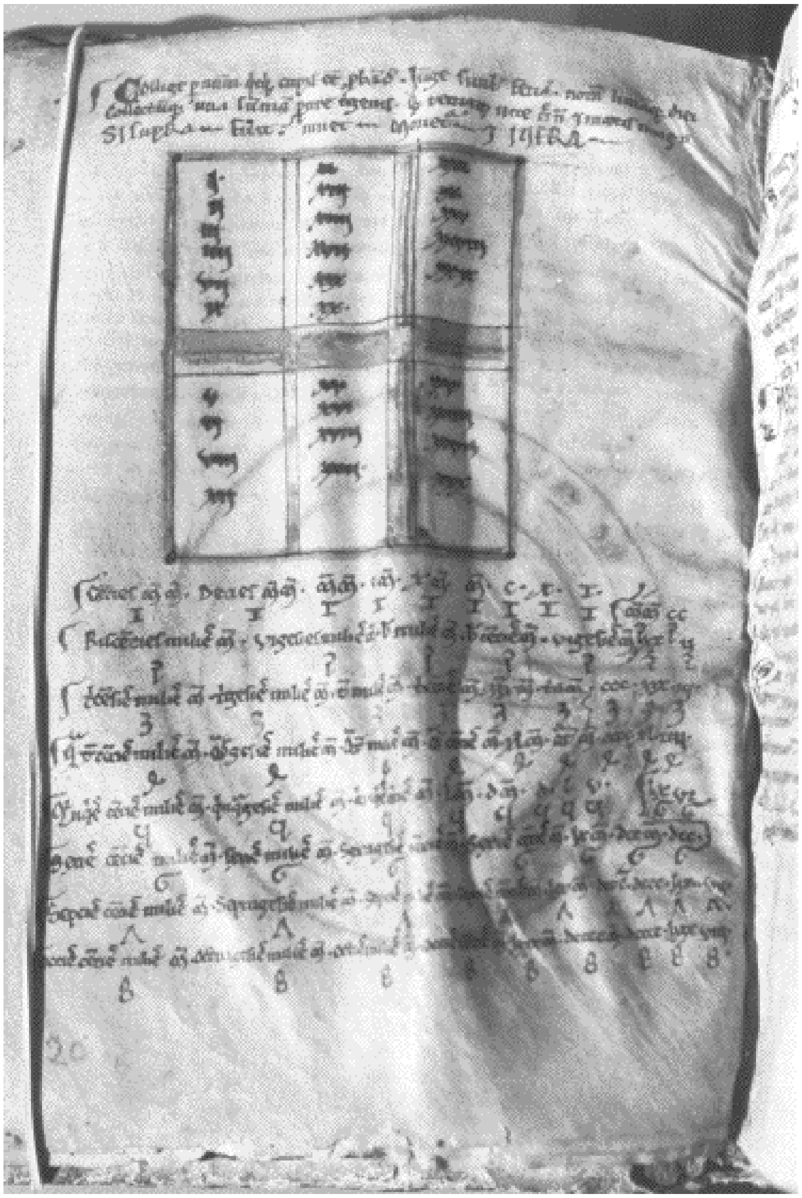

Plate I. Cashel, G. P. A. Bolton Library, Medieval MS 1, p. 20. The first eight lines of the table illustrating Indian numerals and the principle of place-value. Above this is a table for prognosticating whether a sick man will live or die, from the numerical values of the letters of his name, and those of the 'Moon' and the day.

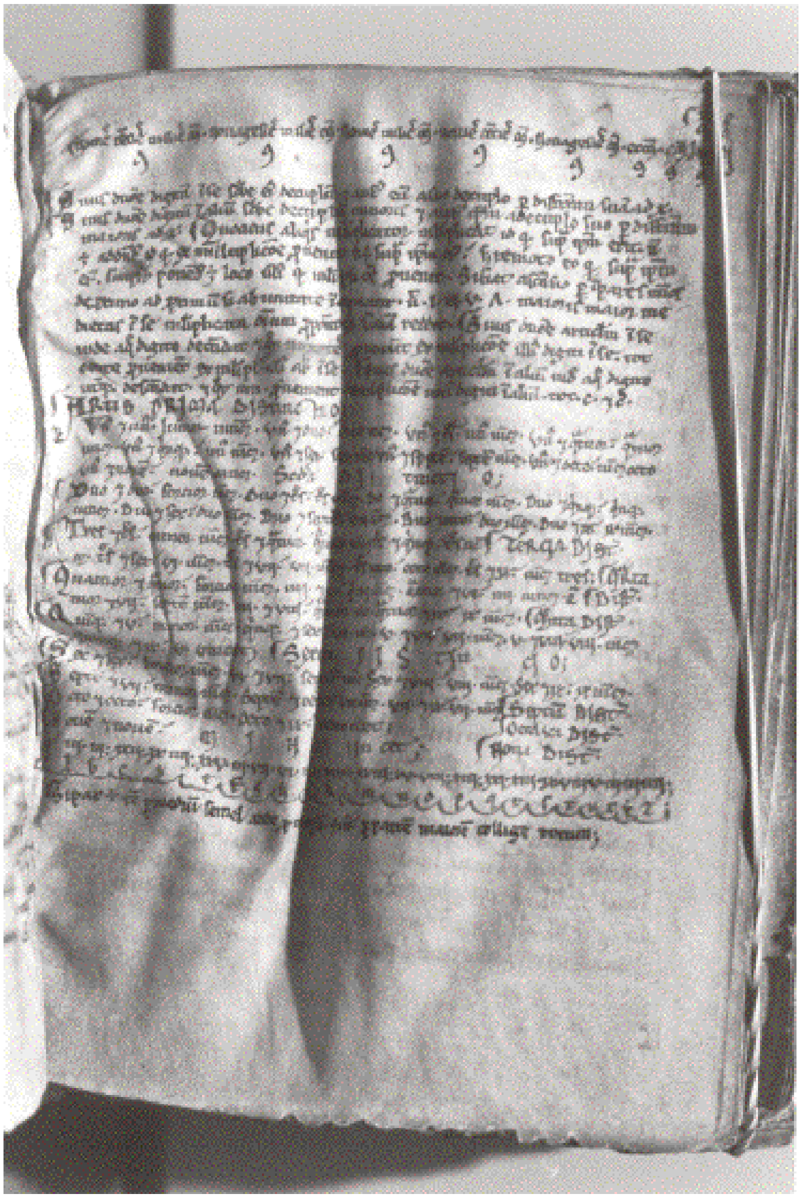

Plate II. Cashel, G. P. A. Bolton Library, Medieval MS 1, p. 21. The last line of the numerical, the basic rules for arithmetic, and the key to the numerical equivalent of Latin letters.