Bulletin of Mathematical Analysis and Applications
ISSN: 1821-1291, URL: http://www.bmathaa.org
Volume 2 Issue 3(2010), Pages 65-73.

STRONG CONVERGENCE RESULTS FOR THE JUNGCK-ISHIKAWA AND JUNGCK-MANN ITERATION PROCESSES

ALFRED OLUFEMI BOSEDE

Abstract. In this paper, we establish some strong convergence results for the Jungck-Ishikawa and Jungck-Mann iteration processes considered in Banach spaces. These results are proved for a pair of nonselfmappings using the Jungck-Ishikawa and Jungck-Mann iterations. Our results improve, generalize and extend some of the known ones in literature especially those of Olatinwo and Imoru [17] and Berinde [2].

1. Introduction

Let (E,d) be a complete metric space, $T : E \longrightarrow E$ a selfmap of E. Suppose that $F_T = \{p \in E : Tp = p\}$ is the set of fixed points of T in E.
Let $\{x_n\}_{n=0}^{\infty} \subset E$ be the sequence generated by an iteration procedure involving the operator T, that is,

$$x_{n+1} = f(T, x_n),\ n = 0, 1, 2, ... \tag{1.1}$$

where $x_0 \in E$ is the initial approximation and f is some function.
If in (1.1),

$$f(T, x_n) = Tx_n,\ n = 0, 1, 2, ... \tag{1.2}$$

then, we have the Picard iteration process, which has been employed to approximate the fixed points of mappings satisfying

$$d(Tx, Ty) \leq ad(x, y),\ \forall x, y \in E,\ a \in [0, 1), \tag{1.3}$$

called the *Banach's contraction condition* and is of great importance in the celebrated Banach's fixed point Theorem [1]. An operator satisfying (1.3) is called a *strict contraction.*
Also, if in (1.1) and E is a Banach space such that for arbitrary $x_0 \in E$,

$$f(T, x_n) = (1 - \alpha_n)x_n + \alpha_n Tx_n,\ n = 0, 1, 2, ..., \tag{1.4}$$

with $\{\alpha_n\}_{n=0}^{\infty}$ a sequence of real numbers in $[0, 1]$, then we have the Mann iteration process. [See Mann [10]].

2000 *Mathematics Subject Classification.* 47H06, 47H09.
Key words and phrases. Strong convergence, Jungck-Ishikawa iteration, Jungck-Mann iteration, nonselfmappings.

Submitted June 2, 2010. Published September, 2010.

For $x_0 \in E$, the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$\begin{aligned} x_{n+1} &= (1-\alpha_n)x_n + \alpha_n T z_n \\ z_n &= (1-\beta_n)x_n + \beta_n T x_n \end{aligned} \tag{1.5}$$

where $\{\alpha_n\}_{n=o}^{\infty}$ and $\{\beta_n\}_{n=o}^{\infty}$ are sequences of real numbers in [0,1], is called the Ishikawa iteration process. [For Example, see Ishikawa [8]].

In 1972, Zamfirescu [25] proved the following result.

Theorem 1.1. *Let (E, d) be a complete metric space and $T : E \longrightarrow E$ be a mapping for which there exist real numbers $a^{'}, b^{'}$ and $c^{'}$ satisfying $0 \leq a^{'} < 1, 0 \leq b^{'} < 0.5$ and $0 \leq c^{'} < 0.5$ such that, for each $x, y \in E$, at least one of the following is true:*
(Z_1) $d(Tx, Ty) \leq a^{'} d(x, y)$;
(Z_2) $d(Tx, Ty) \leq b^{'}[d(x, Tx) + d(y, Ty)]$;
(Z_3) $d(Tx, Ty) \leq c^{'}[d(x, Ty) + d(y, Tx)]$.
Then, T is a Picard mapping.

An operator T satisfying the contractive conditions $(Z_1), (Z_2)$ and (Z_3) in Theorem 1.1 above is called a *Zamfirescu operator*.

Remark 1.1. The proof of this Theorem is contained in Berinde [2].

If

$$\delta = max\{a^{'}, \frac{b^{'}}{1-b^{'}}, \frac{c^{'}}{1-c^{'}}\}, \tag{1.6}$$

in Theorem 1.1, then

$$0 \leq \delta < 1. \tag{1.7}$$

Then, for all $x, y \in E$, and by using Z_2, it was proved in Berinde [2] that

$$d(Tx, Ty) \leq 2\delta d(x, Tx) + \delta d(x, y), \tag{1.8}$$

and using Z_3 gives

$$d(Tx, Ty) \leq 2\delta d(x, Ty) + \delta d(x, y), \tag{1.9}$$

where $0 \leq \delta < 1$ is as defined by (1.6).

Remark 1.2. If $(E, \|.\|)$ is a normed linear space, then (1.8) becomes

$$\|Tx - Ty\| \leq 2\delta \|x - Tx\| + \delta \|x - y\|, \tag{1.10}$$

for all $x, y \in E$ and where $0 \leq \delta < 1$ is as defined by (1.6).

2. Preliminaries

Rhoades [21, 22] employed the Zamfirescu condition (1.10) to establish several interesting convergence results for Mann and Ishikawa iteration processes in a uniformly convex Banach space.

Later, the results of Rhoades [21, 22] were extended by Berinde [2] to an arbitrary Banach space for the same fixed point iteration processes. Several other researchers such as Bosede [3, 4] and Rafiq [19, 20] obtained some interesting convergence results for some iteration procedures using various contractive definitions. Apart from these convergence results, Rhoades [23] used a contractive condition independent of the Zamfirescu condition and obtained some stability results for other iteration processes, such as Mann [10] and Kirk iterations.

Using a new idea, Osilike [18] considered the following contractive definition: there exist $L \geq 0, a \in [0, 1)$ such that for each $x, y \in E$,

$$d(Tx, Ty) \leq Ld(x, Tx) + ad(x, y). \tag{2.1}$$

Imoru and Olatinwo [7] extended the results of Osilike [18] using the following contractive condition: there exist $b \in [0,1)$ and a monotone increasing function $\varphi : \Re^+ \longrightarrow \Re^+$ with $\varphi(0) = 0$ such that for each $x, y \in E$,

$$d(Tx, Ty) \leq \varphi(d(x, Tx)) + bd(x, y). \tag{2.2}$$

Employing a new concept, Singh et al [24] introduced the following iteration to obtain some common fixed points and stability results: Let S and T be operators on an arbitrary set Y with values in E such that $T(Y) \subseteq S(Y)$, $S(Y)$ is a complete subspace of E. For arbitrary $x_o \in Y$, the sequence $\{Sx_n\}_{n=o}^{\infty}$ defined by

$$Sx_{n+1} = (1-\alpha_n)Sx_n + \alpha_n Tx_n,\ n = 0, 1, 2, ..., \tag{2.3}$$

where $\{\alpha_n\}_{n=o}^{\infty}$ is a sequence of real numbers in [0,1], is called the *Jungck-Mann iteration process*.
If in (2.3), $\alpha_n = 1$ and $Y = E$, then we have

$$Sx_{n+1} = Tx_n,\ n = 0, 1, 2, ..., \tag{2.4}$$

which is the *Jungck iteration*. [For example, see Jungck [9]].
Jungck [9] proved that the maps S and T satisfying

$$d(Tx, Ty) \leq ad(Sx, Sy),\ \forall x, y \in E,\ a \in [0,1), \tag{2.5}$$

have a unique common fixed point in a complete metric space E, provided that S and T commute, $T(Y) \subseteq S(Y)$ and S is continuous. Some stability results were also obtained by Singh et al [24] for Jungck and Jungck-Mann iteration processes in metric space using both the contractive definition (2.5) and the following: For $S, T : Y \longrightarrow E$ and some $a \in [0,1)$, we have

$$d(Tx, Ty) \leq ad(Sx, Sy) + Ld(Sx, Tx),\ \forall x, y \in Y. \tag{2.6}$$

Let $(E, \|.\|)$ be a Banach space and Y an arbitrary set. Let $S, T : Y \longrightarrow E$ be two nonselfmappings such that $T(Y) \subseteq S(Y)$, $S(Y)$ is a complete subspace of E and S is injective. Then, for $x_o \in Y$, the sequence $\{Sx_n\}_{n=o}^{\infty}$ defined iteratively by

$$\begin{aligned} Sx_{n+1} &= (1-\alpha_n)Sx_n + \alpha_n Tz_n \\ Sz_n &= (1-\beta_n)Sx_n + \beta_n Tx_n \end{aligned} \tag{2.7}$$

where $\{\alpha_n\}_{n=o}^{\infty}$ and $\{\beta_n\}_{n=o}^{\infty}$ are sequences of real numbers in [0,1], is called the *Jungck-Ishikawa iteration process*. [See Olatinwo [15]].
For $x_o \in Y$, the sequence $\{Sx_n\}_{n=o}^{\infty}$ defined iteratively by

$$\begin{aligned} Sx_{n+1} &= (1-\alpha_n)Sx_n + \alpha_n Tz_n \\ Sz_n &= (1-\beta_n)Sx_n + \beta_n Ty_n \\ Sy_n &= (1-\gamma_n)Sx_n + \gamma_n Tx_n \end{aligned} \tag{2.8}$$

where $\{\alpha_n\}_{n=o}^{\infty}$, $\{\beta_n\}_{n=o}^{\infty}$ and $\{\gamma_n\}_{n=o}^{\infty}$ are sequences of real numbers in [0,1], is called the *Jungck-Noor iteration process*. [See Noor [12, 13] and Olatinwo [16]].
In 2010, Olaleru and Akewe [14] introduced the following Jungck-Multistep iterative scheme to approximate the common fixed points of contractive-like operators: For $x_o \in Y$, the sequence $\{Sx_n\}_{n=o}^{\infty}$ defined iteratively by

$$\begin{aligned} Sx_{n+1} &= (1-\alpha_n)Sx_n + \alpha_n Ty_n^1 \\ Sy_n^1 &= (1-\beta_n^1)Sx_n + \beta_n^1 Ty_n^{i+1},\ i = 1, 2, ..., k-2, \\ Sy_n^{k-1} &= (1-\beta_n^{k-1})Sx_n + \beta_n^{k-1} Tx_n,\ k \geq 2, \end{aligned} \tag{2.9}$$

where $\{\alpha_n\}_{n=o}^{\infty}$, $\{\beta_n^i\}_{n=o}^{\infty}$ are sequences of real numbers in [0,1] such that $\sum_{n=0}^{\infty}\alpha_n = \infty$, is called the *Jungck-Multistep iteration process.* [See Olaleru and Akewe [14]].
Olatinwo [15] used the Jungck-Ishikawa iteration process (2.7) to establish some stability as well as some strong convergence results by employing the following contractive definitions: For two nonselfmappings $S, T : Y \longrightarrow E$ with $T(Y) \subseteq S(Y)$, where $S(Y)$ is a complete subspace of E,
(a) there exist a real number $a \in [0, 1)$ and a monotone increasing function $\varphi : \Re^+ \longrightarrow \Re^+$ such that $\varphi(0) = 0$ and $\forall\ x, y \in Y$, we have,

$$\|Tx - Ty\| \leq \varphi(\|Sx - Tx\| + a\,\|Sx - Sy\|\,; \tag{2.10}$$

(b) there exist real numbers $M \geq 0$, $a \in [0, 1)$ and a monotone increasing function $\varphi : \Re^+ \longrightarrow \Re^+$ such that $\varphi(0) = 0$ and $\forall\ x, y \in Y$, we have,

$$\|Tx - Ty\| \leq \frac{\varphi(\|Sx - Tx\|) + a\,\|Sx - Sy\|}{1 + M\,\|Sx - Tx\|}. \tag{2.11}$$

Using the Jungck-Multistep iteration process (2.9), Olaleru and Akewe [14] approximated the common fixed points of contractive-like operators by employing the same contractive condition (2.10) of Olatinwo [15].
In this paper, we prove some strong convergence results for Jungck-Ishikawa and Jungck-Mann iteration processes considered in Banach spaces by using a contractive condition independent of those of Olatinwo [15] and Olaleru and Akewe [14].
Consequently, the following natural extension of Theorem 1.1, (that is, the Zamfirescu [25] condition) shall be required in the sequel:
Theorem 2.1. *For two nonselfmappings $S, T : Y \longrightarrow E$ with $T(Y) \subseteq S(Y)$, there exist real numbers α, β and γ satisfying $0 \leq \alpha < 1$, $0 \leq \beta, \gamma < 0.5$ such that, for each $x, y \in Y$, at least one of the following is true:*
(gz_1) $d(Tx, Ty) \leq \alpha d(Sx, Sy)$;
(gz_2) $d(Tx, Ty) \leq \beta[d(Sx, Tx) + d(Sy, Ty)]$;
(gz_3) $d(Tx, Ty) \leq \gamma[d(Sx, Ty) + d(Sy, Tx)]$.
The contractive conditions $(gz_1), (gz_2)$ and (gz_3) will be called the *generalized Zamfirescu condition.* [See Olatinwo and Imoru [17]].
Indeed, if

$$\delta = max\{\alpha, \frac{\beta}{1-\beta}, \frac{\gamma}{1-\gamma}\}, \tag{2.12}$$

then

$$0 \leq \delta < 1. \tag{2.13}$$

Therefore, for all $x, y \in Y$, and by using (gz_2), we have

$$d(Tx, Ty) \leq 2\delta d(Sx, Tx) + \delta d(Sx, Sy). \tag{2.14}$$

Using (gz_3), we obtain

$$d(Tx, Ty) \leq 2\delta d(Sx, Ty) + \delta d(Sx, Sy), \tag{2.15}$$

where $0 \leq \delta < 1$ is as defined by (2.12).
Remark 2.1. If $(E, \|.\|)$ is a normed linear space or a Banach space, then (2.14) becomes

$$\|Tx - Ty\| \leq 2\delta\,\|Sx - Tx\| + \delta\,\|Sx - Sy\|\,, \tag{2.16}$$

for all $x, y \in E$ and where $0 \leq \delta < 1$ is as defined by (2.12).
Olatinwo and Imoru [17] proved some convergence results for the Jungck-Mann and the Jungck-Ishikawa iteration processes in the class of generalized Zamfirescu

operators by using the contraction condition (2.12).
Our aim in this paper is to prove some strong convergence results for Jungck-Ishikawa and Jungck-Mann iteration processes considered in Banach spaces by using a contractive condition independent of those of Olatinwo [15] and Olaleru and Akewe [14].
These results are established for a pair of nonselfmappings using the Jungck-Ishikawa and Jungck-Mann iterations for a class of functions more general than those of Olatinwo and Imoru [17], Berinde [2] and many others.
We shall employ the following contractive definition: Let $(E, \|.\|)$ be a Banach space and Y an arbitrary set. Suppose that $S, T : Y \longrightarrow E$ are two nonselfmappings such that $T(Y) \subseteq S(Y)$ and $S(Y)$ is a complete subspace of E. Suppose also that z is a coincidence point of S and T, (that is, $Sz = Tz = p$). There exist a constant $L \geq 0$ such that $\forall\ x, y \in Y$, we have

$$\|Tx - Ty\| \leq e^{L\|Sx-Tx\|}(2\delta \|Sx - Tx\| + \delta \|Sx - Sy\|), \tag{2.17}$$

where $0 \leq \delta < 1$ is as defined by (2.12) and e^x denotes the exponential function of $x \in Y$.
Remark 2.2. The contractive condition (2.17) is more general than those considered by Olatinwo and Imoru [17], Berinde [2] and several others in the following sense: For example, if $L = 0$ in the contractive condition (2.17), then we obtain

$$\|Tx - Ty\| \leq 2\delta \|Sx - Tx\| + \delta \|Sx - Sy\| \tag{2.18}$$

which is the generalized Zamfirescu contraction condition (2.16) used by Olatinwo and Imoru [17], where

$$\delta = max\{\alpha, \frac{\beta}{1-\beta}, \frac{\gamma}{1-\gamma}\}, 0 \leq \delta < 1, \tag{2.19}$$

while constants α, β and γ are as defined in Theorem 2.1 above.

3. Main Results

Theorem 3.1. *Let $(E, \|.\|)$ be a Banach space and Y an arbitrary set. Suppose that $S, T : Y \longrightarrow E$ are two nonselfmappings such that $T(Y) \subseteq S(Y)$, $S(Y)$ is a complete subspace of E. Suppose that z is a coincidence point of S and T, (that is, $Sz = Tz = p$). Suppose also that S and T satisfy the contractive condition (2.17). For $x_0 \in Y$, let $\{Sx_n\}_{n=o}^{\infty}$ be the Jungck-Ishikawa iteration process defined by (2.7), where $\{\alpha_n\}_{n=o}^{\infty}$ and $\{\beta_n\}_{n=o}^{\infty}$ are sequences of real numbers in [0,1] such that $\sum_{k=0}^{\infty} \alpha_k = \infty$.*
Then, the Jungck-Ishikawa iteration process converges strongly to p.
Proof. Let $Sx_{n+1} = (1-\alpha_n)Sx_n + \alpha_n Tq_n,\ n = 0, 1, ...,$ where $Sq_n = (1-\beta_n)Sx_n + \beta_n Tx_n$.
Therefore, using the Jungck-Ishikawa iteration (2.7), the contractive condition

(2.17) and the triangle inequality, we have

$$\begin{aligned}
\|Sx_{n+1}-p\| &= \|(1-\alpha_n)Sx_n+\alpha_nTq_n-p\| \\
&= \|(1-\alpha_n)Sx_n+\alpha_nTq_n-((1-\alpha_n)+\alpha_n)p\| \\
&= \|(1-\alpha_n)(Sx_n-p)+\alpha_n(Tq_n-p)\| \\
&\leq (1-\alpha_n)\|Sx_n-p\|+\alpha_n\|Tq_n-p\| \\
&= 1-\alpha_n)\|Sx_n-p\|+\alpha_n\|Tz-Tq_n\| \\
&\leq (1-\alpha_n)\|Sx_n-p\|+\alpha_n e^{L\|Sz-Tz\|}(2\delta\|Sz-Tz\|+\delta\|Sz-Sq_n\|) \\
&= (1-\alpha_n)\|Sx_n-p\|+\alpha_n e^{L\|p-p\|}(2\delta\|p-p\|+\delta\|p-Sq_n\|) \\
&= (1-\alpha_n)\|Sx_n-p\|+\alpha_n e^{L(0)}(2\delta(0)+\delta\|Sq_n-p\|) \\
&= (1-\alpha_n)\|Sx_n-p\|+\alpha_n\delta\|Sq_n-p\|
\end{aligned} \tag{3.1}$$

We estimate $\|Sq_n-p\|$ in (3.1) as follows:

$$\begin{aligned}
\|Sq_n-p\| &= \|(1-\beta_n)Sx_n+\beta_nTx_n-p\| \\
&= \|(1-\beta_n)(Sx_n-p)+\beta_n(Tx_n-p)\| \\
&\leq (1-\beta_n)\|Sx_n-p\|+\beta_n\|Tx_n-p\| \\
&= (1-\beta_n)\|Sx_n-p\|+\beta_n\|Tz-Tx_n\| \\
&\leq (1-\beta_n)\|Sx_n-p\|+\beta_n e^{L\|Sz-Tz\|}(2\delta\|Sz-Tz\|+\delta\|Sz-Sx_n\|) \\
&= (1-\beta_n)\|Sx_n-p\|+\beta_n e^{L(0)}(2\delta(0)+\delta\|p-Sx_n\|) \\
&= (1-\beta_n)\|Sx_n-p\|+\beta_n\delta\|Sx_n-p\| \\
&= (1-\beta_n+\delta\beta_n)\|Sx_n-p\|
\end{aligned} \tag{3.2}$$

Substitute (3.2) into (3.1) gives

$$\begin{aligned}
\|Sx_{n+1}-p\| &\leq [1-(1-\delta)\alpha_n-(1-\delta)\delta\alpha_n\beta_n]\|Sx_n-p\| \\
&\leq [1-(1-\delta)\alpha_n]\|Sx_n-p\| \\
&\leq \prod_{k=0}^{\infty}[1-(1-\delta)\alpha_k]\|Sx_0-p\| \\
&\leq \prod_{k=0}^{\infty} e^{-[1-(1-\delta)\alpha_k]}\|Sx_0-p\| \\
&= e^{-(1-\delta)}\sum_{k=0}^{\infty}\alpha_k\|Sx_0-p\| \longrightarrow 0
\end{aligned} \tag{3.3}$$

as $n \longrightarrow \infty$. By observing that $\sum_{k=0}^{\infty}\alpha_k=\infty$, $\delta\in[0,1)$ and from (3.3), we get

$$\|Sx_n-p\| \longrightarrow 0 \tag{3.4}$$

as $n \longrightarrow \infty$, which implies that $\{Sx_n\}_{n=o}^{\infty}$ converges strongly to p.
To prove the uniqueness, we take $z_1, z_2 \in C(S,T)$, where $C(S,T)$ is the set of coincidence points of S and T such that $Sz_1=Tz_1=p_1$ and $Sz_2=Tz_2=p_2$. Suppose on the contrary that $p_1 \neq p_2$. Then, using the contractive condition (2.17)

and since $0 \leq \delta < 1$, we have

$$\begin{aligned}
\|p_1 - p_2\| &\leq \|Tz_1 - Tz_2\| \\
&= \|p_1 - Tz_2\| \\
&\leq e^{L\|Sz_1 - Tz_1\|}(2\delta \|Sz_1 - Tz_1\| + \delta \|Sz_1 - Sz_2\|) \\
&= e^{L\|p_1 - p_1\|}(2\delta \|p_1 - p_1\| + \delta \|p_1 - p_2\|) \\
&= e^{L(0)}(2\delta(0) + \delta \|p_1 - p_2\|) \\
&= \delta \|p_1 - p_2\| \\
&< \|p_1 - p_2\|,
\end{aligned}$$

which is a contradiction. Therefore, $p_1 = p_2$.
This completes the proof.

Remark 3.1. Our result in Theorem 3.1 of this paper is a generalization of Theorem 3.1 in Olatinwo and Imoru [17], which itself is a generalization of Berinde [2] and many others in literature.

The next result shows that the Jungck-Mann iteration process converges strongly to p.

Theorem 3.2. *Let E, Y, S, T, z and p be as in Theorem 3.1.*
For arbitrary $x_0 \in Y$, let $\{Sx_n\}_{n=o}^{\infty}$ be the Jungck-Mann iteration process defined by (2.3) where $\{\alpha_n\}_{n=o}^{\infty}$ is a sequence of real numbers in [0,1] such that $\sum_{k=0}^{\infty} \alpha_k = \infty$. Then, the Jungck-Mann iteration process converges strongly to p.

Proof. Using the Jungck-Mann iteration (2.3), the contractive condition (2.17) and the triangle inequality, we have

$$\begin{aligned}
\|Sx_{n+1} - p\| &= \|(1-\alpha_n)Sx_n + \alpha_n Tx_n - p\| \\
&= \|(1-\alpha_n)Sx_n + \alpha_n Tx_n - ((1-\alpha_n) + \alpha_n)p\| \\
&= \|(1-\alpha_n)(Sx_n - p) + \alpha_n(Tx_n - p)\| \\
&\leq (1-\alpha_n)\|Sx_n - p\| + \alpha_n \|Tx_n - p\| \\
&= 1-\alpha_n)\|Sx_n - p\| + \alpha_n \|Tz - Tx_n\| \\
&\leq (1-\alpha_n)\|Sx_n - p\| \\
&+ \alpha_n e^{L\|Sz - Tz\|}(2\delta \|Sz - Tz\| + \delta \|Sz - Sx_n\|) \\
&= (1-\alpha_n)\|Sx_n - p\| + \alpha_n e^{L(0)}(2\delta(0) + \delta \|p - Sx_n\|) \\
&= (1-\alpha_n)\|Sx_n - p\| + \alpha_n \delta \|Sx_n - p\|) \\
&= [1 - (1-\delta)\alpha_n]\|Sx_n - p\| \\
&\leq \prod_{k=0}^{\infty}[1 - (1-\delta)\alpha_k]\|Sx_0 - p\| \\
&\leq \prod_{k=0}^{\infty} e^{-[1-(1-\delta)\alpha_k]}\|Sx_0 - p\| \\
&= e^{-(1-\delta)}\sum_{k=0}^{\infty}\alpha_k \|Sx_0 - p\| \longrightarrow 0
\end{aligned} \tag{3.5}$$

as $n \longrightarrow \infty$. Since $\sum_{k=0}^{\infty} \alpha_k = \infty$, $\delta \in [0, 1)$, therefore from (3.4), we have

$$\|Sx_n - p\| \longrightarrow 0 \tag{3.6}$$

as $n \longrightarrow \infty$, which implies that the Jungck-Mann iteration converges strongly to p. This completes the proof.

Remark 3.2. Theorem 3.2 of this paper is a generalization of Theorem 3.3 in Olatinwo and Imoru [17]. Theorem 3.2 is also a generalization of the results obtained by Berinde [2] and this is also a further improvement to many existing known results in literature.

A special case of Jungck-Mann iteration process is that of Jungck-Krasnoselskij iteration process which is Jungck-Mann iteration, with each $\alpha_n = \lambda$, for some $0 < \lambda < 1$.

For arbitrary $x_o \in Y$, the sequence $\{Sx_n\}_{n=o}^{\infty}$ defined by

$$Sx_{n+1} = (1-\lambda)Sx_n + \lambda Tx_n, \; n = 0, 1, 2, ..., \tag{3.7}$$

for some $0 < \lambda < 1$, is called the *Jungck-Krasnoselskij iteration process.*

Corollary 3.1. *Let E, Y, S, T, z and p be as in Theorem 3.1. For arbitrary $x_0 \in Y$, let $\{Sx_n\}_{n=o}^{\infty}$ be the Jungck-Krasnoselskij iteration process defined by (3.7) for some $0 < \lambda < 1$. Then, the Jungck-Krasnoselskij iteration process converges strongly to p.*

Proof. In Theorem 3.2, set each $\alpha_n = \lambda$.

Acknowledgment. The author would like to thank the anonymous referees for their comments which helped to improve this article.

References

[1] V. Berinde, *Iterative Approximation of Fixed Points*, Editura Efemeride, Baia Mare, (2002).

[2] V. Berinde, *On the convergence of the Ishikawa iteration in the class of quasi-contractive operators*, Acta Math. Univ. Comenianae, **LXXIII(1)**(2004), 119–126.

[3] A. O. Bosede, *Noor iterations associated with Zamfirescu mappings in uniformly convex Banach spaces*, Fasciculi Mathematici, **42**(2009), 29–38.

[4] A. O. Bosede, *Some common fixed point theorems in normed linear spaces, Acta Univ. Palacki. Olomuc.*, Fac. rer. nat., Mathematica, **49**(1)(2010), 19–26.

[5] A. O. Bosede and B. E. Rhoades, *Stability of Picard and Mann iterations for a general class of functions*, Journal of Advanced Mathematical Studies, **3**(2)(2010), 1–3.

[6] C. O. Imoru, G. Akinbo and A. O. Bosede, *On the fixed points for weak compatible type and parametrically $\varphi(\epsilon, \delta; a)$-contraction mappings*, Math. Sci. Res. Journal, **10**(10)(2006), 259–267.

[7] C. O. Imoru and M. O. Olatinwo, *On the stability of Picard and Mann iteration processes*, Carpathian J. Math., **19**(2)(2003), 155–160.

[8] S. Ishikawa, *Fixed point by a new iteration method*, Proc. Amer. Math. Soc., **44**(1)(1974), 147–150.

[9] G. Jungck, *Commuting mappings and fixed points*, Amer. Math. Monthly, **83**(4)(1976), 261–263.

[10] W. R. Mann, *Mean value methods in iterations*, Proc. Amer. Math. Soc., **4**(1953), 506–510.

[11] M. A. Noor, *General variational inequalities*, Appl. Math. Letters., **1**(1988), 119–121.

[12] M. A. Noor, *New approximations schemes for general variational inequalities*, J. Math. Anal. Appl., **251**(2000), 217–229.

[13] M. A. Noor, *Some new developments in general variational inequalities*, Appl. Math. Computation, **152**(2004), 199–277.

[14] J. O. Olaleru and H. Akewe, *On Multistep iterative scheme for approximating the common fixed points of contractive-like operators*, International Journal of Mathematics and Mathematical Sciences, Vol. 2010, (2010), Article ID 530964, 11 pages.

[15] M. O. Olatinwo, *Some stability and strong convergence results for the Jungck-Ishikawa iteration process*, Creat. Math. Inform., **17**(2008), 33–42.

[16] M. O. Olatinwo, *A generalization of some convergence results using a Jungck-Noor three-step iteration process in arbitrary Banach space*, Fasciculi Mathematici, **40**(2008), 37–43.

[17] M. O. Olatinwo and C. O. Imoru, *Some convergence results for the Jungck-Mann and the Jungck-Ishikawa iteration processes in the class of generalized Zamfirescu operators*, Acta Math. Univ. Comenianae, **LXXVII**(2)(2008), 299–304.

[18] M. O. Osilike, *Stability results for fixed point iteration procedures*, J. Nigerian Math. Soc., **14/15**(1995/1996), 17–29.

[19] A. Rafiq, *A convergence theorem for Mann fixed point iteration procedure*, Applied Mathematics E-Notes, **6**(2006), 289–293.

[20] A. Rafiq, *On the convergence of the three-step iteration process in the class of quasi-contractive operators*, Acta Math. Acad. Paedagog Nyiregyhaziensis, **22**(2006), 305–309.

[21] B. E. Rhoades, *Fixed point iteration using infinite matrices*, Trans. Amer. Math. Soc., **196**(1974), 161–176.

[22] B. E. Rhoades, *Comments on two fixed point iteration methods*, J. Math. Anal. Appl., **56**(2)(1976), 741–750.

[23] B. E. Rhoades, *Fixed point theorems and stability results for fixed point iteration procedures*, Indian J. Pure Appl. Math., **21**(1)(1990), 1–9.

[24] S. L. Singh, C. Bhatmagar and S. N. Mishra, *Stability of Jungck-type iterative procedures*, International J. Math. & Math. Sc., **19**(2005), 3035–3043.

[25] T. Zamfirescu, *Fix point theorems in metric spaces*, Arch. Math., **23**(1972), 292–298.

Alfred Olufemi Bosede

Department of Mathematics Lagos State University Ojo, Lagos State, Nigeria

E-mail address: aolubosede@yahoo.co.uk