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FURTHER RESULTS ON FRACTIONAL CALCULUS OF SRIVASTAVA POLYNOMIALS

(COMMUNICATED BY H. M. SRIVASTAVA)

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Abstract. Series expansion methods for fractional integrals are important and useful for treating certain problems of pure and applied mathematics. The aim of the present investigation is to obtain certain new fractional calculus formulae, which involve Srivastava polynomials. Several special cases of our main findings which are also believed to be new have been given. For the sake of illustration, we point out that the fractional calculus formulae obtained by Saigo & Raina (see [12]) follow as particular cases of our findings.

1. Introduction

Srivastava and Saigo (see [16]) have studied in their paper multiplication of fractional calculus operator and boundary value problems involving the Euler-Darboux equation and Ross (see [8]) obtained the fractional integral formulae by using series expansion method. The aim of the present investigation is to obtained further fractional calculus formulae, using series expansion method, for the Srivastava polynomials which were introduced by Srivastava (see [14]) .The name Srivastava polynomials, it self indicates the importance of the results, because we can derive a number of fractional calculus formulae for various classical orthogonal polynomials. The most widely used definition of an integral of fractional order is via an integral transform, called the Riemann-Liouville operator of fractional integration (see [3], [10]).

$$ {}_cR_x^{-\alpha} = {}_cD_x^{\alpha}[f(x)] = \frac{1}{\Gamma(-\alpha)} \int_c^x (x-t)^{-\alpha-1} f(t)dt, \mathrm{Re}(\alpha) > 0. \tag{1.1}$$

$$ = \frac{d^n}{dx^n} {}_cD_x^{\alpha-n}[f(x)], 0 \leq Re(\alpha) < n. \tag{1.2}$$

where f(x) is a locally integrable function and n is a positive integer.
Several authors (see [1]-[4], [6]-[7], [13], and [18]) have defined and studied operators

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of fractional calculus via a series expansion approach.
For β and η be real numbers

$$I_{0,x}^{\alpha,\beta,\eta}[f(x)] = \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} {}_2F_1\left[\begin{array}{l} \alpha+\beta, -\eta \\ \alpha \end{array} ; 1-\frac{t}{x}\right] f(t)dt, \mathrm{Re}(\alpha) > 0 \tag{1.3}$$

$$= \frac{d^n}{dx^n} I_{0,x}^{\alpha+n,\beta-n,\eta-n}[f(x)], \mathrm{Re}(\alpha) \leq 0 \tag{1.4}$$

where $0 < Re(\alpha)+n \leq 1$, n being a positive integer. Which were analyzed by Saigo (see [9]).
The Srivastava polynomials $S_n^m(x)$ introduced by Srivastava (see [14]) is as follows

$$S_n^m(x) = \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k} x^k \tag{1.5}$$

where m is an arbitrary positive integer and the coefficient $A_{n,k}$(n, k$\geq$ 0) are arbitrary constants, real or complex.
By suitable specializing the coefficients $A_{n,k}$ the polynomial set $S_n^m(x)$ reduces to the various classical orthogonal polynomial (see [17]). The particular cases of our main results have come out in terms of the well-known Kampe$'$-de-Fe$'$riet double hypergeometric functions, general triple hypergeometric functions$F^{(3)}[.,.,.]$ and Pathan's quadruple hypergeometric functions $F_P^{(4)}[.,.,.,.]$(see [15],[5])

$$F_{l:m;n}^{p:q;k}\left[\begin{array}{l} (a_j):(b_q);(c_k); \\ (\alpha_l):(\beta_m);(\gamma_n); \end{array} x, y\right]$$

$$= \sum_{r,s=0}^{\infty} \frac{\prod_{j=1}^{p}(a_j)_{r+s} \prod_{j=1}^{q}(b_j)_r \prod_{j=1}^{k}(c_j)_s}{\prod_{j=1}^{l}(\alpha_j)_{r+s} \prod_{j=1}^{m}(\beta_j)_r \prod_{j=1}^{n}(\gamma_j)_s} \frac{x^r y^s}{r!s!}. \tag{1.6}$$

The convergence conditions for which are

$(i) p+q < l+m+1, p+k < l+n+1, |x| < \infty; |y| < \infty;$
$(ii) p+q = l+m+1, p+k = l+n+1; |x|^{1/(p-l)} + |y|^{1/(p-l)} < 1, if p > l;$
$\max\{|x|,|y|\} < 1 if p \leq l.$

and

$$F^{(3)}[x,y,z] \equiv F^{(3)}\left[\begin{array}{l} (a)::(b);(b');(b''):(c);(c');(c''); \\ (e)::(g);(g');(g''):(h);(h');(h''); \end{array} x,y,z\right]$$

$$= \sum_{m,n,p=0}^{\infty} \Lambda(m,n,p) \frac{x^m}{m!} \frac{y^n}{n!} \frac{z^p}{p!}, \tag{1.7}$$

where, for convenience,

$$\Lambda(m,n,p) = \frac{\prod_{j=1}^{A}(a_j)_{m+n+p} \prod_{j=1}^{B}(b_j)_{m+n} \prod_{j=1}^{B'}(b'_j)_{n+p} \prod_{j=1}^{B''}(b''_j)_{p+m}}{\prod_{j=1}^{E}(e_j)_{m+n+p} \prod_{j=1}^{G}(g_j)_{m+n} \prod_{j=1}^{G'}(g'_j)_{n+p} \prod_{j=1}^{G''}(g''_j)_{p+m}} \tag{1.8}$$

$$\frac{\prod_{j=1}^{C}(c_j)_m \prod_{j=1}^{C'}\left(c'_j\right)_n \prod_{j=1}^{C''}\left(c''_j\right)_p}{\prod_{j=1}^{H}(h_j)_m \prod_{j=1}^{H'}\left(h'_j\right)_n \prod_{j=1}^{H''}\left(h''_j\right)_p}$$

provided that $A+B+B''+C \leq E+G+G''+H+1$ etc.; the equality signs holds for $|x|<1, |y|<1, |z|<1$.

$$F_p^{(4)}\left[x_1,x_2,x_3,x_4\right] \equiv F_p^{(4)}\left[\begin{array}{l}(a)::(b);(b');(b'');(b'''):(c);(c');(c'');(c''');\\(e)::(g);(g');(g'');(g'''):(h);(h');(h'');(h''');\end{array}\quad x_1,x_2,x_3,x_4\right]$$

$$=\sum_{m,n,p,q=0}^{\infty}\Lambda(m,n,p,q)\frac{x_1^m}{m!}\frac{x_2^n}{n!}\frac{x_3^p}{p!}\frac{x_4^q}{q!}, \tag{1.9}$$

where, for convenience,

$$\Lambda(m,n,p,q)=\frac{\prod_{j=1}^{A}(a_j)_{m+n+p+q}}{\prod_{j=1}^{E}(e_j)_{m+n+p+q}} \tag{1.10}$$

$$\frac{\prod_{j=1}^{B}(b_j)_{m+n+p}\prod_{j=1}^{B'}\left(b'_j\right)_{n+p+q}\prod_{j=1}^{B''}\left(b''_j\right)_{p+q+m}\prod_{j=1}^{B'''}\left(b'''_j\right)_{q+m+n}}{\prod_{j=1}^{G}(g_j)_{m+n+p}\prod_{j=1}^{G'}\left(g'_j\right)_{n+p+q}\prod_{j=1}^{G''}\left(g''_j\right)_{p+q+m}\prod_{j=1}^{G'''}\left(g'''_j\right)_{q+m+n}}$$

$$\frac{\prod_{j=1}^{C}(c_j)_m\prod_{j=1}^{C'}\left(c'_j\right)_n\prod_{j=1}^{C''}\left(c''_j\right)_p\prod_{j=1}^{C'''}\left(c'''_j\right)_q}{\prod_{j=1}^{H}(h_j)_m\prod_{j=1}^{H'}\left(h'_j\right)_n\prod_{j=1}^{H''}\left(h''_j\right)_p\prod_{j=1}^{H'''}\left(h'''_j\right)_q}$$

provided that denominator parameters are neither zero nor negative integers. We prove the following fractional calculus formula in this paper

$$I_{0,x}^{\alpha,\beta,\eta}\left[x^k(b-ax)^{-\lambda}S_n^m\left(x^{\rho_1}(b-ax)^{-\sigma_1}\right)\right]$$

$$=b^{-\lambda}x^{k-\beta}\sum_{i=0}^{[n/m]}\frac{(-n)_{mi}}{i!}A_{n,i}\Gamma\left[\begin{array}{c}k+\rho_1 i+1,-\beta+\eta+k+\rho_1 i+1\\-\beta+k+\rho_1 i+1,\alpha+\eta+k+\rho_1 i+1\end{array}\right]$$
$$\times\left(\frac{x^{\rho_1}}{b^{\sigma_1}}\right)^i{}_3F_2\left[\begin{array}{c}\lambda+\sigma_1 i,k+\rho_1 i+1,-\beta+\eta+k+\rho_1 i+1\\-\beta+k+\rho_1 i+1,\alpha+\eta+k+\rho_1 i+1\end{array};\frac{ax}{b}\right] \tag{1.11}$$

valid for $\min(k,\lambda,\rho_1,\sigma_1)>0, \left|\frac{ax}{b}\right|<1$ and $k+\rho_1 i>\max\{0,Re(\beta-\eta)\}-1$ and

$$I_{0,x}^{\alpha,\beta,\eta}\left[x^k e^{cx}(b-ax)^{-\lambda}S_n^m\left(x^{\rho_1}(b-ax)^{-\sigma_1}\right)\right]$$

$$=b^{-\lambda}x^{k-\beta}\sum_{i=0}^{[n/m]}\frac{(-n)_{mi}}{i!}A_{n,i}\Gamma\left[\begin{array}{c}k+\rho_1 i+1,-\beta+\eta+k+\rho_1 i+1\\-\beta+k+\rho_1 i+1,\alpha+\eta+k+\rho_1 i+1\end{array}\right]$$
$$\times\left(\frac{x^{\rho_1}}{b^{\sigma_1}}\right)^i F_{2:-;-}^{2:1;-}\left[\begin{array}{c}k+\rho_1 i+1,-\beta+\eta+k+\rho_1 i+1:\lambda+\sigma_1 i;-;\\-\beta+k+\rho_1 i+1,\alpha+\eta+k+\rho_1 i+1:-;-;\end{array}\frac{ax}{b},cx\right] \tag{1.12}$$

valid for $\min(k,\lambda,\rho_1,\sigma_1)>0, \left|\frac{ax}{b}\right|<1$and $k+\rho_1 i>\max\{0,Re(\beta-\eta)\}-1$

$$I_{0,x}^{\alpha,\beta,\eta}\left[x^k(b-ax)^{-\lambda}S_n^m\left(x^{\rho_1}(b-ax)^{-\sigma_1}\right)S_q^p\left(x^l\right)\right]$$

$$=b^{-\lambda}x^{k-\beta}\sum_{i=0}^{[n/m]}\sum_{j=0}^{[q/p]}\frac{(-n)_{mi}(-q)_{pj}}{i!j!}A_{n,i}A_{q,j} \tag{1.13}$$

$$\Gamma\left[\begin{array}{l} k+\rho_1 i+lj+1,-\beta+\eta+k+\rho_1 i+lj+1 \\ -\beta+k+\rho_1 i+lj+1,\alpha+\eta+k+\rho_1 i+lj+1 \end{array}\right]\times\left(\frac{x^{\rho_1}}{b^{\sigma_1}}\right)^i x^{lj}$$

$${}_3F_2\left[\begin{array}{l} \lambda+\sigma_1 i,k+\rho_1 i+lj+1,-\beta+\eta+k+\rho_1 i+lj+1 \\ -\beta+k+\rho_1 i+lj+1,\alpha+\eta+k+\rho_1 i+lj+1 \end{array};\frac{ax}{b}\right]$$

valid for $\min(k,l,\lambda,\rho_1,\sigma_1)>0, \left|\frac{ax}{b}\right|<1$and $k+lj+\rho_1 i>\max\{0,Re(\beta-\eta)-1\}$

$$I_{0,x}^{\alpha,\beta,\eta}\left[x^k e^{cx}(b-ax)^{-\lambda}S_n^m\left(x^{\rho_1}(b-ax)^{-\sigma_1}\right)S_q^p\left(x^l\right)\right] \tag{1.14}$$

$$=b^{-\lambda}x^{k-\beta}\sum_{i=0}^{[n/m]}\sum_{j=0}^{[q/p]}\frac{(-n)_{mi}(-q)_{pj}}{i!j!}A_{n,i}A_{q,j}$$

$$\Gamma\left[\begin{array}{l} k+\rho_1 i+lj+1,-\beta+\eta+k+\rho_1 i+lj+1 \\ -\beta+k+\rho_1 i+lj+1,\alpha+\eta+k+\rho_1 i+lj+1 \end{array}\right]\times\left(\tfrac{x^{\rho_1}}{b^{\sigma_1}}\right)^i x^{lj}$$
$$F_{2:-;-}^{2:1;-}\left[\begin{array}{l} k+\rho_1 i+lj+1,-\beta+\eta+k+\rho_1 i+lj+1:\lambda+\sigma_1 i;-; \\ -\beta+k+\rho_1 i+lj+1,\alpha+\eta+k+\rho_1 i+lj+1:-;-; \end{array}\tfrac{ax}{b},cx\right]$$

valid for $\min(k,l,\lambda,\rho_1,\sigma_1)>0, \left|\frac{ax}{b}\right|<1$and $k+lj+\rho_1 i>\max\{0,Re(\beta-\eta)-1\}$
Proofs: To establish the fractional calculus formulae (1.11) and (1.12), we first express the Srivastava polynomials $S_n^m(x)$ occurring on its left-hand side in the series form given by (1.5) and than making use of the following binomial expansion for$(b-ax)^{-\lambda}$

$$(b-ax)^{-\lambda}=b^{-\lambda}\sum_{l=0}^{\infty}\frac{(\lambda)_l}{l!}\left(\frac{ax}{b}\right)^l,\left|\frac{ax}{b}\right|<1 \tag{1.15}$$

By using series expansion of e^{cx}\{for (1.12)\} and also use of the well known formula as follows, or by using (1.3), taking$f(x)=x^k$, we have

$$I_{0,x}^{\alpha,\beta,\eta}\left[x^k\right]=\frac{\Gamma(k+1)\Gamma(-\beta+\eta+k+1)}{\Gamma(-\beta+k+1)\Gamma(\alpha+\eta+k+1)}x^{k-\beta} \tag{1.16}$$

where $k>\max(0,\beta-\eta)-1$;(see [11]).
Further using the series expansion of ${}_3F_2(.)$,we finally arrive at the desired results.

The fractional calculus formula (1.13) and (1.14) are established with the help of series expansions of $F_{l:m;n}^{p:q;k}[.,.]$ given by (1.6) and than proceeding on lines similar to that of above, we arrive at the required results.

2. Applications

As the special cases of our main results, if we taking $\sigma_1=0$,fractional calculus formula (1.11) readily yields

$$I_{0,x}^{\alpha,\beta,\eta}\left[x^k(b-ax)^{-\lambda}S_n^m\left(x^{\rho_1}\right)\right] \tag{2.1}$$

$$=b^{-\lambda}x^{k-\beta}\textstyle\sum_{i=0}^{[n/m]}\frac{(-n)_{mi}}{i!}A_{n,i}\Gamma\left[\begin{array}{l} k+\rho_1 i+1,-\beta+\eta+k+\rho_1 i+1 \\ -\beta+k+\rho_1 i+1,\alpha+\eta+k+\rho_1 i+1 \end{array}\right]$$
$$\times x^{\rho_1 i}{}_3F_2\left[\begin{array}{l} \lambda,k+\rho_1 i+1,-\beta+\eta+k+\rho_1 i+1 \\ -\beta+k+\rho_1 i+1,\alpha+\eta+k+\rho_1 i+1 \end{array};\tfrac{ax}{b}\right]$$

valid for $\min(k,\lambda,\rho_1)>0, \left|\frac{ax}{b}\right|<1$and $k+\rho_1 i>\max\{0,Re(\beta-\eta)\}-1$.

while (1.12) reduce to

$$I_{0,x}^{\alpha,\beta,\eta}\left[x^k e^{cx}(b-ax)^{-\lambda} S_n^m(x^{\rho_1})\right] \tag{2.2}$$

$$= b^{-\lambda} x^{k-\beta} \sum_{i=0}^{[n/m]} \frac{(-n)_{mi}}{i!} A_{n,i} \Gamma\left[\begin{array}{c} k+\rho_1 i+1, -\beta+\eta+k+\rho_1 i+1 \\ -\beta+k+\rho_1 i+1, \alpha+\eta+k+\rho_1 i+1 \end{array}\right]$$
$$\times x^{\rho_1 i} F_{2:-;-}^{2:1;-}\left[\begin{array}{c} k+\rho_1 i+1, -\beta+\eta+k+\rho_1 i+1 : \lambda; -; \\ -\beta+k+\rho_1 i+1, \alpha+\eta+k+\rho_1 i+1 : -; -; \end{array} \frac{ax}{b}, cx\right]$$

valid for $\min(k,\lambda,\rho_1) > 0, \left|\frac{ax}{b}\right| < 1$ and $k+\rho_1 i > \max\{0, Re(\beta-\eta)\} - 1$.
Similarly (1.13) and (1.14) becomes

$$I_{0,x}^{\alpha,\beta,\eta}\left[x^k (b-ax)^{-\lambda} S_n^m(x^{\rho_1}) S_q^p(x^l)\right] \tag{2.3}$$

$$= b^{-\lambda} x^{k-\beta} \sum_{i=0}^{[n/m]} \sum_{j=0}^{[q/p]} \frac{(-n)_{mi}(-q)_{pj}}{i!j!} A_{n,i} A_{q,j}$$

$$\Gamma\left[\begin{array}{c} k+\rho_1 i+lj+1, -\beta+\eta+k+\rho_1 i+lj+1 \\ -\beta+k+\rho_1 i+lj+1, \alpha+\eta+k+\rho_1 i+lj+1 \end{array}\right] \times x^{\rho_1 i} x^{lj}$$

$${}_3F_2\left[\begin{array}{c} \lambda, k+\rho_1 i+lj+1, -\beta+\eta+k+\rho_1 i+lj+1 \\ -\beta+k+\rho_1 i+lj+1, \alpha+\eta+k+\rho_1 i+lj+1 \end{array} ; \frac{ax}{b}\right]$$

valid for $\min(k,l,\lambda,\rho_1) > 0, \left|\frac{ax}{b}\right| < 1$ and $k+lj+\rho_1 i > \max\{0, Re(\beta-\eta)-1\}$.

$$I_{0,x}^{\alpha,\beta,\eta}\left[x^k e^{cx}(b-ax)^{-\lambda} S_n^m(x^{\rho_1}) S_q^p(x^l)\right] \tag{2.4}$$

$$= b^{-\lambda} x^{k-\beta} \sum_{i=0}^{[n/m]} \sum_{j=0}^{[q/p]} \frac{(-n)_{mi}(-q)_{pj}}{i!j!} A_{n,i} A_{q,j}$$

$$\Gamma\left[\begin{array}{c} k+\rho_1 i+lj+1, -\beta+\eta+k+\rho_1 i+lj+1 \\ -\beta+k+\rho_1 i+lj+1, \alpha+\eta+k+\rho_1 i+lj+1 \end{array}\right] \times x^{\rho_1 i} x^{lj}$$
$$F_{2:-;-}^{2:1;-}\left[\begin{array}{c} k+\rho_1 i+lj+1, -\beta+\eta+k+\rho_1 i+lj+1 : \lambda; -; \\ -\beta+k+\rho_1 i+lj+1, \alpha+\eta+k+\rho_1 i+lj+1 : -; -; \end{array} \frac{ax}{b}, cx\right]$$

valid for $\min(k,l,\lambda,\rho_1) > 0, \left|\frac{ax}{b}\right| < 1$ and $k+lj+\rho_1 i > \max\{0, Re(\beta-\eta)-1\}$.
Out of these two results, (2.1) and (2.2) on specialization of $A_{n,i}$ reduce to a very special results, due to Saigo and Raina (see [12]).
If we take $\lambda = 0$ in (2.1) to (2.4), these formulae reduce to

$$I_{0,x}^{\alpha,\beta,\eta}\left[x^k S_n^m(x^{\rho_1})\right] \tag{2.5}$$

$$= x^{k-\beta} \sum_{i=0}^{[n/m]} \frac{(-n)_{mi}}{i!} A_{n,i} \Gamma\left[\begin{array}{c} k+\rho_1 i+1, -\beta+\eta+k+\rho_1 i+1 \\ -\beta+k+\rho_1 i+1, \alpha+\eta+k+\rho_1 i+1 \end{array}\right] x^{\rho_1 i}$$

valid for $\min(k,\rho_1) > 0$,and $k+\rho_1 i > \max\{0, Re(\beta-\eta)\} - 1$

$$I_{0,x}^{\alpha,\beta,\eta}\left[x^k e^{cx} S_n^m(x^{\rho_1})\right] \tag{2.6}$$

$$= x^{k-\beta} \sum_{i=0}^{[n/m]} \sum_{j=0}^{\infty} \frac{(-n)_{mi}}{i!j!} A_{n,i}$$

$$\Gamma\left[\begin{array}{c} k+\rho_1 i+1, -\beta+\eta+k+\rho_1 i+1 \\ -\beta+k+\rho_1 i+1, \alpha+\eta+k+\rho_1 i+1 \end{array}\right] x^{\rho_1 i}(cx)^j$$

valid for $\min(k,\rho_1)>0$,and $k+\rho_1 i>\max\{0, Re(\beta-\eta)\}-1$

$$I_{0,x}^{\alpha,\beta,\eta}\left[x^k S_n^m\left(x^{\rho_1}\right) S_q^p\left(x^l\right)\right] \tag{2.7}$$

$$= x^{k-\beta}\sum_{i=0}^{[n/m]}\sum_{j=0}^{[q/p]}\frac{(-n)_{mi}(-q)_{pj}}{i!j!}A_{n,i}A_{q,j}$$

$$\Gamma\left[\begin{array}{c} k+\rho_1 i+lj+1, -\beta+\eta+k+\rho_1 i+lj+1 \\ -\beta+k+\rho_1 i+lj+1, \alpha+\eta+k+\rho_1 i+lj+1 \end{array}\right]\times x^{\rho_1 i}x^{lj}$$

valid for $\min(k,l,\rho_1)>0$,and $k+lj+\rho_1 i>\max\{0, Re(\beta-\eta)\}-1$

$$I_{0,x}^{\alpha,\beta,\eta}\left[x^k e^{cx} S_n^m\left(x^{\rho_1}\right) S_q^p\left(x^l\right)\right] \tag{2.8}$$

$$= x^{k-\beta}\sum_{i=0}^{[n/m]}\sum_{j=0}^{[q/p]}\frac{(-n)_{mi}(-q)_{pj}}{i!j!}A_{n,i}A_{q,j}$$

$$\Gamma\left[\begin{array}{c} k+\rho_1 i+lj+1, -\beta+\eta+k+\rho_1 i+lj+1 \\ -\beta+k+\rho_1 i+lj+1, \alpha+\eta+k+\rho_1 i+lj+1 \end{array}\right]\times x^{\rho_{1,i}}x^{lj}$$

valid for $\min(k,l,\rho_1)>0$,and $k+lj+\rho_1 i>\max\{0, Re(\beta-\eta)\}-1$,These formulae believed to be new.

Particular Cases

On account of the most general nature of $S_n^m[x]$ occurring in our main results given by (1.11) to (1.14), a large number of formulae involving simpler functions of one and more variables can be easily obtained as their particular cases. We however have given here only few particular cases by way of illustration

(i) By setting $\rho_1, \sigma_1, b, a, m = 1$, $k=\alpha$ and

$$A_{n,i}=\frac{(1+\alpha+\beta+n)_i(1+\alpha)_n}{n!(1+\alpha)_i}$$

in (1.11) and (2.1), we obtain results involving Jacobi polynomial $P_n^{(\alpha,\beta)}$ and $Kampe'\ de\ Fe'riet$series, given in (1.6)

(ii) By setting $\rho_1, \sigma_1, b, a, c, m = 1$, $k=\alpha$ and

$$A_{n,i}=\frac{(1+\alpha+\beta+n)_i(1+\alpha)_n}{n!(1+\alpha)_i}$$

in (1.12) and (2.2), we obtain results involving Jacobi polynomial $P_n^{(\alpha,\beta)}$ and general triple hypergeometric function $F^{(3)}[.,.,.]$,given in (1.7).

(iii) By putting $\rho_1, \sigma_1, b, a, m, l, p = 1$, $k=\alpha$ and

$$A_{n,i}=\frac{(1+\alpha+\beta+n)_i(1+\alpha)_n}{(1+\alpha)_i}$$

and

$$A_{q,j}=\frac{(1+\alpha)_q}{q!(1+\alpha)_j}$$

in (1.13) and (2.3) yields a results involving the Jacobi polynomial $P_n^{(\alpha,\beta)}$, Laguerre polynomial $P_q^{(\alpha)}$ and general triple hypergeometric function $F^3\left[.,.,.\right]$, given in (1.7).
(iv) By putting $\rho_1, \sigma_1, b, a, m, l, p, c = 1$, $k = \alpha$ and

$$A_{n,i} = \frac{(1+\alpha+\beta+n)_i\,(1+\alpha)_n}{(1+\alpha)_i}$$

and

$$A_{q,j} = \frac{(1+\alpha)_q}{q!\,(1+\alpha)_j}$$

in (1.14) and (2.4) yields a results involving the Jacobi polynomial$P_n^{(\alpha,\beta)}$, Laguerre polynomial $P_q^{(\alpha)}$and Pathan's quadruple hypergeometric functions $F_P^4\left[.,.,.,.\right]$,given in (1.9).
(v) By setting $\rho_1, m = 1$, $k = \alpha$ and

$$A_{n,i} = \frac{(1+\alpha)_n}{n!\,(1+\alpha)_i}$$

in (2.5) and (2.6), we obtain results involving Laguerre polynomial $P_n^{(\alpha)}$ well known Gauss' function. ${}_pF_q(.)$ and Kampe'-de-Fe'riet series, given in (1.6) respectively.
(vi) By putting $\rho_1, m, l, p = 1$, $k = \alpha$ and

$$A_{n,i} = \frac{(1+\alpha)_n}{n!\,(1+\alpha)_i}$$

and

$$A_{q,j} = \frac{(1+\alpha)_q}{q!\,(1+\alpha)_j}$$

in (2.7) and (2.8) yields a results involving the Laguerre polynomials$P_n^{(\alpha)}$, $P_q^{(\alpha)}$ and Kampe'-de-Fe'riet function and general triple hypergeometric functions$F^3\left[.,.,.\right]$, given by (1.6) and (1.7) respectively.

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