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SOME CONVERGENCE RESULTS FOR SEQUENCES OF OPERATORS IN BANACH SPACES

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Abstract. In this paper, we establish some fixed point theorems in connection with sequences of operators in the Banach space setting for Mann and Ishikawa iterative processes. Our results extend some of the results of Berinde, Bonsall, Nadler and Rus from complete metric space to the Banach space setting.

1. Introduction

In this paper, we establish some fixed point theorems in connection with sequences of operators in the Banach space settings for Mann and Ishikawa iterative processes. Our results extend some of the results of Berinde [1, 2], Bonsall [3], Nadler [6] and Rus [8, 9] from complete metric space to the Banach space setting.

Let $(E, ||.||)$ be a Banach space and $T : E \to E$ a selfmap of E. Suppose that $F_T = \{\ p \in E \mid Tp = p\ \}$ is the set of fixed points of T.

In the Banach space setting, we state the following well-known iterative processes which have been employed to approximate the fixed points of various operators over the years:

Define the sequence $\{x_n\}_{n=0}^{\infty}$ by

$$x_{n+1} = (1-\lambda)x_n + \lambda T x_n,\ n = 0, 1, \cdots,\ x_0 \in E, \tag{1}$$

where $\lambda \in [0, 1]$. Then, Eqn, (1) is called the Schaefer iterative process (see Schaefer [10]).

For $x_0 \in E$, the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n T x_n,\ n = 0, 1, \cdots, \tag{2}$$

where $\{\alpha_n\}_{n=0}^{\infty} \subset [0, 1]$, is called the Mann iteration process (see Mann [5]).

For $x_0 \in E$, the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$\left.\begin{array}{l} x_{n+1} = (1-\alpha_n)x_n + \alpha_n T z_n \\ z_n = (1-\beta_n)x_n + \beta_n T x_n \end{array}\right\},\ n = 0, 1, \cdots, \tag{3}$$

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where $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_{\rm n}\}_{\rm n=0}^{\infty}$ are sequences in $[0,1]$, is called the Ishikawa iteration process (see Ishikawa [4]).

Remark 1.1: If $\lambda = 1$ in (1), or, $\alpha_n = 1$ in (2), then we obtain

$$x_{n+1} = Tx_n,\ n = 0, 1, \cdots,\ x_0 \in E, \tag{4}$$

which is called the Picard iteration. See [1, 2, 8, 9] for Picard iteration.

Definition 1.1 [1, 2, 8]**:** (a) A function $\psi\colon \mathbb{R}^+ \to \mathbb{R}^+$ is called a *comparison function* if it satisfies the following conditions:

(i) ψ is monotone increasing; (ii) $\lim\limits_{n\to\infty} \psi^n(t)) \to 0,\ \forall\ t \geq 0$.

(b) A comparison function satisfying $t - \psi(t) \to \infty$ as t $\to \infty$ is called a *strict comparison function.*

Remark 1.2: Every comparison function satisfies $\psi(0) = 0$ and $\psi(t) < t,\ \forall\, t \in \mathbb{R}^+$.

We shall employ the following contractive conditons:

(i) For a selfmapping $T\colon E \to E$, there exist real numbers $L \geq 0$ and $a \in [0, 1)$, such that

$$||Tx - Ty|| \leq \frac{\phi(||x - Tx||) + a||x - y||}{1 + L||x - Tx||},\ \forall\ x,\ y \in E, \tag{5}$$

where $\phi\colon \mathbb{R}^+ \to \mathbb{R}^+$ is a monotone increasing function such that $\phi(0) = 0$.

(ii) For a selfmapping $T\colon E \to E$, there exist a strict comparison function $\psi\colon \mathbb{R}^+ \to \mathbb{R}^+$ and a monotone increasing function $\phi\colon \mathbb{R}^+ \to \mathbb{R}^+$, with $\phi(0) = 0$, such that

$$||Tx - Ty|| \leq \phi(||x - Tx||) + \psi(||x - y||),\ \forall\ x,\ y \in E. \tag{6}$$

2. Main Results

Theorem 2.1. *Let $(E, ||.||)$ be a Banach space and $\{T_n\}_{n=0}^{\infty}$ a sequence of operators $T_n : E \to E$ such that $F_{T_n} = \{x_n^*\}$ for each $n \in \mathbb{N}$. For $x_0 \in E$, let $\{x_n\}_{n=0}^{\infty}$ be the Mann iterative process defined by (2), $\alpha_n \in [0, 1]$. Suppose that the sequence $\{T_n\}_{n=0}^{\infty}$ converges uniformly to a mapping $T : E \to E$ satisfying (5), with $F_T = \{x^*\}$, where $\phi : \mathbb{R}^+ \to \mathbb{R}^+$ is a monotone increasing function such that $\phi(0) = 0$. Then, $x_n^* \to x^*$* as n $\to \infty$.

Proof. Let $\epsilon > 0$ and choose a natural number N such that for $n \geq N$, we have $||T_n x - Tx|| < (1 - a)\epsilon,$ for all x $\in$ E. Then, for $n \geq N$ we have

$$\begin{aligned} ||x_n^* - x^*|| &\leq (1 - \alpha_n)||x_n^* - x^*|| + \alpha_n||Tx^* - T_n x_n^*|| \\ &\leq (1 - \alpha_n)||x_n^* - x^*|| + \alpha_n[||Tx^* - Tx_n^*|| + ||Tx_n^* - T_n x_n^*||] \\ &\leq (1 - \alpha_n)||x_n^* - x^*|| + \alpha_n[\tfrac{\varphi(||x^* - Tx^*||) + a||x^* - x_n^*||}{1 + L||x^* - Tx^*||} + ||Tx_n^* - T_n x_n^*||] \\ &< (1 - \alpha_n)||x_n^* - x^*|| + a\alpha_n||x^* - x_n^*|| + \alpha_n(1 - a)\epsilon, \end{aligned}$$

from which we have $||x_n^* - x^*|| < \epsilon$,

Also, since $\epsilon > 0$ is arbitrary, then $x_n^* \to x^*$ as n $\to \infty$.

□

Proof. The proof of this result is more direct and similar to that of Theorem 2.1.

Theorem 2.2. *Let $(E, ||.||)$ be a Banach space and $\{T_n\}_{n=0}^{\infty}$ a sequence of operators $T_n : E \to E$ such that $F_{T_n} = \{x_n^*\}$ for each $n \in I\!N$. For $x_0 \in E$, let $\{x_n\}_{n=0}^{\infty}$ be the Mann iterative process defined by (2), $\alpha_n \in [0,1]$. Suppose that the sequence $\{T_n\}_{n=0}^{\infty}$ converges uniformly to a mapping $T : E \to E$ satisfying (6), with $F_T = \{x^*\}$, where $\phi : I\!R^+ \to I\!R^+$ is a monotone increasing function such that $\phi(0) = 0$ and $\psi : I\!R^+ \to I\!R^+$ is a strict comparison function. Then, $x_n^* \to x^*$* as n $\to \infty$.

Proof. Let $\epsilon > 0$ and choose a natural number N such that for $n \geq N$, we have $||T_n x - Tx|| < \epsilon,$ for all x $\in$ E. Then, for $n \geq N$ we have

$$\begin{aligned} ||x_n^* - x^*|| &\leq (1-\alpha_n)||x_n^* - x^*|| + \alpha_n ||Tx^* - T_n x_n^*|| \\ &\leq (1-\alpha_n)||x_n^* - x^*|| + \alpha_n [||Tx^* - Tx_n^*|| + ||Tx_n^* - T_n x_n^*||] \\ &\leq (1-\alpha_n)||x_n^* - x^*|| + \alpha_n [\varphi(||x^* - Tx^*||) + \psi(||x^* - x_n^*||) + ||Tx_n^* - T_n x_n^*||] \\ &< (1-\alpha_n)||x_n^* - x^*|| + \alpha_n \psi(||x^* - x_n^*||) + \alpha_n \epsilon, \end{aligned}$$

from which we have $||x_n^* - x^*|| - \psi(||x^* - x_n^*||) < \epsilon,$
leading to $||x_n^* - x^*|| < \epsilon, \ \forall \ n \geq N,$
since ψ is a strict comparison function. Also, since $\epsilon > 0$ is arbitrary, then $x_n^* \to x^*$ as n $\to \infty$.

□

Theorem 2.3. *Let $(E, \ ||.||)$ be a Banach space and $\{T_n\}_{n=0}^{\infty}$ a sequence of operators $T_n : E \to E$ such that $F_{T_n} = \{x_n^*\}$, for each $n \in I\!N$. For $x_0 \in E$, let $\{x_n\}_{n=0}^{\infty}$ be the Ishikawa iterative process defined by (3), $\alpha_n, \ \beta_n \in [0,1]$. Suppose that the sequence $\{T_n\}_{n=0}^{\infty}$ converges uniformly to a mapping $T : E \to E$ satisfying (6), with $F_T = \{x^*\}$, where $\psi : I\!R_+ \to I\!R_+$ is a sublinear, strict comparison function and $\phi : I\!R^+ \to I\!R^+$ is a monotone increasing function such that $\phi(0) = 0$. Then, $x_n^* \to x^*$* as n $\to \infty$.

Proof. Let $\epsilon > 0$ and choose a natural number N such that for $n \geq N$, we have $||T_n x - Tx|| < \frac{\epsilon}{2\beta_n}, \ \beta_n > 0,$ for all x $\in$ E and $||T_n z - Tz|| < \frac{\epsilon}{2},$ for all z $\in$ E,
$z^* = (1-\beta_n)x^* + \beta_n Tx^*, \ z_n^* = (1-\beta_n)x_n^* + \beta_n T_n x_n^*.$
Therefore, we have by using (6) and the fact that $\psi(t) < t \ \forall \ t \in \mathbb{R}_+$ that

$$\begin{aligned} ||x_n^* - x^*|| &\leq (1-\alpha_n)||x_n^* - x^*|| + \alpha_n [||Tz^* - Tz_n^*|| + ||Tz_n^* - T_n z_n^*||] \\ &\leq (1-\alpha_n)||x_n^* - x^*|| + \alpha_n [\varphi(||z^* - Tz^*||) + \psi(||z^* - z_n^*||) + ||Tx_n^* - T_n x_n^*||] \\ &= (1-\alpha_n)||x_n^* - x^*|| + \alpha_n \psi(||z_n^* - z^*||) + \alpha_n ||Tz_n^* - T_n z_n^*|| \\ &\leq (1-\alpha_n)||x_n^* - x^*|| + \alpha_n (1-\beta_n) \psi(||x^* - x_n^*||) \\ &\quad + \alpha_n \beta_n \psi^2(||x^* - x_n^*||) + \alpha_n \beta_n \psi(||Tx_n^* - T_n x_n^*|| \ ) + \alpha_n ||Tz_n^* - T_n z_n^*|| \\ &\leq (1-\alpha_n)||x_n^* - x^*|| + \alpha_n (1-\beta_n) \psi(||x^* - x_n^*||) \\ &\quad + \alpha_n \beta_n \psi(||x^* - x_n^*||) + \alpha_n \beta_n \psi(||Tx_n^* - T_n x_n^*|| \ ) + \alpha_n ||Tz_n^* - T_n z_n^*|| \\ &= (1-\alpha_n)||x_n^* - x^*|| + \alpha_n \psi(||x_n^* - x^*||) + \alpha_n \beta_n \psi(||Tx_n^* - T_n x_n^*||) \\ &\quad + \alpha_n ||Tz_n^* - T_n z_n^*|| \end{aligned}$$

from which we have

$$\alpha_n [\ ||x_n^* - x^*|| - \psi(||x_n^* - x^*||) \ ] \leq \alpha_n \beta_n \psi(||Tx_n^* - T_n x_n^*||) + \alpha_n ||Tz_n^* - T_n z_n^*||,$$

so that

$$\begin{aligned} ||x_n^* - x^*|| - \psi(||x_n^* - x^*||) &\leq \beta_n \psi(||Tx_n^* - T_n x_n^*||) + ||Tz_n^* - T_n z_n^*|| \\ &< \beta_n ||Tx_n^* - T_n x_n^*|| + ||Tz_n^* - T_n z_n^*|| \\ &< \beta_n \tfrac{\epsilon}{2\beta_n} + \tfrac{\epsilon}{2} = \epsilon, \end{aligned}$$

ψ being a strict comparison function leads to $||x_n^* - x^*|| < \epsilon,\ \forall\ n \geq N$. Since $\epsilon > 0$ is arbitrary, then $x_n^* \to x^*$ as n $\to \infty$.

□

Theorem 2.4. *Let $(E,\ ||.||)$ be a Banach space and $\{T_n\}_{n=0}^{\infty}$ a sequence of operators $T_n : E \to E$ such that $F_{T_n} = \{x_n^*\}$, for each $n \in I\!N$. For $x_0 \in E$, let $\{x_n\}_{n=0}^{\infty}$ be the Ishikawa iterative process defined by (3), $\alpha_n,\ \beta_n \in [0,1]$. Suppose that the sequence $\{T_n\}_{n=0}^{\infty}$ converges uniformly to a mapping $T : E \to E$ satisfying (5) with $F_T = \{x^*\}$, where $\phi : I\!R^+ \to I\!R^+$ is a monotone increasing function such that $\phi(0) = 0$. Then, $x_n^* \to x^*$* as n $\to \infty$.

Proof. Let $\epsilon > 0$ and choose a natural number N such that for $n \geq N$, we have $||T_n x - Tx|| < \frac{(1-a)(1+a\beta_n)}{2a\beta_n}\epsilon,\ a > 0,\ \beta_n > 0,$ for all x $\in$ E and $||T_n z - Tz|| < \frac{(1-a)(1+a\beta_n)}{2}\epsilon,$ for all z $\in$ E,
$z^* = (1-\beta_n)x^* + \beta_n Tx^*,\ z_n^* = (1-\beta_n)x_n^* + \beta_n T_n x_n^*.$
Therefore, we have by using (5) that

$$\begin{aligned}
||x_n^* - x^*|| &\leq (1-\alpha_n)||x_n^* - x^*|| + \alpha_n||Tz^* - T_n z_n^*|| \\
&\leq (1-\alpha_n)||x_n^* - x^*|| + \alpha_n[||Tz^* - Tz_n^*|| + ||Tz_n^* - T_n z_n^*||] \\
&\leq (1-\alpha_n)||x_n^* - x^*|| + \alpha_n[\varphi(||z^* - Tz^*||) + a||z^* - z_n^*|| + ||Tx_n^* - T_n x_n^*||] \\
&= (1-\alpha_n)||x_n^* - x^*|| + a\alpha_n||z_n^* - z^*|| + \alpha_n||Tz_n^* - T_n z_n^*|| \\
&\leq (1-\alpha_n + a\alpha_n - a\alpha_n\beta_n)||x^* - x_n^*|| + a\alpha_n\beta_n[\ \varphi(||x^* - Tx^*||) \\
&\quad +a||x^* - x_n^*|| + ||Tx_n^* - T_n x_n^*||\ ] + \alpha_n||Tz_n^* - T_n z_n^*|| \\
&= (1-\alpha_n + a\alpha_n - a\alpha_n\beta_n + a^2\alpha_n\beta_n)||x^* - x_n^*|| + a\alpha_n\beta_n||Tx_n^* - T_n x_n^*|| \\
&\quad +\alpha_n||Tz_n^* - T_n z_n^*||,
\end{aligned}$$

from which we have

$$\alpha_n(1-a)(1+a\beta_n)||x_n^* - x^*|| \leq a\alpha_n\beta_n||Tx_n^* - T_n x_n^*|| + \alpha_n||Tz_n^* - T_n z_n^*||,$$

leading to

$$\begin{aligned}
||x_n^* - x^*|| &\leq \frac{a\beta_n}{(1-a)(1+a\beta_n)}||Tx_n^* - T_n x_n^*|| + \frac{1}{(1-a)(1+a\beta_n)}||Tz_n^* - T_n z_n^*||, \\
&< \frac{a\beta_n}{(1-a)(1+a\beta_n)}\frac{(1-a)(1+a\beta_n)}{2a\beta_n}\epsilon + \frac{1}{(1-a)(1+a\beta_n)}\frac{(1-a)(1+a\beta_n)}{2}\epsilon \\
&= \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon,
\end{aligned}$$

leading to $||x_n^* - x^*|| < \epsilon,\ \forall\, n \geq N$. Since $\epsilon > 0$ is arbitrary, then $x_n^* \to x^*$ as n $\to \infty$.

□

Remark 2.1: Theorem 2.1 - Theorem 2.4 are extensions of both Theorem 3.2 and Theorem 3.6 of Rus [9]. Theorem 3.2 of Rus [9] is itself Theorem 1 of Nadler [6] and Theorem 7.8 of Berinde [1, 2].

We state the following results for pointwise convergence cases for the iterative processes:

Theorem 2.5. *Let $(E,\ ||.||)$ be a Banach space and $\{T_n\}_{n=0}^{\infty}$ a sequence of operators $T_n : E \to E$ such that $F_{T_n} = \{x_n^*\}$, for each $n \in I\!N$. For $x_0 \in E$, let $\{x_n\}_{n=0}^{\infty}$ be the Ishikawa iterative process defined by (3), $\alpha_n,\ \beta_n \in [0,1]$. Suppose that the sequence $\{T_n\}_{n=0}^{\infty}$ converges pointwise to a mapping $T : E \to E$ satisfying (5) with $F_T = \{x^*\}$, where $\phi : I\!R^+ \to I\!R^+$ is a monotone increasing function such that $\phi(0) = 0$. Then, $x_n^* \to x^*$* as n $\to \infty$.

Proof. We shall use the contractive condition (5) and the pointwise convergence of T_n to T, where $z^* = (1-\beta_n)x^* + \beta_n Tx^*,\ z_n^* = (1-\beta_n)x_n^* + \beta_n T_n x_n^*$.
Therefore we have

$$\begin{aligned} ||x_n^* - x^*|| &\le (1-\alpha_n)||x_n^* - x^*|| + \alpha_n ||Tz^* - T_n z_n^*|| \\ &\le (1-\alpha_n)||x_n^* - x^*|| + \alpha_n[\varphi(||z^* - Tz^*||) + a||z^* - z_n^*|| + ||Tx_n^* - T_n x_n^*||] \\ &= (1-\alpha_n)||x_n^* - x^*|| + a\alpha_n ||z_n^* - z^*|| + \alpha_n ||Tz_n^* - T_n z_n^*|| \\ &\le (1-\alpha_n + a\alpha_n - a\alpha_n\beta_n)||x^* - x_n^*|| + a\alpha_n\beta_n[\ ||Tx^* - Tx_n^*|| \\ &\quad +||Tx_n^* - T_n x_n^*||\ ] + \alpha_n||Tz_n^* - T_n z_n^*|| \\ &\le (1-\alpha_n + a\alpha_n - a\alpha_n\beta_n)||x^* - x_n^*|| + a\alpha_n\beta_n[\ \varphi(||x^* - Tx^*||) \\ &\quad +a||x^* - x_n^*|| + ||Tx_n^* - T_n x_n^*||\ ] + \alpha_n||Tz_n^* - T_n z_n^*|| \\ &= (1-\alpha_n + a\alpha_n - a\alpha_n\beta_n + a^2\alpha_n\beta_n)||x^* - x_n^*|| + a\alpha_n\beta_n||Tx_n^* - T_n x_n^*|| \\ &\quad +\alpha_n||Tz_n^* - T_n z_n^*||, \end{aligned}$$

from which we have

$$\alpha_n(1-a)(1+a\beta_n)||x_n^* - x^*|| \le a\alpha_n\beta_n||Tx_n^* - T_n x_n^*|| + \alpha_n||Tz_n^* - T_n z_n^*||,$$

leading to

$$||x_n^* - x^*|| \le [(1-a)(1+a\beta_n)]^{-1}[\ a\beta_n||Tx_n^* - T_n x_n^*|| + ||Tz_n^* - T_n z_n^*||\ ] \to 0 \text{ as n} \to \infty,$$

since T_n converges pointwise to T. Hence, we have that $||x_n^* - x^*|| \to 0$ as n $\to \infty$.
□

Theorem 2.6. *Let $(E, ||.||)$ be a Banach space and $\{T_n\}_{n=0}^\infty$ a sequence of operators $T_n : E \to E$ such that $F_{T_n} = \{x_n^*\}$ for each $n \in N$. For $x_0 \in E$, let $\{x_n\}_{n=0}^\infty$ be the Mann iterative process defined by (2), $\alpha_n \in [0,1]$. Suppose that the sequence $\{T_n\}_{n=0}^\infty$ converges pointwise to a mapping $T : E \to E$ satisfying (5), with $F_T = \{x^*\}$, where $\phi : \mathbb{R}^+ \to \mathbb{R}^+$ is a monotone increasing function such that $\phi(0) = 0$. Then, $x_n^* \to x^*$* as n $\to \infty$.

Proof. We shall use (2), the contractive condition (5) and the pointwise convergence of T_n to T. Therefore we have that

$$||x_n^* - x^*|| \le (1-a)^{-1}||Tx_n^* - T_n x_n^*|| \to 0 \ as\ n \to \infty,$$

since T_n converges pointwise to T.
□

Theorem 2.7. *Let $(E, ||.||)$ be a Banach space and $\{T_n\}_{n=0}^\infty$ a sequence of operators $T_n : E \to E$ such that $F_{T_n} = \{x_n^*\}$ for each $n \in N$. For $x_0 \in E$, let $\{x_n\}_{n=0}^\infty$ be the Mann iterative process defined by (2), $\alpha_n \in [0,1]$. Suppose that the sequence $\{T_n\}_{n=0}^\infty$ converges pointwise to a mapping $T : E \to E$ satisfying (6), with $F_T = \{x^*\}$, where $\psi : \mathbb{R}^+ \to \mathbb{R}^+$ is a strict comparison function and $\phi : \mathbb{R}^+ \to \mathbb{R}^+$ is a monotone increasing function such that $\phi(0) = 0$. Then, $x_n^* \to x^*$* as n $\to \infty$.

Proof. By using (2), the contractive condition (6) and the pointwise convergence of T_n to T, we have that

$$||x_n^* - x^*|| - \psi(||x_n^* - x^*||) \le ||Tx_n^* - T_n x_n^*|| \to 0 \text{ as n} \to \infty,$$

since T_n converges pointwise to T. It follows that $||x_n^* - x^*|| \to 0$ as n $\to \infty$, since ψ is a strict comparison function.
□

Theorem 2.8. *Let $(E, ||.||)$ be a Banach space and $\{T_n\}_{n=0}^{\infty}$ a sequence of operators $T_n : E \to E$ such that $F_{T_n} = \{x_n^*\}$ for each $n \in N$. For $x_0 \in E$, let $\{x_n\}_{n=0}^{\infty}$ be the Mann iterative process defined by (2), where $\alpha_n \in [0, 1]$. Suppose that the sequence $\{T_n\}_{n=0}^{\infty}$ converges pointwise to a mapping $T : E \to E$ satisfying*

$$||Tx - Ty|| \leq L||x - Tx|| + \psi(||x - y||), \ \forall \ x, \ y \in E, \ L \geq 0, \tag{7}$$

with $F_T = \{x^*\}$, where $\psi : I\!R^+ \to I\!R^+$ is a strict comparison function. Then, $x_n^* \to x^*$ as n $\to \infty$.

Proof. The proof of this result is similar to that of Theorem 2.7. □

Theorem 2.9. *Let $(E, ||.||)$ be a Banach space and $\{T_n\}_{n=0}^{\infty}$ a sequence of operators $T_n : E \to E$ such that $F_{T_n} = \{x_n^*\}$ for each $n \in N$. For $x_0 \in E$, let $\{x_n\}_{n=0}^{\infty}$ be the Ishikawa iterative process defined by (3), where $\alpha_n, \ \beta_n \in [0, 1]$. Suppose that the sequence $\{T_n\}_{n=0}^{\infty}$ converges pointwise to a mapping $T : E \to E$ satisfying (6), with $F_T = \{x^*\}$, where $\psi : I\!R^+ \to I\!R^+$ is a sublinear, strict comparison function and $\phi : I\!R^+ \to I\!R^+$ is a monotone increasing function such that $\phi(0) = 0$. Then,* $x_n^* \to x^*$ as n $\to \infty$.

Proof. The proof of this result is similar to that of Theorem 2.7. □

Remark 2.2: Theorem 2.5 - Theorem 2.9 extend a result of Bonsall [3] (which is Theorem 3.1 of Rus [9]). Theorem 2.7 - Theorem 2.9 are also extensions of Theorem 7.2.1 of Rus [8] (which is Theorem 7.9 of Berinde [1, 2]).

Remark 2.3: Corresponding results can also be deduced from our results for the Schaefer's iterative process defined in (1). See [7].

References

[1] V. Berinde; *Iterative Approximation of Fixed Points,* Editura Efemeride (2002).

[2] V. Berinde; *Iterative Approximation of Fixed Points,* Springer-Verlag Berlin Heidelberg (2007).

[3] F. F. Bonsall; *Lectures on Some Fixed Point Theorems of Functional Analysis,* Tata Inst. Fund. Res., Bombay (1962).

[4] S. Ishikawa; *Fixed Point by a New Iteration Method,* Proc. Amer. Math. Soc. 44 (1) (1974), 147-150. 41 (1973), 430-438.

[5] W. R. Mann; *Mean Value Methods in Iteration,* Proc. Amer. Math. Soc. 44 (1953), 506-510.

[6] S. B. Nadler (JR.); *Sequences of Contractions and Fixed points,* Pacific J. Math. 27 (3) (1968), 579-585.

[7] M. O. Olatinwo; *Some Results on Stability and Sequences of Operators for Schaefer's Fixed Point Iterative Algorithm in Hilbert and Normed Linear Spaces,* Proc. Jangeon Math. Soc. (To appear in 2011).

[8] I. A. Rus; *Generalized Contractions and Applications,* Cluj Univ. Press, Cluj Napoca, (2001).

[9] I. A. Rus; *Sequences of Operators and Fixed Points, Fixed Point Theory,* 5 (2) (2004), 349-368.

[10] H. Schaefer; *Uber die Methode Sukzessiver Approximationen,* Jahresber. Deutsch. Math. Verein. 59 (1957), 131-140.

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