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ALMOST CONTACT METRIC MANIFOLDS ADMITTING SEMI-SYMMETRIC NON-METRIC CONNECTION

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Abstract. In this paper, we study some geometrical properties of almost contact metric manifolds equipped with semi-symmetric non-metric connection. In the last, properties of group manifold are given.

1. Introduction

In [16], Friedmann and Schouten introduced the notion of semi-symmetric linear connection on a differentiable manifold. Hayden [17] introduced the idea of semi-symmetric non-metric connection on a Riemannian manifold. The idea of semi-symmetric metric connection on Riemannian manifold was introduced by Yano [5]. Various properties of such connection have been studied by many geometers. Agashe and Chafle [1] defined and studied a semi-symmetric non-metric connection in a Riemannian manifold. This was further developed by Agashe and Chafle [18], Prasad [19], De and Kamilya [10], Tripathi and Kakkar [8], Pandey and Ojha [20], Chaturvedi and Pandey [7] and several other geometers. Sengupta, De and Binh [9], De and Sengupta [21] defined new types of semi-symmetric non-metric connections on a Riemannian manifold and studied some geometrical properties with respect to such connections. Chaubey and Ojha [2], defined new type of semi-symmetric non-metric connection on an almost contact metric manifold. In [6], Chaubey defined a semi-symmetric non-metric connection on an almost contact metric manifold and studied its different geometrical properties. Some properties of such connection have been further studied by Jaiswal and Ojha [14], Chaubey and Ojha [15]. In the present paper, we study the properties of such connection in an almost contact metric manifold. Section 2 is preliminaries in which the basic definitions are given. Section 3 deals with brief account of semi-symmetric non-metric connection. In section 4, some properties of curvature tensors are obtained. It is also shown that a Sasakian manifold, equipped with a semi-symmetric non-metric connection, Weyl projective, conharmonic, concircular and conformal curvature tensors of the

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manifold coincide if and only if it is Ricci flat with respect to the semi-symmetric non-metric connection. In section 5, the group manifold with respect to the semi-symmetric non-metric connection is studied and proved that an almost contact metric manifold equipped with a semi-symmetric non-metric connection to be a group manifold if and only if the manifold is flat with respect to the semi-symmetric non-metric connection and it is cosymplectic. Also a Sasakian manifold equipped with a semi-symmetric non-metric connection for which the manifold is a group manifold, is conformally flat.

2. Preliminaries

If on an odd dimensional differentiable manifold M_n, $n = 2m+1$, of differentiability class C^∞, there exist a vector valued real linear function F , a $1-$form A and a vector field T, satisfying

$$\overline{\overline{X}} + X = A(X)T, \tag{2.1}$$

$$A(\overline{X}) = 0, \tag{2.2}$$

where

$$\overline{X} \overset{\text{def}}{=} FX$$

for arbitrary vector field X, then M_n is said to be an almost contact manifold and the system $\{F, A, T\}$ is said to give an almost contact structure [3], [4] to M_n. In consequence of (2.1) and (2.2), we find

$$A(T) = 1, \tag{2.3}$$

$$\overline{T} = 0 \tag{2.4}$$

and

$$rank\{F\} = n-1.$$

If the associated Riemannian metric g of type $(0,2)$ in M_n satisfy

$$g(\overline{X}, \overline{Y}) = g(X,Y) - A(X)A(Y) \tag{2.5}$$

for arbitrary vector fields X, Y in M_n, then (M_n, g) is said to be an almost contact metric manifold and the structure $\{F, A, T, g\}$ is called an almost contact metric structure [3], [4] to M_n.

Putting T for X in (2.5) and then using (2.3) and (2.4), we find

$$A(X) = g(X,T). \tag{2.6}$$

If we define

$$'F(X,Y) \overset{\text{def}}{=} g(\overline{X}, Y), \tag{2.7}$$

then

$$'F(X,Y) +' F(Y,X) = 0. \tag{2.8}$$

An almost contact metric manifold (M_n, g) is said to be a Sasakian manifold [3], [4] if

$$(D_X{}'F)(Y,Z) = A(Y)g(X,Z) - A(Z)g(X,Y), \tag{2.9}$$

where

$$(D_X{}'F)(Y,Z) \overset{\text{def}}{=} g((D_XF)(Y), Z) \tag{2.10}$$

for arbitrary vector fields X and Y.

On a Sasakian manifold, the following relations hold [3], [4]

$$D_XT = \overline{X}, \tag{2.11}$$

$$(D_X{}'F)(Y,Z) + (D_Y{}'F)(Z,X) + (D_Z{}'F)(X,Y) = 0, \tag{2.12}$$

$$'F(Y,Z) = (D_Y A)(Z), \tag{2.13}$$

and

$$'K(X,Y,Z,T) = (D_Z{}'F)(X,Y) \tag{2.14}$$

for arbitrary vector fields X, Y, Z.

The Weyl projective curvature tensor W, conformal curvature tensor V, conharmonic curvature tensor L and concircular curvature tensor C of the Riemannian connection D are given by

$$W(X,Y,Z) = K(X,Y,Z) - \frac{1}{(n-1)}\left\{Ric(Y,Z)X - Ric(X,Z)Y\right\}, \tag{2.15}$$

$$\begin{aligned} V(X,Y,Z) &= K(X,Y,Z) - \frac{1}{(n-2)}(Ric(Y,Z)X \\ &- Ric(X,Z)Y - g(X,Z)RY + g(Y,Z)RX) \\ &+ \frac{r}{(n-1)(n-2)}\left\{g(Y,Z)X - g(X,Z)Y\right\}, \end{aligned} \tag{2.16}$$

$$\begin{aligned} L(X,Y,Z) &= K(X,Y,Z) - \frac{1}{(n-2)}(Ric(Y,Z)X \\ &- Ric(X,Z)Y - g(X,Z)RY + g(Y,Z)RX), \end{aligned} \tag{2.17}$$

$$C(X,Y,Z) = K(X,Y,Z) - \frac{r}{n(n-1)}\left\{g(Y,Z)X - g(X,Z)Y\right\}, \tag{2.18}$$

where K, Ric and r are respectively the curvature, Ricci and scalar curvature tensors of the Riemannian connection [3] D.

Definition- A vector field T is said to be a harmonic vector field [22], [23] if it satisfies

$$(D_X A)(Y) - (D_Y A)(X) = 0, \tag{2.19}$$

and

$$D_X T = 0, \tag{2.20}$$

for arbitrary vector fields X and Y.

3. Semi-symmetric non-metric connection

A linear connection $\tilde{B}$ on (M_n, g) is said to be a semi-symmetric non-metric connection [6] if the torsion tensor $\tilde{S}$ of the connection $\tilde{B}$ and the Riemannian metric g of type $(0,2)$ satisfy the following conditions

$$\tilde{S}(X,Y) = 2'F(X,Y)T, \tag{3.1}$$

$$(\tilde{B}_X g)(Y,Z) = -A(Y)'F(X,Z) - A(Z)'F(X,Y) \tag{3.2}$$

for any arbitrary vector fields X, Y, Z ; where A is 1-form on (M_n, g) with T as associated vector field.

It is known that [6],

$$\tilde{B}_X Y = D_X Y +' F(X,Y)T, \tag{3.3}$$

$$'\tilde{S}(X,Y,Z) \overset{\text{def}}{=} g(\tilde{S}(X,Y),Z) = 2A(Z)'F(X,Y), \tag{3.4}$$

$$(\tilde{B}_X F)(Y) = (D_X F)(Y) + g(\overline{X},\overline{Y})T, \tag{3.5}$$

$$(\tilde{B}_X A)(Y) = (D_X A)(Y) - g(\overline{X},Y), \tag{3.6}$$

where D is a Riemannian connection.

From (3.4), we have

$$\begin{aligned} {}'\tilde{S}(X,Y,Z) &+ {}'\tilde{S}(Y,Z,X) + {}'\tilde{S}(Z,X,Y) \\ &= 2[A(X)g(\overline{Y},Z) + A(Y)g(X,\overline{Z}) + A(Z)g(\overline{X},Y)]. \end{aligned} \tag{3.7}$$

Covariant derivative of (3.1) gives

$$\begin{aligned} (\tilde{B}_X\tilde{S})(Y,Z) &= 2g((\tilde{B}_XF)(Y),Z)T \\ &- 2A(Z)g(\overline{X},\overline{Y})T + 2g(\overline{Y},Z)D_XT \\ &= 2(\tilde{B}_X{}'F)(Y,Z)T + 2{}'F(Y,Z)D_XT. \end{aligned} \tag{3.8}$$

Let us put (3.3) as

$$\tilde{B}_XY = D_XY + H(X,Y), \tag{3.9}$$

where

$$H(X,Y) = {}'F(X,Y)T \tag{3.10}$$

being a tensor field of type (1,2).

Putting

$${}'H(X,Y,Z) \stackrel{\text{def}}{=} g(H(X,Y),Z) = A(Z){}'F(X,Y), \tag{3.11}$$

then from (3.4) and (3.11), we obtain

$${}'\tilde{S}(X,Y,Z) = 2{}'H(X,Y,Z). \tag{3.12}$$

Theorem 3.1. *An almost contact metric manifold M_n with a semi-symmetric non-metric connection $\tilde{B}$ is a nearly cosymplectic manifold if and only if*

$$(\tilde{B}_X\tilde{S})(Y,Z) - (\tilde{B}_Y\tilde{S})(Z,X) = 2[g(\overline{Y},Z)D_XT + g(\overline{X},Z)D_YT] \tag{3.13}$$

and is cosymplectic if and only if

$$(\tilde{B}_Y\tilde{S})(X,Z) - (\tilde{B}_Y\tilde{S})(Z,X) = 4g(\overline{X},Z)D_YT. \tag{3.14}$$

Proof. Covariant differentiation of (3.1) gives

$$(\tilde{B}_X\tilde{S})(Y,Z) = 2g((D_XF)(Y),Z)T + 2g(\overline{Y},Z)D_XT. \tag{3.15}$$

Interchanging X and Y in (3.15) and then subtracting from (3.15), we obtain

$$\begin{aligned} (\tilde{B}_X\tilde{S})(Y,Z) &- (\tilde{B}_Y\tilde{S})(X,Z) = 2g((D_XF)(Y) \\ &- (D_YF)(X),Z)T + 2[g(\overline{Y},Z)D_XT - g(\overline{X},Z)D_YT]. \end{aligned} \tag{3.16}$$

From (3.15) we have

$$\begin{aligned} (\tilde{B}_X\tilde{S})(Y,Z) &- (\tilde{B}_Y\tilde{S})(Z,X) = 2[g(\overline{Y},Z)D_XT - g(\overline{Z},X)D_YT] \\ &+ 2\{(D_X{}'F)(Y,Z) - (D_Y{}'F)(Z,X)\}T. \end{aligned} \tag{3.17}$$

An almost contact metric manifold M_n with a Riemannian connection D is a nearly cosymplectic manifold [3], if

$$(D_X{}'F)(Y,Z) = (D_Y{}'F)(Z,X).$$

Using this result in (3.17), we find (3.13). Converse part is obvious from (3.13) and (3.17). Again, equations (3.16) and (3.17) gives

$$(\tilde{B}_Y\tilde{S})(X,Z) - (\tilde{B}_Y\tilde{S})(Z,X) = 4(D_Y{}'F)(X,Z)T + 4g(\overline{X},Z)D_YT. \tag{3.18}$$

An almost contact metric manifold M_n with the Remannian connection D is cosymplectic [3] if

$$(D_X{}'F)(Y,Z) = 0.$$

In consequence of above equation, (3.18) gives (3.14). Again equations (3.14) and (3.18) gives the converse part. □

Theorem 3.2. *An almost contact metric manifold admitting a semi-symmetric non-metric connection $\tilde{B}$ is a quasi-Sasakian manifold if*

$$(\tilde{B}_X{}'F)(Y,Z) + (\tilde{B}_Y{}'F)(Z,X) + (\tilde{B}_Z{}'F)(X,Y) = 0.$$

Proof. We have [6],

$$(\tilde{B}_X{}'F)(Y,Z) = (D_X{}'F)(Y,Z). \tag{3.19}$$

An almost contact metric manifold (M_n, g) is said to be a quasi-Sasakian manifold if $'F$ is closed, i.e., (2.12) hold ([3], [4]).

Using (3.19) in (2.12), we obtain the result. □

4. Curvature tensor with respect to semi-symmetric non-metric connection

Curvature tensor of M_n with respect to semi-symmetric non-metric connection $\tilde{B}$ is defined as

$$R(X,Y,Z) = \tilde{B}_X\tilde{B}_Y Z - \tilde{B}_Y\tilde{B}_X Z - \tilde{B}_{[X,Y]}Z. \tag{4.1}$$

In view of (3.3), (4.1) becomes

$$\begin{aligned} R(X,Y,Z) &= K(X,Y,Z) + g(\overline{Y},Z)D_X T \\ &- g(\overline{X},Z)D_Y T + g((D_X F)(Y) - (D_Y F)(X), Z)T, \end{aligned} \tag{4.2}$$

where

$$K(X,Y,Z) = D_X D_Y Z - D_Y D_X Z - D_{[X,Y]}Z$$

is the curvature tensor with respect to the Riemannian connection [3].

In case of Sasakian manifold, (4.2) becomes

$$\begin{aligned} R(X,Y,Z) &= K(X,Y,Z) + g(\overline{Y},Z)\overline{X} \\ &- g(\overline{X},Z)\overline{Y} + g((D_X F)(Y) - (D_Y F)(X), Z)T. \end{aligned} \tag{4.3}$$

Using (2.10), (2.12), (4.3) and Bianchi's first identity with respect to Riemannian connection D, the Bianchi's first identity with respect to semi-symmetric non-metric connection $\tilde{B}$ is

$$\begin{aligned} R(X,Y,Z) &+ R(Y,Z,X) + R(Z,X,Y) \\ &= 2[g(\overline{X},Y)\overline{Z} + g(\overline{Y},Z)\overline{X} + g(X,\overline{Z})\overline{Y}]. \end{aligned} \tag{4.4}$$

If we define

$$'R(X,Y,Z,W) = g(R(X,Y,Z),W), \tag{4.5}$$

then

$$'R(X,Y,Z,W) +' R(Y,X,Z,W) = 0. \tag{4.6}$$

This shows that the curvature tensor with respect to semi-symmetric non-metric connection $\tilde{B}$ is skew-symmetric in the first two slots.

$$\begin{aligned} (\tilde{B}_X\tilde{S})(Y,Z) &+ (\tilde{B}_Y\tilde{S})(Z,X) + (\tilde{B}_Z\tilde{S})(X,Y) \\ &= 2[g(\overline{Y},Z)\overline{X} + g(\overline{Z},X)\overline{Y} + g(\overline{X},Y)\overline{Z}]. \end{aligned} \tag{4.7}$$

In consequence of (4.4) and (4.7), we obtain

$$\begin{aligned} (\tilde{B}_X\tilde{S})(Y,Z) &+ (\tilde{B}_Y\tilde{S})(Z,X) + (\tilde{B}_Z\tilde{S})(X,Y) \\ &= R(X,Y,Z) + R(Y,Z,X) + R(Z,X,Y). \end{aligned} \tag{4.8}$$

Now, we prove the following theorems

Theorem 4.1. *Let M_n be an almost contact metric manifold admitting a semi-symmetric non-metric connection $\tilde{B}$ whose curvature tensor vanishes. Then M_n is flat if and only if*

$$(\tilde{B}_X\tilde{S})(Y,Z) = (\tilde{B}_Y\tilde{S})(X,Z). \tag{4.9}$$

Proof. In consequence of (3.16) and (4.2), we find

$$R(X,Y,Z) = K(X,Y,Z) + \frac{1}{2}\left\{(\tilde{B}_X\tilde{S})(Y,Z) - (\tilde{B}_Y\tilde{S})(X,Z)\right\}. \tag{4.10}$$

If the curvature tensor with respect to semi-symmetric non-metric connection $\tilde{B}$ vanishes, then (4.10) becomes

$$K(X,Y,Z) = \frac{1}{2}\left\{(\tilde{B}_Y\tilde{S})(X,Z) - (\tilde{B}_X\tilde{S})(Y,Z)\right\}. \tag{4.11}$$

If $K(X,Y,Z) = 0$, then (4.11) gives (4.9). Converse part is obvious from (4.9) and (4.11). □

Theorem 4.2. *Let M_n be an almost contact metric manifold admitting a semi-symmetric non-metric connection $\tilde{B}$ whose curvature tensor vanishes. Then M_n is an almost cosymplectic manifold if and only if the vector field T is harmonic.*

Proof. In consequence of (2.10) and (2.20), (4.2) becomes

$$R(X,Y,Z) = K(X,Y,Z) + \{(D_X{'F})(Y,Z) + (D_Y{'F})(Z,X)\}\,T. \tag{4.12}$$

If $R(X,Y,Z) = 0$, then (4.12) gives

$$K(X,Y,Z) = -\{(D_X{'F})(Y,Z) + (D_Y{'F})(Z,X)\}\,T. \tag{4.13}$$

Taking cyclic sum of (4.13) in X, Y and Z and then using Bianchi's first identity with respect to D, we get

$$(D_X{'F})(Y,Z) + (D_Y{'F})(Z,X) + (D_Z{'F})(X,Y) = 0, \tag{4.14}$$

which show that the manifold M_n is quasi-Sasakian manifold [3], [4].

An almost contact metric manifold on which the fundamental $2-$form $'F$ and contact form A are both closed, i.e.

$$(i) \qquad d'F = 0 \qquad (ii) \qquad du = 0, \tag{4.15}$$

where d denotes the exterior derivative, has been called an almost contact metric manifold [24]. Thus in view of (2.19) and (4.14), we find the necessary part of the theorem. Converse part is obvious from (2.10), (4.2) and (4.15). □

Theorem 4.3. *Let M_n be an almost contact metric manifold admitting a semi-symmetric non-metric connection $\tilde{B}$ whose curvature tensor vanishes. Then M_n is flat if and only if*

$$(\tilde{B}_XH)(Y,Z) = (\tilde{B}_YH)(X,Z). \tag{4.16}$$

Proof. From (3.10), we have

$$H(Y,Z) ='F(Y,Z)T.$$

Covariant derivative of last result with respect to X gives

$$(\tilde{B}_XH)(Y,Z) = (\tilde{B}_X{'F})(Y,Z)T +' F(Y,Z)\tilde{B}_XT. \tag{4.17}$$

In view of (2.2), (2.6), (2.7), (3.3) and (3.19), (4.17) becomes

$$(\tilde{B}_X H)(Y,Z) = (D_X{}'F)(Y,Z)T +' F(Y,Z)D_X T. \tag{4.18}$$

Interchanging X and Y in (4.18) and then subtracting from (4.18), we have

$$\begin{aligned}(\tilde{B}_X H)(Y,Z) \quad &- \quad (\tilde{B}_Y H)(X,Z) \\ &= \quad (D_X{}'F)(Y,Z)T +' F(Y,Z)D_X T \\ &- \quad (D_Y{}'F)(X,Z)T -' F(X,Z)D_Y T.\end{aligned} \tag{4.19}$$

In view of $R(X,Y,Z) = 0$, (2.7) and (4.19), (4.2) becomes

$$K(X,Y,Z) = (\tilde{B}_Y H)(X,Z) - (\tilde{B}_X H)(Y,Z). \tag{4.20}$$

If M_n is flat, then (4.20) gives (4.16). Converse part is obvious from (4.16) and (4.20). □

Theorem 4.4. *In a Sasakian manifold M_n, equipped with a semi-symmetric non-metric connection $\tilde{B}$, the Weyl projective, conharmonic, concircular and conformal curvature tensors of the manifold coincide if and only if it is Ricci flat with respect to $\tilde{B}$.*

Proof. In view of (2.9) and (4.3), we have

$$\begin{aligned}R(X,Y,Z) \quad &= \quad K(X,Y,Z) - A(X)g(Y,Z)T \\ &+ \quad A(Y)g(X,Z)T + g(\overline{Y},Z)\overline{X} - g(\overline{X},Z)\overline{Y}.\end{aligned} \tag{4.21}$$

Contracting above with respect to X, we obtain

$$\tilde{Ric}(Y,Z) = Ric(Y,Z), \tag{4.22}$$

therefore

$$\tilde{R}Y = RY \tag{4.23}$$

and

$$\tilde{r} = r. \tag{4.24}$$

Here $\tilde{Ric}$; Ric and $\tilde{r}$; r are the Ricci tensors and scalar curvatures of the connections $\tilde{B}$ and D respectively. In consequence of (2.16) and (2.17), we obtain $r = 0$. Again, in view of (2.15), (2.18) and $r = 0$, we find

$$Ric(Y,Z) = 0. \tag{4.25}$$

The equations (4.22) and (4.25) gives the first part of the theorem. Converse part is obvious from (4.22), (2.15), (2.16), (2.17) and (2.18). □

Theorem 4.5. *Let M_n be a Sasakian manifold admitting a semi-symmetric non-metric connection $\tilde{B}$ whose curvature tensor vanishes. Then M_n is flat if and only if*

$$\begin{aligned}(\tilde{B}_X{}'H)(Y,Z,U) \quad &- \quad (\tilde{B}_Y{}'H)(X,Z,U) \\ &= \quad {}'F(X,Z){}'F(Y,U) -' F(Y,Z){}'F(X,U).\end{aligned} \tag{4.26}$$

Proof. Using (4.3) in (4.5), we get

$$\begin{aligned}{}'R(X,Y,Z,U) \quad &= \quad {}'K(X,Y,Z,U) \\ &+ \quad {}'F(Y,Z){}'F(X,U) -' F(X,Z){}'F(Y,U) \\ &+ \quad A(U)\left\{(D_X{}'F)(Y,Z) - (D_Y{}'F)(X,Z)\right\}.\end{aligned} \tag{4.27}$$

Taking covariant derivative of (3.11) and then using (2.13),(2.14) and (3.19), we get

$$(\tilde{B}_U{}'H)(X,Y,Z) = A(Z)'K(X,Y,U,T). \tag{4.28}$$

Using (2.14) and (4.28) in (4.27), we obtain

$$\begin{aligned} 'R(X,Y,Z,U) &= \ 'K(X,Y,Z,U) \\ &+ \ 'F(Y,Z)'F(X,U) -' F(X,Z)'F(Y,U) \\ &+ \ (\tilde{B}_X{}'H)(Y,Z,U) - (\tilde{B}_Y{}'H)(X,Z,U). \end{aligned} \tag{4.29}$$

If $'R(X,Y,Z,U) = 0$ and $'K(X,Y,Z,U) = 0$, then from (4.29) we easily find (4.26). Conversely, when (4.26) is satisfied and $'R(X,Y,Z,U) = 0$, then from (4.29) the manifold is flat. □

5. Group manifold of the semi-symmetric non-metric connection $\tilde{B}$

A manifold satisfying

$$R(X,Y,Z) = 0 \tag{5.1}$$

and

$$(\tilde{B}_X\tilde{S})(Y,Z) = 0 \tag{5.2}$$

is called a group manifold [5].

Theorem 5.1. *An almost contact metric manifold M_n with a semi-symmetric non-metric connection $\tilde{B}$ is a group manifold if and only if M_n is flat with respect to $\tilde{B}$ and M_n is cosymplectic.*

Proof. From (3.15) and (5.2) , we have

$$(\tilde{B}_X\tilde{S})(Y,Z) = 0 \Leftrightarrow (D_X{}'F)(Y,Z)T +' F(Y,Z)D_XT = 0$$

$$\Leftrightarrow (D_X{}'F)(Y,Z)T = -'F(Y,Z)D_XT. \tag{5.3}$$

Inner product of equation (5.3) with T gives

$$(a) \quad D_X{}'F = 0 \quad and \quad (b) \quad D_XT = 0. \tag{5.4}$$

Equations (5.1) and (5.4) (a) state the necessary part of the theorem. Sufficient part is obvious from (3.15), (5.1), (5.2) and (5.4). □

Corollary 5.2. *A group manifold equipped with a semi-symmetric non-metric connection $\tilde{B}$ is flat.*

Proof is obvious from (4.2), (5.1) and (5.4).

Theorem 5.3. *A Sasakian manifold M_n admitting a semi-symmetric non-metric connection $\tilde{B}$ for which the manifold is a group manifold, is conformally flat.*

Proof is obvious from (4.22), (5.1) and theorem (4.4) .

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