Bulletin of Mathematical Analysis and Applications
ISSN: 1821-1291, URL: http://www.bmathaa.org
Volume 3 Issue 4 (2011), Pages 180-188

A FIXED POINT RESULT INVOLVING A GENERALIZED WEAKLY CONTRACTIVE CONDITION IN G-METRIC SPACES

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Abstract. In this paper, we prove a fixed point result for a self-mapping on a G-metric space satisfying (ψ, φ)-weakly contractive conditions. Besides this, a non-trivial example is presented.

1. Introduction

Some generalizations of the notion of a metric space have been proposed by some authors. In 2006, Mustafa in collaboration with Sims introduced a new notion of generalized metric space called G-metric space [12]. In fact, Mustafa et al. studied many fixed point results for a self-mapping in G-metric space under certain conditions, see [11, 12, 13, 14, 15]. For other results on G-metric spaces, see [1, 2, 3, 4, 5, 16, 17, 18, 19].
In the present work, we study some fixed point results for a self-mapping in a complete G-metric space X under weakly contractive conditions related to altering distance functions.

Definition 1.1. (altering distance functions [9]) A mapping $f : [0, +\infty[\to [0, +\infty[$ is called an altering distance function if the following properties are satisfied:
(a) f is continuous and non-decreasing.
(b) $f(t) = 0 \Longleftrightarrow t = 0$.

We present now the necessary definitions and results in G-metric spaces, which will be useful for the rest.

Definition 1.2. [12] Let X be a nonempty set, and let $G : X \times X \times X \longrightarrow R_+$ be a function satisfying the following properties:
(G1) $G(x, y, z) = 0$ if $x = y = z$,
(G2) $G(x, x, y) > 0$ for all $x, y \in X$, with $x \neq y$,
(G3) $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$, with $y \neq z$,
(G4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = ...$, (symmetry in all three variables),
(G5) $G(x, y, z) \leq G(x, , a, a) + G(a, y, z)$, for all $x, y, z, a \in X$, (rectangle inequality).

2000 *Mathematics Subject Classification.* 47B10, 46G99, 46G25.
Key words and phrases. G-metric space, fixed point, weak contraction, altering distance.

Submitted February 4, 2011. Published November 13, 2011.

Then the function G is called a generalized metric, or more specially a G-metric on X, and the pair (X,G) is called a G-metric space.

Definition 1.3. [12] Let (X,G) be a G-metric space and let (x_n) be a sequence of points of X, a point $x \in X$ is said to be the limit of the sequence (x_n), if $\lim_{n,m\to+\infty} G(x,x_n,x_m) = 0$, and we say that the sequence (x_n) is G-convergent to x or (x_n) G-converges to x.

Thus, $x_n \to x$ in a G-metric space (X,G) if for any $\varepsilon > 0$ there exists $k \in \mathbb{N}$ such that $G(x,x_n,x_m) < \varepsilon$ for all $m,n \geq k$.

Proposition 1.4. [12] Let (X,G) be a G-metric space. Then, the following are equivalent
(1) $\{x_n\}$ is G-convergent to x
(2) $G(x_n,x_n,x) \to 0$ as $n \to +\infty$
(3) $G(x_n,x,x) \to 0$ as $n \to +\infty$
(4) $G(x_n,x_m,x) \to 0$ as $n,m \to +\infty$.

Definition 1.5. [12] Let (X,G) be a G-metric space. A sequence (x_n) is is called a G-Cauchy sequence if for any $\varepsilon > 0$ there exists $k \in \mathbb{N}$ such that $G(x_n,x_m,x_l) < \varepsilon$ for all $m,n,l \geq k$, that is $G(x_n,x_m,x_l) \to 0$ as $n,m,l \to +\infty$.

Proposition 1.6. [13] Let (X,G) be a G-metric space. Then, the following are equivalent:
(1) the sequence (x_n) is is G-Cauchy
(2) for any $\varepsilon > 0$ there exists $k \in \mathbb{N}$ such that $G(x_n,x_m,x_m) < \varepsilon$ for all $m,n \geq k$.

Proposition 1.7. [12] *Let (X,G) be a G-metric space. Then $f : X \to X$ is G-continuous at $x \in X$ if and only if it is G-sequentially continuous at x, that is, whenever (x_n) is G-convergent to x, $(f(x_n))$ is G-convergent to $f(x)$.*

Proposition 1.8. [12] *Let (X,G) be a G-metric space. Then the function $G(x,y,z)$ is jointly continuous in all three of its variables.*

Definition 1.9. [12] A G-metric space (X,G) is called G-complete if every G-Cauchy sequence is G-convergent in (X,G).

In [6], Alber and Guerre-Delabriere introduced the concept of weak contraction in Hilbert spaces. After this, Dutta and Choudhuty [8] obtained on a complete ordinary metric space a fixed point result for a self map involving a (ψ,φ)- weakly contractive condition. It is the following

Theorem 1.10. [6] *Let (X,d) be a complete metric space. Suppose the map $f : X \to X$ satisfies for all $x,y \in X$*

$$\psi\bigg(d(fx,fy)\bigg) \leq \psi\bigg(d(x,y)\bigg) - \varphi\bigg(d(x,y)\bigg), \tag{1.1}$$

where ψ and φ are altering distance functions given in Definition 1.1. Then f has a unique fixed point.

Motivated by the above result, we address the same question on G-metric spaces. More precisely, taking a self-mapping on a complete G-metric space satisfying a generalized weak contraction condition given by (2.1), we establish a fixed point result. In the second part of the paper, an example is also presented.

2. MAIN RESULTS

Our first main result is the following

Theorem 2.1. *Let X be a complete G-metric space. Suppose the map $T : X \to X$ satisfies for all $x, y, z \in X$*

$$\psi\Big(G(Tx,Ty,Tz)\Big) \le \psi\Big(G(x,y,z)\Big) - \varphi\Big(G(x,y,z)\Big), \tag{2.1}$$

where ψ and φ are altering distance functions given in Definition 1.1. Then T has a unique fixed point (say u) and T is G-continuous at u.

Proof. Let x_0 be an arbitrary point in X, and let $x_{n+1} = Tx_n$ for any $n \in \mathbb{N}$. Assume $x_n \neq x_{n-1}$. For $n \in \mathbb{N}$, we have thanks to (2.1) and definition of φ

$$\begin{aligned}\psi\Big(G(x_n,x_{n+1},x_{n+1})\Big) &= \psi\Big(G(Tx_{n-1},Tx_n,Tx_n)\Big)\\ &\le \psi\Big(G(x_{n-1},x_n,x_n)\Big) - \varphi\Big(G(x_{n-1},x_n,x_n)\Big) \qquad (2.2)\\ &\le \psi\Big(G(x_{n-1},x_n,x_n)\Big).\end{aligned}$$

Since ψ is non-decreasing, we get that

$$G(x_n,x_{n+1},x_{n+1}) \le G(x_{n-1},x_n,x_n). \tag{2.3}$$

If we take $t_n = G(x_n,x_{n+1},x_{n+1})$, then from (2.3), we get $0 \le t_n \le t_{n-1}$, so the sequence (t_n) is non-increasing, hence it converges to some $r \ge 0$. Letting this in (2.2), then as $n \longrightarrow +\infty$

$$\psi(r) \le \psi(r) - \varphi(r),$$

using the continuity of ψ and φ. Then, we find $\varphi(r) = 0$, hence by a property of φ, we have $r = 0$. We rewrite this as

$$\lim_{n\to+\infty} G(x_n,x_{n+1},x_{n+1}) = 0. \tag{2.4}$$

Next, we prove that (x_n) is a G-Cauchy sequence. We argue by contradiction. Assume that (x_n) is not a G-Cauchy sequence. Then, following Proposition 1.6, there exists $\varepsilon > 0$ for which we can find subsequences $(x_{m(k)})$ and $(x_{n(k)})$ of (x_n) with $n(k) > m(k) > k$ such that

$$G(x_{n(k)},x_{m(k)},x_{m(k)}) \ge \varepsilon. \tag{2.5}$$

Further, corresponding to $m(k)$, we can choose $n(k)$ in such a way that it is the smallest integer with $n(k) > m(k)$ and satisfying (2.5). Then

$$G(x_{n(k)-1},x_{m(k)},x_{m(k)}) < \varepsilon. \tag{2.6}$$

We have, using (2.6) and the condition (G5), that

$$\begin{aligned}\varepsilon \le G(x_{n(k)},x_{m(k)},x_{m(k)}) &\le G(x_{n(k)},x_{n(k)-1},x_{n(k)-1}) + G(x_{n(k)-1},x_{m(k)},x_{m(k)})\\ &< \varepsilon + G(x_{n(k)},x_{n(k)-1},x_{n(k)-1}).\end{aligned} \tag{2.7}$$

In other words, from the conditions (G3)-(G4)

$$0 \le G(x_{n(k)}, x_{n(k)-1}, x_{n(k)-1}) = G(x_{n(k)-1}, x_{n(k)-1}, x_{n(k)}) \le G(x_{n(k)-1}, x_{n(k)}, x_{n(k)}).$$

Letting $k \to +\infty$, and using (2.4), we find $G(x_{n(k)}, x_{n(k)-1}, x_{n(k)-1}) \to 0$. We take this in (2.7)

$$\lim_{k\to+\infty} G(x_{n(k)}, x_{m(k)}, x_{m(k)}) = \varepsilon. \tag{2.8}$$

Moreover, we have thanks to condition (G4)

$$\begin{aligned} G(x_{n(k)}, x_{m(k)}, x_{m(k)}) \le& G(x_{n(k)}, x_{n(k)-1}, x_{n(k)-1}) + G(x_{n(k)-1}, x_{m(k)-1}, x_{m(k)-1}) \\ &+G(x_{m(k)-1}, x_{m(k)}, x_{m(k)}), \end{aligned}$$

$$\begin{aligned} G(x_{n(k)-1}, x_{m(k)-1}, x_{m(k)-1}) \le& G(x_{n(k)-1}, x_{n(k)}, x_{n(k)}) + G(x_{n(k)}, x_{m(k)}, x_{m(k)}) \\ &+G(x_{m(k)}, x_{m(k)-1}, x_{m(k)-1}). \end{aligned}$$

Letting $k \longrightarrow +\infty$ in the two above inequalities and using (2.4)-(2.8)

$$\lim_{k\to+\infty} G(x_{n(k)-1}, x_{m(k)-1}, x_{m(k)-1}) = \varepsilon. \tag{2.9}$$

Setting $x = x_{n(k)-1}$ and $y = y_{m(k)-1}$ in (2.1) and using (2.5), we obtain thanks to the fact that ψ is increasing

$$\begin{aligned} \psi(\varepsilon) \le& \psi(G(x_{n(k)}, x_{m(k)}, x_{m(k)})) = \psi(G(Tx_{n(k)-1}, Tx_{m(k)-1}, Tx_{m(k)-1}) \\ \le& \psi(G(x_{n(k)-1}, x_{m(k)-1}, x_{m(k)-1}) - \varphi(G(x_{n(k)-1}, x_{m(k)-1}, x_{m(k)-1}). \end{aligned}$$

Letting $k \to +\infty$, then using (2.9) and the continuity of ψ and φ, we get

$$\psi(\varepsilon) \le \psi(\varepsilon) - \varphi(\varepsilon),$$

yielding that $\varphi(\varepsilon) = 0$, which is a contradiction since $\varepsilon > 0$. This shows that (x_n) is a G-Cauchy sequence and since X is a G-complete space, hence (x_n) is G-convergent to some $u \in X$, that is from Proposition 1.6

$$\lim_{n\to+\infty} G(x_n, x_n, u) = G(x_n, u, u) = 0. \tag{2.10}$$

We show now that u is a fixed point of the map T. From (2.1),

$$\begin{aligned} \psi\Bigg(G(x_{n+1}, x_{n+1}, Tu)\Bigg) =& \psi\Bigg(G(Tx_n, Tx_n, Tu)\Bigg) \\ \le& \psi\Bigg(G(x_n, x_n, u)\Bigg) - \varphi\Bigg(G(x_n, x_n, u)\Bigg). \end{aligned}$$

Thanks to (2.10) and the continuity of ψ and φ, we find

$$\lim_{n\to+\infty} G(x_{n+1}, x_{n+1}, Tu) = 0. \tag{2.11}$$

Again, using the conditions (G4) and (G5) given by Definition 1.2, one can write

$$G(u, u, Tu) \le G(u, u, x_{n+1}) + G(x_{n+1}, x_{n+1}, Tu).$$

Letting $n \to +\infty$ in the above inequality and having in mind (2.10) and (2.11), one finds $G(u, u, Tu) = 0$, and then $Tu = u$. Hence u is a fixed point of T. Let us show its uniqueness. Let v be another fixed point of T, then

$$\begin{aligned}\psi\Bigg(G(u,u,v)\Bigg) =&\psi\Bigg(G(Tu,Tu,Tv)\Bigg)\\ \leq&\psi\Bigg(G(u,u,v)\Bigg)-\varphi\Bigg(G(u,u,v)\Bigg).\end{aligned}$$

It follows that $\varphi\Bigg(G(u,u,v)\Bigg)=0$, and then $G(u,u,v)=0$, yielding that $u=v$. Following Proposition 1.7, to show that T is G-continuous at u, let (y_n) be any sequence in X such that (y_n) is G-convergent to u. For $n\in\mathbb{N}$, we have

$$\begin{aligned}\psi\Bigg(G(u,u,Ty_n)\Bigg) =&\psi\Bigg(G(Tu,Tu,Ty_n)\Bigg)\\ \leq&\psi\Bigg(G(u,u,y_n)\Bigg)-\varphi\Bigg(G(u,u,y_n)\Bigg).\end{aligned}$$

Letting $n\to+\infty$ and using again the continuity of ψ and φ, the right-hand side of the above inequality tends to 0, then we obtain

$$\lim_{n\to+\infty} G(u,u,Ty_n)=0.$$

Hence $(Ty_n)_n$ is G-convergent to $u=Tu$, so T is G-continuous at u.

□

As an application of Theorem 2.1, we have the following corollaries.

Corollary 2.2. *Let X be a complete G-metric space. Suppose the map $T:X\to X$ satisfies for $m\in\mathbb{N}$ and $x,y,z\in X$*

$$\psi\Bigg(G(T^mx,T^my,T^mz)\Bigg)\leq\psi\Bigg(G(x,y,z)\Bigg)-\varphi\Bigg(G(x,y,z)\Bigg), \tag{2.12}$$

where ψ and φ are altering distance functions given in Definition 1.1. Then T has a unique fixed point (say u), and T is G-continuous at u.

Proof. From Theorem 2.1, we conclude that T^m has a unique fixed point say u. Since

$$Tu=T(T^mu)=T^{m+1}u=T^m(Tu),$$

we have that Tu is also a fixed point to T^m. By uniqueness of u, we get $Tu=u$. □.

Corollary 2.3. *Let X be a complete G-metric space. Suppose the map $T:X\to X$ satisfies for all $x,y,z\in X$*

$$G(Tx,Ty,Tz)\leq kG(x,y,z), \tag{2.13}$$

where $k\in[0,1)$, then T has a unique fixed point (say u), and T is G-continuous at u.

Proof. It suffices to take in Theorem 2.1, $\psi(t)=t$ and $\varphi(t)=1-k$ for $k\in[0,1)$. □.

Remark 2.4. Corollary 2.3 corresponds to Theorem 2.1 in [11].

3. Example

We give in this section an example illustrating Theorem 2.1.
Let $X = [0,1] \cup \{2,3,4,...\}$ and

$$G(x,y,z) = \begin{cases} \max\{|x-y|,|y-z|,|z-x|\} \quad if \quad x,y,z \in [0,1] \\ \quad \text{and at least} \quad x \neq y \quad \text{or} \quad y \neq z \quad \text{or} \quad z \neq x \\ x+y+z \quad \text{if at leat} \quad x \quad \text{or} \quad y \quad \text{or} \quad z \notin [0,1] \\ \quad \text{and at least} \quad x \neq y \quad \text{or} \quad y \neq z \quad \text{or} \quad z \neq x \\ 0 \quad \text{if} \quad x = y = z. \end{cases}$$

It is a simple exercise that (X, G) is a G-metric space. We claim that it is a G-complete space. To do this, let $\{x_n\}$ be a G-Cauchy sequence in X. By proposition 1.6, for any $\varepsilon > 0$ there exists $n_0 \in \mathbb{N}$ such that for any $n \geq n_0$ and $m \geq n_0$, we have $G(x_n, x_m, x_m) < \varepsilon$. We distinguish two cases.

(1) If $x_n = x_m$.
Here $G(x_n, x_m, x_m) = 0$ for any $m, n \geq n_0$. In particular, $G(x_n, x_{n_0}, x_{n_0}) = 0$, which gives that the sequence $\{x_n\}$ G-converges to x_{n_0}.

(2) If $x_n \neq x_m$.
The sequence $\{x_n\}$ is G-Cauchy, hence by definition of G, we have necessarily x_n and x_m are in $[0,1]$. As a consequence,

$$|x_n - x_m| =: G(x_n, x_m, x_m) < \varepsilon,$$

for any $m, n \geq n_0$. We find that $\{x_n\}$ is a Cauchy sequence in $[0,1]$, which is complete with respect to the metric $|\ .\ |$. Hence, there exists $x \in [0,1]$ such that $|x_n - x| \to 0$ as $n \to +\infty$. There are two possibilities, that are $x = x_n$ and then $G(x_n, x, x) = 0$, or $x \neq x_n$ and so $G(x_n, x, x) = |x_n - x|$. Always, we obtain

$$\lim_{n \to +\infty} G(x_n, x, x) = 0,$$

meaning that $\{x_n\}$ G-converges to x. In the two cases we have the completeness of (X, G).

Now, let $\psi : [0, +\infty[\to [0, +\infty[$ such that

$$\psi(t) = \begin{cases} t \quad \text{if} \quad 0 \leq t \leq 1 \\ t^2 \quad \text{if} \quad t > 1. \end{cases}$$

Again, we define $\varphi : [0, +\infty[\longrightarrow [0, +\infty[$ such that

$$\varphi(t) = \begin{cases} \frac{1}{2}t^2 \quad \text{if} \quad 0 \leq t \leq 1 \\ \frac{1}{2} \quad \text{if} \quad t > 1. \end{cases}$$

Moreover, let $T : X \longrightarrow X$ be defined as

$$Tx = \begin{cases} x - \frac{1}{2}x^2 \quad \text{if} \quad 0 \leq x \leq 1 \\ x - 1 \quad \text{if} \quad x \in \{2,3,4,...\} \end{cases}$$

It is obvious that ψ and φ verifies hypotheses of Theorem 2.1. Without loss of generality, we assume that $x > y > z$ and discuss the following cases:

Case 1: $x \in [0,1]$
Here, necessarily $y > z$ and $y, z \in [0,1]$. By definition of G, we have

$$G(x,y,z) = \max\{|x-y|, |y-z|, |z-x|\} = x - z \in [0,1].$$

It follows that by definition of ψ and φ

$$\begin{aligned} \psi(G(x,y,z)) - \varphi(G(x,y,z)) =& G(x,y,z) - \frac{1}{2}G^2(x,y,z) \\ =&(x-z) - \frac{1}{2}(x-z)^2 \\ \geq&(x-z) - \frac{1}{2}(x^2-z^2). \end{aligned} \tag{3.1}$$

Again, by definition of T, we get $Tx, Ty, Tz \in [0,1]$ and

$$Tx = x - \frac{1}{2}x^2 > Ty = y - \frac{1}{2}y^2 > Tz = z - \frac{1}{2}z^2,$$

and hence

$$G(Tx,Ty,Tz) = \max\{|Tx-Ty|, |Ty-Tz|, |Tz-Tx|\} = Tx-Tz = [(x-z)-\frac{1}{2}(x^2-z^2)] \in [0,1].$$

It follows that

$$\psi(G(Tx,Ty,Tz)) = G(Tx,Ty,Tz) = (x-z) - \frac{1}{2}(x^2-z^2),$$

(3.1) gives us

$$\psi(G(Tx,Ty,Tz)) \leq \psi(G(x,y,z)) - \varphi(G(x,y,z)).$$

Then the inequality (2.1) holds.
Case 2: $x \in \{3,4,...\}$
Since $x > y > z$, hence y may be in $\{3,4,...\}$ or in $[0,1]$. We start with the case when $y \in \{3,4,...\}$. Even here, we have two cases for z, indeed $z \in \{3,4,...\}$ or in $[0,1]$.
• If $z \in \{3,4,...\}$. Here, $Tx = x-1 > Ty = y-1 = Tz = z-1 \geq 2$. Then

$$G(Tx,Ty,Tz) = Tx + Ty + Tz = x + y + z - 3 > 1.$$

We deduce then

$$\psi(G(Tx,Ty,Tz)) = G^2(Tx,Ty,Tz) = (x+y+z-3)^2. \tag{3.2}$$

In other words, $G(x,y,z) = x + y + z > 1$, so

$$\begin{aligned} \psi(G(x,y,z)) - \varphi(G(x,y,z)) =& G^2(x,y,z) - \frac{1}{2} \\ =&(x+y+z)^2 - \frac{1}{2}. \end{aligned} \tag{3.3}$$

Comparing (3.2) to (3.3) we find

$$\psi(G(Tx,Ty,Tz)) \leq \psi(G(x,y,z)) - \varphi(G(x,y,z)),$$

meaning that (2.1) holds.
• If $z \in [0,1]$. Here, we have $G(x,y,z) = x + y + z > 1$, then

$$\begin{aligned} \psi(G(x,y,z)) - \varphi(G(x,y,z)) =& G^2(x,y,z) - \frac{1}{2} \\ =&(x+y+z)^2 - \frac{1}{2}. \end{aligned} \tag{3.4}$$

Again, since $Tz = z - \frac{1}{2}z^2 \in [0,1]$ and $Tx = x - 1 \neq Ty = y - 1 > 1$, then $G(Tx,Ty,Tz) = Tx + Ty + Tz = x + y + z - 2 - \frac{1}{2}z^2 > 1$. Therefore,

$$\psi(G(Tx,Ty,Tz)) = G^2(Tx,Ty,Tz) = (x+y+z-2-\frac{1}{2}z^2)^2. \tag{3.5}$$

We compare (3.4) to (3.5) to get that (2.1) holds.
Let us now do the case where $y \in [0,1]$. Here, $y > z \in [0,1]$. The same strategy yields that

$$\begin{aligned}\psi(G(Tx,Ty,Tz)) = \psi\left(G(x-1, y-\frac{1}{2}y^2, z-\frac{1}{2}z^2)\right) \\ =&\psi(x+y+z-1-\frac{1}{2}y^2-\frac{1}{2}z^2) \\ =&(x+y+z-1-\frac{1}{2}y^2-\frac{1}{2}z^2)^2.\end{aligned} \tag{3.6}$$

Moreover,

$$\psi(G(x,y,z)) - \varphi(G(x,y,z)) = (x+y+z)^2 - \frac{1}{2}.$$

We deduce then

$$\psi(G(Tx,Ty,Tz)) \leq \psi(G(x,y,z)) - \varphi(G(x,y,z)),$$

that is the inequality (2.1).
Case 3: $x = 2$
In this case, since $x > y > z$, we have necessarily $y, z \in [0,1]$. Here, we have

$$\begin{aligned}\psi(G(Tx,Ty,Tz)) = \psi\left(G(1, y-\frac{1}{2}y^2, z-\frac{1}{2}z^2)\right) \\ =&\psi(1-(z-\frac{1}{2}z^2)) \\ =&1-(z-\frac{1}{2}z^2).\end{aligned} \tag{3.7}$$

Again,

$$\psi(G(x,y,z)) - \varphi(G(x,y,z)) = \psi(2+y+z) - \varphi(2+y+z) = (2+y+z)^2 - \frac{1}{2}.$$

It is clear that (2.1) holds.
As a conclusion, the hypotheses of Theorem 2.1 are verified, and then we find that $u = 0$ is the unique fixed point of T in X.

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