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A NEW THEOREM ON LOCAL PROPERTIES OF FACTORED FOURIER SERIES

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ŞEBNEM YILDIZ

Abstract. In this paper, a general theorem dealing with the local property of $|A, p_n; \delta|_k$ summability of factored Fourier series, which generalizes some known results, has been proved. This new theorem also includes several known and new results.

1. Introduction

Let $\sum a_n$ be a given infinite series with partial sums (s_n). Let (p_n) be a sequence of positive numbers such that

$$P_n = \sum_{v=0}^{n} p_v \to \infty \quad as \quad n \to \infty, \quad (P_{-i} = p_{-i} = 0, \quad i \geq 1). \tag{1.1}$$

The sequence-to-sequence transformation

$$\sigma_n = \frac{1}{P_n} \sum_{v=0}^{n} p_v s_v \tag{1.2}$$

defines the sequence (σ_n) of the $(\bar{N}, p_n)$ means of the sequence (s_n), generated by the sequence of coefficients (p_n) (see [19]).
The series $\sum a_n$ is said to be summable $|\bar{N}, p_n|_k$, $k \geq 1$, if (see [3])

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n} \right)^{k-1} |\Delta \sigma_{n-1}|^k < \infty \tag{1.3}$$

and it is said to be summable $|\bar{N}, p_n; \delta|_k$, $k \geq 1$ and $\delta \geq 0$, if (see [6])

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n} \right)^{\delta k + k - 1} |\Delta \sigma_{n-1}|^k < \infty. \tag{1.4}$$

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where

$$\Delta\sigma_{n-1} = -\frac{p_n}{P_n P_{n-1}} \sum_{v=1}^{n} P_{v-1} a_v, \quad n \geq 1.$$

In the special case, when $p_n = 1$ for all values of n (resp. $\delta = 0$), $|\bar{N}, p_n; \delta|_k$ summability is the same as $|C, 1; \delta|_k$ (resp. $|\bar{N}, p_n|_k$) summability. Also, if we take $\delta = 0$, $k = 1$ and $p_n = 1/(n+1)$ (resp.$k = 1$ and $\delta = 0$) summability $|\bar{N}, p_n; \delta|_k$ becomes $|R, logn, 1|$ (resp. $|\bar{N}, p_n|$) summability.
Let $A = (a_{nv})$ be a normal matrix, i.e., a lower triangular matrix of nonzero diagonal entries. Then A defines the sequence-to-sequence transformation, mapping the sequence $s = (s_n)$ to $As = (A_n(s))$, where

$$A_n(s) = \sum_{v=0}^{n} a_{nv} s_v, \quad n = 0, 1, ... \tag{1.5}$$

The series $\sum a_n$ is said to be summable $|A|_k$, $k \geq 1$, if (see [40])

$$\sum_{n=1}^{\infty} n^{k-1} |\bar{\Delta} A_n(s)|^k < \infty, \tag{1.6}$$

and it is said to be summable $|A, p_n; \delta|_k$, $k \geq 1$ and $\delta \geq 0$, if (see [31])

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{\delta k + k - 1} |\bar{\Delta} A_n(s)|^k < \infty, \tag{1.7}$$

where

$$\bar{\Delta} A_n(s) = A_n(s) - A_{n-1}(s).$$

If we take $p_n = 1$ for all n and $\delta = 0$, then $|A, p_n; \delta|_k$ summability reduces to $|A|_k$ summability. Also, if we take $\delta = 0$, then $|A, p_n; \delta|_k$ summability reduces to $|A, p_n|_k$ summability (see [37]). Furthermore, if we take $a_{nv} = \frac{p_v}{P_n}$, then $|A, p_n; \delta|_k$ summability is the same as $|\bar{N}, p_n; \delta|_k$ summability.
A sequence (λ_n) is said to be convex if $\Delta^2 \lambda_n \geq 0$ for every positive integer n, where $\Delta^2 \lambda_n = \Delta(\Delta\lambda_n)$ and $\Delta\lambda_n = \lambda_n - \lambda_{n+1}$.
Let $f(t)$ be a periodic function with period 2π, and integrable (L) over $(-\pi, \pi)$. Without any loss of generality we may assume that the constant term in the Fourier series of $f(t)$ is zero, so that

$$\int_{-\pi}^{\pi} f(t) dt = 0 \tag{1.8}$$

and

$$f(t) \sim \sum_{n=1}^{\infty} (a_n cosnt + b_n sinnt) \quad = \quad \sum_{n=1}^{\infty} C_n(t), \tag{1.9}$$

where (a_n) and (b_n) denote the Fourier coefficients. It is well known (see [41]) that convergence of the Fourier series at a point is a local property, that is to say, however small $\sigma > 0$ may be the behavior of $\{s_n(x)\}$ the nth partial sums of the series $\sum C_n(x)$, depends upon the nature of the generating function in the interval $(x - \sigma, x + \sigma)$ only and it is not affected by the values it takes outside the interval. On the other hand it is known that absolute convergence of a Fourier series is not a local property. Also Bosanquet and Kestelman (see [18]) showed that even

summability $|C,1|$ of a Fourier series a given point is not a local property of the generating function.
Also it is known that the convergence of the Fourier series can be ensured by local hypothesis, that is to say, the behavior of the convergence of Fourier series for a particular value of x depends on the behavior of the function in the immediate neighborhood of this point only.

2 Known Results

Mohanty (see [26]) demonstrated that the $|R, logn, 1|$ summability of the factored Fourier series

$$\sum \frac{C_n(t)}{log(n+1)} \tag{1.10}$$

at $t = x$, is a local property of the generating function of $\sum C_n(t)$. Later on Matsumoto (see [22]) improved this result by replacing the series (1.10) by

$$\sum \frac{C_n(t)}{\{loglog(n+1)\}^{1+\epsilon}}, \quad \epsilon > 0. \tag{1.11}$$

Generalizing the above result Bhatt (see [2]) proved the following theorem.

Theorem 2.1 If (λ_n) is a convex sequence such that $\sum n^{-1}\lambda_n$ is convergent, then the summability $|R, logn, 1|$ of the series $\sum C_n(t)\lambda_n logn$ at a point can be ensured by a local property.
The local property problem of the factored Fourier series have been studied by several authors (see [1, 4-17, 20-21, 23-25, 27-36, 38-39]). Few of them are given above.
Among them, Bor proved the following theorem in which the conditions on the sequence (λ_n) are somewhat more general than Theorem 2.1.

Theorem 2.2 ([10]) Let $k \geq 1$. If (λ_n) is a non-negative and non-increasing sequence such that $\sum p_n\lambda_n$ is convergent, then the summability $|\bar{N}, p_n|_k$ of the series $\sum C_n(t)\lambda_n P_n$ at a point is a local property of the generating function f.

3 Main Results

The aim of this paper is to prove a more general theorem which includes some of the above-mentioned result as special cases.
Before stating the main theorem, we must first introduce some further notations.
Given a normal matrix $A = (a_{nv})$, we associate two lower semimatrices $\bar{A} = (\bar{a}_{nv})$ and $\hat{A} = (\hat{a}_{nv})$ as follows:

$$\bar{a}_{nv} = \sum_{i=v}^{n} a_{ni}, \quad n, v = 0, 1, ... \tag{1.12}$$

and

$$\hat{a}_{00} = \bar{a}_{00} = a_{00}, \quad \hat{a}_{nv} = \bar{a}_{nv} - \bar{a}_{n-1,v}, \quad n = 1, 2, ... \tag{1.13}$$

It may be noted that $\bar{A}$ and $\hat{A}$ are the well-known matrices of series-to-sequence and series-to-series transformations, respectively. Then, we have

$$A_n(s) = \sum_{v=0}^{n} a_{nv}s_v = \sum_{v=0}^{n} \bar{a}_{nv}a_v \tag{1.14}$$

and

$$\bar{\Delta} A_n(s) = \sum_{v=0}^{n} \hat{a}_{nv} a_v. \tag{1.15}$$

Now, we shall prove the following theorem.

Theorem 3.1 Let $k \geq 1$ and $0 \leq \delta < 1/k$. If $A = (a_{nv})$ is a positive normal matrix such that

$$\overline{a}_{no} = 1,\ n = 0, 1, ..., \tag{1.16}$$

$$a_{n-1,v} \geq a_{nv},\ \ for\ n \geq v+1, \tag{1.17}$$

$$a_{nn} = O\left(\frac{p_n}{P_n}\right). \tag{1.18}$$

If all the conditions of Theorem 2.2 and the conditions

$$\sum_{n=1}^{m} \left(\frac{P_n}{p_n}\right)^{\delta k} p_n \lambda_n \quad = \quad O(1) \quad as \quad m \to \infty, \tag{1.19}$$

$$\sum_{n=1}^{m} \left(\frac{P_n}{p_n}\right)^{\delta k} P_n \Delta\lambda_n \quad = \quad O(1) \quad as \quad m \to \infty, \tag{1.20}$$

$$\sum_{n=v+1}^{m+1} \left(\frac{P_n}{p_n}\right)^{\delta k} |\Delta_v(\hat{a}_{nv})| = O\left\{\left(\frac{P_v}{p_v}\right)^{\delta k-1}\right\} \quad as \quad m \to \infty, \tag{1.21}$$

$$\sum_{n=v+1}^{m+1} \left(\frac{P_n}{p_n}\right)^{\delta k} |\hat{a}_{n,v+1}| = O\left\{\left(\frac{P_v}{p_v}\right)^{\delta k}\right\} \quad as \quad m \to \infty \tag{1.22}$$

are satisfied, then the summability $|A, p_n; \delta|_k$ of the series $\sum C_n(t) P_n \lambda_n$ at a point is a local property of the generating function f.

Remark It should be noted that if we take $\delta = 0$ in this theorem, then the condition (1.20) is satisfied by Lemma 3.2. Condition (1.19) is satisfied by a hypothesis of the theorem when $\delta = 0$. Also in this case condition (1.21) and (1.22) are obvious. We need the following lemmas for the proof of our theorem.

Lemma 3.2 (**[10]**) If (λ_n) is a non-negative and non-increasing sequence such that $\sum p_n \lambda_n$ is convergent, where (p_n) is a sequence of positive numbers such that $P_n \to \infty$ as $n \to \infty$, then $P_n \lambda_n = O(1)$ as $n \to \infty$ and $\sum P_n \Delta\lambda_n < \infty$.

Lemma 3.3 Let $s_n = O(1)$. If (λ_n) is a non-negative and non-increasing sequence such that $\sum p_n \lambda_n$ is convergent and the conditions (1.16)-(1.22) of Theorem 3.1 are satisfied, then the series $\sum a_n \lambda_n P_n$ is summable $|A, p_n; \delta|_k$, $k \geq 1$ and $0 \leq \delta < 1/k$.

Proof of Lemma 3.3 Let (I_n) denote the A-transform of the series $\sum_{n=1}^{\infty} a_n \lambda_n P_n$. Then, by (1.14) and (1.15), we have

$$\bar{\Delta} I_n \quad = \quad \sum_{v=1}^{n} \hat{a}_{nv} a_v P_v \lambda_v.$$

Applying Abel's transformation to this sum, we get that

$$\begin{aligned}
\bar{\Delta} I_n &= \sum_{v=1}^{n} \hat{a}_{nv} a_v P_v \lambda_v \\
&= \sum_{v=1}^{n-1} \Delta_v(\hat{a}_{nv}\lambda_v P_v) \sum_{r=1}^{v} a_r + \hat{a}_{nn}\lambda_n P_n \sum_{v=1}^{n} a_v \\
&= \sum_{v=1}^{n-1} \Delta_v(\hat{a}_{nv}\lambda_v P_v) s_v + a_{nn}\lambda_n P_n s_n \\
&= \sum_{v=1}^{n-1} \Delta_v(\hat{a}_{nv})\lambda_v P_v s_v + \sum_{v=1}^{n-1} \hat{a}_{n,v+1}\Delta\lambda_v P_v s_v - \sum_{v=1}^{n-1} \hat{a}_{n,v+1} p_{v+1}\lambda_{v+1} s_v + a_{nn}\lambda_n P_n s_n \\
&= I_{n,1} + I_{n,2} + I_{n,3} + I_{n,4}.
\end{aligned}$$

Since

$$|I_{n,1} + I_{n,2} + I_{n,3} + I_{n,4}|^k \le 4^k(|I_{n,1}|^k + |I_{n,2}|^k + |I_{n,3}|^k + |I_{n,4}|^k),$$

to complete the proof of Lemma 3.3, it is sufficient to show that

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{\delta k+k-1} |I_{n,r}|^k < \infty, \quad for \quad r = 1, 2, 3, 4. \tag{1.23}$$

First, by applying Hölder's inequality with indices k and k', where $k > 1$ and $\frac{1}{k} + \frac{1}{k'} = 1$, we have that

$$\begin{aligned}
\sum_{n=2}^{m+1} \left(\frac{P_n}{p_n}\right)^{\delta k+k-1} |I_{n,1}|^k &\le \sum_{n=2}^{m+1} \left(\frac{P_n}{p_n}\right)^{\delta k+k-1} \left\{\sum_{v=1}^{n-1} |\Delta_v(\hat{a}_{nv})| \lambda_v P_v |s_v|\right\}^k \\
&= O(1) \sum_{n=2}^{m+1} \left(\frac{P_n}{p_n}\right)^{\delta k+k-1} \left\{\sum_{v=1}^{n-1} |\Delta_v(\hat{a}_{nv})| \lambda_v^k P_v^k |s_v|^k\right\} \times \left\{\sum_{v=1}^{n-1} |\Delta_v(\hat{a}_{nv})|\right\}^{k-1} \\
&= O(1) \sum_{n=2}^{m+1} \left(\frac{P_n}{p_n}\right)^{\delta k+k-1} a_{nn}^{k-1} \sum_{v=1}^{n-1} |\Delta_v(\hat{a}_{nv})| \lambda_v^k P_v^k \\
&= O(1) \sum_{v=1}^{m} \lambda_v^k P_v^k \sum_{n=v+1}^{m+1} \left(\frac{P_n}{p_n}\right)^{\delta k} |\Delta_v(\hat{a}_{nv})| \\
&= O(1) \sum_{v=1}^{m} \lambda_v^k P_v^k \left(\frac{P_v}{p_v}\right)^{\delta k-1} \\
&= O(1) \sum_{v=1}^{m} \left(\frac{P_v}{p_v}\right)^{\delta k} (P_v\lambda_v)^{k-1} p_v \lambda_v \\
&= O(1) \sum_{v=1}^{m} \left(\frac{P_v}{p_v}\right)^{\delta k} p_v \lambda_v \\
&= O(1) \quad as \quad m \to \infty,
\end{aligned}$$

by virtue of the hypotheses of Lemma 3.3 and Lemma 3.2.
Now, using Hölder's inequality we have that

$$
\begin{aligned}
\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}|I_{n,2}|^k &\leq \sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}\left\{\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}|\Delta\lambda_v P_v|s_v|\right\}^k \\
&= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}\left\{\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}|^k\Delta\lambda_v P_v|s_v|^k\right\} \\
&\quad \times\left\{\sum_{v=1}^{n-1}\Delta\lambda_v P_v\right\}^{k-1} \\
&= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}\left\{\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}||\hat{a}_{n,v+1}|^{k-1}\Delta\lambda_v P_v\right\} \\
&= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}a_{nn}^{k-1}\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}|\Delta\lambda_v P_v \\
&= O(1)\sum_{v=1}^{m}\Delta\lambda_v P_v\sum_{n=v+1}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k}|\hat{a}_{n,v+1}| \\
&= O(1)\sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^{\delta k}\Delta\lambda_v P_v \\
&= O(1) \quad as \quad m\to\infty,
\end{aligned}
$$

by virtue of the hypotheses of Lemma 3.3 and Lemma 3.2.
Again, we have that

$$
\begin{aligned}
\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}|I_{n,3}|^k &\leq \sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}\left\{\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}|p_{v+1}\lambda_{v+1}s_v\right\}^k \\
&= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}\left\{\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}|^k p_v\lambda_v\right\}\times\left\{\sum_{v=1}^{n-1}p_v\lambda_v\right\}^{k-1} \\
&= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}\left\{\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}|^{k-1}|\hat{a}_{n,v+1}|p_v\lambda_v\right\} \\
&= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}a_{nn}^{k-1}\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}|p_v\lambda_v \\
&= O(1)\sum_{v=1}^{m}p_v\lambda_v\sum_{n=v+1}^{m+1}\left(\frac{P_n}{p_n}\right)^{\delta k}|\hat{a}_{n,v+1}| \\
&= O(1)\sum_{v=1}^{m}\left(\frac{P_v}{p_v}\right)^{\delta k}p_v\lambda_v \\
&= O(1) \quad as \quad m\to\infty.
\end{aligned}
$$

Finally, since $P_n\lambda_n = O(1)$ as $n \to \infty$, we have that

$$\begin{aligned}\sum_{n=1}^{m}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}|I_{n,4}|^k &= \sum_{n=1}^{m}\left(\frac{P_n}{p_n}\right)^{\delta k+k-1}|a_{nn}\lambda_n P_n s_n|^k\\ &= O(1)\sum_{n=1}^{m}\left(\frac{P_n}{p_n}\right)^{\delta k}(P_n\lambda_n)^{k-1}\lambda_n p_n\\ &= O(1) \quad as \quad m\to\infty,\end{aligned}$$

by virtue of the hypotheses of Lemma 3.3 and Lemma 3.2.

This completes the proof of Lemma 3.3.

4 Proof of Theorem 3.1

The convergence of the Fourier series at $t = x$ is a local property of f (i.e., it depends only on the behaviour of f in an arbitrarily small neighbourhood of x), and hence the summability of the Fourier series at $t = x$ by any regular linear summability method is also a local property of f. Since the behaviour of the Fourier series, as far as convergence is concerned, for a particular value of x depends on the behaviour of the function in the immediate neighbourhood of this point only, hence the truth of Theorem 3.1 is a consequence of Lemma 3.3.

If we take $\delta = 0$ in this theorem, then we get a local property result concerning the $|A, p_n|_k$ summability. Also, if we take $\delta = 0$ and $a_{nv} = \frac{p_v}{P_n}$ in this theorem, then we get a local property result concerning the $|\bar{N}, p_n|_k$ summability.

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Şebnem YILDIZ

Department of Mathematics, Ahi Evran University, Kırşehir, Turkey

E-mail address: sebnemyildiz@ahievran.edu.tr