Bulletin of Mathematical Analysis and Applications
ISSN: 1821-1291, URL: http://www.bmathaa.org
Volume 9 Issue 2(2017), Pages 37-44.

SOME SUBCLASSES OF BI-UNIVALENT FUNCTIONS ASSOCIATED WITH SRIVASTAVA-ATTIYA OPERATOR

SHAHID KHAN, NAZAR KHAN, SAQIB HUSSAIN, QAZI ZAHOOR AHMAD, MUHAMMAD ASAD ZAIGHUM

Abstract. In this paper, we introduce certain new subclasses of bi-univalent functions in open unit disk associated with the Srivastava-Attiya operator. We obtain coefficient bounds $|a_2|$ and $|a_3|$ for the functions belonging to these new classes.

1. Introduction

Let $\mathcal{A}$ denote the class of functions of the form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \qquad (z \in E) \tag{1.1}$$

which are analytic in the open unit disk $E = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}$. Further, by $\mathcal{S}$, we shall denote the class of all functions in $\mathcal{A}$ which are univalent in E.

A function $f \in \mathcal{A}$ is in the class $\mathcal{S}^*(\beta)$ of starlike functions of order $\beta\,(0 \leqq \beta < 1)$ if the following condition is satisfied:

$$\Re\left(\frac{zf'(z)}{f(z)} - \beta\right) > 0 \qquad (z \in E).$$

Moreover, a function $f \in \mathcal{A}$ is in the class $\mathcal{C}(\beta)$ of convex functions of order $\beta\,(0 \leqq \beta < 1)$ if the following condition is satisfied:

$$\Re\left(1 + \frac{zf''(z)}{f'(z)} - \beta\right) > 0 \qquad (z \in E).$$

For two analytic functions f given by (1.1) and g given by

$$g(z) = z + \sum_{n=2}^{\infty} b_n z^n \qquad (z \in E).$$

2010 *Mathematics Subject Classification.* Primary 05A30, 30C45; Secondary 11B65, 47B38.
Key words and phrases. bi-univalent function, hadamard product, coefficient bounds, multiplier fractional differential operator.

Submitted November 17, 2016. Published May 26, 2017.
Communicated by H. M. Srivastava.

Their convolution (Hadamard product) is defined by

$$(f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n. \tag{1.2}$$

It is well known that every univalent function f has an inverse f^{-1}, defined by

$$f^{-1}(f(z)) = z \qquad (z \in E)$$

and

$$f(f^{-1}(w)) = w \qquad \left(|w| < r_0(f);\ r_0(f) \geqq \frac{1}{4}\right)$$

where

$$f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \ldots.$$

A function $f \in \mathcal{A}$ is said to be bi-univalent in E if both f and f^{-1} are univalent in E. The class of all such functions is denoted by Σ.

The work of Srivastava et al. [10] essentially revived the investigation of various subclasses of the bi-univalent function class Σ in recent years. In a considerably large number of sequels to the aforementioned work of Srivastava et al. [10], several different subclasses of the bi-univalent function class Σ were introduced and studied analogously by many authors (see, for example, [2], [5], [11], [12], [13], [15] and [16]), but only non-sharp estimates on the initial coefficients $|a_2|$ and $|a_3|$ in the Taylor-Maclaurin expansion (1.1) were obtained in these recent papers.

Furthermore, generalized Hurwitz-Lerch Zeta function $\phi(u, b, z)$ is defined by

$$\begin{aligned} \phi(\mu, b, z) &= \sum_{n=0}^{\infty} \frac{z^n}{(n+b)^{\mu}}, \\ &= b^{-\mu} + \frac{z}{(1+b)^{\mu}} + \sum_{n=2}^{\infty} \frac{z^n}{(n+b)^{\mu}}, \end{aligned}$$

where $b \in \mathbb{C}$ with $b \neq 0, -1, -2, \ldots\ldots, \mu \in \mathbb{C}$, $\Re(\mu) > 1$ and $z \in E$.
Using Hurwitz-Lerch zeta functions with the convolution of an analytic functions, Srivastava and Attiya [14] introduced a family of linear operators $J_{\mu,b} : \mathcal{A} \longrightarrow \mathcal{A}$ as:

$$J_{\mu,b} f(z) = G_{\mu,b} * f(z) = z + \sum_{n=2}^{\infty} \left(\frac{1+b}{n+b}\right)^{\mu} a_n z^n, \tag{1.3}$$

where $b \in \mathbb{C}$ with $b \neq 0, -1, -2, \ldots\ldots, \mu \in \mathbb{C}$, $z \in E$ and $G_{\mu,b} \in \mathcal{A}$ given by

$$\begin{aligned} G_{\mu,b} &= (1+b)^{\mu} \left[\phi(\mu, b, z) - b^{-\mu}\right], \\ &= z + \sum_{n=2}^{\infty} \left(\frac{1+b}{n+b}\right)^{\mu} z^n. \end{aligned} \tag{1.4}$$

The following recursive relation can easily be obtained by using (1.3) and (1.4)

$$z\left[J_{\mu,b} f(z)\right]' = (1+b) J_{\mu-1,b} f(z) - b J_{\mu,b} f(z).$$

Remark 1: $J_{0,b}$ and $J_{-\mu,b}$ give the identity and inverse operator of $J_{\mu,b}$ respectively.
Remark 2: Srivastava-Attiya operator defined in (1.3) generalizes many known operators for example:

(i) For $\mu = 1$ and $b = 0$, (1.3) reduces to the well-known operator defined earlier by Alexander [1].

ii) For $\mu = 1$ and $b = 1$, (1.3) reduces to the well-known operator defined by Libera [8].

(iii) For $\mu = 1$ and $b = \gamma > -1$, $\gamma \in \mathbb{N}$, (1.3) reduces to the Bernardi integral operator defined by Bernardi [3].

(iv) For $\mu = \sigma > 0$ and $b = 1$, (1.3) reduces to Jung–Kim–Srivastava integral operator [6].

The object of the present work is to introduce two new subclasses of the function class Σ and find estimates on the coefficients $|a_2|$ and $|a_3|$ for functions in these new subclasses of the function class Σ using the technique of Srivastava et al. [10] (see, also[7]).

Here we recall a lemma which we will use in our main results.

Lemma 1 [9]. If $h \in P$, then $|c_n| \leqq 2$ for each n, where P is the family of all functions h, analytic in E, for which

$$\Re\left(h\left(z\right)\right) > 0, \qquad z \in E,$$

where

$$h(z) = 1 + c_1 z + c_2 z^2 ...., \qquad z \in E.$$

2. Coefficient bounds for the function class $M_\Sigma(\mu, b, \alpha, \lambda)$

Definition 1. A function f defined by (1.1) is said to be in the class $M_\Sigma(\mu, b, \alpha, \lambda)$ if the following condition are satisfied:

$$\left|\arg\left(\frac{z\left[J_{\mu,b}f(z)\right]'}{(1-\lambda)z + \lambda J_{\mu,b}f(z)}\right)\right| < \frac{\alpha\pi}{2}, \;\; 0 < \alpha \leqq 1; 0 \leqq \lambda \leqq 1; z \in E, \tag{2.1}$$

and

$$\left|\arg\left(\frac{w\left[J_{\mu,b}g(w)\right]'}{(1-\lambda)w + \lambda J_{\mu,b}g(w)}\right)\right| < \frac{\alpha\pi}{2}, \;\; 0 < \alpha \leqq 1; 0 \leqq \lambda \leqq 1; w \in E, \tag{2.2}$$

where the function g is given by

$$g(w) = f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + ... \tag{2.3}$$

That is, the extension of f^{-1} to E.

Special Cases:

i) For $\mu = 0$, $\lambda = 0$ and $b = 0$ in (2.1) and (2.2) we have the class $M_\Sigma(0, 0, \alpha, 0) = \mathcal{H}_\Sigma^\alpha$, defined by Srivastava et.al [10].

ii) For $\mu = 0$, $\lambda = 1$ and $b = 0$ in (2.1) and (2.2) we have the class $M_\Sigma(0, 0, \alpha, 1) = \delta_\Sigma^*(\alpha)$ defined by Brannan and Taha [4].

Theorem 1. Let the function f defined by (1.1) be in the class $M_\Sigma(\mu, b, \alpha, \lambda)$ $(0 < \alpha \leqq 1; 0 \leqq \lambda \leqq 1)$. Then

$$|a_2| \leqq \frac{2\alpha}{\sqrt{\left\{\{2\alpha(\lambda^2 - 2\lambda) - (\alpha - 1)(2-\lambda)^2\}\left(\frac{1+b}{2+b}\right)^{2\mu} + 2\alpha(3-\lambda)\left(\frac{1+b}{3+b}\right)^{\mu}\right\}}}, \tag{2.4}$$

$$|a_3| \leqq \frac{4\alpha^2}{(2-\lambda)^2\left(\frac{1+b}{2+b}\right)^{2\mu}} + \frac{2\alpha}{(3-\lambda)\left(\frac{1+b}{3+b}\right)^{\mu}}. \tag{2.5}$$

Proof. From (2.1) and (2.2) we have

$$\frac{z\left[J_{\mu,b}f(z)\right]'}{(1-\lambda)z+\lambda J_{\mu,b}f(z)}=\left[p(z)\right]^{\alpha}, \tag{2.6}$$

$$\frac{w\left[J_{\mu,b}f(w)\right]'}{(1-\lambda)w+\lambda J_{\mu,b}f(w)}=\left[q(w)\right], \tag{2.7}$$

where $p(z)$ and $q(w)$ have the following forms:

$$p(z)=1+p_1z+p_2z^2..., \tag{2.8}$$

and

$$q(w)=1+q_1w+q_2w^2.... \tag{2.9}$$

Now, equating the coefficients in (2.6) and (2.7), we have

$$(2-\lambda)\left(\frac{1+b}{2+b}\right)^{\mu}a_2=\alpha p_1, \tag{2.10}$$

$$(\lambda^2-2\lambda)\left(\frac{1+b}{2+b}\right)^{2\mu}a_2^2+(3-\lambda)\left(\frac{1+b}{3+b}\right)^{\mu}a_3=\frac{1}{2}\left[\alpha(\alpha-1)p_1^2+2\alpha p_2\right], \tag{2.11}$$

$$-(2-\lambda)\left(\frac{1+b}{2+b}\right)^{\mu}a_2=\alpha q_1, \tag{2.12}$$

and

$$(\lambda^2-2\lambda)\left(\frac{1+b}{2+b}\right)^{2\mu}a_2^2+(3-\lambda)\left(\frac{1+b}{3+b}\right)^{\mu}(2a_2^2-a_3)=\frac{1}{2}\left[\alpha(\alpha-1)q_1^2+2\alpha q_2\right]. \tag{2.13}$$

From (2.10) and (2.12), we have

$$2(2-\lambda)^2\left(\frac{1+b}{2+b}\right)^{2\mu}a_2^2=\alpha^2(p_1^2+q_1^2), \tag{2.14}$$

and

$$p_1=-q_1. \tag{2.15}$$

From (2.11), (2.13), (2.14) and (2.15), we have

$$\begin{aligned}&\left\{\left\{2\alpha(\lambda^2-2\lambda)-(\alpha-1)(2-\lambda)^2\right\}\left(\frac{1+b}{2+b}\right)^{2\mu}+2\alpha(3-\lambda)\left(\frac{1+b}{3+b}\right)^{\mu}\right\}a_2^2\\ =\;&\alpha^2(p_2+q_2)\end{aligned} \tag{2.16}$$

Applying Lemma 1 on (2.16), we have

$$|a_2|\leqq\frac{2\alpha}{\sqrt{\left\{\left\{2\alpha(\lambda^2-2\lambda)-(\alpha-1)(2-\lambda)^2\right\}\left(\frac{1+b}{2+b}\right)^{2\mu}+2\alpha(3-\lambda)\left(\frac{1+b}{3+b}\right)^{\mu}\right\}}}.$$

This gives the bound on $|a_2|$ as given in (2.4).
Next, to find the bound on $|a_3|$, by subtracting (2.13) from (2.11), we have

$$2(3-\lambda)\left(\frac{1+b}{3+b}\right)^{\mu}a_3-2(3-\lambda)\left(\frac{1+b}{3+b}\right)^{\mu}a_2^2=\alpha(p_2-q_2)+\frac{\alpha(\alpha-1)}{2}(p_1^2-q_1^2) \tag{2.17}$$

From (2.14), (2.15) and (2.17), we have

$$a_3 = \left[\frac{\alpha^2 p_1^2}{(2-\lambda)^2 \left(\frac{1+b}{2+b}\right)^{2\mu}} + \frac{\alpha(p_2 - q_2)}{2(3-\lambda)\left(\frac{1+b}{3+b}\right)^{\mu}} \right] \tag{2.18}$$

Applying Lemma 1 once again on (2.18) for the coefficients p_2 and q_2, we have

$$|a_3| \leqq \frac{4\alpha^2}{(2-\lambda)^2 \left(\frac{1+b}{2+b}\right)^{2\mu}} + \frac{2\alpha}{(3-\lambda)\left(\frac{1+b}{3+b}\right)^{\mu}}.$$

This completes the proof.
For $\lambda = 0$, $\mu = 0$ and $b = 0$ in Theorem 1 we have the following corollary due to Srivastava et al. [10].

Corollary 1. Let f given by (1.1) be in the class $\mathcal{H}_\Sigma^\alpha$. Then

$$|a_2| \leqq \alpha\sqrt{\frac{2}{\alpha+2}} \qquad \text{and} \qquad |a_3| \leqq \frac{\alpha(3\alpha+2)}{3}.$$

Corollary 2. Let the function f defined by (1.1) be in the class $M_\Sigma(1, 0, \alpha, \lambda)$ for $0 < \alpha \leqq 1; 0 \leqq \lambda \leqq 1$. Then

$$|a_2| \leqq \frac{2\alpha}{\sqrt{\{\lambda^2(\alpha+1) - 4(\alpha+\lambda-1)\}\,\frac{1}{4} + \frac{2}{3}\alpha(3-\lambda)}}, \quad |a_3| \leqq \frac{4\alpha^2}{\frac{1}{4}(2-\lambda)^2} + \frac{2\alpha}{\frac{1}{3}(3-\lambda)}.$$

Corollary 3. Let the function f defined by (1.1) be in the class $M_\Sigma(1, 1, \alpha, \lambda)$ for $0 < \alpha \leqq 1; 0 \leqq \lambda \leqq 1$. Then

$$|a_2| \leqq \frac{2\alpha}{\sqrt{\{\lambda^2(\alpha+1) - 4(\alpha+\lambda-1)\}\,\frac{4}{9} + \alpha(3-\lambda)}}, \quad |a_3| \leqq \frac{4\alpha^2}{\frac{4}{9}(2-\lambda)^2} + \frac{2\alpha}{\frac{1}{2}(3-\lambda)}$$

Corollary 4. Let the function f defined by (1.1) be in the class $M_\Sigma(1, \gamma, \alpha, \lambda)$ for $0 < \alpha \leqq 1; 0 \leqq \lambda \leqq 1$. Then

$$|a_2| \leqq \frac{2\alpha}{\sqrt{\{\lambda^2(\alpha+1) - 4(\alpha+\lambda-1)\}\left(\frac{1+\gamma}{2+\gamma}\right)^2 + 2\alpha(3-\lambda)\left(\frac{1+\gamma}{3+\gamma}\right)}},$$

$$|a_3| \leqq \frac{4\alpha^2}{(2-\lambda)^2\left(\frac{1+\gamma}{2+\gamma}\right)^2} + \frac{2\alpha}{(3-\lambda)\left(\frac{1+\gamma}{3+\gamma}\right)^{\mu}}.$$

3. Coefficient bounds for the function class $M_\Sigma(\mu, b, \beta, \lambda)$

Definition 2. A function f defined by (1.1) is said to be in the class $M_\Sigma(\mu, b, \beta, \lambda)$ if the following condition is satisfied:

$$\Re\left(\frac{z\,[J_{\mu,b}f(z)]'}{(1-\lambda)z + \lambda J_{\mu,b}f(z)}\right) > \beta, \quad 0 \leqq \beta < 1; 0 \leqq \lambda \leqq 1; z \in E, \tag{3.1}$$

and

$$\Re\left(\frac{w\,[J_{\mu,b}g(w)]'}{(1-\lambda)w + \lambda J_{\mu,b}g(w)}\right) > \beta, \quad 0 \leqq \beta < 1; 0 \leqq \lambda \leqq 1; w \in E, \tag{3.2}$$

where the function g is given in (2.3).
Special Cases:

i) For $\mu = b = \lambda = 0$, (3.1) and (3.2) reduced to the class $\mathcal{H}_\Sigma(\beta)$ defined by Srivastava et.al [10].
ii) For $\mu = b = 0$ and $\lambda = 1$, (3.1) reduced to the well-known starlike function of order β, see [9].
Theorem 2. Let $f \in \mathcal{A}$ defined by (1.1) be in the class $M_\Sigma(\mu, b, \beta, \lambda)$ for $0 \leqq \beta < 1; 0 \leqq \lambda \leqq 1$. Then

$$|a_2| \leqq \frac{\sqrt{2(1-\beta)}}{\sqrt{\left\{(\lambda^2 - 2\lambda)\left(\frac{1+b}{2+b}\right)^{2\mu} + (3-\lambda)\left(\frac{1+b}{3+b}\right)^{\mu}\right\}}}, \tag{3.3}$$

$$|a_3| \leqq (1-\beta)\left\{\frac{4(1-\beta)}{(2-\lambda)^2\left(\frac{1+b}{2+b}\right)^{2\mu}} + \frac{2}{(3-\lambda)\left(\frac{1+b}{3+b}\right)^{\mu}}\right\}. \tag{3.4}$$

Proof. From (3.1) and (3.2), we have

$$\frac{z\left[J_{\mu,b}f(z)\right]'}{(1-\lambda)z + \lambda J_{\mu,b}f(z)} = \beta + (1-\beta)p(z), \tag{3.5}$$

$$\frac{w\left[J_{\mu,b}f(w)\right]'}{(1-\lambda)w + \lambda J_{\mu,b}f(w)} = \beta + (1-\beta)q(w), \tag{3.6}$$

where $p(z)$ and $q(w)$ are given in (2.8) and (2.9) respectively. Equating the coefficients in (3.5) and (3.6), we obtain

$$(2-\lambda)\left(\frac{1+b}{2+b}\right)^{\mu} a_2 = (1-\beta)p_1, \tag{3.7}$$

$$(\lambda^2 - 2\lambda)\left(\frac{1+b}{2+b}\right)^{2\mu} a_2^2 + (3-\lambda)\left(\frac{1+b}{3+b}\right)^{\mu} a_3 = (1-\beta)p_2, \tag{3.8}$$

$$-(2-\lambda)\left(\frac{1+b}{2+b}\right)^{\mu} a_2 = (1-\beta)q_1, \tag{3.9}$$

and

$$(\lambda^2 - 2\lambda)\left(\frac{1+b}{2+b}\right)^{2\mu} a_2^2 + (3-\lambda)\left(\frac{1+b}{3+b}\right)^{\mu}(2a_2^2 - a_3) = (1-\beta)q_2. \tag{3.10}$$

From (3.7) and (3.9), we have

$$2(2-\lambda)^2\left(\frac{1+b}{2+b}\right)^{2\mu} a_2^2 = (1-\beta)^2\left(p_2^2 + q_2^2\right), \tag{3.11}$$

and

$$p_1 = -q_1. \tag{3.12}$$

Adding (3.8) and (3.10), we have

$$\left\{2(\lambda^2 - 2\lambda)\left(\frac{1+b}{2+b}\right)^{2\mu} + 2(3-\lambda)\left(\frac{1+b}{3+b}\right)^{\mu}\right\} a_2^2 = (1-\beta)(p_2 + q_2). \tag{3.13}$$

Applying Lemma 1 on (3.13), we have

$$|a_2| \leqq \frac{\sqrt{2(1-\beta)}}{\sqrt{\left\{(\lambda^2-2\lambda)\left(\frac{1+b}{2+b}\right)^{2\mu}+(3-\lambda)\left(\frac{1+b}{3+b}\right)^{\mu}\right\}}}.$$

This gives the bound on $|a_2|$ as given in (3.3).
Next, in order to find the bound on $|a_3|$, by subtracting (3.10) from (3.8), we have

$$2(3-\lambda)\left(\frac{1+b}{3+b}\right)^{\mu}a_3-2(3-\lambda)\left(\frac{1+b}{3+b}\right)^{\mu}a_2^2=(1-\beta)(p_2-q_2) \qquad (3.14)$$

Substitution the value of a_2^2 from (3.11) in (3.14), we have

$$a_3=\left(\frac{(1-\beta)^2\left(p_2^2+q_2^2\right)}{2(2-\lambda)^2\left(\frac{1+b}{2+b}\right)^{2\mu}}\right)+\frac{(1-\beta)(p_2-q_2)}{2(3-\lambda)\left(\frac{1+b}{3+b}\right)^{\mu}} \qquad (3.15)$$

Applying Lemma 1 on (3.15) for the coefficient p_2 and q_2, we have

$$|a_3| \leqq (1-\beta)\left\{\frac{4(1-\beta)}{(2-\lambda)^2\left(\frac{1+b}{2+b}\right)^{2\mu}}+\frac{2}{(3-\lambda)\left(\frac{1+b}{3+b}\right)^{\mu}}\right\}.$$

This completes the proof.
Corollary 5 [10]. Let $f(z)$ be given by (1.1) be in the function class $\mathcal{H}_\Sigma(\beta)$ $(0 \leqq \beta < 1)$. Then

$$|a_2| \leqq \sqrt{\frac{2(1-\beta)}{3}} \quad \text{and} \quad |a_3| \leqq \frac{(1-\beta)(5-3\beta)}{3}.$$

Corollary 6. Let the function $f(z)$ defined by (1.1) be in the class $M_\Sigma(0,0,\beta,1)$ $(0 \leqq \beta < 1)$. Then

$$|a_2| \leqq \sqrt{2(1-\beta)} \quad \text{and} \quad |a_3| \leqq (1-\beta)(5-4\beta).$$

Corollary 7. Let the function $f(z)$ defined by (1.1) be in the class $M_\Sigma(1,1,\beta,\lambda)$ $(0 \leqq \beta < 1; 0 \leqq \lambda \leqq 1)$. Then

$$|a_2| \leqq \frac{\sqrt{2(1-\beta)}}{\sqrt{\frac{4}{9}\left\{(\lambda^2-2\lambda)+\frac{1}{2}(3-\lambda)\right\}}}.$$

$$|a_3| \leqq (1-\beta)\left\{\frac{4(1-\beta)}{\frac{4}{9}(2-\lambda)^2}+\frac{2}{\frac{1}{2}(3-\lambda)}\right\}.$$

Acknowledgments

The authors are thankful to referees for their valuable suggestions. This research is carried out under the HEC projects grant No. 21-1235/SRGP/R&D/HEC/2016 and 21-1193/SRGP/R&D/HEC/2016.

References

[1] J. W. Alexander, Functions which map the interior of the unit circle upon simple regions, Annals of Mathematics (Series 2), 17 (1915) 12–22.
[2] S. Altınkaya, S. Yalçın, Faber polynomial coefficient bounds for a subclass of bi-univalent functions, C. R. Acad. Sci. Paris, Ser.I, 353 (2015) 1075–1080.
[3] D. A. Brannan, J. Clunie, W. E. Kirwan, Coefficient estimates for a class of starlike functions, Canad. J. Math. 22 (1970) 476–485.
[4] D. A. Brannan, T. S. Taha, On some classes of bi-univalent functions, in: S.M. Mazhar, A. Hamoui, N.S. Faour (Eds.), Math. Anal. and Appl., Kuwait; February 18–21, 1985, in: KFAS Proceedings Series, vol. 3, Pergamon Press, Elsevier Science Limited, Oxford, 1988, pp. 53–60. see also Studia Univ. Babes-Bolyai Math. 31 (2) (1986) 70–77.
[5] S. Bulut, Coefficient estimates for a class of analytic and bi-univalent functions, Novi Sad J. Math. 43.2 (2013) 59–65.
[6] I. B. Jung, Y. C. Kim, H. M. Srivastava, The Hardy space of analytic functions associated with certain one-parameter families of integral operators, J. Math. Anal. Appl, 176 (1993) 138–147.
[7] X-F. Li, A. P. Wang, Two new subclasses of bi-univalent functions, Int. Math. Forum. 7 (2012) 1495–1504.
[8] R. J. Libera, Some classes of regular univalent functions, Proc. Amer. Math. Soc, 135 (1969) 429–449.
[9] Ch. Pommerenke, Univalent Functions, Vanderhoeck and Ruprecht, Gottengen.1975.
[10] H. M. Srivastava, A. K. Mishra, P. Gochhayat, Certain subclasses of analytic and bi-univalent functions, Appl. Math. Lett. 23 (2010) 1188–1192.
[11] H. M. Srivastava, D. Bansal, Coefficient estimates for a subclass of analytic and bi-univalent functions, J. Egyptian Math. Soc. 23 (2015) 242–246.
[12] H. M. Srivastava, S. Bulut, M. Caglar, N. Yagmur, Coefficient estimates for a general subclass of analytic and bi-univalent functions, Filomat 27 (2013) 831–842.
[13] H. M. Srivastava, S. S umer Eker, R. M. Ali, Coefficient Bounds for a certain class of analytic and bi-univalent functions, Filomat 29 (2015) 1839–1845.
[14] H. M. Srivastava, A. A. Attiya, An integral operator associated with the Hurwitz-Lerch zeta function and differential subordination, Integral Transforms Spec. Funct. 18 (2007) 207–216.
[15] Q.-H. Xu, Y.-C. Gui, H. M. Srivastava, Coefficient estimates for a certain subclass of analytic and bi-univalent functions, Appl. Math. Lett. 25 (2012) 990–994.
[16] Q.-H. Xu, H.-G. Xiao, H. M. Srivastava, A certain general subclass of analytic and bi-univalent functions and associated coefficient estimate problems, Appl. Math. Comput. 218 (2012) 11461–11465.

Shahid Khan
Department of Mathematics Riphah International University Islamabad, Pakistan.
E-mail address: shahidmath761@gmail.com

Nazar Khan
Department of Mathematics Abbottabad University of Science and Technology, Abbottabad, Pakistan.
E-mail address: nazarmaths@gmail.com

Saqib Hussain
Department of Mathematics COMSATS Institute of Information Technology Abbottabad, Pakistan.
E-mail address: saqib_math@yahoo.com

Qazi Zahoor Ahmad
Department of Mathematics Abbottabad University of Science and Technology, Abbottabad, Pakistan.
E-mail address: zahoorqazi5@gmail.com

Muhammad Asad Zaighum
Department of Mathematics Riphah International University Islamabad, Pakistan.
E-mail address: asadzaighum@gmail.com