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On viruses in graphs and digraphs

Virus en grafos y digrafos

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Abstract

A virus is a local configuration that, if present in a graph or a digraph, forbids these graphs or digraphs to have a specific property. The aim of this article is to sketch the evolution of the virus theory from its birth in 1991. Moreover some new results and open questions are given. The properties with its known viruses, that will be discussed in this work, are the following: hamiltonian, traceable, k-connected, k-edge-connected, strongly connected and have a perfect matching.
Key words and phrases: virus, graph, digraph, graph property, digraph property.

Resumen

Un virus es una configuración local que estando presente en un grafo o en un digrafo, impide que este tenga una propiedad específica. El objetivo del presente artículo es presentar la evolución que ha tenido la teoría de virus desde su nacimiento en 1991. Además damos resultados y problemas abiertos. Las propiedades con virus conocidos que

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discutiremos en este trabajo son: hamiltoniano, traceable, k-conectado, k-lado-conectado, fuertemente conexo y contiene un matching perfecto.
Palabras y frases clave: virus, grafo, digrafo, propiedades de grafos, propiedades de digrafos.

1 Introduction

The virus theory was born in 1991 from the continuing concern of the graph theoretical group of Centro ISYS of the Universidad Central de Venezuela with the automatic generation of "pseudo-random" graph with certain properties, for use in interactive graphical systems called AMDI [9] and GREAT [8]. Of course, it makes about equal sense to force or to forbid any given graph or digraph property. When it comes to "almost sure" properties, i.e. properties that a random graph possesses with probability tending to 1 as the number of vertices tends to infinity, there is no problem in getting the graph with the desired property. For the hard side of the problem in getting a graph without the property, the notion of "virus" is introduced as a local configuration in the graph, whose characterization is the main goal in virus theory, which forces the graph not to possess the forbidden property. It is the right way to proceed, in the sense that most graphs without the considered property, contain indeed the corresponding local configuration. This implies that the imposition of the configuration provides graphs which are nearly random within the set of all graphs without the property.

Notice that the problem of identifying viruses is much more difficult when the co-problem does not have a certification identifying it. For example, the bipartite co-problem is the presence of odd length circuits.

Let P be a property defined on all graphs or digraphs and let $\mathbb{N}$ be the set of nonnegative integers. In [2] a graph virus is a structure (H, T, f) composed of a graph H and a function f defined on a subset T of its vertex set, with values in $\mathcal{N}$. The graph virus (H, T, f) is present in a graph G if it contains a proper induced subgraph H^1, isomorphic to H, such that, for all $x \in T$, $d_G(x) - d_{H^1}(x) = f(x)$, where $d_G(x)$ and $d_{H^1}(x)$ denote the degree of vertex x in the graphs G and H^1, respectively (H^1 and H are assimilated). For digraphs a similar definitions was given in [2].

In order to clarify the applicability of the virus notion, consider first that we wish to randomly generate a graph G of order n, that does not possess a given property P. If we have a family of viruses characterizing the absence of P, then we can randomly select a member (H, T, f) from the family of viruses for P compatible with the order of G (thus $|V(H)| + max_{x \in V(G)} f(x) \leq n$)

then a random graph on $n - |V(H)|$ vertices, and then add randomly edges between $V(H)$ and the remaining vertices subset, in a way compatible with f, thus obtaining the desired graph.

On the other hand assume that we wish to decide if a given graph G possesses property P. For certain important properties (as for instance, "G is hamiltonian" or "G is traceable") such decision problems can be very hard to solve, the exact solution sometimes requiring a computational effort more than polynomial on the order of the input graph. If a virus V for P is known, and if the presence or absence of V can be inexpensively determined (in terms of computational time), then an approximate solution to the problem can be given simply by checking the presence/absence of V in G. If V is present in G, then we know that G does not have P. On the contrary, if V is not present in G, we can say that G has P with a relatively high probability. The probability will depend on how close the presence of V comes to characterize the absence of P, with respect to the uniform distribution on family of graphs of a given order.

Another fact in favor of the importance of the virus theory, given in [11], is the following: it is well known that the problem to decide when a digraph is hamiltonian is NP-complete [13]. A "yes" answer to the hamiltonicity problem for a given digraph can be verified by checking in polynomial time that a sequence of vertices given by an oracle is a hamiltonian circuit. In case of non-hamiltonian digraphs, as stated in [7] pages 28, 29, there is no known way of verifying a "yes" answer to the complementary problem of deciding if a digraph is non-hamiltonian. A solution to this problem is to provide a hamiltonian virus, whose presence in the digraph can also be checked in polynomial time. In case of the non-hamiltonian virus-free digraphs D, they must hold the following particular structure given in [11]: for each vertex x the remaining subdigraph $D - x$ has a covering by vertex disjoint paths $P_1, \cdots, P_r$ such that each one of them makes a circuit with x.

In [2] the following "metaconjecture" is presented:

For many important properties there exist viruses of small order in "almost all" instances in which the property is not present.

The term "almost all" should be understood here in its usual probabilistic sense meaning a proportion tending to 1 as the number of vertices tends to infinity.

The study of this metaconjecture is useful because the presence (absence) of a virus of order k inside a graph of order n can be detected by a procedure with an execution time bounded by $O(n^{k+1})$. If k is a small number (say, less than or equal to 3) then, an almost surely correct answer to the question "does

G have property P?" can be given with a procedure of low computational cost. There are two examples in favor of this metaconjecture. It is proved for non strongly connected graphs in [2] and graphs without perfect matching in [3] that "almost all" of them contain a virus of smallest order of the class.

Some properties are characterized by viruses; more precisely, a property P is characterized by a class of viruses $\mathcal{V}$ if the following assertions are equivalent:

(i) G does not have the property P

(ii) some virus $V \in \mathcal{V}$ is present in G

In general, the class of all viruses for property P gives only $(ii) \to (i)$. In [2], it is proven that the property "G is strongly connected" are characterized by its property viruses.

On the other hand, in [10] it is shown that the viruses for the property "G is bipartite" do not characterize this property. Recall that a graph G is bipartite if and only if does not contain cycles of order odd. In a similar way in [10] it is shown that the viruses for the property "G has a perfect matching" do not characterize this property; moreover, it gives a characteristic property of the graphs without perfect matching and no viruses for the property "G has a perfect matching". However in [2] the virus for the property "G has a perfect matching" are characterized.

In this paper, we show that the hamiltonian property is not characterized by its viruses, at least in the direct case. We give an example of a non-hamiltonian digraph without viruses and we characterize such graphs and digraphs. But we have found no non-hamiltonian graph without viruses. Nevertheless in [11] we show that a balanced bipartite digraphs are hamiltonian if and only if they are hamiltonian virus-free. Moreover, we characterized the viruses for "G is traceable". We did not prove that the viruses for "G is traceable" characterize the property. So it is an open problem. At the present, we did not find non-traceable graphs or digraphs having no viruses. We give some structure properties are given for such graphs or digraphs, sufficient to show that no very small such graph can exist.

The situation is quite different for the properties "G is k-connected" and "G is k-edge-connected" since these properties are characterized by their viruses, and even by a reduced class of viruses, but the viruses themselves are not characterized and it constitutes also an open problem.

1.1 Terminology

Our terminology is generally standard [1]. We give several definitions, introduced in [2, 4], relevant to follow the discussion.

Definition 1. *A graph virus is a triple (H, T, f) where H is a graph, T a subset of the vertex set of H, and f is a mapping from T to $\mathbb{N}$.*

A graph virus (H, T, f) is present in a graph G if it has a proper induced subgraph H^1 isomorphic to H (for convenience we identify H^1 with H), such that on each vertex $x \in T$ the equality $d_G(x) = f(x) + d_H(x)$ holds.

A graph virus V is a virus for a graph property P if every graph where V is present lacks the property P.

Now we give the similar definitions for digraph viruses.

Definition 2. *A digraph virus is a 5-tuple (H, T^+, T^-, f^+, f^-) where H is a digraph, T^+, T^- are subsets of the vertex set of H, and f^+, f^- are mappings from T^+, T^- respectively to $\mathbb{N}$.*

A digraph virus (H, T^+, T^-, f^+, f^-) is present in a digraph G if it has a proper induced subdigraph H^1 isomorphic to H (for convenience we identify H^1 with H), such that the equalities $d_G^+(x) = f^+(x) + d_H^+(x)$ on each vertex x of T^+ and $d_G^-(x) = f^-(x) + d_H^-(x)$ on each vertex x of T^- hold.

A digraph virus V is a virus for a digraph property P if every digraph where V is present lacks the property P.

Lemma 1. *If a graph (or digraph) of order n has no virus of cardinality $h < n$ for some property P, then it has no virus of cardinality less than h for P.*

The virus theory and some of its applications, has been present in some international congress, for example in [5, 6, 8, 12].

This paper besides of the introduction and conclusions, contains three main sections dedicated to the traceable property, hamiltonian property, k-connected property and k-edge-connected property.

2 Virus for "traceable" property

Theorem 1. *Let (H, T, f) be a graph virus. It is a virus for the property "G is traceable" if and only if every set of disjoint paths $P_1, P_2, \cdots, P_r$ covering $V(H)$, one at least of the two following conditions occurs:*

1. *There exist at least three paths P_j such that:*
   - *if P_j consists of just one vertex $\{x_j^1\}$ then $f(x_j^1) \leq 1$.*
   - *if $P_j = (x_j^1, \cdots, x_j^{q(j)})$ then $f(x_j^1) = 0$ or $f(x_j^{q(j)}) = 0$.*

2. *There exists a path* $P_j = (x_j^1, \cdots, x_j^{q(j)})$ *such that* $f(x_j^1) = 0$ *and* $f(x_j^{q(j)}) = 0$.

Proof. **Necessity**: Assume (H, T, f) is given and suppose there exists r disjoint paths P_j, $1 \leq j \leq r$ covering $V(H)$, ordered in such a way that:

i. for $1 < j < r$

- if $P_j = (x_j^1)$ then $f(x_j^1) \geq 2$ or $x^1 \notin T$
- if $q(j) > 1$ and $P_j = (x_j^1, \cdots, x_j^{q(j)})$ then $f(x_j^1) \geq 1$ or $x_j^1 \notin T$ and $f(x_j^{q(j)}) \geq 1$ or $x_j^{q(j)} \notin T$.

ii. for $j = 1$

- if $P_1 = (x_1^1)$ then $f(x_1^1) \geq 1$ or $x_1^1 \notin T$
- if $q(1) > 1$ and $P_1 = (x_1^1, \cdots, x_1^{q(1)})$ then $f(x_1^{q(1)}) \geq 1$ or $x^{q(1)} \notin T$.

iii. for $j = r$

- if $P_r = (x_r^1)$ then $f(x_r^1) \geq 1$ or $x_r^1 \notin T$.
- if $q(r) > 1$ and $P_r = (x_r^1, \cdots, x_r^{q(r)})$ then $f(x_r^1) \geq 1$ or $x_r^1 \notin T$.

We will build a traceable graph G containing a hamiltonian path P where the paths P_j are subpaths of P and (H, T, f) is present.

If $r > 2$, we add $r - 1$ edges $y_i z_i$, $1 \leq i \leq r - 1$, and edges connecting y_i to $x_i^{q(i)}$ and z_i to x_{i+1}^1 for $2 \leq i \leq r - 1$.

If $r = 1$, we add one vertex connected by an edge to x_1^1 or $x_1^{q(1)}$.

Obviously, G has a hamiltonian path, but (H, T, f) may be not present in G. Therefore, we replace the edges $y_i z_i$ by complete graphs $K_{n(i)}$ with $n(i) \geq 2$ large enough to connect $x_i^{q(i)}$ to $f(x_i^{q(i)})$ vertices of $K_{n(i)}$ and x_{i+1}^1 to $f(x_{i+1}^1)$ other vertices of $K_{n(i)}$ if these values are defined. We may also have to add complete graphs connected to x_1^1 and $x_r^{q(r)}$.

Sufficiency: Let G be a traceable graph where (H, T, f) is present. Let P be a hamiltonian path of G. Let $P_1, P_2, \cdots, P_r$ be the successive paths induced by H on P. Clearly these paths constitute a covering of $V(H)$. For all $2 \leq i \leq r - 1$ we have:

i. if $P_i = (x_i^1)$ then $d_G(x_i^1) \geq d_H(x_i^1) + 2$ hence $f(x_i^1) \geq 2$.

ii. if $P_i = (x_i^1, \cdots, x_i^{q(i)})$ then $d_G(x_i^1) \geq d_H(x_i^1) + 1$ and $d_G(x_i^{q(i)}) \geq d_H(x_i^{q(i)}) + 1$. Hence $f(x_i^1) \neq 0$ and $f(x_i^{q(i)}) \neq 0$.

For P_i with $i = 1, r$ we have $f(x_i^1) \neq 0$ or $f(x_i^{q(i)}) \neq 0$. Thus (H, T, f) does not satisfy the conditions of the theorem.

□

Corollary 1. *A graph G without virus for the traceable property has for each vertex x the following condition: G is covered by paths ending at x and cycles going through x, all disjoints in $G \setminus x$, and at most two paths (perhaps none) appear.*

Corollary 2. *A non traceable graph G with minimum degree less than or equal to 3 contains a virus for the traceable property.*

Proof. Let us reason ab absurdo. Let G be a non-traceable graph, containing no virus for the traceable property and let x a vertex of G of degree ≤ 3. Then let $V = (H, T, f)$ with H the graph induced on $V(G) \setminus x$, $T = V(H)$ and $f = 1$ on the neighbors of x in G and $f = 0$ on the other vertices of $G \setminus x$. The structure V is present in G and is not a virus for traceable property.

Since V is not a virus for traceable property, it has a partition into paths such that:

- at most two paths contains a extremity y with $f(y) = 0$ and

- no path has both ends with $f = 0$.

Since G is not traceable, at least 3 paths appear in that partition. This implies that at least 4 vertices have $f = 1$, thus the degree of x in G is at least 4. This contradicts the degree of x.

□

Corollary 3. *A non-traceable graph with order at most 9 contains a virus for traceable property.*

Proof. Let us reason ab absurdo. Let G be a non-traceable graph, containing no virus for the traceable property. By Corollary 2 the minimum degree of G is at least 4. Following, [1] Corollary 2.3 page 23, we know that every graph of order n and minimum degree at least $\frac{n-1}{2}$ is traceable; thus G is traceable. A contradiction.

□

The following theorem provides a characterization of a virus for the digraph property "D is traceable".

The directed paths P_i used below will be lists of vertices (x_i^j), $1 \leq j \leq q(i)$, with $q(i) \geq 1$, such that all 2-tuples (x_i^j, x_i^{j+1}) are arcs.

Theorem 2. *Let $(H, T^+, T^-, f^+.f^-)$ be a digraph virus. It is a virus for the digraph property "D is traceable" if and only if for every set of disjoint directed paths $P_1, \cdots, P_r$ covering $V(H)$, one at least of the following conditions occurs:*

1. *there exists at least two paths P_i and P_j such that:*

   - $f^-(x_j^1) = 0$ *and* $f^-(x_i^1) = 0$ *or*
   - $f^+(x_j^{q(j)}) = 0$ *and* $f^+(x_i^{q(i)}) = 0$

2. *there exists at least a path P_i such that $f^+(x_i^{q(i)}) = 0$ and $f^-(x_i^1) = 0$.*

Proof. The spirit of the proof is similar to the one of Theorem 1. □

Corollary 4. *A digraph D without virus for the traceable property has for each vertex x the following condition: D is covered by cycles and perhaps one path going through x, all disjoint in $D \setminus x$.*

3 Virus for "hamiltonian" property

Theorem 3 ([2])**.** *A graph virus (H, T, f) is a virus for the property "G is hamiltonian" if and only if for every set of disjoint paths $P_1, \cdots, P_r$ covering $V(H)$ there exists a path P_j such that: if P_j consists of just one vertex $\{x_j^1\}$ then $f(x_j^1) \leq 1$ and if $V(P_j) = \{x_j^1, \cdots, x_j^{q(j)}\}$ then $f(x_j^1) = 0$ or $f(x_j^{q(j)}) = 0$.*

Some viruses are made of a stable S on s vertices, and a complete graph of less that $s+1$ vertices all connected to all vertices in S, moreover $T = S$, and $f = 0$ on T.

The Petersen virus is build from a circuit on 6 vertices, and 3 other vertices, each connected by two edges to two opposite vertices of the circuit. T contains the 9 vertices and $f = 0$ on the vertices of the cycle, and $f = 1$ on the three other vertices. It is induced by the removal of one vertex of Petersen graph, and prevents Petersen graph to be hamiltonian.

Theorem 4. *The digraph virus (H, T^+, T^-, f^+, f^-) is a virus for the property "D is hamiltonian" if and only if for every set of disjoint paths $P_1, \cdots, P_r$ covering $V(H)$ there exists a path $P_j = (x_j^1, \cdots, x_j^{q(j)})$, with $q(j) \geq 1$ such that either $f^-(x_j^1) = 0$ or $f^+(x_j^{q(j)}) = 0$.*

This theorem is an extension of the one in [2] about viruses with $T^+ = T^-$.

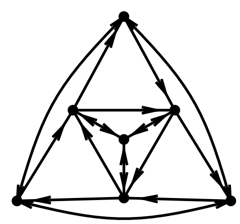

Figure 1: A non-hamiltonian digraph W without viruses

The digraph W in Figure 1 shows that there exist non hamiltonian digraphs without any virus. Owing to the symmetries of W, the removal of any vertex of W leaves a subdigraph of some hamiltonian digraph and by Lemma 1 W does not have viruses of cardinality less that 6.

Corollary 5 ([11]). *A graph or digraph G without virus for the hamiltonian property has the following structure: for each vertex x the remaining $G \setminus x$ has a covering by vertex disjoint paths $P_1, \cdots, P_r$ such that each of them makes a circuit with x.*

Proof. Directly from Lemma 1 and Theorem 3 and Theorem 4.

□

Corollary 6. *A non-hamiltonian graph with minimum degree less that or equal to 3 contains always a virus for the hamiltonian property.*

Proof. Let us reason ab absurdo. Let G be a non-hamiltonian graph, containing no virus for the hamiltonian property and let x a vertex of G of degree ≤ 3. Then let $V = (H, T, f)$ with H the graph induced on $V(G) \setminus x$, $T = V(H)$ and $f = 1$ on the neighbors of x in G and $f = 0$ on the other vertices of $G \setminus x$. The structure V is present in G and is not a virus for hamiltonian property. Since V is not a virus for hamiltonian property, it has a partition into paths such that each extremity y of these paths has $f(y) > 0$ and vertices z having $f \geq 2$. Such vertices z cannot occur, because $f \leq 1$ and since G is not hamiltonian, the number of the paths is at least 2. This implies that at least 4 vertices have $f = 1$, thus the degree of x in G is at least 4. This contradicts the degree of x. □

The Petersen virus is obtained by the procedure described above from the notoriously non-hamiltonian and cubic Petersen graph.

Corollary 7. *A graph with al least* 3 *vertices, that contains no virus for the hamiltonian property and has a vertex* x *with degree at most* 5 *is traceable.*

Proof. Notice that the graph cannot have isolated vertices (they are viruses for the hamiltonian property). Like in the former proof, we consider the virus V induced on $G \setminus x$; since it is not a virus for the hamiltonian property, it has a partition into paths whose extremities are adjacent to x. At most two of these paths can exist because of the degree of x; we have then a hamiltonian path in G by concatenation of the first path, the vertex x and the second path. □

Corollary 8. *A non-hamiltonian graph with order less than or equal to* 8 *contains a virus for the hamiltonian property.*

Proof. Let us reason ab absurdo. Let G be a non-hamiltonian graph, containing no virus for the hamiltonian property. By Corollary 6 the minimum degree is at least 4. According to the theorem of Dirac [1] Theorem 2.1, page 20, every graph of order n and minimum degree at least $\frac{n}{2}$ is hamiltonian; hence G is hamiltonian. A contradiction.

□

Some viruses for the property "D is hamiltonian" are made of a stable S with s vertices and a complete graph K with less than s vertices all arcs between S and K in both directions. T^+ is S and $f^+ = 0$ on T^+ and T^- is empty.

4 Viruses for "k-connected" and "k-edge-connected" properties

Some related results were already known:

Theorem 5 ([10])**.** *The graph virus* (H, T, f) *is a virus for the property "G is connected" if and only if for every vertex* $x \in T = V(H)$*, the equality* $f(x) = 0$ *holds. Moreover, the viruses for the property "G is connected" characterize this property.*

Theorem 6 ([2])**.** *The digraph virus* (H, T^+, T^-, f^+, f^-) *is a virus for the property "D is strongly connected" if and only if*

- *either* $T^+ = V(H)$ *and* $f^+(x) = 0$ *on* T^+
- *or* $T^- = V(H)$ *and* $f^-(x) = 0$ *on* T^-.

Moreover, the viruses for the property "D is strongly connected" characterize this property.

Some viruses for property "G is connected" are the triples (H, T, f) with H a complete graph and $f = 0$ on $T = V(H)$. Some viruses for property "D is strongly connected" are the 5-tuples (H, T^+, T^-, f^+, f^-) with H a complete graph and either $T^+ = V(H)$ and $f^+(x) = 0$ on T^+ or $T^- = V(H)$ and $f^-(x) = 0$ on T^-.

Now we give our characterization for k-connectivity, that generalizes the results above.

Theorem 7. *Some viruses for the property "k-connected" are the* (H, T, f) *such that there are at most* $k-1$ *vertices of* H *that either do not belong to* T *or have* $f > 0$*, and there is at least one vertex in* T *with* $f = 0$.

This class of viruses is sufficient to characterize the property "k-connected" but for the complete graphs.

Proof. If such a virus is present in a graph G, the set of the vertices of H that either do not belong to T or have $f > 0$ constitute a cutset of G (removing these vertices leaves a graph that is not connected).

On the other hand, if G has a cutset C with at most $k-1$ vertices, then the virus induced by C and one of the connected components of $G \setminus C$ fulfils our description and is present in G.

□

We may note that K_n with $n < k$ is not k-connected and contains no virus for the property "k-connected". We note also that there exist viruses that are not in the class described above; for example (H, T, f) where H and T have one vertex with $f = 1$ is a virus for "2-connected". Some for the property "G is k-connected" are made of a complete graph K with at least k vertices, with a nonempty part T where $f = 0$ and $K \setminus T$ has at most $k-1$ vertices.

Theorem 8. *Some viruses for the property k-edge-connected are the* (H, T, f) *such that* $T = V(H)$ *and* $\sum_T f \leq k-1$.

Proof. If such a virus is present in a graph G, the edges between H^1 and the remaining part of G constitute an edge-cutset of G with at most $k-1$ edges. Hence G is not k-edge-connected.

On the other hand, if G is not k-edge-connected, it has an edge-cutset with at most $k-1$ edges. One connected component of the graph obtained by deleting this cutset induces a virus complying with our description.

□

These viruses characterize the property, but there are other viruses for it: for example, if H is a path of length 2, with $T = V(H)$ and $f = 0$ on the middle vertex, then (H, T, f) is a virus for "3-edge-connected".

5 Conclusions

As it was mentioned in the Introduction, the origin of virus theory concerns the requirements for computational environments [8], [9] for assistance to researchers in graph theory, developed at the ISYS research center of the Universidad Central de Venezuela. An important functionality of these systems is a tool for automatic pseudo-random generation of digraphs, imposing given conditions on graph properties, some of which forbid a certain property.

Moreover, this theory can be also useful in the construction of approximate algorithms for NP-complete or polynomial problems, requiring a considerable computational effort.

Natural consequences of this research are the following problems:

1. For given property P, decide whether or not a triple (H, T, f) or a 5-tuple (H, T^+, T^-, f^+, f^-) is a virus for P.

   Evaluate the complexity of the former problem.

2. Find viruses for properties in graphs that do not satisfy that property. The Theorem 1, 2, 3 and 4 are related to this problem, in case of "hamiltonian" and "traceable" properties.

3. What properties are characterized by their viruses? For such properties, the detection of viruses in a graph that does no have property P and their destruction by minimal changes (addition or removal or arcs, for example) could lead to a moderate change to the graph that gives it the property.

4. Evaluate how close are the property P and the property "G has no virus of order $< n$ for P". This evaluation is useful to design approximate algorithms, with their asymptotical error probability.

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