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The control agglomerations of an internet network with delay feedback[1]

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Abstract

In this paper we obtain the results in the control agglomerations of an internet network with delay feedback, via Browder theorem.

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1 Introduction

Be (X, d) a complete metric space, $f : X \to X$ and $\varphi : \mathbb{R}_+ \to \mathbb{R}_+$ thus

$$d(f(x), f(y)) \leq \varphi(d(x, y)), \text{ for all } x, y \in X$$

We construct the sequence of successive approximations

$$x_{n+1} = f(x_n),\ n \in \mathbb{N}$$

corresponding to a point $x_0 \in X$ and we discuss the conditions in which upon φ in this sequence is convergent and has as a limit a fixed point of f. We have:

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$$d(x_{n+p}, x_n) \leq \sum_{k=n}^{n+p-1} \varphi^k(d(x_0, f(x_0))).$$

So the convergency of the series, $\sum_{k=1}^{\infty} \varphi^k(r)$, provides convergency of the sequence of successive approximations. In this way we have the following theorems:

Theorem 1.1. *(Boyd- Wong)[3] Be (X, d) a complete metric space and $f : X \to X$ a application with the property that exists a function $\varphi : \mathbb{R}_+ \to \mathbb{R}_+$ upper semi continuous thus $\varphi(r) < r$ for $r > 0$ and*

$$d(f(x), f(y)) \leq \varphi(d(x, y)), \quad \textit{for all} \quad x, y \in X.$$

In this conditions application f has a unique fixed point, point that can be obtained through the method of successive approximations, starting with any element from X.

Theorem 1.2. *[3] (Browder) Be (X, d) a complete metric space, limited and $f : X \to X$ a application with the property that it exists $\varphi : \mathbb{R}_+ \to \mathbb{R}_+$ continue at right, non-decreasing and $\varphi(r) < r$, for $r > 0$ thus*

$$d(f(x), f(y)) \leq \varphi(d(x, y)), \quad \textit{for all } x, y \in X$$

In these conditions application f has a unique fixed point, point that can be obtained through the method of successive approximations, starting with any element from X.

2 Main result

The mechanism of the control agglomerations of an internet network were studied in [1],[2]. In this paragraph we consider a model for an internet network formed by $n(n \geq 3)$ sequences and a source, which formulates itself like a agglomeration system with delay feedback.

The model is describe trough the retarded differential equations system:

$$(1) \qquad {x_i}'(t) = k[w - af(x_i(t-\tau) - b\sum_{j=1}^{n} f(x_j(t-\tau)))],\ t \in [0,T],$$

where k is a real, positive parameter, $x_i(t)$ is the dispatch rate from the source at the t moment to the user i, τ the sum of delay sendings and returnings , ω is the source, the function characterizes the agglomeration and it is a growing and unindent null function. Function f characterizes the feedback agglomeration of the send signals. System (1) corresponds with the algorithm of the sending rates, which are used for the adjustment of these. The initial conditions of the retarded differential equations system (1) are:

$$(2) \qquad x(\theta) = \psi(\theta),\ \theta \in [-\tau, 0],$$

where $\psi : [\tau, 0] \to \mathbb{R}^n$ is a continue function. We assume that function $f : \mathbb{R}_+ \to \mathbb{R}_+$ is nonlinear and the third order of derivative exists and is continuous.
System (1) is written:

$$(3) \qquad x'(t) = kwh - kBF(x(t-\tau)),\ t \in [0,T]$$

where:

$$(4) \qquad h = (1,1,...,1)^T, \quad B = \begin{pmatrix} a & b... & b \\ b & a... & b \\ b & b... & a \end{pmatrix},$$

$$F(x(t-\tau)) = (f(x_1(t-\tau),...,f(x_n(t-\tau)))^T, x(t) = (x_1(t),...,x_n(t))^T.$$

Equation (3) is equivalent with the next integral equation:

$$(5) \qquad x(t) = \begin{cases} \psi(0) + \int_0^t [kwh - kBF(x(s-\tau))]ds,\ t \in [0,T] \\ \psi(t),\ t \in [-\tau, 0] \end{cases}$$

Theorem 2.1. *We assume that*

(i) there exists φ a function like in Theorem 1.2;

(ii) $\|F(u)-F(v)\|_{\mathbb{R}^n} \leq \varphi(\|u-v\|_{\mathbb{R}^n})$, for all $u, v \in \mathbb{R}^n$;

(iii) $|kB|T \leq 1$.

Then the equation (5) has a unique solution x in $C([-\tau, T], |\cdot|_\infty)$

Proof. Be $S : (C[-\tau, T], |\cdot|_\infty) \to (C[-\tau, T], |\cdot|_\infty)$

$$S(x)(t) = \begin{cases} \psi(0) + \int\limits_0^t [kwh - kBF(x(s-\tau))]ds, \ t \in [0, T] \\ \psi(t), \ t \in [-\tau, 0] \end{cases}$$

Then for any $x, y \in C[-\tau, T]$ we have

$$\|S(x)(t) - S(y)(t)\|_{\mathbb{R}^n} \leq \int\limits_0^t |kB|\|F(x(s-\tau) - F(y(s-\tau))\|_{\mathbb{R}^n} ds \leq$$

$$\leq \int\limits_0^t |kB|\varphi(\|x(s-\tau) - y(s-\tau)\|_{\mathbb{R}^n})|ds \leq$$

$$\leq |kB|T\varphi(|x-y|_\infty) \leq \varphi(|x-y|_\infty)$$

Using the Theorem 1.2 we obtain that equation (5) has a unique solution.

For the date dependence we use the following theorem:

Theorem 2.2. *([2], [3]) Be (X, d) a complete metric space and $f_1, f_2 : X \longrightarrow X$ such that:*

(i) it exists φ_1, φ_2 like in Theorem 1.2 for f_1, f_2;

(ii) there exists $\eta > 0$ such that

$$d(f_1(x), f_2(x)) \leq \eta, \quad \textit{for all} \ \ x \in X.$$

Then

$$d(\overline{x}_1, \overline{x}_2) \leq \min\{\varphi_{1\eta}, \varphi_{2\eta}\},$$

where $\varphi_{i\eta} = \sup\{t \in \mathbb{R}_+ \mid t - \varphi_i(t) \leq \eta\}$.

Theorem 2.3. *We consider equation (5) with F_1, F_2 data with the conditions from Theorem 2.1. Moreover, we assume that there $\eta > 0$ such that*

$$\|F_1(u) - F_2(u)\|_{\mathbb{R}^n} \leq \eta, \text{ for all } u \in \mathbb{R}^n$$

then

$$d(\overline{x}_1, \overline{x}_2) \leq \min\{\varphi_{1\eta}, \varphi_{2\eta}\}.$$

Proof. We apply Theorem 2.2 for the operator S defined in the proof of Theorem 2.1 with data F_1 and F_2.

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