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Application of the Variational Iteration Method for Solving Differential - Difference Equations

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Abstract

In this paper, an application of variational iteration method is applied to solve differential - difference equations (DDEs). Comparisons are made between exact solution and the variational iteration method. To illustrate the ability and reliability of the method, some examples are given, revealing its effectiveness and simplicity.

Keywords: *Variational iteration method; differential- difference equations*
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1 Introduction

Recently, the variational iteration method [1–3] has been favorably applied to various kinds of nonlinear problems, for example, fractional differential equations

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[4,5], nonlinear differential equations [6], nonlinear thermo elasticity [7], nonlinear wave equations [8,9]. In this Letter we apply the method to solve differential - difference equations (DDEs).
To illustrate the method, consider the following general functional equation

$$Lu(x)+N(x)=g(x), \tag{1}$$

Where L is a linear operator, N is a non-linear operator and $g(t)$ is a known analytical function. According to the variational iteration method, we can construct the following correction functional

$$u_{n+1}(x)=u_n(x)+\int_0^x \lambda(\xi)\left\{Lu_n(\xi)+N\tilde{u}_n(\xi)-g(\xi)\right\}d\xi, \tag{2}$$

Where λ is a general Lagrange multiplier which can be identified optimally via variational theory, u_0 is an initial approximation with possible unknowns, and $\tilde{u}_n$ is considered as restricted variation, i.e., $\delta\tilde{u}_n=0$.

2 numerical examples

In this section, we applied the method presented in this paper to two examples to show the efficiency of the approach.

Example1. Consider the third order linear constant coefficient DDE [10]

$$y'''(x)-y''(x)-y(x)+(e-2)y''(x-1)+y'(x-1)+y(x-1)=2e-7 \tag{3}$$

With conditions

$$y(0)=1, y'(0)=0, y''(0)=1$$

The analytical solution of the above problem is given by,

$$y(x)=x^2+x+2-e^{-x} \tag{4}$$

In the view of the variational iteration method, we construct a correction functional in the following form:

$$\begin{aligned} y_{n+1}(x)=y_n(x)+\int_0^x \lambda\{&y'''(\xi)-y''(\xi)-y(\xi)+(e-2)y''(\xi-1) \\ &+y'(\xi-1)+y(\xi-1)-2e+7\}d\xi \end{aligned} \tag{5}$$

To find the optimal $\lambda(s)$, calculation variation with respect to y_n, we have the following stationary conditions:

$$\begin{aligned}
&\delta y_n : \lambda'''(\xi)=0,\\
&\delta y_n'' : \lambda(\xi)\big|_{\xi=x}=0,\\
&\delta y_n' : \lambda'(\xi)\big|_{\xi=x}=0,\\
&\delta y_n : 1-\lambda''(\xi)\big|_{\xi=x}=0.
\end{aligned}$$

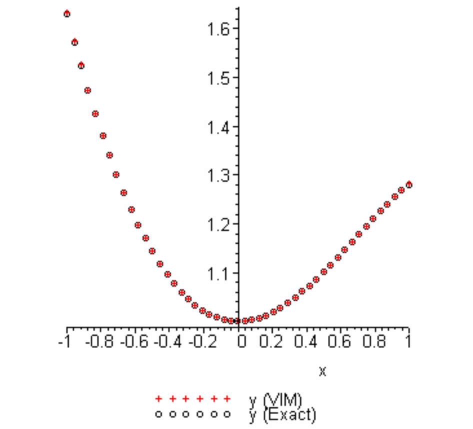
The Lagrange multiplier, therefore, can identified as follows:

$$\lambda=\frac{-(x-\xi)^2}{2}.$$

Substituting the identified multiplier into Eq.(5), we have the following iteration formula:

$$\begin{aligned}
y_{n+1}(x)=y_n(x)-\frac{1}{2}\int_0^x (x-\xi)^2\{&y'''(\xi)-y''(\xi)-y(\xi)+(e-2)y''(\xi-1)\\
&+y'(\xi-1)+y(\xi-1)-2e+7\}d\xi
\end{aligned}\tag{6}$$

Starting with the initial approximation $y_0=1+\frac{x^2}{2}$ in Eq. (6) successive approximations $y_i(x)$'s will be achieved. The plot of exact solution Eq.(3), the 5th order of approximate solution obtained using the VIM and absolute error between the exact and numerical solutions of this example are shown in Fig. 1.



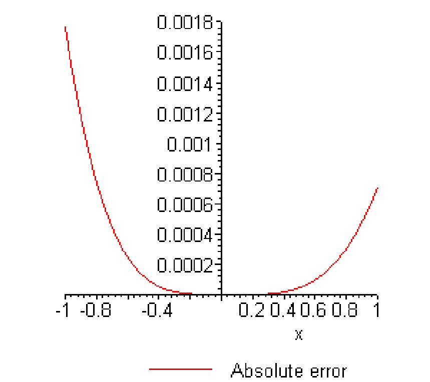


Fig. 1. The plots of approximate solution, exact solution and absolute error for Example 1.

Example 2. Now we consider the third order linear constant coefficient DDE[11]

$$y'''(x)-\cos(x)y'(x)-\sin(x)y'\left(x-\frac{\pi}{2}\right)+\sqrt{2}y\left(x-\frac{\pi}{4}\right)=\sin(x)-2\cos(x)-1 \quad (7)$$

With conditions:

$$y(0)=0, y'(0)=1, y''(0)=0$$

The analytical solution of the above problem is given by,

$$y(x)=\sin(x). \tag{8}$$

In the view of the variational iteration method, we construct a correction functional in the following form:

$$\begin{aligned} y_{n+1}(x)=y_n(x)-\frac{1}{2}\int_0^x y'''(x)-\cos(x)y'(x)-\sin(x)y'\left(x-\frac{\pi}{2}\right) \\ +\sqrt{2}y\left(x-\frac{\pi}{4}\right)-\sin(x)+2\cos(x)+1\Big\} d\xi \end{aligned} \tag{9}$$

Starting with the initial approximation $y_0=x$ in Eq. (9) successive approximations $y_i(x)$'s will be achieved. The plot of exact solution Eq. (7), the 5th order of approximate solution obtained using the VIM and absolute error between the exact and numerical solutions of this example are shown in Fig. 2.

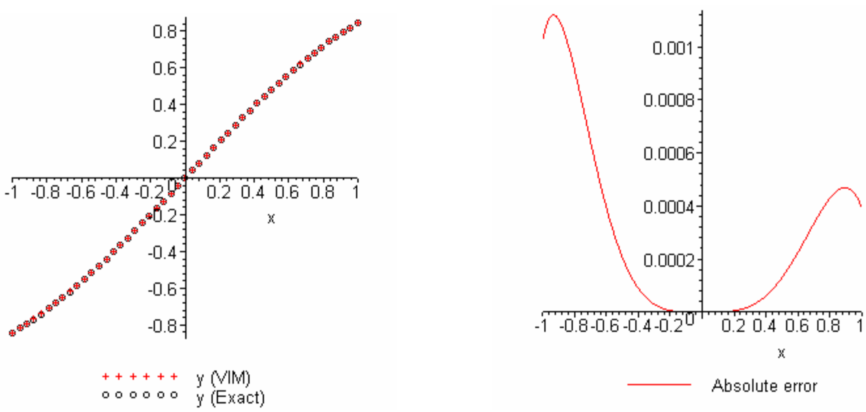


Fig. 2. The plots of approximate solution, exact solution and absolute error for Example 2.

3 Conclusion

The variational iteration method is an efficient method for solving various kinds of problems. In this Letter, we apply the variational iteration method to differential - difference equations. Since this method does not need to the discretization of the variables, there is no computation round off errors. Also this method is useful for finding an accurate approximation of the exact solution. The obtained results showed that this approach can solve the problem effectively and it needs less CPU time. The computations associated with the examples in this paper were performed using maple 13.

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