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Rotation Effect on MHD Flow Past an Impulsively Started Vertical Plate with Variable Mass Diffusion

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Abstract

Rotation effects on MHD flow past an impulsively started vertical plate with variable mass diffusion is studied here. The governing equations involved in the present analysis are solved by the Laplace-transform technique. The velocity, concentration and skin friction are studied for different parameters like mass Grashof number, Schmidt number, magnetic field parameter, rotation parameter and time.

Keywords: *Rotation effects, MHD, mass diffusion.*

1 Introduction

Study of MHD flow with heat and mass transfer plays an important role in chemical, mechanical and biological Sciences. Some important applications are cooling of nuclear reactors, liquid metals fluid, power generation system and aero dynamics. The response of laminar skin friction and heat transfer to fluctuations in the stream velocity was studied by Lighthill [8]. Free convection effects on the oscillating flow past an infinite vertical porous plate with constant suction - I, was studied by Soundalgekar [16] which was further improved by Vajravelu et al. [18]. Further researches in these areas were done by Gupta et al.[3], jaiswal et al.[6]and Soundalgekar et al. [17] by taking different models.

Some effects like radiation and mass transfer on MHD flow were studied by Muthucumaraswamy et al. [10] to [11] and Prasad et al. [12]. Radiation effects on mixed convection along a vertical plate with uniform surface temperature were studied by Hossain and Takhar [5]. Mass transfer effects on the flow past an exponentially accelerated vertical plate with constant heat flux was studied by Jha, Prasad and Rai [7].

On the other hand, Radiation and free convection flow past a moving plate was considered by Raptis and Perdikis [15]. MHD flow past an impulsively started vertical plate with variable temperature and mass diffusion were studied by Rajput and Kumar [13]. Further Rajput and Kumar [14] considered rotation and radiation effects on MHD flow past an impulsively started vertical plate with variable temperature. We are considering the rotation and radiation effects on MHD flow past an impulsively started vertical plate with variable temperature. The results are shown with the help of graphs (Figure-1 to Figure-8) and table-1.

2 Mathematical Analysis

In this paper we have consider the unsteady MHD flow of an electrically conducting fluid induced by viscous incompressible fluid past an impulsively started vertical plate with variable mass diffusion. The fluid and the plate rotate as a rigid body with a uniform angular velocity Ω' about z'-axis in the presence of an imposed uniform magnetic field B_0 normal to the plate. Initially, the concentration of the fluid near the plate are assumed to be C'_∞. At time $t' > 0$, the plate starts moving with a velocity $u' = u_0$ in its own plane and the concentration from the plate is raised to C'_w. Since the plate occupying the plane $z' = 0$ is of infinite extent, all the physical quantities depend only on z' and t'. It is assumed that the induced magnetic field is negligible so that $\vec{B}_0 = (0, 0, B_0)$. Under the above assumptions, the flow is governed by the following set of equations:

$$\frac{\partial u'}{\partial t'} - 2\Omega' v' = g\beta^* \left(C' - C'_\infty\right) + \nu \frac{\partial^2 u'}{\partial z'^2} - \frac{\sigma B_0^2 u'}{\rho} \quad , \tag{1}$$

$$\frac{\partial v'}{\partial t'} + 2\Omega' u' = \nu \frac{\partial^2 v'}{\partial z'^2} - \frac{\sigma B_0^2 v'}{\rho} \quad , \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial z'^2} \quad , \tag{3}$$

with boundary conditions :

$$\left.\begin{array}{l} t' \leq 0 : u' = 0,\ C' = C'_\infty \quad \text{for all the values of} \quad z' \\ t' > 0 : u' = u_0,\ C' = C'_\infty + \left(C'_w - C'_\infty\right) At', \text{ at } z' = 0, \\ \qquad u' \to 0,\ C' \to C'_\infty \text{ as } z' \to \infty \quad , \end{array}\right\} \tag{4}$$

where $A = \frac{u_0^2}{\nu}$.

Where the symbols are, B_0- external magnetic field, $C^{'}-$ species concentration in the fluid, $C_w^{'}-$ concentration of the fluid, $C_\infty^{'}-$ concentration in the fluid far away from the plate, $D-$ chemical molecular diffusivity, $g-$ acceleration due to gravity, $t^{'}-$ time, $u^{'}-$ primary velocity of the fluid, $v^{'}-$ secondary velocity of the fluid, u_0- velocity of the fluid, $z^{'}-$ coordinate axis normal to the plate, β^*- volumetric coefficient of expansion with concentration, $\sigma-$ Stefan–Boltzmann constant, $\rho-$ density $\nu-$ kinematic viscosity and $\Omega^{'}-$ rotation parameter.

Introducing the following non - dimensional quantities:

$$\left.\begin{array}{lll} u = \frac{u^{'}}{u_0}, & v = \frac{v^{'}}{u_0}, & t = \frac{t^{'} u_0^2}{\nu}, \\ z = \frac{z^{'} u_0}{\nu}, & G_m = \frac{g\beta^*\nu\left(C_w^{'}-C_\infty^{'}\right)}{u_0^3}, & \Omega = \frac{\Omega^{'}\nu}{u_0^2}, \\ C = \frac{\left(C^{'}-C_\infty^{'}\right)}{\left(C_w^{'}-C_\infty^{'}\right)}, & M = \frac{\sigma B_0^2 \nu}{\rho u_0^2} \text{ and} & S_c = \frac{\nu}{D}, \end{array}\right\}$$

where u is dimensionless velocity along $x-$ axis, $v-$ dimensionless velocity along y-axis, $z-$ dimensionless coordinate axis normal to the plate, $C-$ dimensionless concentration, G_m- mass Grashof number, S_c- Schmidt number, $t-$ dimensionless time, $\Omega-$ dimensionless rotation parameter and M is magnetic field parameter; equations (1), (2) and (3) reduces to:

$$\frac{\partial u}{\partial t} - 2\Omega v = G_m C + \frac{\partial^2 u}{\partial z^2} - Mu, \tag{5}$$

$$\frac{\partial v}{\partial t} + 2\Omega u = \frac{\partial^2 v}{\partial z^2} - Mv, \tag{6}$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial z^2}. \tag{7}$$

The non- dimensional boundary conditions are given by:

$$\left.\begin{array}{r} t \leq 0 : u = 0,\ C = 0 \text{ for all the values of } z \\ t > 0 : u = 1,\ C = t, \text{ at } z = 0, \\ u \to 0,\ C \to 0 \text{ as } z \to \infty. \end{array}\right\} \tag{8}$$

Let us assume $q = u + iv$, then from equations (5) and (6), we get,

$$\frac{\partial q}{\partial t} = G_m C + \frac{\partial^2 q}{\partial z^2} - mq, \tag{9}$$

where $m = M + 2i\Omega$.

Also, the non- dimensional boundary conditions (8) are reduced to:

$$\left.\begin{array}{r} t \leq 0 : q(z,0) = 0,\ C(z,0) = 0 \text{ for all the values of } z \\ t > 0 : q(0,t) = 1,\ C(0,t) = t, \text{ at } z = 0, \\ q(z,t) \to 0,\ C(z,t) \to 0 \text{ as } z \to \infty. \end{array}\right\} \tag{10}$$

The dimensionless governing equations (7) and (9), subject to the boundary conditions (10), are solved by the usual Laplace transform technique with some help from [1], [2] and [4], the solutions are derived as follows :

$$\begin{aligned} q(z,t) &= q_1 e^{-z\sqrt{m}} erfc\left(\eta-\sqrt{mt}\right)+q_2 e^{z\sqrt{m}} erfc\left(\eta+\sqrt{mt}\right) \\ &-\tfrac{G_2 e^{bt}}{2b^2}\left[e^{-z\sqrt{bS_c}} erfc\left(\eta\sqrt{S_c}-\sqrt{bt}\right)+e^{z\sqrt{bS_c}} erfc\left(\eta\sqrt{S_c}+\sqrt{bt}\right)\right] \\ &+\tfrac{G_2}{b^2} erfc\left(\eta\sqrt{S_c}\right)+\tfrac{tG_2}{b}\left[(1+2\eta^2 S_c) erfc\left(\eta\sqrt{S_c}\right)-\tfrac{2\eta\sqrt{S_c}}{\sqrt{\pi}}e^{-\eta^2 S_c}\right] \\ &+\tfrac{G_2 e^{bt}}{2b^2}\left[e^{-z\sqrt{m_2}} erfc\left(\eta-\sqrt{m_2 t}\right)+e^{z\sqrt{m_2}} erfc\left(\eta+\sqrt{m_2 t}\right)\right]. \end{aligned} \tag{11}$$

$$C(z,t)=t\left[(1+2\eta^2 S_c) erfc(\eta\sqrt{S_c})-\frac{2\eta\sqrt{S_c}}{\sqrt{\pi}}e^{-\eta^2 S_c}\right], \tag{12}$$

where $q_1=\frac{G_1}{2}-\frac{G_2}{2b}(t-\frac{z}{2\sqrt{m}}),\ q_2=\frac{G_1}{2}-\frac{G_2}{2b}(t+\frac{z}{2\sqrt{m}})$,
$m_2=m+b$ and $\eta=\frac{z}{2\sqrt{t}}$.

Where $erf\,(a+ib)$ is given as [9]:

$$\begin{aligned} erf\,(a+ib) &= erf(a)+\tfrac{e^{-a^2}}{2a\pi}\left[1-\cos(2ab)+i\sin(2ab)\right] \\ &+\tfrac{2e^{-a^2}}{\pi}\sum_{n=1}^{\infty}\tfrac{e^{-n^2/4}}{n^2+4a^2}\left[f_n(a,b)+ig_n(a,b)\right]+\epsilon(a,b) \end{aligned}$$

with $f_n=2a-2a\cosh(nb)\cos(2ab)+n\sinh(nb)\sin(2ab)$,
$g_n=2a\cosh(nb)\sin(2ab)+n\sinh(nb)\cos(2ab)$ and $\epsilon(a,b)\approx 10^{-16}|erf(a+ib)|$.

3 Skin Friction

The Skin-friction components τ_x and τ_y are obtained as:

$$\tau_x+i\tau_y=-\left(\frac{\partial q}{\partial z}\right)_{z=0}. \tag{13}$$

Therefore,using equation (11), we obtain:

$$\begin{aligned} \tau_x+i\tau_y &= \tau_1 erf\left(\sqrt{mt}\right)+\tfrac{\tau_2}{\sqrt{\pi t}}e^{-mt}-\tfrac{G_2 e^{bt}\sqrt{m+b}}{b^2} erf\left(\sqrt{(m+b)t}\right) \\ &-\tfrac{G_2 e^{bt}\sqrt{S_c}}{b^{3/2}} erf\left(\sqrt{bt}\right)+\tfrac{2G_2\sqrt{tS_c}}{b\sqrt{\pi}}, \end{aligned} \tag{14}$$

where $\tau_1=G_1-\frac{t\sqrt{m}G_2}{b}-\frac{G_2}{2b\sqrt{m}},\quad \tau_2=\frac{1}{2}\left(G_1-\frac{tG_2}{b}\right)^2-\frac{G_2}{b^2}$.

4 Results and Discussion

The velocity profiles for different parameters $M,\ S_c,\ G_m,\ \Omega$ and t are shown by figures-1 to 8.

Primary velocity profiles are shown in figures-1 to 4. From figure-1, it is clear that the primary velocity increases when magnetic field parameter M

is decreased (keeping other parameters $S_c = 2.01$, $G_m = 5$, $\Omega = 0.5$, $t = 0.2$ constant). Primary velocity profile for different values of mass Grashof number G_m is shown in figure-2. It shows that primary velocity increases with increasing mass Grashof number G_m. It is clear from figure-3 that the primary velocity increases when time t is increased. But in figure-4 the primary velocity decreases when rotation parameter Ω is increased.

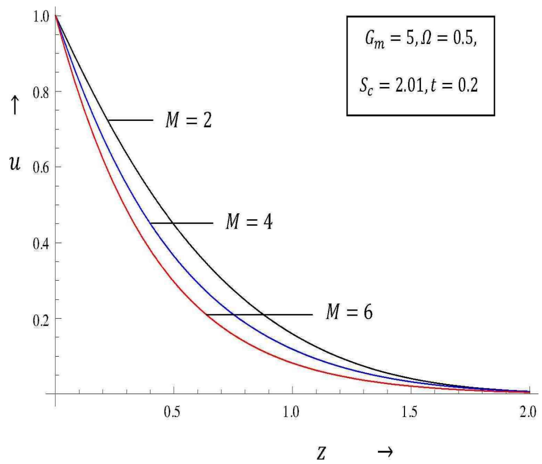

Figure 1: Primary velocity Profiles

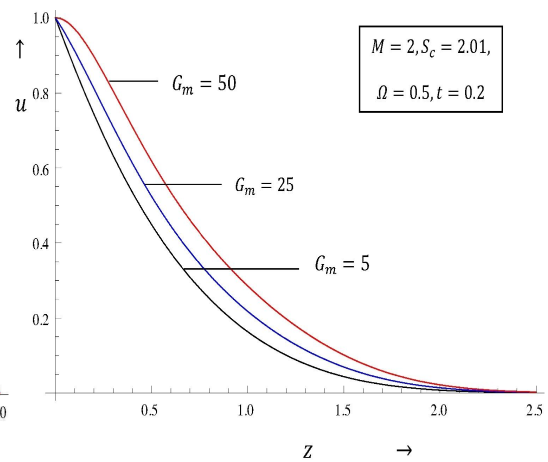

Figure 2: Primary velocity Profiles

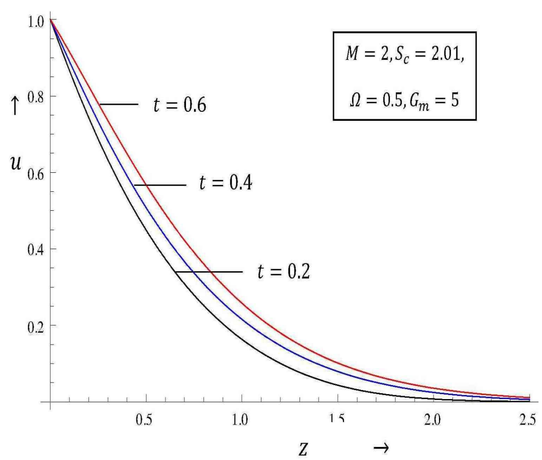

Figure 3: Primary velocity Profiles

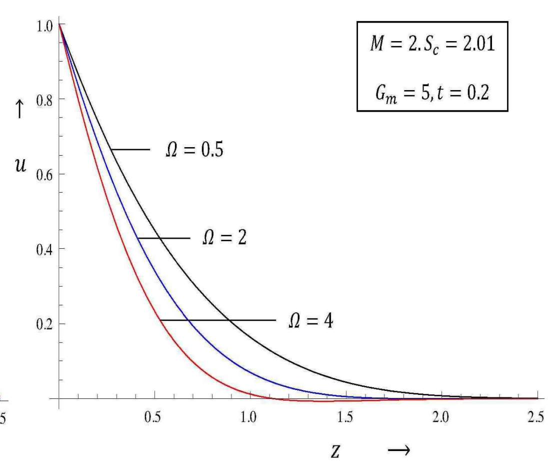

Figure 4: Primary velocity Profiles

Secondary velocity profile is shown in figures-5 to 8. In figure-5 it is observed that secondary velocity increases when magnetic field parameter M is increased. Similarly in figure-6, the secondary velocity increases when the value of G_m is increased. But in figure-7 and figure-8, the secondary velocity

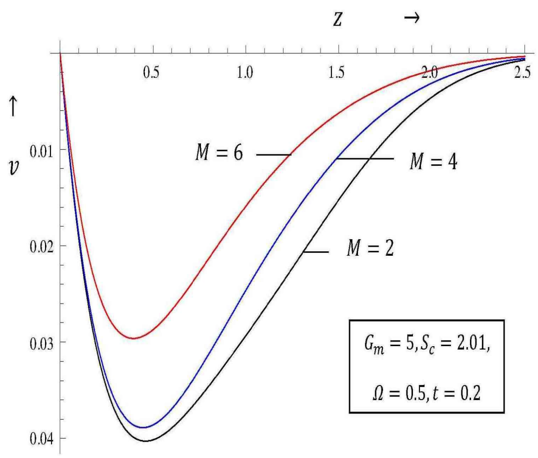

Figure 5: Secondary velocity Profiles

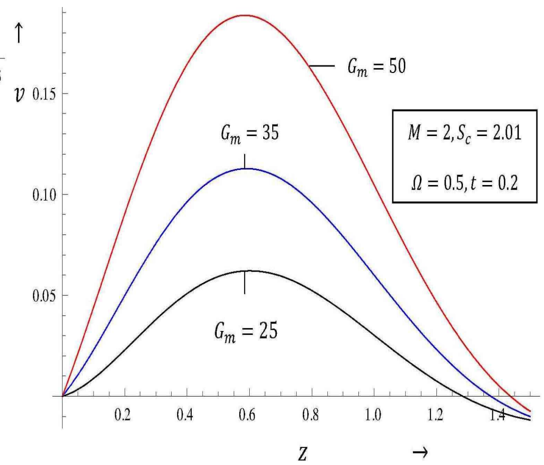

Figure 6: Secondary velocity Profiles

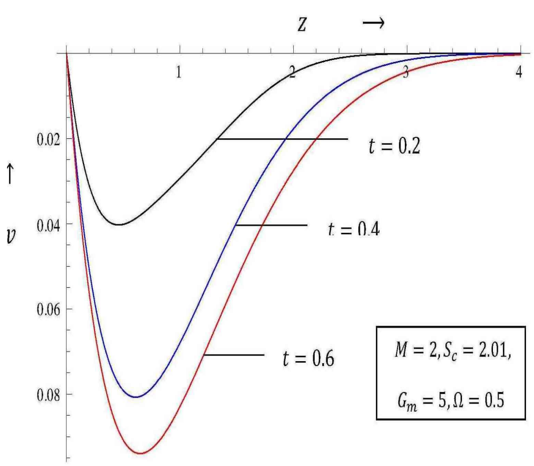

Figure 7: Secondary velocity Profiles

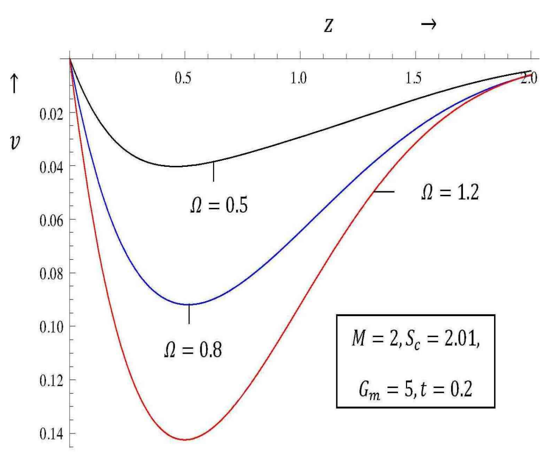

Figure 8: Secondary velocity Profiles

Table 1: Skin friction for different parameters

G_m	M	Ω	S_c	t	τ_x	τ_y
5.0	2.0	0.5	2.01	0.2	1.427	0.246
5.0	2.0	0.5	2.01	0.4	1.020	0.356
5.0	2.0	0.5	4.0	0.2	1.097	0.287
5.0	2.0	2.0	2.01	0.2	1.793	0.8371
5.0	4.0	2.0	2.01	0.2	1.996	0.189
10	2.0	0.5	2.01	0.2	1.112	0.269
-5.0	2.0	0.5	2.01	0.2	2.057	0.199

decreases when time t and rotation parameter Ω are increased, respectively.

The values of skin friction are tabulated in Table-(1) for different parameters. When the values of M and Ω are increased (keeping other parameters constant), the value of τ_x is also increased. But if values of S_c, G_m and t are increased, the value of τ_x gets decreased. Similarly, when the values of t, G_m, S_c and Ω are increased (keeping other parameters constant), the value of τ_y is increased. But if M is increased, the value of τ_y gets decreased.

5 Conclusion

In this paper a theoretical analysis has been done to study the rotation effects on MHD flow past an impulsively started vertical plate with variable mass diffusion. Solutions for the model have been derived by using Laplace - transform technique. Some conclusions of the study are as below :

- Primary velocity (u) increases with the increase in G_m and t, and decreases with increase in Ω and M.
- Secondary velocity (v) increases with the increase in M and G_m, and decreases with increase in t and Ω.
- Skin friction :
  - i. τ_x increases when magnetic field parameter and rotation parameter are increased but decreases when mass Grashof number, Schmidt number and time are increased.
  - ii. τ_y increases when mass Grashof number, Schmidt number, rotation parameter and t are increased but decreases when magnetic field is increased.

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