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Profit Evaluation of a Repairable System with Three-Stage Deterioration

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Abstract

This paper deals with the cost analysis of a repairable system subject to deterioration. The system works in three different modes: the normal, deterioration and failure. The deterioration can be minor, medium or major. The failure can be type I in which the system is perfectly repair or type II in which the system is minimally repaired. We analyzed the system using linear first order differential equations and developed explicit expressions for availability, busy period due to failure, busy period due to minor deterioration, busy period due to major deterioration and profit function for the system. Based on assumed numerical values given to the system parameters, some particular cases have also been discussed graphically to see the effect of deterioration, failure and repair rates on profit. The results have indicated that deterioration and failure rates decrease the profit while repair, minor and major maintenance rates increase the profit.

Keywords: *Deterioration, repairable system, maintenance.*

1 Introduction

Failure is an unavoidable phenomenon which can be dangerous and costly and bring about less production and profit. Proper maintenance planning plays a role in achieving high system reliability, availability and production output. Many researchers have studied reliability problem of different systems [see, for instance Satyavati (2011**)**].It is therefore important to keep the equipments/systems always available and to lay emphasis on system availability at the highest order. System availability represents the percentage of time the system is available to users. As the age of equipment increases, the equipment slowly deteriorates correspondingly. Deterioration failure is still the inevitable fate of the equipment. In many manufacturing situation, the condition of the system has significant impact on the quantity and quality of the unit produced. Most of these systems are subjected to random deterioration which can results in unexpected failures and disastrous effect on safety and the economy it is therefore important to find a way to slow down the deterioration rate, and to prolong equipment’s service life. Maintenance policies are vital in the analysis of deterioration and deteriorating systems as they help in improving reliability and availability of the systems. Maintenance models assume perfect repair [see for example Yusuf and Hussaini (2012)], minimal repair and imperfect repair which between perfect and minimal repair.

Large volumes of literature exist on the issue relating to deterioration and prediction of availability of various systems under different maintenance policies. Yusuf and Bala (2012) deal with stochastic modeling of two unit parallel system under two types of failures, where the system works in normal mode, deterioration (slow, mild, or fast) in model I and normal and failure modes in model II. Yusuf et al (2012) dealt with modeling the reliability and availability characteristics of a system with three Stages of deterioration. Marcous *et al* (2002) deal with the modeling bridge deterioration, Wirahadikusumah *et al* (2001) model deterioration of combined sewers. Cost analysis of redundant system working in normal and failure modes are numerous. Various models are developed concerning the cost analysis of a redundant system. Mokaddis and Matta (2010) studied the cost analysis of two dissimilar unit cold standby redundant systems subject to inspection and random change of units. Using semi Markov process technique various measures of system effectiveness are obtained. El-said (2008) studied cost analysis of a system with preventive maintenance. Haggag (2009) studied Cost analysis of a system involving common cause failures and preventive maintenance. Haggag (2009) dealt with Cost analysis of K-out-of- n repairable system with dependent failure and standby support. The problem considered in this paper is more general than the work of Yusuf *et al* (2012). We consider a system with three modes: normal, deterioration and failure where deterioration stages could be minor, medium or major whereas the failure is of two types [Yusuf *et al* (2012)]. Type I failure is control by perfect repair while type II failure is control by minimal repair. Our objectives are to develop the explicit expressions for availability, busy period due to failure, busy period due to major maintenance, busy period due to minor maintenance and profit function and capture the effect of failure rates, deterioration rates, minor and major maintenance rates, perfect and minimal repairs on profit based on assumed numerical values given to the system parameters.

2 Notations

$\beta_{12}, \beta_{13}, \beta_{14}$: Minor, medium and major deterioration rates respectively from S_1 to S_2, S_3 and S_4

β_{23}, β_{34}: Deterioration rates from S_2 to S_3 and S_3 to S_4 respectively

$\beta_{15}, \beta_{25}, \beta_{35}, \beta_{45}$: Failure rates

α_{51}, α_{54} : Perfect and minimal repair rates

$\alpha_{21}, \alpha_{31}, \alpha_{41}$: Major maintenance rates

α_{32}, α_{43}: Minor maintenance rates

$A_V(\infty), B_F(\infty), B_M(\infty), B_N(\infty), P_R$: System availability, busy period due to failure, busy period due to major maintenance, busy period due to minor maintenance, profit function.

3 Assumptions and Description of the System

1. State of the system can be: Perfect (S_1), minor deterioration (S_2), medium deterioration (S_3), major (S_4) or failed state (S_5)
2. At any given time *t* the system is either in the operating state, deteriorating states or in the failed state.
3. The units operate simultaneously
4. State S_5 can be access from the previous state
5. The state of the system changes as time progresses
6. System/units work in S_1, S_2, S_3 and S_4
7. The deteriorate stages can be minor, medium or major

Table 1: Transition rate table of the system

	S_1	S_2	S_3	S_4	S_5
S_1		β_{12}	β_{13}	β_{14}	β_{15}
S_2	α_{21}		β_{23}		β_{25}
S_3	α_{31}	α_{32}		β_{34}	β_{35}
S_4	α_{41}		α_{43}		β_{45}
S_5	α_{51}			α_{54}	

4 Availability Analysis

From table 1, let $P_i(t)$ be the probability that the system at time $t \geq 0$ is in state S_i. Let $P(t)$ be the probability row vector at time t with the initial conditions:

$$
\begin{aligned}
P(0) &= [P_1(0), P_2(0), P_3(0), P_4(0), P_5(0)] \\
&= [1, 0, 0, 0, 0]
\end{aligned}
$$

$$
\begin{aligned}
\frac{dP_1(t)}{dt} &= -(\beta_{12}+\beta_{13}+\beta_{14}+\beta_{15})P_1(t)+\alpha_{21}P_2(t)+\alpha_{31}P_3(t)+\alpha_{41}P_4(t)+\alpha_{51}P_5(t) \\
\frac{dP_2(t)}{dt} &= -(\alpha_{21}+\beta_{23}+\beta_{25})P_2(t)+\beta_{12}P_1(t)+\alpha_{32}P_3(t) \\
\frac{dP_3(t)}{dt} &= -(\alpha_{31}+\alpha_{32}+\beta_{34}+\beta_{35})P_3(t)+\beta_{13}P_1(t)+\beta_{23}P_2(t)+\alpha_{43}P_4(t) \\
\frac{dP_4(t)}{dt} &= -(\alpha_{41}+\alpha_{43}+\beta_{45})P_4(t)+\beta_{14}P_1(t)+\beta_{34}P_3(t)+\alpha_{54}P_5(t) \\
\frac{dP_5(t)}{dt} &= -(\alpha_{51}+\alpha_{54})P_5(t)+\beta_{15}P_1(t)+\beta_{25}P_2(t)+\beta_{35}P_3(t)+\beta_{45}P_4(t) \qquad (1)
\end{aligned}
$$

The differential equations above can be expressed as:

$$\dot{P} = AP \qquad (2)$$

Where

$$
A = \begin{bmatrix}
y_1 & \alpha_{21} & \alpha_{31} & \alpha_{41} & \alpha_{51} \\
\beta_{12} & y_2 & \alpha_{32} & 0 & 0 \\
\beta_{13} & \beta_{23} & y_3 & \alpha_{43} & 0 \\
\beta_{14} & 0 & \beta_{34} & y_4 & \alpha_{54} \\
\beta_{15} & \beta_{25} & \beta_{35} & \beta_{45} & y_5
\end{bmatrix}
$$

And

$$y_1 = -(\beta_{12}+\beta_{13}+\beta_{14}+\beta_{15}),\ y_2 = -(\alpha_{21}+\beta_{23}+\beta_{25}),$$
$$y_3 = -(\alpha_{31}+\alpha_{32}+\beta_{34}+\beta_{35}),\ y_4 = -(\alpha_{41}+\alpha_{43}+\beta_{45}),\ y_5 = -(\alpha_{51}+\alpha_{54})$$

The differential equations in (1) above can be expressed in matrix form as:

$$\begin{bmatrix} \dot{P}_1 \\ \dot{P}_2 \\ \dot{P}_3 \\ \dot{P}_4 \\ \dot{P}_5 \end{bmatrix} = \begin{bmatrix} y_1 & \alpha_{21} & \alpha_{31} & \alpha_{41} & \alpha_{51} \\ \beta_{12} & y_2 & \alpha_{32} & 0 & 0 \\ \beta_{13} & \beta_{23} & y_3 & \alpha_{43} & 0 \\ \beta_{14} & 0 & \beta_{34} & y_4 & \alpha_{54} \\ \beta_{15} & \beta_{25} & \beta_{35} & \beta_{45} & y_5 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix}$$

The system availability can be obtained from the solutions for $P_i(t)$, $i = 1, 2, \ldots, 5$. The state 1, 2, 3 and 4 are the working states of the system. Following El-Said (2008), Haggag (2009) and Wang et al (2006), the steady-state availability is given by:

$$A_V(\infty) = P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) \tag{3}$$

In the steady state, the derivatives of the state probabilities become zero, hence

$$AP = 0 \tag{4}$$

Which in matrix form is

$$\begin{bmatrix} y_1 & \alpha_{21} & \alpha_{31} & \alpha_{41} & \alpha_{51} \\ \beta_{12} & y_2 & \alpha_{32} & 0 & 0 \\ \beta_{13} & \beta_{23} & y_3 & \alpha_{43} & 0 \\ \beta_{14} & 0 & \beta_{34} & y_4 & \alpha_{54} \\ \beta_{15} & \beta_{25} & \beta_{35} & \beta_{45} & y_5 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Using the following normalizing condition

$$P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) + P_5(\infty) = 1 \tag{5}$$

We substitute (5) in the last row of in (4) to give

$$\begin{bmatrix} y_1 & \alpha_{21} & \alpha_{31} & \alpha_{41} & \alpha_{51} \\ \beta_{12} & y_2 & \alpha_{32} & 0 & 0 \\ \beta_{13} & \beta_{23} & y_3 & \alpha_{43} & 0 \\ \beta_{14} & 0 & \beta_{34} & y_4 & \alpha_{54} \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

And solve for $P_1(\infty), P_2(\infty), P_3(\infty), P_4(\infty)$

The steady-state availability is given by:

$$A_V(\infty) = \frac{N_1}{D_1}$$

5 Busy Period Analysis

Using the same initial conditions

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0)]$$
$$= [1, 0, 0, 0, 0, 0, 0]$$

Using the above differential equations expressed in matrix form as:

$$\begin{bmatrix} \dot{P}_1 \\ \dot{P}_2 \\ \dot{P}_3 \\ \dot{P}_4 \\ \dot{P}_5 \end{bmatrix} = \begin{bmatrix} y_1 & \alpha_{21} & \alpha_{31} & \alpha_{41} & \alpha_{51} \\ \beta_{12} & y_2 & \alpha_{32} & 0 & 0 \\ \beta_{13} & \beta_{23} & y_3 & \alpha_{43} & 0 \\ \beta_{14} & 0 & \beta_{34} & y_4 & \alpha_{54} \\ \beta_{15} & \beta_{25} & \beta_{35} & \beta_{45} & y_5 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix}$$

In the steady state, the derivatives of the state probabilities become zero so that

$$AP = 0$$

Which in matrix form is

$$\begin{bmatrix} y_1 & \alpha_{21} & \alpha_{31} & \alpha_{41} & \alpha_{51} \\ \beta_{12} & y_2 & \alpha_{32} & 0 & 0 \\ \beta_{13} & \beta_{23} & y_3 & \alpha_{43} & 0 \\ \beta_{14} & 0 & \beta_{34} & y_4 & \alpha_{54} \\ \beta_{15} & \beta_{25} & \beta_{35} & \beta_{45} & y_5 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Using the following normalizing condition in (6) below:

$$P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) + P_5(\infty) = 1 \quad (6)$$

We substitute (6) in any of the redundant rows in (5) to give

$$\begin{bmatrix} y_1 & \alpha_{21} & \alpha_{31} & \alpha_{41} & \alpha_{51} \\ \beta_{12} & y_2 & \alpha_{32} & 0 & 0 \\ \beta_{13} & \beta_{23} & y_3 & \alpha_{43} & 0 \\ \beta_{14} & 0 & \beta_{34} & y_4 & \alpha_{54} \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The steady-state Busy period due to failure is given by:

$$B_F(\infty) = P_5(\infty) = \frac{N_2}{D_1}$$

The steady-state Busy period due to major maintenance is given by:

$$B_M(\infty) = P_2(\infty) + P_3(\infty) + P_4(\infty) = \frac{N_3}{D_1}$$

The steady-state Busy period due to minor maintenance is given by:

$$B_N(\infty) = P_3(\infty) + P_4(\infty) = \frac{N_4}{D_1}$$

6 Profit Analysis

The system is subjected to perfect repair, minimal repair, minor and major maintenance respectively. The repairman performed, major and minor maintenance, perfect and minimal repairs to the system in states 2, 3, 4 and 5. Let C_0, C_1, C_2 and C_3 be the revenue generated when the system is in working state and no income when in failed state and cost due repair (corrective maintenance), cost due major maintenance and cost due minor maintenance respectively. Following El-said [12] and Haggag [13], the expected total profit per unit time incurred to the system in the steady-state is:

Profit=total revenue generated – cost incurred due to failure – cost incurred due major maintenance - cost incurred due to minor maintenance

Thus,

$$P_R = C_0 A_V - C_1 B_F - C_2 B_M - C_3 B_N$$

7 Numerical Simulations

In this section, we numerically obtained the results for mean time to system failure (MTSF) for the developed model. For the model analysis the following set of parameters values are fixed throughout the simulations for consistency:

$\alpha_{21} = 0.3, \alpha_{31} = 0.2, \alpha_{41} = 0.45, \alpha_{51} = 0.8, \alpha_{32} = 0.7, \alpha_{43} = 0.6, \alpha_{54} = 0.9,$
$\beta_{12} = 0.1, \beta_{13} = 0.4, \beta_{14} = 0.6, \beta_{15} = 0.1, \beta_{23} = 0.4, \beta_{24} = 0.5, \beta_{25} = 0.55,$

$\beta_{35} = 0.2$, $\beta_{34} = 0.7$, $\beta_{45} = 0.4$, $C_0 = 1{,}000$, $C_1 = 200$, $C_3 = 50$ and vary the parameter in question.

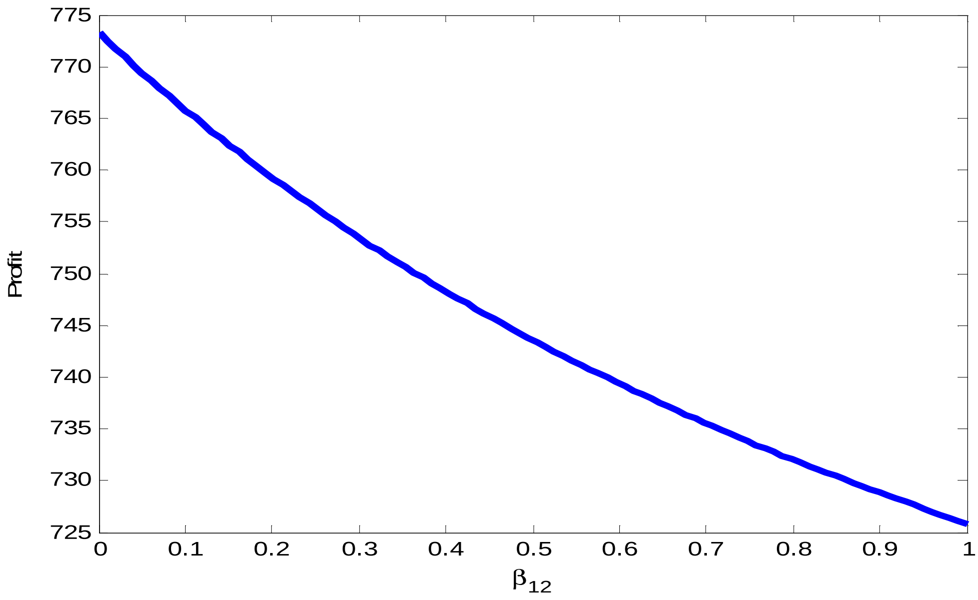

Fig. 1: Effect of β_{12} on Profit

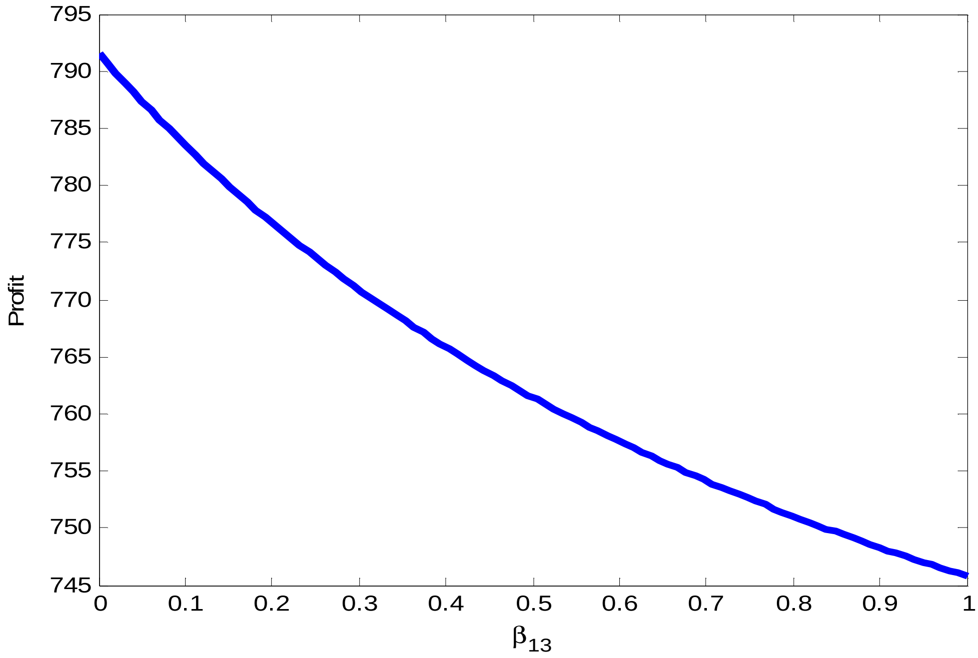

Fig. 2: Effect of β_{13} on Profit

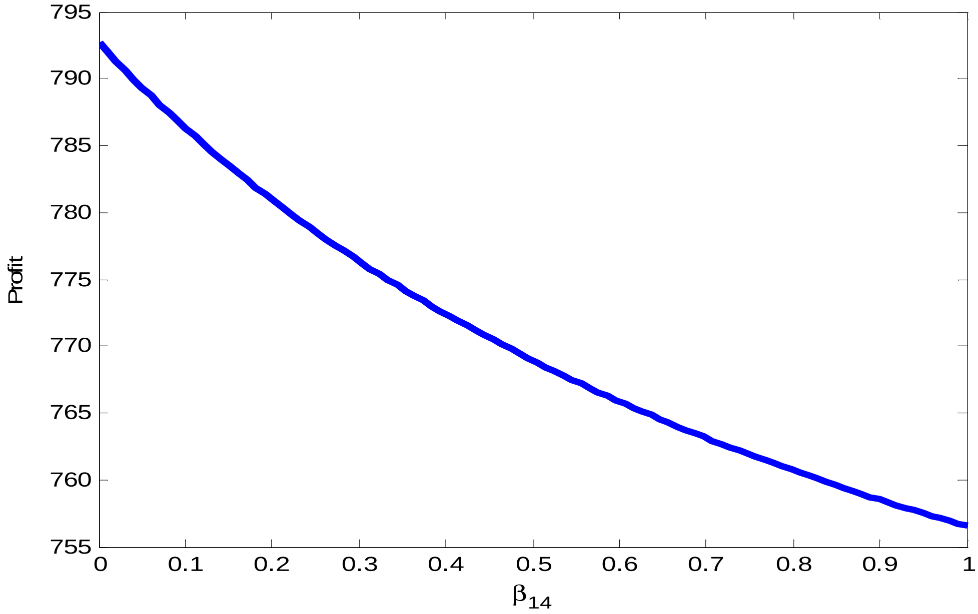

Fig. 3: Effect of β_{14} on Profit

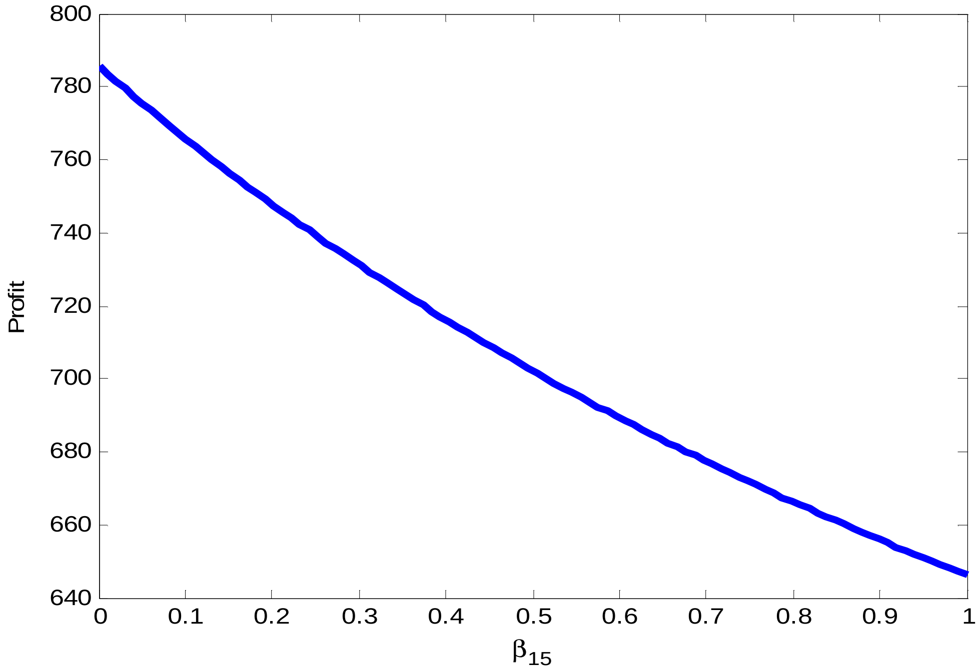

Fig. 4: Effect of β_{15} on Profit

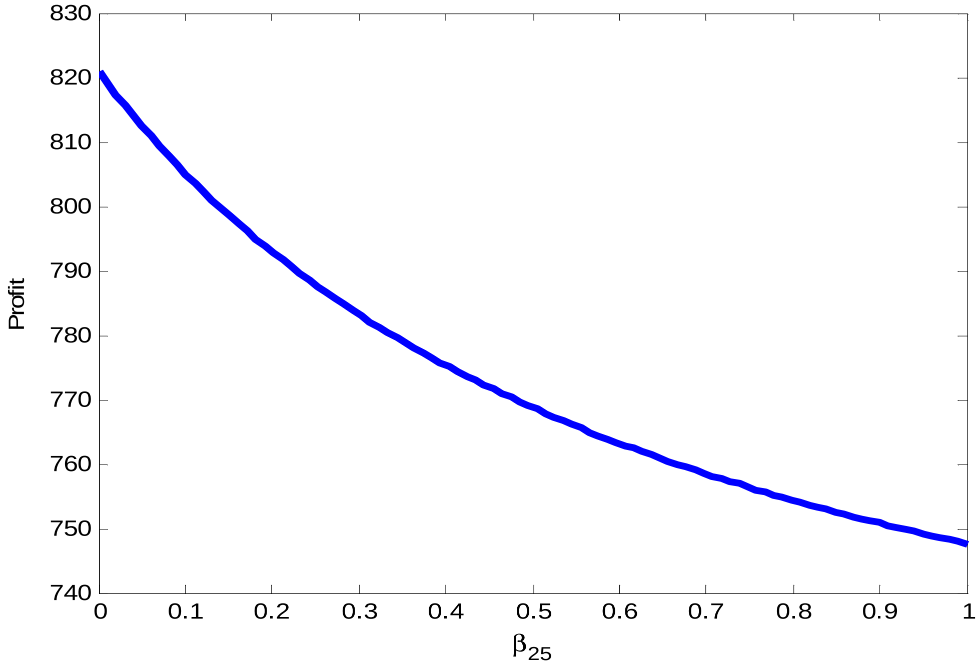

Fig. 5: Effect of β_{25} on Profit

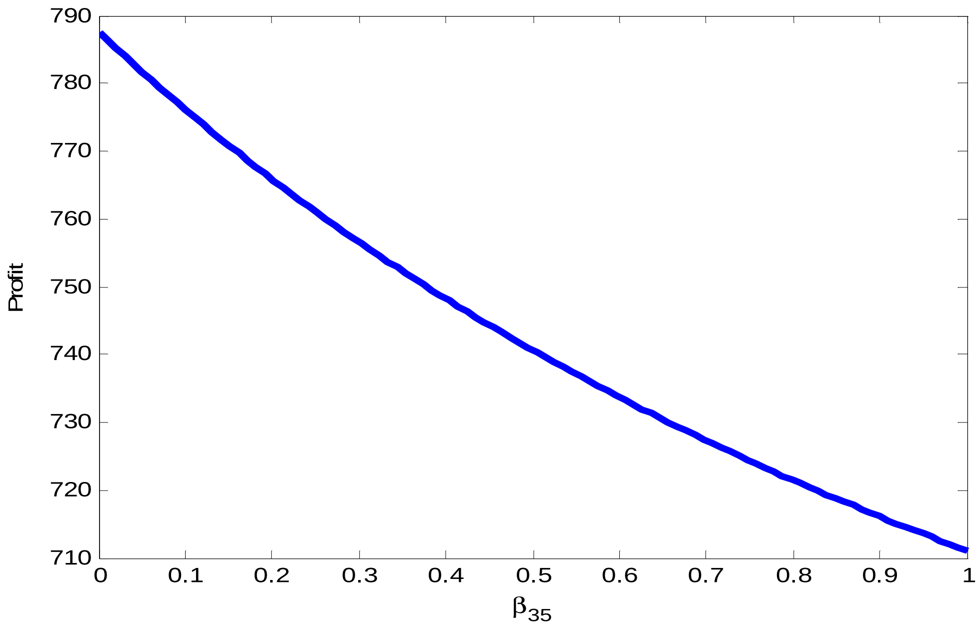

Fig. 6: Effect of β_{35} on Profit

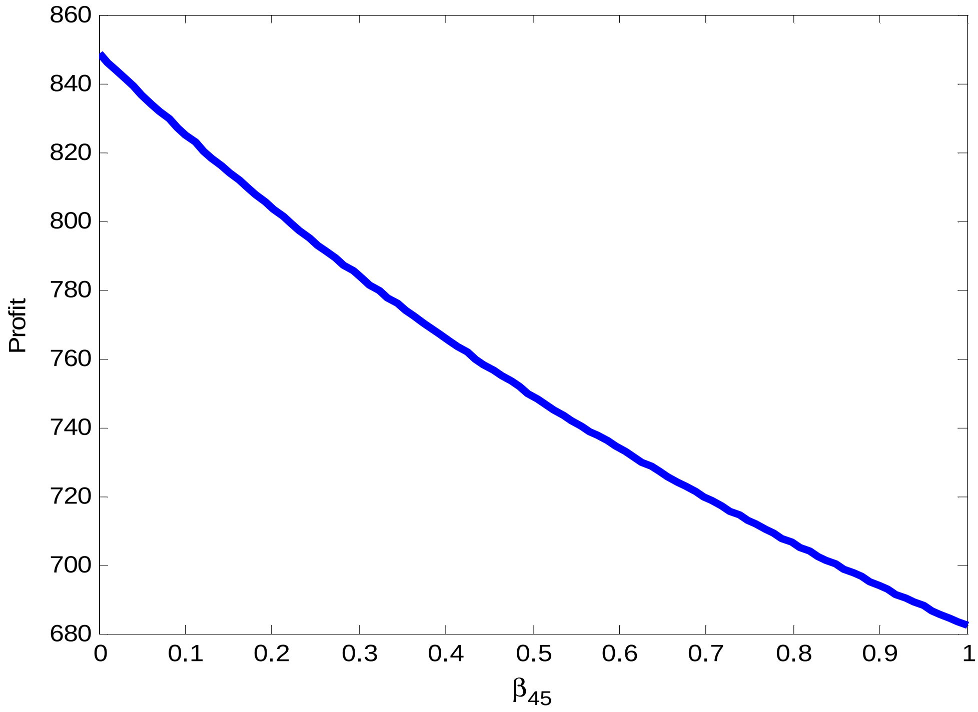

Fig. 7: Effect of β_{45} on Profit

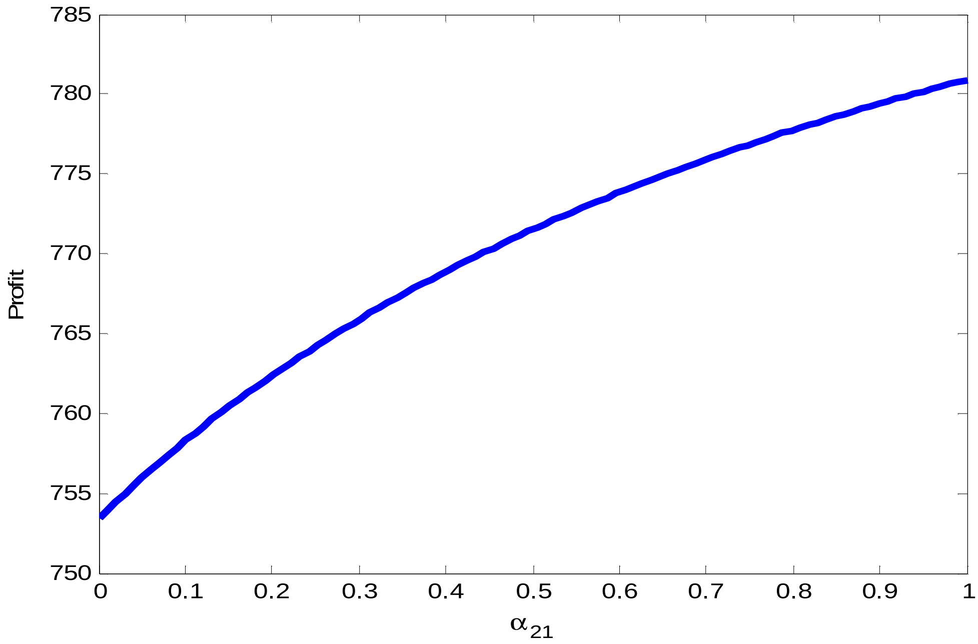

Fig. 8: Effect of α_{21} on Profit

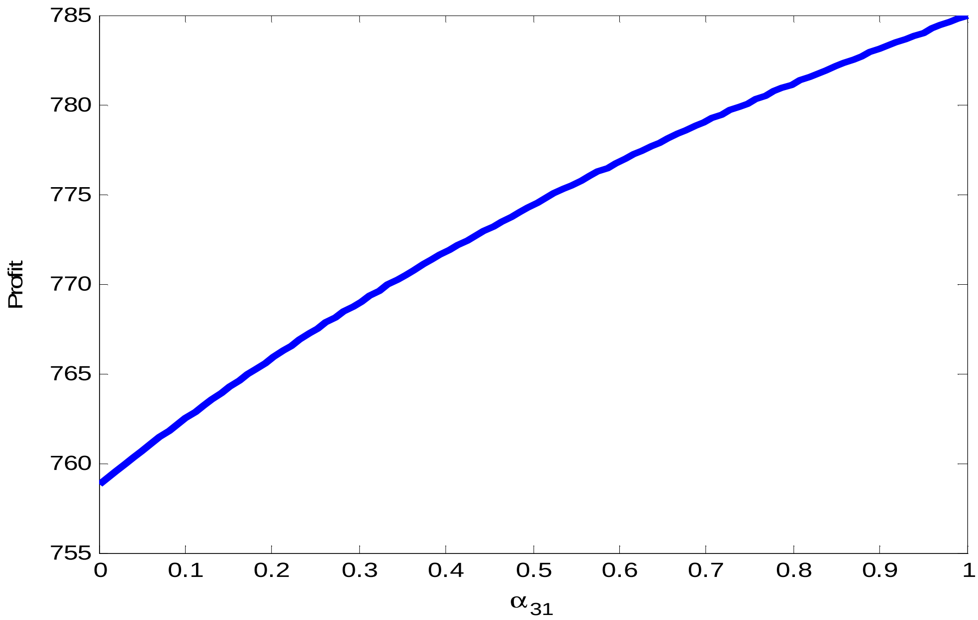

Fig. 9: Effect of α_{31} on Profit

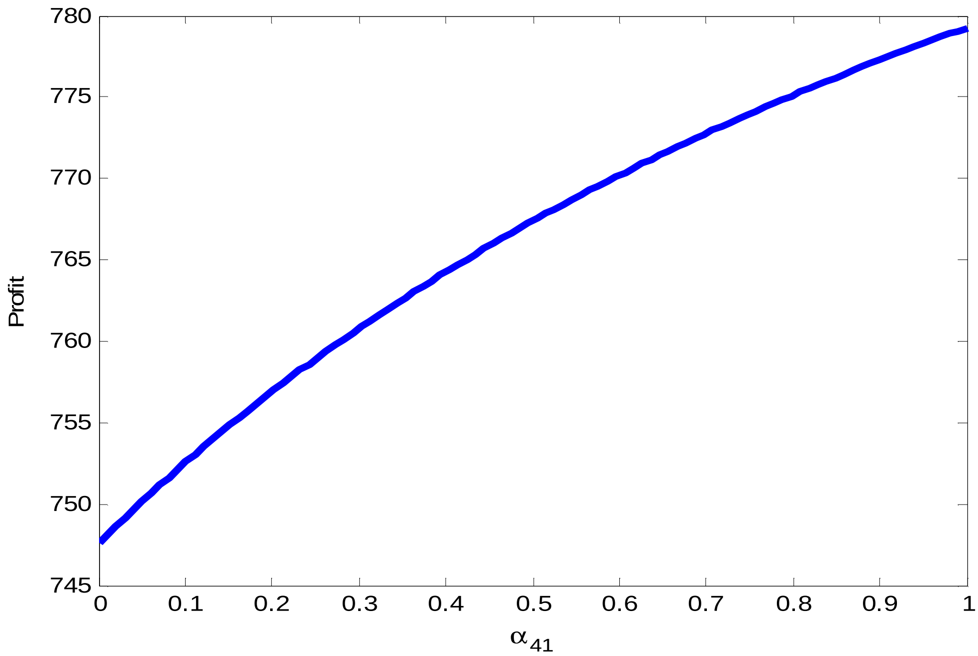

Fig. 10: Effect of α_{41} on Profit

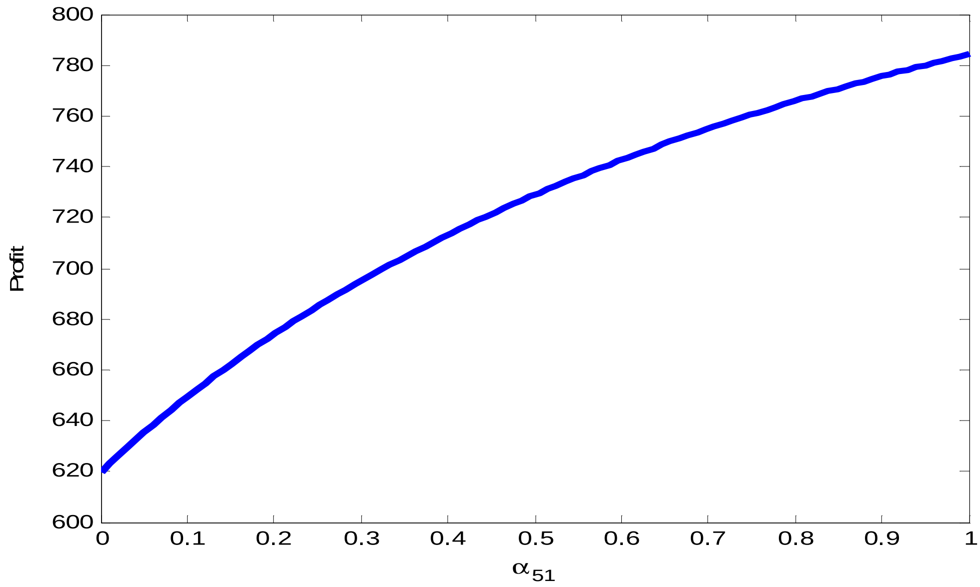

Fig. 11: Effect of α_{51} on Profit

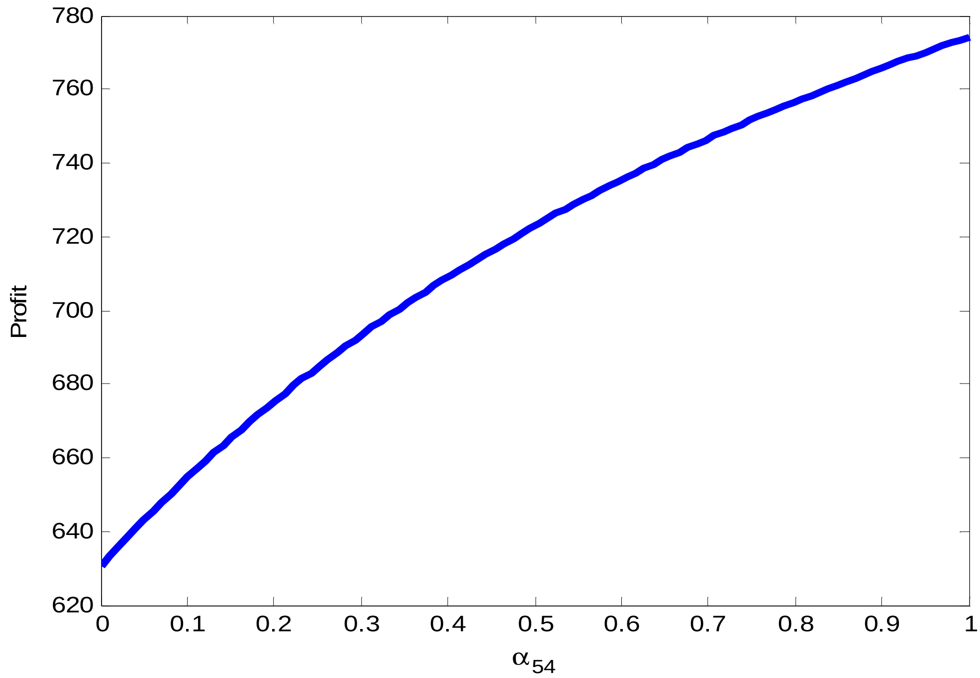

Fig. 12: Effect of α_{54} on Profit

It is evident from figures 1 – 7 that the increase in deterioration and failure rates induces the decrease in profit, while from figures 8 – 12, the increase in minor, major maintenance, perfect and minimal repair rates induces increase in profit.

8 Conclusion

In this paper, we developed the explicit expressions for the availability, busy period due to failure of the system, busy period due to major maintenance, busy due to minor maintenance and profit function. The effect of both minor and major maintenance, perfect and minimal repair, deterioration rates have been capture. The results have shown that both failure and deterioration rates decreases the profit while perfect, minimal, minor and major maintenance rates increase the profit.

Appendix

$$
\begin{aligned}
&N_1 = (\alpha_{21}\alpha_{41}\alpha_{54}\beta_{34} + \alpha_{41}\alpha_{54}\beta_{23}\beta_{34} + \alpha_{41}\alpha_{54}\beta_{25}\beta_{34} + \alpha_{21}\alpha_{41}\alpha_{51}\beta_{34} + \alpha_{21}\alpha_{51}\beta_{45}\beta_{34} + \alpha_{41}\alpha_{51}\beta_{23}\beta_{34} + \alpha_{51}\beta_{23}\beta_{45}\beta_{34} + \\
&\alpha_{41}\alpha_{51}\beta_{25}\beta_{34} + \alpha_{51}\beta_{25}\beta_{45}\beta_{34} + \alpha_{31}\alpha_{21}\alpha_{41}\alpha_{54} + \alpha_{31}\alpha_{41}\alpha_{54}\beta_{23} + \alpha_{31}\alpha_{21}\alpha_{43}\alpha_{54} + \alpha_{41}\alpha_{54}\beta_{23}\beta_{35} + \alpha_{31}\alpha_{43}\alpha_{54}\beta_{23} + \\
&\alpha_{31}\alpha_{41}\alpha_{54}\beta_{25} + \alpha_{31}\alpha_{54}\alpha_{43}\beta_{25} + \alpha_{32}\alpha_{21}\alpha_{41}\alpha_{54} + \alpha_{32}\alpha_{21}\alpha_{43}\alpha_{54} + \alpha_{32}\alpha_{41}\alpha_{54}\beta_{25} + \alpha_{21}\alpha_{41}\alpha_{54}\beta_{35} + \alpha_{41}\alpha_{54}\beta_{25}\beta_{35} + \\
&\alpha_{31}\alpha_{21}\alpha_{43}\alpha_{51} + \alpha_{31}\alpha_{43}\alpha_{51}\beta_{23} + \alpha_{31}\alpha_{51}\beta_{23}\beta_{45} + \alpha_{31}\alpha_{41}\alpha_{51}\beta_{25} + \alpha_{31}\alpha_{43}\alpha_{51}\beta_{25} + \alpha_{31}\alpha_{51}\beta_{25}\beta_{45} + \alpha_{31}\alpha_{41}\alpha_{51}\beta_{23} + \\
&\alpha_{32}\alpha_{21}\alpha_{41}\alpha_{51} + \alpha_{32}\alpha_{21}\alpha_{43}\alpha_{51} + \alpha_{32}\alpha_{21}\alpha_{51}\beta_{45} + \alpha_{32}\alpha_{41}\alpha_{51}\beta_{25} + \alpha_{32}\alpha_{43}\alpha_{51}\beta_{25} + \alpha_{32}\alpha_{51}\beta_{25}\beta_{45} + \alpha_{21}\alpha_{41}\alpha_{51}\beta_{35} + \\
&\alpha_{21}\alpha_{43}\alpha_{51}\beta_{35} + \alpha_{21}\alpha_{51}\beta_{35}\beta_{45} + \alpha_{41}\alpha_{51}\beta_{35}\beta_{23} + \alpha_{43}\alpha_{51}\beta_{35}\beta_{23} + \alpha_{51}\beta_{35}\beta_{23}\beta_{45} + \alpha_{41}\alpha_{51}\beta_{35}\beta_{25} + \alpha_{43}\alpha_{51}\beta_{35}\beta_{25} + \\
&\alpha_{51}\beta_{35}\beta_{25}\beta_{45} + \alpha_{31}\alpha_{21}\alpha_{41}\alpha_{51} + \alpha_{31}\alpha_{21}\alpha_{51}\beta_{45}) + (\alpha_{32}\alpha_{43}\alpha_{54}\beta_{14} + \alpha_{31}\alpha_{41}\alpha_{54}\beta_{12} + \alpha_{31}\alpha_{43}\alpha_{54}\beta_{12} + \alpha_{32}\alpha_{41}\alpha_{54}\beta_{12} + \\
&\alpha_{32}\alpha_{43}\alpha_{54}\beta_{12} + \alpha_{41}\alpha_{54}\beta_{12}\beta_{35} + \alpha_{32}\alpha_{43}\alpha_{51}\beta_{14} + \alpha_{31}\alpha_{41}\alpha_{51}\beta_{12} + \alpha_{31}\alpha_{43}\alpha_{51}\beta_{12} + \alpha_{31}\alpha_{51}\beta_{12}\beta_{45} + \alpha_{32}\alpha_{41}\alpha_{51}\beta_{12} + \\
&\alpha_{32}\alpha_{43}\alpha_{51}\beta_{12} + \alpha_{32}\alpha_{51}\beta_{12}\beta_{45} + \alpha_{41}\alpha_{51}\beta_{35}\beta_{12} + \alpha_{43}\alpha_{51}\beta_{35}\beta_{12} + \alpha_{51}\beta_{35}\beta_{12}\beta_{45} + \alpha_{32}\alpha_{43}\alpha_{54}\beta_{15} + \alpha_{41}\alpha_{54}\beta_{34}\beta_{12} + \\
&\alpha_{41}\alpha_{51}\beta_{34}\beta_{12} + \alpha_{51}\beta_{34}\beta_{12}\beta_{45} + \alpha_{32}\alpha_{43}\alpha_{54}\beta_{13} + \alpha_{32}\alpha_{41}\alpha_{51}\beta_{13} + \alpha_{32}\alpha_{43}\alpha_{51}\beta_{13} + \alpha_{32}\alpha_{51}\beta_{13}\beta_{45} + \alpha_{32}\alpha_{41}\alpha_{54}\beta_{13}) + \\
&(\alpha_{43}\alpha_{54}\beta_{23}\beta_{14} + \alpha_{41}\alpha_{54}\beta_{12}\beta_{23} + \alpha_{43}\alpha_{54}\beta_{23}\beta_{12} + \alpha_{21}\alpha_{43}\alpha_{54}\beta_{14} + \alpha_{43}\alpha_{54}\beta_{14}\beta_{25} + \alpha_{43}\alpha_{51}\beta_{23}\beta_{14} + \alpha_{54}\alpha_{43}\beta_{23}\beta_{15} + \\
&\alpha_{43}\alpha_{54}\beta_{25}\beta_{15} + \alpha_{43}\alpha_{51}\beta_{25}\beta_{14} + \alpha_{41}\alpha_{51}\beta_{23}\beta_{12} + \alpha_{43}\alpha_{51}\beta_{12}\beta_{23} + \alpha_{51}\beta_{23}\beta_{12}\beta_{45} + \alpha_{43}\alpha_{54}\beta_{25}\beta_{12} + \alpha_{21}\alpha_{43}\alpha_{54}\beta_{15} + \\
&\alpha_{21}\alpha_{43}\alpha_{51}\beta_{14} + \alpha_{51}\beta_{13}\beta_{25}\beta_{45} + \alpha_{41}\alpha_{54}\beta_{13}\beta_{25} + \alpha_{43}\alpha_{54}\beta_{12}\beta_{23} + \alpha_{43}\alpha_{51}\beta_{13}\beta_{23} + \alpha_{41}\alpha_{51}\beta_{13}\beta_{25} + \alpha_{21}\alpha_{43}\alpha_{54}\beta_{13} + \\
&\alpha_{21}\alpha_{43}\alpha_{51}\beta_{13} + \alpha_{41}\alpha_{54}\beta_{13}\beta_{23} + \alpha_{21}\alpha_{41}\alpha_{54}\beta_{13} + \alpha_{43}\alpha_{51}\beta_{13}\beta_{25} + \alpha_{21}\alpha_{41}\alpha_{51}\beta_{12} + \alpha_{21}\alpha_{51}\beta_{13}\beta_{45} + \alpha_{51}\beta_{13}\beta_{23}\beta_{45} + \\
&\alpha_{43}\alpha_{54}\beta_{13}\beta_{25} + \alpha_{41}\alpha_{51}\beta_{13}\beta_{23}) + (\alpha_{54}\alpha_{31}\alpha_{21}\beta_{14} + \alpha_{31}\alpha_{54}\beta_{14}\beta_{23} + \alpha_{31}\alpha_{54}\beta_{14}\beta_{25} + \alpha_{54}\alpha_{32}\alpha_{21}\beta_{14} + \alpha_{32}\alpha_{54}\beta_{14}\beta_{25} + \\
&\alpha_{21}\alpha_{54}\beta_{14}\beta_{34} + \alpha_{54}\beta_{14}\beta_{34}\beta_{23} + \alpha_{54}\beta_{14}\beta_{34}\beta_{25} + \alpha_{21}\alpha_{54}\beta_{14}\beta_{35} + \alpha_{54}\beta_{14}\beta_{35}\beta_{23} + \alpha_{54}\beta_{14}\beta_{35}\beta_{25} + \alpha_{54}\beta_{23}\beta_{34}\beta_{12} + \\
&\alpha_{21}\alpha_{54}\beta_{13}\beta_{34} + \alpha_{54}\beta_{13}\beta_{34}\beta_{23} + \alpha_{54}\beta_{13}\beta_{34}\beta_{25} + \alpha_{51}\beta_{23}\beta_{34}\beta_{14} + \alpha_{54}\beta_{23}\beta_{34}\beta_{15} + \alpha_{54}\beta_{12}\beta_{23}\beta_{35} + \alpha_{31}\alpha_{51}\beta_{14}\beta_{23} + \\
&\alpha_{31}\alpha_{54}\beta_{25}\beta_{12} + \alpha_{32}\alpha_{54}\beta_{25}\beta_{12} + \alpha_{21}\alpha_{31}\alpha_{54}\beta_{15} + \alpha_{21}\alpha_{32}\alpha_{54}\beta_{15} + \alpha_{31}\alpha_{54}\beta_{23}\beta_{15} + \alpha_{54}\beta_{25}\beta_{35}\beta_{12} + \alpha_{31}\alpha_{54}\beta_{25}\beta_{15} + \\
&\alpha_{32}\alpha_{54}\beta_{25}\beta_{15} + \alpha_{21}\alpha_{54}\beta_{35}\beta_{15} + \alpha_{21}\alpha_{51}\beta_{14}\beta_{35} + \alpha_{21}\alpha_{31}\alpha_{51}\beta_{14} + \alpha_{54}\beta_{23}\beta_{35}\beta_{15} + \alpha_{51}\beta_{23}\beta_{14}\beta_{35} + \alpha_{54}\beta_{25}\beta_{35}\beta_{15} + \\
&\alpha_{32}\alpha_{51}\beta_{25}\beta_{14} + \alpha_{51}\beta_{25}\beta_{14}\beta_{35} + \alpha_{31}\alpha_{51}\beta_{25}\beta_{14} + \alpha_{32}\alpha_{21}\alpha_{51}\beta_{14} + \alpha_{51}\beta_{34}\beta_{12}\beta_{23} + \alpha_{51}\beta_{34}\beta_{25}\beta_{14} + \alpha_{21}\alpha_{54}\beta_{34}\beta_{15} + \\
&\alpha_{54}\beta_{34}\beta_{25}\beta_{12} + \alpha_{54}\beta_{34}\beta_{25}\beta_{15} + \alpha_{21}\alpha_{51}\beta_{34}\beta_{14} + \alpha_{21}\alpha_{51}\beta_{13}\beta_{34} + \alpha_{51}\beta_{13}\beta_{34}\beta_{25} + \alpha_{32}\alpha_{54}\beta_{13}\beta_{25} + \alpha_{21}\alpha_{54}\beta_{13}\beta_{35} + \\
&\alpha_{54}\beta_{13}\beta_{35}\beta_{23} + \alpha_{54}\beta_{13}\beta_{35}\beta_{25} + \alpha_{51}\beta_{13}\beta_{23}\beta_{34})
\end{aligned}
$$

$$
\begin{aligned}
&N_2 = \alpha_{21}\alpha_{31}\beta_{14}\beta_{45} + \alpha_{31}\beta_{14}\beta_{23}\beta_{45} + \alpha_{31}\beta_{14}\beta_{25}\beta_{45} + \alpha_{21}\alpha_{32}\beta_{14}\beta_{45} + \alpha_{32}\alpha_{43}\beta_{14}\beta_{25} + \alpha_{32}\beta_{14}\beta_{25}\beta_{45} + \alpha_{21}\beta_{4}\beta_{34}\beta_{45} + \\
&\beta_{14}\beta_{23}\beta_{34}\beta_{45} + \beta_{14}\beta_{25}\beta_{34}\beta_{45} + \alpha_{21}\alpha_{43}\beta_{14}\beta_{35} + \alpha_{21}\beta_{14}\beta_{35}\beta_{45} + \alpha_{43}\beta_{14}\beta_{23}\beta_{35} + \beta_{14}\beta_{23}\beta_{35}\beta_{45} + \alpha_{43}\beta_{14}\beta_{25}\beta_{35} + \\
&\beta_{14}\beta_{25}\beta_{35}\beta_{45} + \beta_{12}\beta_{23}\beta_{34}\beta_{45} + \alpha_{21}\beta_{13}\beta_{34}\beta_{45} + \beta_{13}\beta_{23}\beta_{34}\beta_{45} + \beta_{13}\beta_{25}\beta_{34}\beta_{45} + \alpha_{41}\beta_{15}\beta_{23}\beta_{34} + \beta_{15}\beta_{23}\beta_{34}\beta_{45} +
\end{aligned}
$$

$$\alpha_{41}\beta_{12}\beta_{23}\beta_{35} + \alpha_{43}\beta_{12}\beta_{23}\beta_{35} + \beta_{12}\beta_{23}\beta_{35}\beta_{45} + \alpha_{21}\alpha_{41}\beta_{13}\beta_{35} + \alpha_{21}\alpha_{43}\beta_{13}\beta_{35} + \alpha_{21}\beta_{13}\beta_{35}\beta_{45} + \alpha_{41}\beta_{13}\beta_{23}\beta_{35} +$$
$$\alpha_{43}\beta_{13}\beta_{23}\beta_{35} + \beta_{13}\beta_{23}\beta_{35}\beta_{45} + \alpha_{32}\alpha_{41}\beta_{13}\beta_{25} + \alpha_{41}\beta_{13}\beta_{25}\beta_{35} + \alpha_{32}\alpha_{43}\beta_{13}\beta_{25} + \alpha_{43}\beta_{13}\beta_{25}\beta_{35} + \alpha_{32}\beta_{13}\beta_{25}\beta_{45} +$$
$$\beta_{13}\beta_{25}\beta_{35}\beta_{45} + \alpha_{31}\alpha_{41}\beta_{12}\ \beta_{25} + \alpha_{31}\alpha_{43}\beta_{12}\beta_{25} + \alpha_{31}\beta_{12}\beta_{25}\beta_{45} + \alpha_{32}\alpha_{41}\beta_{12}\beta_{25} + \alpha_{32}\alpha_{43}\beta_{12}\beta_{25} + \alpha_{21}\alpha_{31}\alpha_{41}\beta_{15} +$$
$$\alpha_{31}\alpha_{41}\beta_{15}\beta_{23} + \alpha_{31}\alpha_{41}\beta_{15}\beta_{25} + \alpha_{21}\alpha_{31}\alpha_{43}\beta_{15} + \alpha_{31}\alpha_{43}\beta_{15}\beta_{23} + \alpha_{31}\alpha_{43}\beta_{15}\beta_{25} + \alpha_{21}\alpha_{31}\beta_{15}\beta_{45} + \alpha_{31}\beta_{15}\beta_{23}\beta_{45} +$$
$$\alpha_{31}\beta_{15}\beta_{25}\beta_{45} + \alpha_{21}\alpha_{32}\alpha_{41}\beta_{15} + \alpha_{32}\alpha_{41}\beta_{15}\beta_{25} + \alpha_{21}\alpha_{32}\alpha_{43}\beta_{15} + \alpha_{32}\alpha_{43}\beta_{15}\beta_{25} + \alpha_{21}\alpha_{32}\beta_{15}\beta_{45} + \alpha_{32}\beta_{15}\beta_{25}\beta_{45} +$$
$$\alpha_{21}\alpha_{41}\beta_{15}\beta_{35} + \alpha_{41}\beta_{15}\beta_{23}\beta_{35} + \alpha_{41}\beta_{15}\beta_{25}\beta_{35} + \alpha_{21}\alpha_{43}\beta_{15}\beta_{35} + \alpha_{43}\beta_{23}\beta_{35}\beta_{43} + \alpha_{43}\beta_{15}\beta_{25}\beta_{35} + \alpha_{21}\beta_{15}\beta_{36}\beta_{45} +$$
$$\beta_{15}\beta_{23}\beta_{35}\beta_{45} + \beta_{15}\beta_{25}\beta_{35}\beta_{45} + \alpha_{32}\beta_{12}\beta_{25}\beta_{45} + \alpha_{41}\beta_{12}\beta_{25}\beta_{35} + \alpha_{43}\beta_{12}\beta_{25}\beta_{35} + \beta_{12}\beta_{25}\beta_{35}\beta_{45} + \beta_{15}\beta_{25}\beta_{34}\beta_{45} +$$
$$\alpha_{21}\alpha_{41}\beta_{15}\beta_{34} + \alpha_{21}\beta_{15}\beta_{34}\beta_{45} + \beta_{12}\beta_{25}\beta_{34}\beta_{45} + \alpha_{41}\beta_{12}\beta_{25}\beta_{34} + \alpha_{41}\beta_{15}\beta_{25}\beta_{34}$$

$$N_3 = (\alpha_{32}\alpha_{43}\alpha_{54}\beta_{14} + \alpha_{31}\alpha_{41}\alpha_{54}\beta_{12} + \alpha_{31}\alpha_{43}\alpha_{54}\beta_{12} + \alpha_{32}\alpha_{41}\alpha_{54}\beta_{12} + \alpha_{32}\alpha_{43}\alpha_{54}\beta_{12} + \alpha_{41}\alpha_{54}\beta_{12}\beta_{35} + \alpha_{32}\alpha_{43}\alpha_{51}\beta_{14} +$$
$$\alpha_{32}\alpha_{41}\alpha_{51}\beta_{12} + \alpha_{32}\alpha_{43}\alpha_{51}\beta_{12} + \alpha_{32}\alpha_{51}\beta_{12}\beta_{45} + \alpha_{32}\alpha_{43}\alpha_{54}\beta_{15} + \alpha_{41}\alpha_{51}\beta_{12}\beta_{35} + \alpha_{43}\alpha_{51}\beta_{12}\beta_{35} + \alpha_{51}\beta_{12}\beta_{35}\beta_{45} +$$
$$\alpha_{31}\alpha_{41}\alpha_{51}\beta_{12} + \alpha_{31}\alpha_{43}\alpha_{51}\beta_{12} + \alpha_{31}\alpha_{51}\beta_{12}\beta_{45} + \alpha_{41}\alpha_{51}\beta_{12}\beta_{34} + \alpha_{51}\beta_{12}\beta_{34}\beta_{45} + \alpha_{41}\alpha_{54}\beta_{12}\beta_{34} + \alpha_{32}\alpha_{41}\alpha_{54}\beta_{13} +$$
$$\alpha_{32}\alpha_{51}\beta_{13}\beta_{45} + \alpha_{32}\alpha_{41}\alpha_{51}\beta_{13} + \alpha_{32}\alpha_{43}\alpha_{51}\beta_{13} + \alpha_{32}\alpha_{43}\alpha_{54}\beta_{13}) +$$
$$(\alpha_{43}\alpha_{54}\beta_{14}\beta_{23} + \alpha_{41}\alpha_{54}\beta_{12}\beta_{23} + \alpha_{43}\alpha_{54}\beta_{12}\beta_{23} +$$
$$\alpha_{21}\alpha_{43}\alpha_{54}\beta_{14} + \alpha_{43}\alpha_{54}\beta_{14}\beta_{25} + \alpha_{43}\alpha_{54}\beta_{12}\beta_{25} + \alpha_{21}\alpha_{43}\alpha_{54}\beta_{15} + \alpha_{21}\alpha_{43}\alpha_{51}\beta_{14} + \alpha_{41}\alpha_{51}\beta_{12}\beta_{23} + \alpha_{43}\alpha_{51}\beta_{12}\beta_{23} +$$

$$\alpha_{51}\beta_{12}\beta_{23}\beta_{45} + \alpha_{43}\alpha_{54}\beta_{15}\beta_{23} + \alpha_{43}\alpha_{54}\beta_{15}\beta_{25} + \alpha_{43}\alpha_{51}\beta_{14}\beta_{25} + \alpha_{43}\alpha_{51}\beta_{14}\beta_{23} + \alpha_{51}\beta_{13}\beta_{23}\beta_{45} + \alpha_{51}\beta_{13}\beta_{25}\beta_{45} +$$
$$\alpha_{43}\alpha_{51}\beta_{13}\beta_{23} + \alpha_{41}\alpha_{51}\beta_{13}\beta_{23} + \alpha_{41}\alpha_{54}\beta_{13}\beta_{23} + \alpha_{43}\alpha_{51}\beta_{13}\beta_{25} + \alpha_{41}\alpha_{51}\beta_{13}\beta_{25} + \alpha_{21}\alpha_{41}\alpha_{51}\beta_{13} + \alpha_{41}\alpha_{54}\beta_{13}\beta_{25} +$$
$$\alpha_{21}\alpha_{41}\alpha_{54}\beta_{13} + \alpha_{21}\alpha_{43}\alpha_{51}\beta_{13} + \alpha_{21}\alpha_{51}\beta_{13}\beta_{45} + \alpha_{21}\alpha_{43}\alpha_{54}\beta_{13} + \alpha_{43}\alpha_{54}\beta_{13}\beta_{23} + \alpha_{43}\alpha_{54}\beta_{13}\beta_{25}) +$$
$$(\alpha_{21}\alpha_{31}\beta_{14}\beta_{45} +$$
$$\alpha_{31}\beta_{14}\beta_{23}\beta_{45} + \alpha_{31}\beta_{14}\beta_{25}\beta_{45} + \alpha_{21}\alpha_{32}\beta_{14}\beta_{45} + \alpha_{32}\alpha_{43}\beta_{14}\beta_{25} + \alpha_{32}\beta_{14}\beta_{25}\beta_{45} + \alpha_{21}\beta_{14}\beta_{34}\beta_{45} + \beta_{14}\beta_{23}\beta_{34}\beta_{45} +$$

$$\beta_{13}\beta_{25}\beta_{34}\beta_{45} + \alpha_{21}\alpha_{43}\beta_{14}\beta_{35} + \alpha_{21}\beta_{14}\beta_{35}\beta_{45} + \alpha_{43}\beta_{14}\beta_{23}\beta_{35} + \beta_{14}\beta_{23}\beta_{35}\beta_{45} + \alpha_{43}\beta_{14}\beta_{25}\beta_{35} + \beta_{14}\beta_{25}\beta_{35}\beta_{45} +$$

$$\beta_{12}\beta_{23}\beta_{34}\beta_{45} + \alpha_{21}\beta_{13}\beta_{34}\beta_{45} + \beta_{13}\beta_{23}\beta_{34}\beta_{45} + \beta_{13}\beta_{25}\beta_{34}\beta_{45} + \alpha_{41}\beta_{15}\beta_{23}\beta_{34} + \beta_{15}\beta_{23}\beta_{34}\beta_{45} + \alpha_{41}\beta_{12}\beta_{23}\beta_{35} +$$

$$\alpha_{43}\beta_{12}\beta_{23}\beta_{35} + \beta_{12}\beta_{23}\beta_{35}\beta_{45} + \alpha_{21}\alpha_{41}\beta_{13}\beta_{35} + \alpha_{21}\alpha_{43}\beta_{13}\beta_{35} + \alpha_{21}\beta_{13}\beta_{5}\beta_{45} + \alpha_{41}\beta_{13}\beta_{23}\beta_{35} + \alpha_{43}\beta_{13}\beta_{23}\beta_{35} +$$

$$\beta_{13}\beta_{23}\beta_{35}\beta_{45} + \alpha_{32}\alpha_{41}\beta_{13}\beta_{25} + \alpha_{41}\beta_{13}\beta_{25}\beta_{35} + \alpha_{32}\alpha_{43}\beta_{13}\beta_{25} + \alpha_{43}\beta_{13}\beta_{25}\beta_{35} + \alpha_{32}\beta_{13}\beta_{25}\beta_{45} + \beta_{13}\beta_{25}\beta_{35}\beta_{45} +$$

$$\alpha_{31}\alpha_{41}\beta_{12}\beta_{25} + \alpha_{21}\alpha_{43}\beta_{12}\beta_{25} + \alpha_{31}\beta_{12}\beta_{25}\beta_{45} + \alpha_{32}\alpha_{41}\beta_{12}\beta_{25} + \alpha_{32}\alpha_{43}\beta_{12}\beta_{25} + \alpha_{21}\alpha_{31}\alpha_{41}\beta_{15} + \alpha_{31}\alpha_{41}\beta_{15}\beta_{23} +$$

$$\alpha_{31}\alpha_{41}\beta_{15}\beta_{25} + \alpha_{21}\alpha_{31}\alpha_{43}\beta_{15} + \alpha_{31}\alpha_{43}\beta_{15}\beta_{23} + \alpha_{31}\alpha_{43}\beta_{15}\beta_{25} + \alpha_{21}\alpha_{31}\beta_{15}\beta_{45} + \alpha_{31}\beta_{15}\beta_{23}\beta_{45} + \alpha_{31}\beta_{15}\beta_{25}\beta_{45} +$$

$$\alpha_{21}\alpha_{32}\alpha_{41}\beta_{15} + \alpha_{32}\alpha_{41}\beta_{15}\beta_{25} + \alpha_{21}\alpha_{32}\alpha_{43}\beta_{15} + \alpha_{32}\alpha_{43}\beta_{15}\beta_{25} + \alpha_{21}\alpha_{32}\beta_{15}\beta_{45} + \alpha_{32}\beta_{15}\beta_{25}\beta_{45} + \alpha_{21}\alpha_{41}\beta_{15}\beta_{35} +$$

$$\alpha_{41}\beta_{15}\beta_{23}\beta_{35} + \alpha_{41}\beta_{15}\beta_{25}\beta_{35} + \alpha_{21}\alpha_{43}\beta_{15}\beta_{35} + \alpha_{43}\beta_{15}\beta_{23}\beta_{35} + \alpha_{43}\beta_{15}\beta_{25}\beta_{35} + \alpha_{21}\beta_{15}\beta_{35}\beta_{45} + \beta_{15}\beta_{23}\beta_{35}\beta_{45} +$$

$$\beta_{15}\beta_{25}\beta_{35}\beta_{45} + \alpha_{32}\beta_{12}\beta_{25}\beta_{45} + \alpha_{41}\beta_{12}\beta_{25}\beta_{35} + \alpha_{43}\beta_{12}\beta_{25}\beta_{35} + \beta_{12}\beta_{25}\beta_{35}\beta_{45} + \beta_{15}\beta_{25}\beta_{34}\beta_{45} + \alpha_{21}\alpha_{41}\beta_{15}\beta_{34} +$$

$$\alpha_{21}\beta_{15}\beta_{34}\beta_{45} + \beta_{12}\beta_{25}\beta_{34}\beta_{45} + \alpha_{41}\beta_{12}\beta_{25}\beta_{34} + \alpha_{41}\beta_{15}\beta_{25}\beta_{34})$$

$N_4 =$

$$\alpha_{21}\alpha_{43}\alpha_{54}\beta_{14} + \alpha_{43}\alpha_{54}\beta_{14}\beta_{25} + \alpha_{43}\alpha_{54}\beta_{12}\beta_{25} + \alpha_{21}\alpha_{43}\alpha_{54}\beta_{15} + \alpha_{21}\alpha_{43}\alpha_{51}\beta_{14} + \alpha_{41}\alpha_{51}\beta_{12}\beta_{23} + \alpha_{43}\alpha_{51}\beta_{12}\beta_{23} +$$

$$\alpha_{51}\beta_{12}\beta_{23}\beta_{45} + \alpha_{43}\alpha_{54}\beta_{15}\beta_{23} + \alpha_{43}\alpha_{54}\beta_{15}\beta_{25} + \alpha_{43}\alpha_{51}\beta_{14}\beta_{25} + \alpha_{43}\alpha_{51}\beta_{14}\beta_{23} + \alpha_{51}\beta_{13}\beta_{23}\beta_{45} + \alpha_{51}\beta_{13}\beta_{25}\beta_{45} +$$

$$\alpha_{43}\alpha_{51}\beta_{13}\beta_{23} + \alpha_{41}\alpha_{51}\beta_{13}\beta_{23} + \alpha_{41}\alpha_{54}\beta_{13}\beta_{23} + \alpha_{43}\alpha_{51}\beta_{13}\beta_{25} + \alpha_{41}\alpha_{51}\beta_{13}\beta_{25} + \alpha_{21}\alpha_{41}\alpha_{51}\beta_{13} + \alpha_{41}\alpha_{54}\beta_{13}\beta_{25} +$$

$$\alpha_{21}\alpha_{41}\alpha_{54}\beta_{13} + \alpha_{21}\alpha_{43}\alpha_{51}\beta_{13} + \alpha_{21}\alpha_{51}\beta_{13}\beta_{45} + \alpha_{21}\alpha_{43}\alpha_{54}\beta_{13} + \alpha_{43}\alpha_{54}\beta_{13}\beta_{23} + \alpha_{43}\alpha_{54}\beta_{13}\beta_{25}) +$$

$$(\alpha_{21}\alpha_{31}\beta_{14}\beta_{45} +$$

$$\alpha_{31}\beta_{14}\beta_{23}\beta_{45} + \alpha_{31}\beta_{14}\beta_{25}\beta_{45} + \alpha_{21}\alpha_{32}\beta_{14}\beta_{45} + \alpha_{32}\alpha_{43}\beta_{14}\beta_{25} + \alpha_{32}\beta_{14}\beta_{25}\beta_{45} + \alpha_{21}\beta_{14}\beta_{34}\beta_{45} + \beta_{14}\beta_{23}\beta_{34}\beta_{45} +$$

$$\beta_{13}\beta_{25}\beta_{34}\beta_{45} + \alpha_{21}\alpha_{43}\beta_{14}\beta_{35} + \alpha_{21}\beta_{14}\beta_{35}\beta_{45} + \alpha_{43}\beta_{14}\beta_{23}\beta_{35} + \beta_{14}\beta_{23}\beta_{35}\beta_{45} + \alpha_{43}\beta_{14}\beta_{25}\beta_{35} + \beta_{14}\beta_{25}\beta_{35}\beta_{45} +$$

$$\beta_{12}\beta_{23}\beta_{34}\beta_{45} + \alpha_{21}\beta_{13}\beta_{34}\beta_{45} + \beta_{13}\beta_{23}\beta_{34}\beta_{45} + \beta_{13}\beta_{25}\beta_{34}\beta_{45} + \alpha_{41}\beta_{15}\beta_{23}\beta_{34} + \beta_{15}\beta_{23}\beta_{34}\beta_{45} + \alpha_{41}\beta_{12}\beta_{23}\beta_{35} +$$

$$\alpha_{43}\beta_{12}\beta_{23}\beta_{35} + \beta_{12}\beta_{23}\beta_{35}\beta_{45} + \alpha_{21}\alpha_{41}\beta_{13}\beta_{35} + \alpha_{21}\alpha_{43}\beta_{13}\beta_{35} + \alpha_{21}\beta_{13}\beta_{5}\beta_{45} + \alpha_{41}\beta_{13}\beta_{23}\beta_{35} + \alpha_{43}\beta_{13}\beta_{23}\beta_{35} +$$

$$\beta_{13}\beta_{23}\beta_{35}\beta_{45} + \alpha_{32}\alpha_{41}\beta_{13}\beta_{25} + \alpha_{41}\beta_{13}\beta_{25}\beta_{35} + \alpha_{32}\alpha_{43}\beta_{13}\beta_{25} + \alpha_{43}\beta_{13}\beta_{25}\beta_{35} + \alpha_{32}\beta_{13}\beta_{25}\beta_{45} + \beta_{13}\beta_{25}\beta_{35}\beta_{45} +$$

$$\alpha_{31}\alpha_{41}\beta_{12}\beta_{25} + \alpha_{21}\alpha_{43}\beta_{12}\beta_{25} + \alpha_{31}\beta_{12}\beta_{25}\beta_{45} + \alpha_{32}\alpha_{41}\beta_{12}\beta_{25} + \alpha_{32}\alpha_{43}\beta_{12}\beta_{25} + \alpha_{21}\alpha_{31}\alpha_{41}\beta_{15} + \alpha_{31}\alpha_{41}\beta_{15}\beta_{23} +$$

$$\alpha_{31}\alpha_{41}\beta_{15}\beta_{25} + \alpha_{21}\alpha_{31}\alpha_{43}\beta_{15} + \alpha_{31}\alpha_{43}\beta_{15}\beta_{23} + \alpha_{31}\alpha_{43}\beta_{15}\beta_{25} + \alpha_{21}\alpha_{31}\beta_{15}\beta_{45} + \alpha_{31}\beta_{15}\beta_{23}\beta_{45} + \alpha_{31}\beta_{15}\beta_{25}\beta_{45} +$$

$$\alpha_{21}\alpha_{32}\alpha_{41}\beta_{15} + \alpha_{32}\alpha_{41}\beta_{15}\beta_{25} + \alpha_{21}\alpha_{32}\alpha_{43}\beta_{15} + \alpha_{32}\alpha_{43}\beta_{15}\beta_{25} + \alpha_{21}\alpha_{32}\beta_{15}\beta_{45} + \alpha_{32}\beta_{15}\beta_{25}\beta_{45} + \alpha_{21}\alpha_{41}\beta_{15}\beta_{35} +$$

$$\alpha_{41}\beta_{15}\beta_{23}\beta_{35} + \alpha_{41}\beta_{15}\beta_{25}\beta_{35} + \alpha_{21}\alpha_{43}\beta_{15}\beta_{35} + \alpha_{43}\beta_{15}\beta_{23}\beta_{35} + \alpha_{43}\beta_{15}\beta_{25}\beta_{35} + \alpha_{21}\beta_{15}\beta_{35}\beta_{45} + \beta_{15}\beta_{23}\beta_{35}\beta_{45} +$$

$$\beta_{15}\beta_{25}\beta_{35}\beta_{45} + \alpha_{32}\beta_{12}\beta_{25}\beta_{45} + \alpha_{41}\beta_{12}\beta_{25}\beta_{35} + \alpha_{43}\beta_{12}\beta_{25}\beta_{35} + \beta_{12}\beta_{25}\beta_{35}\beta_{45} + \beta_{15}\beta_{25}\beta_{34}\beta_{45} + \alpha_{21}\alpha_{41}\beta_{15}\beta_{34} +$$

$$\alpha_{21}\beta_{15}\beta_{34}\beta_{45} + \beta_{12}\beta_{25}\beta_{34}\beta_{45} + \alpha_{41}\beta_{12}\beta_{25}\beta_{34} + \alpha_{41}\beta_{15}\beta_{25}\beta_{34} + \alpha_{43}\alpha_{54}\beta_{14}\beta_{23} + \alpha_{41}\alpha_{54}\beta_{12}\beta_{23} + \alpha_{43}\alpha_{54}\beta_{12}\beta_{23}$$

$$D_1 = \alpha_{31}\alpha_{21}\alpha_{54}\beta_{14} + \alpha_{31}\alpha_{21}\beta_{14}\beta_{45} + \alpha_{31}\beta_{14}\beta_{23}\beta_{45} + \alpha_{31}\beta_{14}\beta_{25}\beta_{45} + \alpha_{21}\alpha_{32}\beta_{14}\beta_{45} + \alpha_{32}\alpha_{43}\beta_{14}\beta_{25} + \alpha_{32}\beta_{14}\beta_{25}\beta_{45} +$$

$$\alpha_{21}\beta_{14}\beta_{34}\beta_{45} + \beta_{14}\beta_{34}\beta_{23}\beta_{45} + \beta_{14}\beta_{34}\beta_{25}\beta_{45} + \alpha_{31}\alpha_{41}\beta_{12}\beta_{25} + \alpha_{31}\alpha_{43}\beta_{12}\beta_{25} + \alpha_{31}\beta_{12}\beta_{45}\beta_{25} + \alpha_{32}\alpha_{41}\beta_{12}\beta_{25} +$$

$$\alpha_{32}\alpha_{43}\beta_{12}\beta_{25} + \alpha_{32}\beta_{12}\beta_{45}\beta_{25} + \alpha_{41}\beta_{35}\beta_{12}\beta_{25} + \alpha_{43}\beta_{35}\beta_{12}\beta_{25} + \alpha_{51}\beta_{34}\beta_{12}\beta_{23} + \alpha_{51}\beta_{34}\beta_{25}\beta_{14} + \alpha_{41}\beta_{34}\beta_{12}\beta_{25} +$$

$$\alpha_{41}\alpha_{21}\beta_{34}\beta_{15} + \alpha_{41}\alpha_{54}\beta_{34}\beta_{12} + \alpha_{21}\alpha_{54}\beta_{34}\beta_{15} + \alpha_{54}\beta_{34}\beta_{25}\beta_{12} + \alpha_{21}\beta_{34}\beta_{15}\beta_{45} + \beta_{34}\beta_{12}\beta_{45}\beta_{25} + \alpha_{41}\alpha_{51}\beta_{34}\beta_{12} +$$

$$\alpha_{41}\beta_{34}\beta_{15}\beta_{25} + \alpha_{51}\beta_{34}\beta_{12}\beta_{45} + \alpha_{54}\beta_{34}\beta_{25}\beta_{15} + \alpha_{21}\alpha_{51}\beta_{34}\beta_{14} + \beta_{34}\beta_{15}\beta_{45}\beta_{25} + \alpha_{51}\beta_{13}\beta_{25}\beta_{45} + \alpha_{32}\alpha_{43}\alpha_{54}\beta_{13} +$$

$$\alpha_{41}\alpha_{51}\beta_{13}\beta_{32} + \alpha_{41}\alpha_{54}\beta_{13}\beta_{25} + \alpha_{54}\beta_{13}\beta_{35}\beta_{25} + \alpha_{51}\beta_{13}\beta_{23}\beta_{34} + \alpha_{43}\alpha_{54}\beta_{13}\beta_{23} + \alpha_{43}\alpha_{51}\beta_{13}\beta_{23} + \alpha_{41}\alpha_{51}\beta_{13}\beta_{25} +$$

$$\alpha_{32}\alpha_{43}\alpha_{51}\beta_{13} + \alpha_{21}\alpha_{51}\beta_{13}\beta_{34} + \alpha_{21}\alpha_{43}\alpha_{54}\beta_{13} + \alpha_{21}\alpha_{43}\alpha_{51}\beta_{13} + \alpha_{41}\alpha_{54}\beta_{13}\beta_{23} + \alpha_{21}\alpha_{41}\alpha_{54}\beta_{13} + \alpha_{43}\alpha_{51}\beta_{13}\beta_{25} +$$
$$\alpha_{51}\beta_{13}\beta_{34}\beta_{25} + \alpha_{21}\alpha_{41}\alpha_{51}\beta_{13} + \alpha_{21}\alpha_{51}\beta_{13}\beta_{45} + \alpha_{32}\alpha_{51}\beta_{13}\beta_{45} + \alpha_{51}\beta_{13}\beta_{23}\beta_{45} + \alpha_{32}\alpha_{41}\alpha_{54}\beta_{13} + \alpha_{43}\alpha_{54}\beta_{13}\beta_{25} +$$
$$\alpha_{41}\alpha_{51}\beta_{13}\beta_{23} + \alpha_{32}\alpha_{54}\beta_{13}\beta_{25} + \alpha_{21}\alpha_{54}\beta_{13}\beta_{35} + \alpha_{54}\beta_{13}\beta_{35}\beta_{23} + \alpha_{21}\alpha_{43}\beta_{14}\beta_{35} + \alpha_{21}\beta_{14}\beta_{35}\beta_{45} + \alpha_{43}\beta_{14}\beta_{35}\beta_{23} +$$
$$\beta_{14}\beta_{35}\beta_{23}\beta_{45} + \alpha_{43}\beta_{14}\beta_{35}\beta_{25} + \beta_{14}\beta_{35}\beta_{25}\beta_{45} + \beta_{23}\beta_{34}\beta_{12}\beta_{45} + \alpha_{21}\beta_{13}\beta_{34}\beta_{45} + \beta_{13}\beta_{34}\beta_{23}\beta_{45} + \beta_{13}\beta_{34}\beta_{25}\beta_{45} +$$
$$\alpha_{31}\alpha_{54}\beta_{14}\beta_{23} + \alpha_{31}\alpha_{54}\beta_{14}\beta_{25} + \alpha_{54}\alpha_{32}\alpha_{21}\beta_{14} + \alpha_{32}\alpha_{54}\beta_{14}\beta_{25} + \alpha_{21}\alpha_{54}\beta_{14}\beta_{34} + \alpha_{54}\beta_{14}\beta_{34}\beta_{23} + \alpha_{54}\beta_{14}\beta_{34}\beta_{25} +$$
$$\alpha_{21}\alpha_{54}\beta_{14}\beta_{35} + \alpha_{54}\beta_{14}\beta_{35}\beta_{23} + \alpha_{54}\beta_{14}\beta_{35}\beta_{25} + \alpha_{54}\beta_{23}\beta_{34}\beta_{12} + \alpha_{21}\alpha_{54}\beta_{13}\beta_{34} + \alpha_{54}\beta_{13}\beta_{34}\beta_{23} + \alpha_{54}\beta_{13}\beta_{34}\beta_{25} +$$
$$\alpha_{41}\beta_{23}\beta_{34}\beta_{15} + \beta_{23}\beta_{34}\beta_{15}\beta_{45} + \alpha_{51}\beta_{23}\beta_{34}\beta_{14} + \alpha_{54}\beta_{23}\beta_{34}\beta_{15} + \alpha_{41}\beta_{35}\beta_{12}\beta_{23} + \alpha_{43}\beta_{35}\beta_{12}\beta_{23} + \beta_{35}\beta_{12}\beta_{45}\beta_{23} +$$
$$\alpha_{21}\alpha_{41}\beta_{13}\beta_{35} + \alpha_{21}\alpha_{43}\beta_{13}\beta_{35} + \alpha_{21}\beta_{13}\beta_{45}\beta_{35} + \alpha_{41}\beta_{23}\beta_{13}\beta_{35} + \alpha_{43}\beta_{23}\beta_{13}\beta_{35} + \beta_{23}\beta_{13}\beta_{45}\beta_{35} + \alpha_{41}\alpha_{32}\beta_{25}\beta_{13} +$$
$$\alpha_{41}\beta_{25}\beta_{13}\beta_{35} + \alpha_{43}\alpha_{32}\beta_{25}\beta_{13} + \alpha_{43}\beta_{25}\beta_{13}\beta_{35} + \alpha_{32}\beta_{25}\beta_{13}\beta_{45} + \beta_{25}\beta_{13}\beta_{45}\beta_{35} + \alpha_{21}\alpha_{41}\alpha_{54}\beta_{34} + \alpha_{41}\alpha_{54}\beta_{23}\beta_{34} +$$
$$\alpha_{41}\alpha_{54}\beta_{25}\beta_{34} + \alpha_{21}\alpha_{41}\alpha_{51}\beta_{34} + \alpha_{21}\alpha_{51}\beta_{45}\beta_{34} + \alpha_{41}\alpha_{51}\beta_{23}\beta_{34} + \alpha_{51}\beta_{23}\beta_{45}\beta_{34} + \alpha_{41}\alpha_{51}\beta_{25}\beta_{34} + \alpha_{51}\beta_{25}\beta_{45}\beta_{34} +$$
$$\alpha_{31}\alpha_{21}\alpha_{41}\alpha_{54} + \alpha_{31}\alpha_{41}\alpha_{54}\beta_{23} + \alpha_{31}\alpha_{21}\alpha_{43}\alpha_{54} + \alpha_{32}\alpha_{43}\alpha_{54}\beta_{14} + \alpha_{41}\alpha_{54}\beta_{23}\beta_{35} + \alpha_{43}\alpha_{54}\beta_{23}\beta_{14} + \alpha_{31}\alpha_{43}\alpha_{54}\beta_{23} +$$
$$\alpha_{31}\alpha_{41}\alpha_{54}\beta_{25} + \alpha_{31}\alpha_{43}\alpha_{54}\beta_{25} + \alpha_{31}\alpha_{41}\alpha_{54}\beta_{12} + \alpha_{31}\alpha_{43}\alpha_{54}\beta_{12} + \alpha_{32}\alpha_{21}\alpha_{41}\alpha_{54} + \alpha_{32}\alpha_{21}\alpha_{43}\alpha_{54} + \alpha_{32}\alpha_{41}\alpha_{54}\beta_{25} +$$
$$\alpha_{32}\alpha_{41}\alpha_{54}\beta_{12} + \alpha_{32}\alpha_{43}\alpha_{54}\beta_{12} + \alpha_{21}\alpha_{41}\alpha_{54}\beta_{35} + \alpha_{41}\alpha_{54}\beta_{25}\beta_{35} + \alpha_{41}\alpha_{54}\beta_{12}\beta_{35} + \alpha_{41}\alpha_{54}\beta_{12}\beta_{23} + \alpha_{43}\alpha_{54}\beta_{23}\beta_{12} +$$
$$\alpha_{54}\beta_{12}\beta_{23}\beta_{35} + \alpha_{21}\alpha_{43}\alpha_{54}\beta_{14} + \alpha_{43}\alpha_{54}\beta_{14}\beta_{25} + \alpha_{31}\alpha_{21}\alpha_{43}\alpha_{51} + \alpha_{32}\alpha_{43}\alpha_{51}\beta_{14} + \alpha_{43}\alpha_{51}\beta_{23}\beta_{14} + \alpha_{31}\alpha_{43}\alpha_{51}\beta_{23} +$$
$$\alpha_{31}\alpha_{51}\beta_{23}\beta_{45} + \alpha_{31}\alpha_{41}\alpha_{51}\beta_{25} + \alpha_{31}\alpha_{43}\alpha_{51}\beta_{25} + \alpha_{31}\alpha_{51}\beta_{25}\beta_{45} + \alpha_{31}\alpha_{41}\alpha_{51}\beta_{12} + \alpha_{31}\alpha_{43}\alpha_{51}\beta_{12} + \alpha_{31}\alpha_{51}\beta_{12}\beta_{45} +$$
$$\alpha_{31}\alpha_{51}\beta_{14}\beta_{23} + \alpha_{31}\alpha_{41}\alpha_{51}\beta_{23} + \alpha_{32}\alpha_{21}\alpha_{41}\alpha_{51} + \alpha_{32}\alpha_{21}\alpha_{43}\alpha_{51} + \alpha_{32}\alpha_{21}\alpha_{51}\beta_{45} + \alpha_{32}\alpha_{41}\alpha_{51}\beta_{25} + \alpha_{32}\alpha_{43}\alpha_{51}\beta_{25} +$$
$$\alpha_{32}\alpha_{51}\beta_{25}\beta_{45} + \alpha_{32}\alpha_{41}\alpha_{51}\beta_{12} + \alpha_{32}\alpha_{43}\alpha_{51}\beta_{12} + \alpha_{32}\alpha_{51}\beta_{12}\beta_{45} + \alpha_{21}\alpha_{41}\alpha_{51}\beta_{35} + \alpha_{21}\alpha_{43}\alpha_{51}\beta_{35} + \alpha_{21}\alpha_{51}\beta_{35}\beta_{45} +$$
$$\alpha_{31}\alpha_{54}\beta_{25}\beta_{12} + \alpha_{32}\alpha_{54}\beta_{25}\beta_{12} + \alpha_{21}\alpha_{31}\alpha_{54}\beta_{15} + \alpha_{21}\alpha_{32}\alpha_{54}\beta_{15} + \alpha_{31}\alpha_{54}\beta_{23}\beta_{15} + \alpha_{54}\beta_{25}\beta_{35}\beta_{12} + \alpha_{31}\alpha_{54}\beta_{25}\beta_{15} +$$
$$\alpha_{43}\alpha_{54}\beta_{23}\beta_{15} + \alpha_{43}\alpha_{54}\beta_{25}\beta_{15} + \alpha_{43}\alpha_{51}\beta_{25}\beta_{14} + \alpha_{32}\alpha_{54}\beta_{25}\beta_{15} + \alpha_{41}\alpha_{51}\beta_{35}\beta_{23} + \alpha_{43}\alpha_{51}\beta_{35}\beta_{23} + \alpha_{51}\beta_{35}\beta_{23}\beta_{45} +$$
$$\alpha_{41}\alpha_{51}\beta_{35}\beta_{25} + \alpha_{43}\alpha_{51}\beta_{35}\beta_{25} + \alpha_{51}\beta_{35}\beta_{25}\beta_{45} + \alpha_{41}\alpha_{51}\beta_{35}\beta_{12} + \alpha_{43}\alpha_{51}\beta_{35}\beta_{12} + \alpha_{51}\beta_{35}\beta_{12}\beta_{45} + \alpha_{41}\alpha_{51}\beta_{23}\beta_{12} +$$
$$\alpha_{43}\alpha_{51}\beta_{23}\beta_{12} + \alpha_{51}\beta_{23}\beta_{12}\beta_{45} + \alpha_{21}\alpha_{54}\beta_{35}\beta_{15} + \alpha_{21}\alpha_{51}\beta_{14}\beta_{35} + \alpha_{21}\alpha_{31}\alpha_{51}\beta_{14} + \alpha_{54}\beta_{23}\beta_{35}\beta_{15} + \alpha_{51}\beta_{23}\beta_{14}\beta_{35} +$$
$$\alpha_{54}\beta_{25}\beta_{35}\beta_{15} + \alpha_{32}\alpha_{51}\beta_{25}\beta_{14} + \alpha_{51}\beta_{25}\beta_{14}\beta_{35} + \alpha_{31}\alpha_{51}\beta_{25}\beta_{14} + \alpha_{32}\alpha_{43}\alpha_{54}\beta_{15} + \alpha_{43}\alpha_{54}\beta_{25}\beta_{12} + \alpha_{21}\alpha_{43}\alpha_{54}\beta_{15} +$$
$$\alpha_{21}\alpha_{43}\alpha_{51}\beta_{14} + \alpha_{31}\alpha_{21}\alpha_{41}\alpha_{51} + \alpha_{32}\alpha_{21}\alpha_{51}\beta_{14} + \alpha_{31}\alpha_{21}\alpha_{51}\beta_{45} + \beta_{35}\beta_{12}\beta_{45}\beta_{25} + \alpha_{31}\alpha_{41}\alpha_{21}\beta_{15} + \alpha_{31}\alpha_{41}\beta_{15}\beta_{23} +$$
$$\alpha_{31}\alpha_{41}\beta_{15}\beta_{25} + \alpha_{31}\alpha_{43}\alpha_{21}\beta_{15} + \alpha_{31}\alpha_{43}\beta_{15}\beta_{23} + \alpha_{31}\alpha_{43}\beta_{15}\beta_{25} + \alpha_{31}\alpha_{21}\beta_{15}\beta_{45} + \alpha_{31}\beta_{15}\beta_{45}\beta_{23} + \alpha_{31}\beta_{15}\beta_{45}\beta_{25} +$$
$$\alpha_{32}\alpha_{41}\alpha_{21}\beta_{15} + \alpha_{32}\alpha_{41}\beta_{15}\beta_{25} + \alpha_{32}\alpha_{43}\alpha_{21}\beta_{15} + \alpha_{32}\alpha_{43}\beta_{15}\beta_{25} + \alpha_{32}\alpha_{21}\beta_{15}\beta_{45} + \alpha_{32}\beta_{15}\beta_{45}\beta_{25} + \alpha_{41}\alpha_{21}\beta_{35}\beta_{15} +$$
$$\alpha_{41}\beta_{35}\beta_{15}\beta_{23} + \alpha_{41}\beta_{35}\beta_{15}\beta_{25} + \alpha_{43}\alpha_{21}\beta_{35}\beta_{15} + \alpha_{43}\beta_{35}\beta_{15}\beta_{23} + \alpha_{43}\beta_{35}\beta_{15}\beta_{25} + \alpha_{21}\beta_{35}\beta_{15}\beta_{45} + \beta_{35}\beta_{15}\beta_{45}\beta_{23} +$$
$$\beta_{35}\beta_{15}\beta_{45}\beta_{25}$$

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