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Numerical Quenching Solutions of a Parabolic Equation Modeling Electrostatic Mems

N'guessan Koffi[1] and Diabate Nabongo[2]

[1,2]Universite Alassane Ouattara
UFR-SED, 16 BP 372 Abidjan 16, Cote d'Ivoire
[1]E-mail: nkrasoft@yahoo.fr
[2]E-mail: nabongo_diabate@yahoo.fr

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Abstract

In this paper, we study the semidiscrete approximation for the following initial-boundary value problem

$$\begin{cases} u_t(x,t) = u_{xx}(x,t) + \lambda f(x)(1-u(x,t))^{-p}, & -l < x < l, \quad t > 0, \\ u(-l,t) = 0, \quad u(l,t) = 0, & t > 0, \\ u(x,0) = u_0(x) \geq 0, & -l \leq x \leq l, \end{cases}$$

where $p > 1$, $\lambda > 0$ *and* $f(x) \in C^1([-l,l])$, *symmetric and nondecreasing on the interval* $(-l,0)$, $0 < f(x) \leq 1$, $f(-l) = 0$, $f(l) = 0$ *and* $l = \frac{1}{2}$. *We find some conditions under which the solution of a semidiscrete form of above problem quenches in a finite time and estimate its semidiscrete quenching time. Moreover, we prove that the semidiscrete solution must quench near the maximum point of the function* $f(x)$, *for* λ *sufficiently large. We also establish the convergence of the semidiscrete quenching time to the theoretical one when the mesh size tends to zero. Finally, we give some numerical experiments for a best illustration of our analysis.*

Keywords: *Convergence, electrostatic MEMS, parabolic equation, semidiscretizations, semidiscrete quenching time.*

1 Introduction

We consider the following initial-boundary value problem

$$u_t(x,t) = u_{xx}(x,t) + \lambda f(x)(1-u(x,t))^{-p}, \quad -l < x < l, \quad t > 0, \tag{1}$$

$$u(-l,t) = 0, \quad u(l,t) = 0, \quad t > 0, \tag{2}$$

$$u(x,0) = u_0(x) \geq 0, \quad -l \leq x \leq l, \tag{3}$$

where $p > 1$, $\lambda > 0$ and $f(x) \in C^1([-l,l])$, symmetric and nondecreasing on the interval $(-l,0)$, $0 < f(x) \leq 1$, $f(-l) = 0$, $f(l) = 0$, $l = \frac{1}{2}$, and $u_0(x)$ is a function which is bounded and symmetric. In addition, $u_0(x)$ is nondecreasing on the interval $(-l,0)$ and $u_0''(x) + \lambda f(x)(1-u_0(x))^{-p} \geq 0$ on $(-l,l)$.

Definition 1.1 *We say that the classical solution u of (1)–(3) quenches in a finite time if there exists a finite time T_q such that $\|u(\cdot,t)\|_\infty < 1$ for $t \in [0,T_q)$ but*

$$\lim_{t\to T_q} \|u(\cdot,t)\|_\infty = 1,$$

where $\|u(\cdot,t)\|_\infty = \max_{-l\leq x\leq l} |u(x,t)|$. The time T_q is called the quenching time of the solution u.

The above problem models the dynamic deflection of an elastic membrane in a simple electrostatic Micro-Electromechanical System(MEMS) device. The parameter λ characterizes the relative strengh of the electrostatic and mechanical forces in the system and is given in terms of applied voltage. The function $f(x)$ represent the varying dielectric permittivity profile. Micro-Electromechanical System(MEMS) is the integration of mechanical elements, sensors, actuators, and electronics on a common silicon substrate through microfabrication technology. Micro-Electromechanical System(MEMS) is arguably the hottest topic in ingineering today. Four decades of advances in this direction, including the development of planar batch fabrication methods, the invention of the scanning-tunnelling and atomic-forces microscopes, and the discovery of the carbon nanotube. Typically a Micro-Electromechanical System(MEMS) device consists of an elastic membrane held at a constant voltage and suspended above a rigid ground plate placed in series with a fixed voltage source. The voltage difference causes a deflection of the membrane, which in turn generates an elastic field in the region between the plate and the membrane. An important nonlinear phenomenon in electrostatically deflected membranes in the so called "pull-in" instability. For moderate voltages, the system is in the stable operation regime: the membrane approaches a steady state and remains separate from the ground plate. When the voltage is increased beyond a critical-value, there is no longer an equilibrium configuration of the membrane.

As a result, the membrane collapses onto the ground plate. This phenomenon is also known as "touchdown" or quenching. The critical value of the voltage required for touchdown to occur is termed the pull-in value. (see [23], [24] and the references therein).

The theoretical analysis of quenching solutions for parabolic equations has been investigated by many authors (see [3], [6], [9], [11], [12],[13], [16], [17] and the references cited therein). Local in time existence and the uniqueness of a classical solution have been proved. In particular in [13], the authors have considered the problem (1)–(3) on a bounded domain Ω of $\mathbb{R}^N$ with $p = 2$. They have proved that under some assumptions, the solution of problem quenches in a finite time and the quenching time is estimated. This paper concerns the numerical study of the phenomenon of quenching, using a semidiscrete form of the problem (1)–(3). We obtain some conditions, under which, the solution of a semidiscrete form of (1)–(3) quenches in a finite time and estimate its semidiscrete quenching time. We also establish the convergence of the semidiscrete quenching time to the theoretical one when the mesh size tends to zero. One may find in [2],[20] and [21], some results concerning the numerical approximations of quenching solutions. A similar study has been undertaken in [1] for the phenomenon of blow-up(we say that a solution blows up in a finite time if it attains the value infinity in a finite time) where the authors have considered the problem (1)–(3) in the case where the reaction term $\lambda f(x)(1-u(x,t))^{-p}$ is replaced by $(u(x,t))^p$ with $p > 1$. In the same way in [2] the numerical extinction has been studied using some discrete and semidiscrete schemes (we say that a solution u extincts in a finite time if it reaches the value zero in a finite time).

Our paper is structured as follows. In the next section, we give some lemmas which will be used throughout the paper. In the third section, under some hypotheses, we show that the solution of the semidiscrete problem quenches in a finite time and estimate its semidiscrete quenching time. In the fourth section, we give a result about the convergence of the semidiscrete quenching time to the theoretical one when the mesh size goes to zero. Finally, in the last section, we give some numerical results to illustrate our analysis.

2 Properties of the Semidiscrete Scheme

In this section, we give some lemmas which will be used throughout the paper. Let us begin with the construction of a semidiscrete scheme. Let I be a positive integer, and consider the grid $x_i = -l + ih$, $0 \leq i \leq I$, where $h = 2l/I$. We approximate the solution u of (1)–(3) by the solution

$U_h(t) = (U_0(t), U_1(t), \ldots, U_I(t))^T$ of the following semidiscrete equations

$$\frac{dU_i(t)}{dt} = \delta^2 U_i(t) + \lambda b_i (1 - U_i(t))^{-p}, \quad 1 \le i \le I-1, \quad t \in (0, T_q^h), \tag{4}$$

$$U_0(t) = 0, \quad U_I(t) = 0, \quad t \in (0, T_q^h), \tag{5}$$

$$U_i(0) = \varphi_i \ge 0, \quad 0 \le i \le I, \tag{6}$$

where b_i is an approximation of $f(x_i)$, $0 \le i \le I$, $b_0 = 0$, $b_I = 0$, $0 < b_i \le 1$, $1 \le i \le I-1$ and

$$b_{I-i} = b_i, \quad 1 \le i \le I-1, \quad \delta^+ b_i > 0, \quad 1 \le i \le E[\frac{I}{2}] - 1,$$

$E[\frac{I}{2}]$ is the integer part of the number $I/2$,

$$\delta^2 U_i(t) = \frac{U_{i+1}(t) - 2U_i(t) + U_{i-1}(t)}{h^2}, \quad 1 \le i \le I-1,$$

$$\varphi_0 = 0, \quad \varphi_I = 0, \quad \varphi_i = \varphi_{I-i}, \quad 0 \le i \le I, \quad \delta^+ \varphi_i > 0, \quad 0 \le i \le E[\frac{I}{2}] - 1,$$

$$\delta^+ \varphi_i = \frac{\varphi_{i+1} - \varphi_i}{h}.$$

Here, $(0, T_q^h)$ is the maximal time interval on which $\|U_h(t)\|_\infty < 1$ where

$$\|U_h(t)\|_\infty = \max_{0 \le i \le I} |U_i(t)|.$$

When the time T_q^h is finite, then we say that the solution $U_h(t)$ of (4)–(6) quenches in a finite time, and the time T_q^h is called the quenching time of the solution $U_h(t)$.
The following lemma is a semidiscrete form of the maximum principle.

Lemma 2.1 *Let $\alpha_h \in C^0([0,T], \mathbb{R}^{I+1})$ and let $V_h \in C^1([0,T), \mathbb{R}^{I+1})$ be such that*

$$\frac{dV_i(t)}{dt} - \delta^2 V_i(t) + \alpha_i(t) V_i(t) \ge 0, \quad 1 \le i \le I-1, \quad t \in (0, T), \tag{7}$$

$$V_0(t) \ge 0, \quad V_I(t) \ge 0, \quad t \in (0, T), \tag{8}$$

$$V_i(0) \ge 0, \quad 0 \le i \le I. \tag{9}$$

Then $V_i(t) \ge 0$, $0 \le i \le I$, $t \in (0, T)$.

Proof. Let $T_0 < T$ and introduce the vector $Z_h(t) = e^{\lambda t}V_h(t)$ where λ is such that $\alpha_i(t) - \lambda > 0$ for $t \in [0, T_0]$, $0 \leq i \leq I$. Let

$$m = \min_{0 \leq i \leq I, 0 \leq t \leq T_0} Z_i(t).$$

For $i = 0, ..., I$, the function $Z_i(t)$ is continue on the compact $[0, T_0]$. Then there exists $i_0 \in \{0, 1, ..., I\}$ and $t_0 \in [0, T_0]$ such that $m = Z_{i_0}(t_0)$.
If $i_0 = 0$ or $i_0 = I$, then $m \geq 0$. If $i_0 \in \{0, 1, ..., I-1\}$, we observe that

$$\frac{dZ_{i_0}(t_0)}{dt} = \lim_{k \to 0} \frac{Z_{i_0}(t_0) - Z_{i_0}(t_0 - k)}{k} \leq 0, \tag{10}$$

$$\delta^2 Z_{i_0}(t_0) = \frac{Z_{i_0+1}(t_0) - 2Z_{i_0}(t_0) + Z_{i_0-1}(t_0)}{h^2} \geq 0. \tag{11}$$

Due to (7), a straightforward computation reveals that

$$\frac{dZ_{i_0}(t_0)}{dt} - \delta^2 Z_{i_0}(t_0) + (\alpha_{i_0}(t_0) - \lambda) Z_{i_0}(t_0) \geq 0. \tag{12}$$

It follows from (10)–(11) that $(\alpha_{i_0}(t_0) - \lambda) Z_{i_0}(t_0) \geq 0$ which implies that $Z_{i_0}(t_0) \geq 0$ because $\alpha_{i_0}(t_0) - \lambda > 0$. We deduce that $V_h(t) \geq 0$ for $t \in [0, T_0]$ and the proof is complete.
Another form of the maximum principle for semidiscrete equations is the comparison lemma below.

Lemma 2.2 *Let $g \in C^0(\mathbb{R} \times \mathbb{R}, \mathbb{R})$. If $V_h, W_h \in C^1([0, T], \mathbb{R}^{I+1})$ are such that*

$$\frac{dV_i(t)}{dt} - \delta^2 V_i(t) + g(V_i(t), t) < \frac{dW_i(t)}{dt} - \delta^2 W_i(t) + g(W_i(t), t), \tag{13}$$

$$1 \leq i \leq I-1, \quad t \in (0, T),$$

$$V_0(t) < W_0(t), \quad V_I(t) < W_I(t), \quad t \in (0, T), \tag{14}$$

$$V_i(0) < W_i(0), \quad 0 \leq i \leq I, \quad t \in (0, T), \tag{15}$$

then $V_i(t) < W_i(t)$ for $0 \leq i \leq I$, $t \in (0, T)$.

Proof. Let $Z_h(t) = W_h(t) - V_h(t)$ and let t_0 be the first $t > 0$ such that $Z_i(t) > 0$ for $t \in [0, t_0)$, $0 \leq i \leq I$, but $Z_{i_0}(t_0) = 0$ for a certain $i_0 \in \{0, ..., I\}$.
If $i_0 = 0$ or $i_0 = I$, we have a contradiction because of (14).
If $i_0 \in \{1, ..., I-1\}$, we obtain

$$\frac{dZ_{i_0}(t_0)}{dt} = \lim_{k \to 0} \frac{Z_{i_0}(t_0) - Z_{i_0}(t_0 - k)}{k} \leq 0,$$

and

$$\delta^2 Z_{i_0}(t_0) = \frac{Z_{i_0+1}(t_0) - 2Z_{i_0}(t_0) + Z_{i_0-1}(t_0)}{h^2} \geq 0,$$

which implies that

$$\frac{dZ_{i_0}(t_0)}{dt} - \delta^2 Z_{i_0}(t_0) + g(W_{i_0}(t_0), t_0) - g(V_{i_0}(t_0), t_0) \leq 0.$$

This inequality contradicts (13) which ends the proof.
The next lemma shows that when i is between 1 and $I-1$, then $U_i(t)$ is positive where $U_h(t)$ is the solution of the semidiscrete problem.

Lemma 2.3 *Let U_h be the solution of (4)–(6). Then, we have*

$$U_i(t) > 0, \quad 1 \leq i \leq I-1, \quad t \in (0, T_q^h).$$

Proof. Assume that there exists a time $t_0 \in (0, T_q^h)$ such that $U_{i_0}(t_0) = 0$ for a certain $i_0 \in \{1, ..., I-1\}$. We observe that

$$\frac{dU_{i_0}(t_0)}{dt} = \lim_{k \to 0} \frac{U_{i_0}(t_0) - U_{i_0}(t_0 - k)}{k} \leq 0, \tag{16}$$

$$\delta^2 U_{i_0}(t_0) = \frac{U_{i_0+1}(t_0) - 2U_{i_0}(t_0) + U_{i_0-1}(t_0)}{h^2} \geq 0, \tag{17}$$

which implies that

$$\frac{dU_{i_0}(t_0)}{dt} - \delta^2 U_{i_0}(t_0) - \lambda b_{i_0}(1 - U_{i_0}(t_0))^{-p} < 0. \tag{18}$$

But this contradicts (4).

Lemma 2.4 *Let U_h be the solution of (4)–(6). Then we have*

$$\frac{d}{dt} U_i(t) > 0, \quad 1 \leq i \leq I-1, \quad t \in (0, T_q^h). \tag{19}$$

Proof. Setting $W_i(t) = \frac{d}{dt} U_i(t)$, $1 \leq i \leq I-1$, it is not hard to see that

$$\frac{d}{dt} W_i(t) = \delta^2 W_i(t) + \lambda b_i p (1 - U_i(t))^{-p-1} W_i(t), \ 1 \leq i \leq I-1, \ t \in (0, T_q^h), \tag{20}$$

$$W_0(t) = 0, \quad W_I(t) = 0, \quad t \in (0, T_q^h), \tag{21}$$

$$W_i(0) > 0, \quad 1 \leq i \leq I-1, \tag{22}$$

Let t_0 be the first $t > 0$ such that $W_{i_0}(t_0) = 0$ for a certain $i_0 \in \{1, ..., I-1\}$. Without out loss of generality, we may suppose that i_0 is the smallest i_0 which ensures the equality. We get

$$\frac{dW_{i_0}(t_0)}{dt} = \lim_{k \to 0} \frac{W_{i_0}(t_0) - W_{i_0}(t_0 - k)}{k} \leq 0, \tag{23}$$

$$\delta^2 W_{i_0}(t_0) = \frac{W_{i_0+1}(t_0) - 2W_{i_0}(t_0) + W_{i_0-1}(t_0)}{h^2} \geq 0, \tag{24}$$

which guarantees that

$$\frac{dW_{i_0}(t_0)}{dt} - \delta^2 W_{i_0}(t_0) - \lambda b_{i_0} p(1 - U_{i_0}(t_0))^{-p-1} W_{i_0}(t_0) < 0. \tag{25}$$

Therefore, we have a contradiction because of (20).
The following lemma reveals that the solution $U_h(t)$ of the semidiscrete problem is symmetric and $\delta^+ U_i(t)$ is positive when i is between 1 and $E[\frac{I}{2}] - 1$.

Lemma 2.5 *Let U_h be the solution of (4)–(6). Then, we have for $t \in (0, T_q^h)$*

$$U_{I-i}(t) = U_i(t),\ 0 \leq i \leq I, \quad \delta^+ U_i(t) > 0,\ 0 \leq i \leq E[\frac{I}{2}] - 1. \tag{26}$$

Proof. Consider the vector V_h defined as follows $V_i(t) = U_{I-i}(t)$ for $0 \leq i \leq I$. For $i = 0$, then we have $V_0(t) = U_{I-0}(t) = U_I(t) = 0$, and $i = I$, then we also have $V_I(t) = U_{I-I}(t) = U_0(t) = 0$. For $i \in \{1, ..., I-1\}$, it follows that

$$\frac{dU_{I-i}(t)}{dt} = \delta^2 U_{I-i}(t) + \lambda b_{I-i}(1 - U_{I-i}(t))^{-p}, \quad 1 \leq i \leq I-1, \quad t \in (0, T_q^h).$$

If we replace $U_{I-i}(t)$ by $V_i(t)$ and use the fact that $b_{I-i} = b_i$, we obtain

$$\frac{dV_i(t)}{dt} - \delta^2 V_i(t) = \lambda b_i(1 - V_i(t))^{-p}, \quad 1 \leq i \leq I-1, \quad t \in (0, T_q^h),$$

which implies that $V_h(t)$ is a solution of (4)–(6).
Define the vector $W_h(t)$ such that $W_h(t) = U_h(t) - V_h(t)$. It is not hard to see that there exists $\theta_i \in (U_i, V_i)$ such that

$$\frac{dW_i}{dt} - \delta^2 W_i + p\lambda b_i(1 - \theta_i(t))^{-p-1} W_i = 0,\ 1 \leq i \leq I-1,\ t \in (0, T_q^h),$$

$$W_0(t) = 0, \quad W_I(t) = 0, \quad t \in (0, T_q^h),$$

$$W_i(0) = 0, \quad 0 \leq i \leq I.$$

From Lemma 2.1, it follows that

$$W_i(t) = 0 \quad \text{for} \quad 0 \leq i \leq I, \quad t \in (0, T_q^h),$$

which implies that $V_h(t) = U_h(t)$.
Now, define the vector $Z_h(t)$ such that

$$Z_i(t) = U_{i+1}(t) - U_i(t), \quad 0 \leq i \leq E[\frac{I}{2}] - 1,$$

and let t_0 be the first $t > 0$ such that $Z_i(t) > 0$ for $t \in [0, t_0)$ but $Z_{i_0}(t_0) = 0$. Without loss of the generality, we assume that i_0 is the smallest integer such that $Z_{i_0}(t_0) = 0$.
If $i_0 = 0$ then we have $U_1(t_0) = U_0(t_0) = 0$, which is a contradiction because from Lemma 2.3. $U_1(t_0) > 0$.
If $i_0 = 1, ..., E[\frac{I}{2}] - 2$, we have

$$\frac{dZ_{i_0}(t_0)}{dt} = \lim_{k\to 0} \frac{Z_{i_0}(t_0) - Z_{i_0}(t_0 - k)}{k} \leq 0, \quad \text{if} \quad 1 \leq i_0 \leq E[\frac{I}{2}] - 1. \tag{27}$$

and

$$\delta^2 Z_{i_0}(t_0) = \frac{(U_{i_0+2}(t_0) - U_{i_0+1}(t_0)) - 2(U_{i_0+1}(t_0) - U_{i_0}(t_0)) + (U_{i_0}(t_0) - U_{i_0-1}(t_0))}{h^2} > 0,$$

which implies that

$$\frac{dZ_{i_0}(t_0)}{dt} - \delta^2 Z_{i_0}(t_0) - \lambda b_{i_0+1}(1 - U_{i_0+1}(t_0))^{-p} + \lambda b_{i_0}(1 - U_{i_0}(t_0))^{-p} < 0,$$

But this contradicts (4).
If $i_0 = E[\frac{I}{2}] - 1$,

$$U_{i_0+2}(t_0) = U_{E[\frac{I}{2}]+1}(t_0) = U_{I-E[\frac{I}{2}]-1}(t_0).$$

-If I is even then $U_{i_0+2}(t_0) = U_{E[\frac{I}{2}]-1} = U_{i_0}(t_0)$ which implies that $\delta^2 Z_{i_0}(t_0) = \frac{(U_{i_0} - U_{i_0-1})(t_0)}{h^2} = \frac{Z_{i_0-1}(t_0)}{h^2} > 0$.
-If I is odd then $U_{i_0+2}(t_0) = U_{I-E[\frac{I-1}{2}]-1} = U_{E[\frac{I+1}{2}]-1}(t_0) = U_{i_0+1}(t_0)$, which leads to $\delta^2 Z_{i_0}(t_0) = \frac{(U_{i_0} - U_{i_0-1})(t_0)}{h^2} = \frac{Z_{i_0-1}(t_0)}{h^2} > 0$. It is easy to see that

$$\frac{dZ_{i_0}(t_0)}{dt} - \delta^2 Z_{i_0}(t_0) - \lambda b_{i_0+1}(1 - U_{i_0+1}(t_0))^{-p} + \lambda b_{i_0}(1 - U_{i_0}(t_0))^{-p} < 0,$$

which contradicts (4). This ends the proof.
The following lemma is the discrete version of the Green's formula.

Lemma 2.6 *Let $U_h, V_h \in \mathbb{R}^{I+1}$ be two vectors such that $U_0 = 0$, $U_I = 0$, $V_0 = 0$, $V_I = 0$. Then, we have*

$$\sum_{i=1}^{I-1} hU_i \delta^2 V_i = \sum_{i=1}^{I-1} hV_i \delta^2 U_i. \tag{28}$$

Proof. A routine calculation yields

$$\sum_{i=1}^{I-1} hU_i \delta^2 V_i = \sum_{i=1}^{I-1} hV_i \delta^2 U_i + \frac{V_I U_{I-1} - U_I V_{I-1} + V_0 U_1 - U_0 V_1}{h},$$

and using the assumptions of the lemma, we obtain the desired result.
Now, let us state a result on the operator δ^2.

Lemma 2.7 *Let $U_h \in \mathbb{R}^{I+1}$ be such that $\|U_h\|_\infty < 1$ and let p be a positive constant. Then, we have*

$$\delta^2(1-U_i)^{-p} \geq p(1-U_i)^{-p-1}\delta^2 U_i \quad for \quad 1 \leq i \leq I-1.$$

Proof. Using Taylor's expansion, we get

$$\begin{aligned}\delta^2(1-U_i)^{-p} = p(1-U_i)^{-p-1}\delta^2 U_i + (U_{i+1}-U_i)^2\frac{p(p+1)}{2h^2}(1-\theta_i)^{-p-2} \\ +(U_{i-1}-U_i)^2\frac{p(p+1)}{2h^2}(1-\eta_i)^{-p-2}\text{if} \quad 1 \leq i \leq I-1,\end{aligned}$$

where θ_i is an intermediate value between U_i and U_{i+1} and η_i the one between U_i and U_{i-1}. The result follows taking into account the fact that $\|U_h\|_\infty < 1$. To end this section, let us give another property of the operator δ^2.

Lemma 2.8 *Let $U_h, V_h \in \mathbb{R}^{I+1}$. If*

$$\delta^+(U_i)\delta^+(V_i) \geq 0, \quad and \quad \delta^-(U_i)\delta^-(V_i) \geq 0, \quad 1 \leq i \leq I-1. \tag{29}$$

then

$$\delta^2(U_iV_i) \geq U_i\delta^2(V_i) + V_i\delta^2(U_i), \quad 1 \leq i \leq I-1,$$

where $\delta^+(U_i) = \frac{U_{i+1}-U_i}{h}$ and $\delta^-(U_i) = \frac{U_{i-1}-U_i}{h}$.

Proof. A straightforward computation yields

$$\begin{aligned} h^2\delta^2(U_iV_i) &= U_{i+1}V_{i+1} - 2U_iV_i + U_{i-1}V_{i-1} \\ &= (U_{i+1}-U_i)(V_{i+1}-V_i) + V_i(U_{i+1}-U_i) + U_i(V_{i+1}-V_i) \\ &+ U_iV_i - 2U_iV_i + (U_{i-1}-U_i)(V_{i-1}-V_i) + (U_{i-1}-U_i)V_i \\ &+ U_i(V_{i-1}-V_i) + U_iV_i, \quad 1 \leq i \leq I-1, \end{aligned}$$

which implies that

$$\delta^2(U_iV_i) = \delta^+(U_i)\delta^+(V_i) + \delta^-(U_i)\delta^-(V_i) + U_i\delta^2(V_i) + V_i\delta^2(U_i), \quad 1 \leq i \leq I-1.$$

Using (29), we obtain the desired result.

3 Quenching Solutions

In this section, we show that under some assumptions, the solution U_h of (4)–(6) quenches in a finite time and estimate its semidiscrete quenching time.

Theorem 3.1 *Let U_h be the solution of (4)–(6) and assume that there exists a constant $A \in (0, 1]$ such that the initial datum at (6) satisfies*

$$\delta^2\varphi_i + \lambda b_i(1-\varphi_i)^{-p} \geq A\sin(ih\pi)(1-\varphi_i)^{-p}, \quad 0 \leq i \leq I, \tag{30}$$

and

$$1 - \frac{2\pi^2}{A(p+1)}(1-\|\varphi_h\|_\infty)^{p+1} > 0. \tag{31}$$

Then, the solution U_h quenches in a finite time T_q^h and we have the following estimate

$$T_q^h \leq -\frac{1}{\pi^2}\ln(1 - \frac{2\pi^2}{A(p+1)}(1-\|\varphi_h\|_\infty)^{p+1}).$$

Proof. Let $(0, T_q^h)$ be the maximal time interval on which $\|U_h(t)\|_\infty < 1$. To prove the finite time quenching, we consider the function $J_h(t)$ defined as follows

$$J_i(t) = \frac{dU_i(t)}{dt} - C_i(t)(1-U_i(t))^{-p}, \quad 0 \leq i \leq I,$$

where $C_i(t) = Ae^{-\lambda_h t}\sin(ih\pi)$, with $\lambda_h = \frac{2-2\cos(\pi h)}{h^2}$. It is not hard to see that

$$\frac{d}{dt}C_i(t) - \delta^2 C_i(t) = 0, \tag{32}$$

$$C_{I-i}(t) = C_i(t), \quad 0 \leq i \leq I, \quad C_{i+1}(t) > C_i(t), \quad 0 \leq i \leq E[\frac{I}{2}] - 1. \tag{33}$$

From (26), (33), we get

$$\delta^+((1-U_i)^{-p})\delta^+(C_i) \geq 0, \quad \textit{and} \quad \delta^-((1-U_i)^{-p})\delta^-(C_i) \geq 0. \tag{34}$$

A straightforward computation reveals that

$$\begin{aligned} \frac{dJ_i}{dt} - \delta^2 J_i &= \frac{d}{dt}(\frac{dU_i}{dt} - \delta^2 U_i) - (1-U_i)^{-p}\frac{dC_i}{dt} - pC_i(1-U_i)^{-p-1}\frac{dU_i}{dt} \\ &+ \delta^2(C_i(1-U_i)^{-p}), \quad 1 \leq i \leq I-1. \end{aligned}$$

From (34), Lemmas 2.7 and 2.8, the last term on the right hand side of the equality $\delta^2(C_i(1-U_i)^{-p})$ is bounded from below by $(1-U_i)^{-p}\delta^2 C_i + p(1-U_i)^{-p-1}C_i\delta^2 U_i$. We deduce that

$$\begin{aligned} \frac{dJ_i(t)}{dt} - \delta^2 J_i(t) &\geq \frac{d}{dt}(\frac{dU_i(t)}{dt} - \delta^2 U_i(t)) - (1-U_i)^{-p}(\frac{dC_i(t)}{dt} - \delta^2 C_i(t)) \\ &- pC_i(t)(1-U_i)^{-p-1}(\frac{dU_i(t)}{dt} - \delta^2 U_i(t)), \ 1 \leq i \leq I-1. \end{aligned}$$

Using (4) and (32), we find that

$$\begin{aligned}\frac{dJ_i}{dt}-\delta^2 J_i &\geq b_i\lambda p(1-U_i)^{-p-1}\frac{dU_i}{dt}-b_i\lambda p(1-U_i)^{-p-1}C_i(1-U_i)^{-p},\\ &\quad 1\leq i\leq I-1,\end{aligned}$$

$$\begin{aligned}\frac{dJ_i}{dt}-\delta^2 J_i &\geq b_i\lambda p(1-U_i)^{-p-1}(\frac{dU_i}{dt}-C_i(1-U_i)^{-p}),\\ &\quad 1\leq i\leq I-1,\end{aligned}$$

We deduce that

$$\frac{dJ_i}{dt}-\delta^2 J_i\geq \lambda b_i(1-U_i)^{-p-1}J_i,\quad 1\leq i\leq I-1,\quad t\in(0,T_q^h).$$

It is not hard to see that $J_0(t)=0$, $J_I(t)=0$ and the relation (30) implies that $J_h(0)\geq 0$. It follows from Lemma 2.1 that $J_h(t)\geq 0$, which implies that

$$\frac{dU_i}{dt}\geq C_i(1-U_i)^{-p},\quad 0\leq i\leq I,\quad t\in(0,T_q^h).$$

From Lemma 2.5, $U_{E[\frac{I}{2}]}\geq U_i$ for $1\leq i\leq E[\frac{I}{2}]-1$. We also remark that $C_{E[\frac{I}{2}]}\geq C_i$ for $1\leq i\leq E[\frac{I}{2}]-1$. Using Taylor's expansion, we find that $\cos(h\pi)\geq 1-\pi^2\frac{h^2}{2}$, which implies that $\lambda_h\leq\pi^2$. Obviously $\sin(E[\frac{I}{2}]h\pi)\geq\frac{1}{2}$. We deduce that

$$\frac{dU_{E[\frac{I}{2}]}}{dt}\geq\frac{A}{2}e^{-\pi^2 t}(1-U_{E[\frac{I}{2}]})^{-p},\quad t\in(0,T_q^h).$$

This inequality can be rewritten as

$$(1-U_{E[\frac{I}{2}]})^p dU_{E[\frac{I}{2}]}\geq\frac{A}{2}e^{-\pi^2 t}dt,\quad t\in(0,T_q^h). \tag{35}$$

A simple integration of the inequality (35) over $(0,T_q^h)$ yields

$$\frac{A(1-e^{-\pi^2 T_q^h})}{2\pi^2}\leq\frac{(1-U_{E[\frac{I}{2}]}(0))^{p+1}}{p+1},$$

which implies that

$$e^{-\pi^2 T_q^h}\geq 1-\frac{2\pi^2}{A(p+1)}(1-U_{E[\frac{I}{2}]}(0))^{p+1}.$$

By using the inequality (31), we obtain

$$T_q^h\leq-\frac{1}{\pi^2}\ln(1-\frac{2\pi^2}{A(p+1)}(1-\|\varphi_h\|_\infty)^{p+1}).$$

We have the desired result.

Remark 3.2 *Therefore by integrating the inequality (35) over interval* (t_0, T_q^h), *we have*

$$T_q^h - t_0 \leq -\frac{1}{\pi^2}\ln(1-\frac{2\pi^2}{A(p+1)}e^{\pi^2 t_0}(1-\|U_h(t_0)\|_\infty)^{p+1}).$$

The Remark 3.2 is crucial to prove the convergence of the semidiscrete quenching time.
When the initial data is null(The membrane is initially undeflected and the voltage is suddenly applied to the upper surface of the membrane at time $t = 0$.), the hypotheses of Theorem 3.1 are satisfied if the parameter λ is large enough. The theorem below also show that λ is large enough, then the semidiscrete solution quenches in a finite time. In addition, in this case the restriction on λ is better than the one of Theorem 3.1.

Theorem 3.3 *Suppose that* $\lambda > \lambda_h \frac{p^p}{b_1(p+1)^{p+1}}$ *with* $\lambda_h = \frac{2-2\cos(\pi h)}{h^2}$. *Then the solution* $U_h(t)$ *of (4)–(6) quenches in a finite time* T_q^h *which is estimated as follows*

$$\frac{1}{\lambda(p+1)} \leq T_q^h \leq \frac{(p+1)^p}{b_1\lambda(p+1)^{p+1} - \lambda_h p^p}.$$

Proof. Since $(0, T_q^h)$ is the maximal time interval on which $\|U_h(t)\|_\infty < 1$, our aim is to show that T_q^h is finite and satisfies the above inequality. From (4), we observe that

$$\frac{dU_i(t)}{dt} \geq \delta^2 U_i(t) + b_1\lambda(1-U_i(t))^{-p}, \quad 1 \leq i \leq I-1, \quad t \in (0, T_q^h).$$

Let a vector $W_h(t)$ such that

$$\frac{dW_i(t)}{dt} = \delta^2 W_i(t) + b_1\lambda(1-W_i(t))^{-p}, \quad 1 \leq i \leq I-1, \quad t \in (0, T_w^h).$$

$$W_0(t) = 0, \quad W_I(t) = 0, \quad t \in (0, T_w^h),$$

$$W_i(0) = 0, \quad 0 \leq i \leq I,$$

where T_w^h is the maximal existence time of $W_h(t)$.
Introduce the function $v(t)$ defined as follows

$$v(t) = \sum_{i=1}^{I-1} \tan(\frac{\pi}{2}h)\sin(i\pi h)W_i(t).$$

Take the derivative of v with respect to t and use (4) to obtain

$$v^{'}(t) = \sum_{i=1}^{I-1} \tan(\frac{\pi}{2}h)\sin(i\pi h)(\delta^2 W_i(t) + b_1\lambda(1-W_i(t))^{-p}.$$

We observe that $\delta^2 \sin(i\pi h) = -\lambda_h \sin(i\pi h)$. From the above equality and Lemma 2.5, we arrive at

$$v^{'}(t) = -\lambda_h v(t) + b_1\lambda \sum_{i=1}^{I-1} \tan(\frac{\pi}{2}h)\sin(i\pi h)(1-W_i(t))^{-p}.$$

By a routine calculation, we find that $\sum_{i=1}^{I-1}\tan(\frac{\pi}{2}h)\sin(i\pi h)$ equals one. Due to the Jensen's Inequality, we get

$$v^{'}(t) \geq -\lambda_h v(t) + b_1\lambda(1-v(t))^{-p}.$$

It is not hard to see that $v(t)(1-v(t))^p$ is bounded from above by $\sup_{0\leq s\leq 1} s(1-s)^p = \frac{p^p}{(p+1)^{p+1}}$. We deduce that

$$v^{'}(t) \geq (b_1\lambda - \frac{\lambda_h p^p}{(p+1)^{p+1}})(1-v(t))^{-p},$$

which implies that

$$(1-v(t))^p dv \geq (b_1\lambda - \frac{\lambda_h p^p}{(p+1)^{p+1}})dt.$$

Integrating this inequality over $(0, T_w^h)$, we find $T_w^h \leq \frac{(p+1)^p}{b_1\lambda(p+1)^{p+1}-\lambda_h p^p}$. The maximum principle implies that $W_i(t) \leq U_i(t)$, $0 \leq i \leq I$, $t \in (0, T_0)$ where $T_0 = \min\{T_w^h, T_q^h\}$. Therefore, we have $T_w^h \geq T_q^h$ and $T_q^h \leq \frac{(p+1)^p}{b_1\lambda(p+1)^{p+1}-\lambda_h p^p}$.
To obtain the lower bound of the semidiscrete quenching time T_q^h, we consider the following differential equation

$$\begin{cases} \chi^{'}(t) = \lambda(1-\chi(t))^{-p}, \quad t>0, \quad p>1, \\ \chi(0) = 0. \end{cases}$$

The function $\chi(t)$ quenches in a finite time $T_\chi = \frac{1}{\lambda(p+1)}$. Introduce the vector $V_h(t)$ such that $V_i(t) = \chi(t)$, $0 \leq i \leq I$, $t \in (0, T_\chi)$. Setting $Z_h(t) = V_h(t) - U_h(t)$. It is not hard to see that there exists $\theta_i \in (U_i, V_i)$ such that

$$\frac{dZ_i(t)}{dt} - \delta^2 Z_i(t) + p\lambda b_i(1-\theta_i(t))^{-p-1}Z_i(t) \geq 0,\ 1 \leq i \leq I-1,\ t \in (0, T_1),$$

$$Z_0(t) \geq 0, \quad Z_I(t) \geq 0, \quad t \in (0, T_1),$$

$$Z_i(0) \geq 0, \quad 0 \leq i \leq I.$$

where $T_1 = \min\{T_\chi, T_q^h\}$. From Lemma 2.1, it follows that $V_h(t) \geq U_h(t)$ for $0 \leq i \leq I$, $t \in (0, T_1)$. Therefore, we have $T_\chi \leq T_q^h$ and $T_q^h \geq \frac{1}{\lambda(p+1)}$.

4 Convergence of Semidiscrete Quenching Times

In this section, under adequate hypotheses, we show the convergence of the semidiscrete quenching time to the theoretical one when the mesh size goes to zero. We denote by

$$u_h(t) = (u(x_0,t),...,u(x_I,t))^T, \ f_h = (f(x_0),...,f(x_I))^T \text{ and } b_h = (b_0,...,b_I)^T.$$

In order to prove this result, firstly, we need the following theorem.

Theorem 4.1 *Assume that (1)-(3) has a solution* $u \in C^{4,1}([-l,l]\times[0,T-\tau])$ *such that* $\sup_{t\in[0,T-\tau]}\|u(\cdot,t)\|_\infty = \alpha < 1$ *with* $\tau \in (0,T)$. *Suppose that the initial datum at (6) and the exponent at (4) satisfy respectively*

$$\|\varphi_h - u_h(0)\|_\infty = o(1) \quad and \quad \|b_h - f_h\|_\infty = o(1) \quad as \quad h \to 0. \tag{36}$$

Then, for h sufficiently small, the problem (4)–(6) has a unique solution $U_h \in C^1([0,T_q^h),\mathbb{R}^{I+1})$ *such that*

$$\max_{0\le t\le T-\tau}\|U_h(t)-u_h(t)\|_\infty = O(\|\varphi_h - u_h(0)\|_\infty + \|b_h - f_h\|_\infty + h^2) \quad as \quad h\to 0.$$

Proof. Let $K>0$, $L>0$ and $M>0$ such that

$$\frac{\|u_{xxxx}\|_\infty}{12} \le K,\ p\lambda\|b_h\|_\infty(1-\frac{\alpha}{2})^{-p-1} \le M, \lambda(1-\frac{\alpha}{2})^{-p-1} \le L. \tag{37}$$

The problem (4)–(6) has for each h, a unique solution $U_h \in C^1([0,T_q^h),\mathbb{R}^{I+1})$. Let $t(h) \le \min\{T-\tau, T_q^h\}$ be the greatest value of $t>0$. There exists a positive real β (with $\alpha<\beta<1$) such that

$$\|U_h(t)-u_h(t)\|_\infty < \frac{\beta-\alpha}{2} \quad \text{for} \quad t\in(0,t(h)). \tag{38}$$

From (36), we deduce that $t(h)>0$ for h sufficiently small. By the triangle inequality, we obtain

$$\|U_h(t)\|_\infty \le \|u(\cdot,t)\|_\infty + \|U_h(t)-u_h(t)\|_\infty \quad \text{for} \quad t\in(0,t(h)),$$

which implies that

$$\|U_h(t)\|_\infty < \alpha + \frac{\beta-\alpha}{2} = \frac{\beta+\alpha}{2} < 1 \quad \text{for} \quad t\in(0,t(h)). \tag{39}$$

Let $e_h(t) = U_h(t) - u_h(t)$ be the error of discretization. Using Taylor's expansion, we have for $t\in(0,t(h))$,

$$\begin{aligned}\frac{de_i(t)}{dt} - \delta^2 e_i(t) &= \frac{h^2}{12}u_{xxxx}(\widetilde{x}_i,t) + p\lambda b_i(1-\xi_i)^{-p-1}e_i(t)\\ &+ \lambda(b_i - f(x_i))(1-u(x_i,t))^{-p}, \quad 1\le i\le I-1,\end{aligned}$$

where ξ_i is an intermediate value between $U_i(t)$ and $u(x_i,t)$). Using (37) and (39), we arrive at

$$\frac{de_i(t)}{dt} - \delta^2 e_i(t) \le M|e_i(t)| + L\|b_h - f_h\|_\infty + Kh^2, \quad 1 \le i \le I-1. \tag{40}$$

Let $z_h(t)$ the vector defined by

$$z_i(t) = e^{(M+1)t}(\|\varphi_h - u_h(0)\|_\infty + L\|b_h - f_h\|_\infty + Kh^2), \ 0 \le i \le I.$$

A direct calculation yields

$$\frac{dz_i(t)}{dt} - \delta^2 z_i(t) > M|z_i(t)| + L\|b_h - f_h\|_\infty + Kh^2, \quad 1 \le i \le I-1, \quad t \in (0, t(h)),$$

$$z_0(t) > e_0(t), \quad z_I(t) > e_I(t), \quad t \in (0, t(h)),$$

$$z_i(0) > e_i(0), \quad 0 \le i \le I.$$

It follows from Lemma 2.2 that $z_i(t) > e_i(t)$ for $t \in (0, t(h))$, $0 \le i \le I$. By the same reasoning, we also prove that $z_i(t) > -e_i(t)$ for $t \in (0, t(h))$, $0 \le i \le I$, which implies that

$$z_i(t) > |e_i(t)|, \quad 0 \le i \le I, \quad t \in (0, t(h)).$$

We deduce that

$$\|U_h(t) - u_h(t)\|_\infty \le e^{(M+1)t}\|\varphi_h - u_h(0)\|_\infty + L\|b_h - f_h\|_\infty + K^2h), \quad t \in (0, t(h)).$$

In order to show that $t(h) = \min\{T - \tau, T_q^h\}$, we argue by contradiction. Suppose that $t(h) < \min\{T - \tau, T_q^h\}$. From (38), we obtain

$$\frac{\beta - \alpha}{2} \le \|U_h(t(h)) - u_h(t(h))\|_\infty \le e^{(M+1)T}(\|\varphi_h - u_h(0)\|_\infty + L\|b_h - p_h\|_\infty + Kh^2). \tag{41}$$

We remark that when h tends to zero, $\frac{\beta-\alpha}{2} \le 0$, which is impossible. Consequently $t(h) = \min\{T - \tau, T_q^h\}$. Let us show that $t(h) = T - \tau$. Suppose that $t(h) = T_q^h < T - \tau$. Arguing as above, we obtain a contradiction, which leads us to the desired result.
Now, we prove the main result of this section, the convergence of the quenching time.

Theorem 4.2 *Suppose that the problem (1)–(3) has a solution u which quenches in a finite time T_q such that $u \in C^{4,1}([-l, l] \times [0, T_q))$ and the initial datum at (6) and the exponent at (4) satisfy the hypothesis (36). Under the assumptions of Theorem 3.1, the problem (4)–(6) has a solution U_h which quenches in a finite time T_q^h and $\lim_{h \to 0} T_q^h = T_q$.*

Proof. Let $0 < \varepsilon < \frac{T_q}{2}$. There exists $\gamma = \beta - \alpha$ (with $0 < \alpha < \beta < 1$) such that

$$-\frac{1}{\pi^2} \ln(1 - \frac{2\pi^2}{A(p+1)} e^{\pi^2 T_q} (1-y)^{p+1}) \leq \frac{\varepsilon}{2} \quad \text{for} \quad y \in [1-\gamma, 1). \tag{42}$$

Since $\lim_{t \to T_q} \|u(\cdot,t)\|_\infty = 1$, there exist $T_1 < T_q$ and $|T_q - T_1| < \frac{\varepsilon}{2}$ such that $1 > \|u(\cdot,t)\|_\infty \geq 1 - \frac{\gamma}{2}$ for $t \in [T_1, T_q)$. From Theorem 4.1, the problem (4)–(6) has for h sufficiently small, the unique solution $U_h(t)$ such that $\|U_h(t) - u_h(t)\|_\infty < \frac{\gamma}{2}$ for $t \in [0, T_2]$ where $T_2 = \frac{T_1 + T_q}{2}$. Using the triangle inequality, we get

$$\|U_h(t)\|_\infty \geq \|u(\cdot,t)\|_\infty - \|U_h(t) - u_h(t)\|_\infty \geq 1 - \frac{\gamma}{2} - \frac{\gamma}{2} \quad \text{for} \quad t \in [T_1, T_2],$$

which implies that

$$\|U_h(t)\|_\infty \geq 1 - \gamma \quad \text{for} \quad t \in [T_1, T_2].$$

From Theorem 3.1, $U_h(t)$ quenches at time T_q^h. Using inequality (39) and the Remark 3.2, we arrive at

$$|T_q^h - T_1| \leq -\frac{1}{\pi^2} \ln(1 - \frac{2\pi^2}{A(p+1)} e^{\pi^2 T_1} (1 - \|U_h(T_1)\|_\infty)^{p+1}) \leq \frac{\varepsilon}{2},$$

it follows that

$$|T_q^h - T_q| \leq |T_q^h - T_1| + |T_1 - T_q| \leq \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon.$$

This complete the proof.

5 Numerical Results

In this section, we present some numerical approximations to the quenching time of the problem (1)–(3) in the case where $u_0(x) = 0$, $\lambda = 10$ and $f(x) = 16(x^2 - \frac{1}{4})^2$. Firstly, we consider the following explicit scheme

$$\frac{U_i^{(n+1)} - U_i^{(n)}}{\Delta t_n^e} = \frac{U_{i+1}^{(n)} - 2U_i^{(n)} + U_{i-1}^{(n)}}{h^2} + b_i \lambda (1 - U_i^{(n)})^{-p}, \quad 1 \leq i \leq I-1,$$

$$U_0^{(n)} = 0, \quad U_I^{(n)} = 0,$$

$$U_i^0 = 0, \quad 0 \leq i \leq I,$$

and secondly, we use the following implicit scheme

$$\frac{U_i^{(n+1)} - U_i^{(n)}}{\Delta t_n} = \frac{U_{i+1}^{(n+1)} - 2U_i^{(n+1)} + U_{i-1}^{(n+1)}}{h^2} + b_i\lambda(1 - U_i^{(n)})^{-p}, \quad 1 \leq i \leq I-1,$$

$$U_0^{(n+1)} = 0, \quad U_I^{(n+1)} = 0,$$

$$U_i^0 = 0, \quad 0 \leq i \leq I,$$

where $n \geq 0$, $b_i = 16(x_i^2 - \frac{1}{4})^2$, $\Delta t_n = h^2(1 - \|U_h^{(n)}\|_\infty)^{p+1}$, $\Delta t_n^e = \min\{\frac{h^2}{2}, \Delta t_n\}$ and $T^n = \sum_{j=0}^{n-1} \Delta t_j$.
In the following tables, in rows, we present the numerical quenching times, the numbers of iterations, CPU times and the orders of the approximations corresponding to meshes of 16, 32, 64, 128, 256, 512, 1024. The numerical quenching time $T^n = \sum_{j=0}^{n-1} \Delta t_j$ is computed at the first time when

$$|T^{n+1} - T^n| \leq 10^{-16}.$$

The order(s) of the method is computed from

$$s = \frac{\log((T_{4h} - T_{2h})/(T_{2h} - T_h))}{\log(2)}.$$

Table 1: Numerical quenching times, numbers of iterations, CPU times (seconds) and orders of the approximations obtained with the explicit Euler method

I	T^n	n	CPU_t	s
16	0.042302	281	-	-
32	0.041856	1102	-	-
64	0.041748	4262	-	2.04
128	0.041721	16401	-	2.02
256	0.041714	62919	2	2.01
512	0.041713	240716	13	2.01
1024	0.041712	918406	93	2.00

Table 2: Numerical quenching times, numbers of iterations, CPU times (seconds) and orders of the approximations obtained with the implicit Euler method

I	T^n	n	CPU_t	s
16	0.043417	279	1	-
32	0.042124	1090	1	-
64	0.041814	4216	1	2.04
128	0.041737	16216	1	2.01
256	0.041718	62178	3	2.01
512	0.041714	237750	20	2.01
1024	0.041713	906544	144	2.00

In the following, we also give some plots to illustrate our analysis. For the different plots, we used both explicit and implicit schemes in the case where $I = 16$. In figures 1 and 2 we can appreciate that the discrete solution is nondecreasing and reaches the value one at the middle node. In figures 3 and 4 we see that the approximation of $u(x, T)$ is nondecreasing and reaches the value one at the middle node. Here, T is the quenching time of the solution u. In figures 5 and 6 we observe that the approximation of $u(x, T)$ is nondecreasing and reaches the value one at the maximum point of $f(x)$. In figures 7 and 8, we remark that the numerical time decreases for the large values of λ.

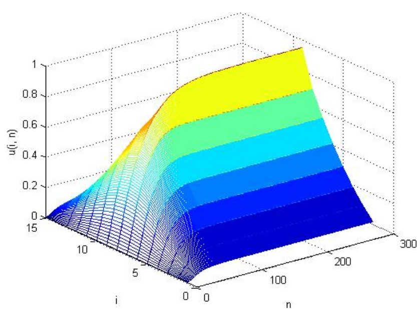

Figure 1: Evolution of the discrete solution(Explicit scheme).

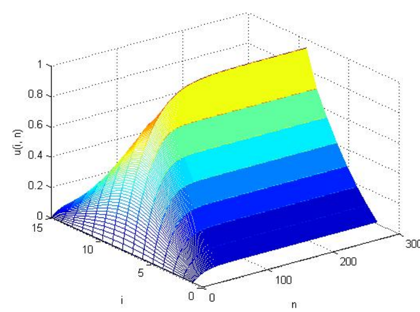

Figure 2: Evolution of the discrete solution(Implicit scheme).

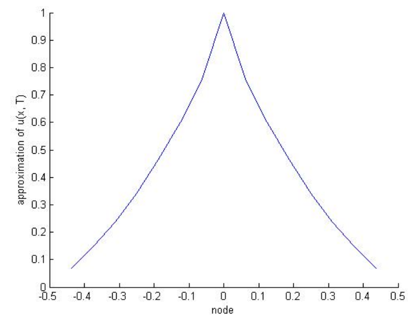

Figure 3: Profil of the approximation of $u(x,T)$ where, T is the quenching time (Explicit scheme).

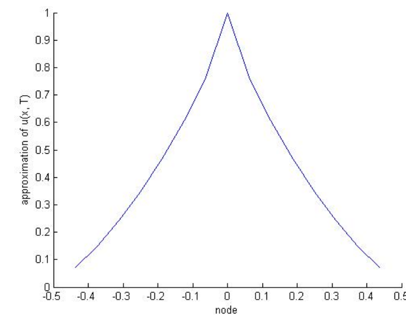

Figure 4: Profil of the approximation of $u(x,T)$ where, T is the quenching time (Implicit scheme).

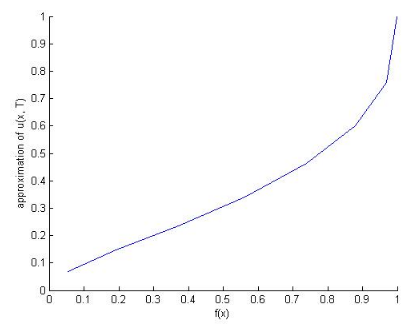

Figure 5: Graph of U against $f(x)$ (Explicit scheme).

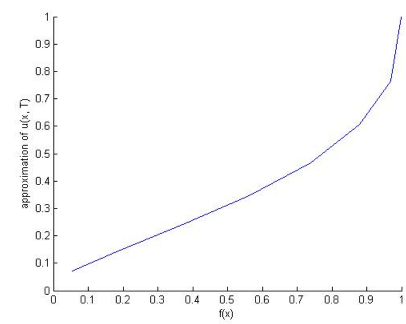

Figure 6: Graph of U against $f(x)$ (Implicit scheme).

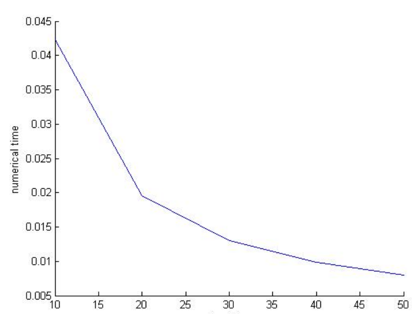

Figure 7: Graph of T against lambda where, T is the quenching time (Explicit scheme).

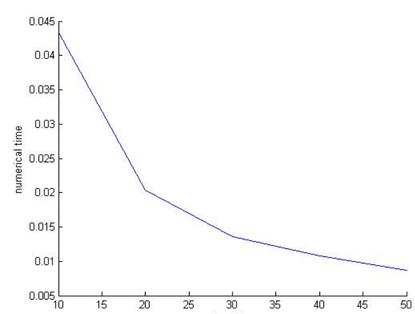

Figure 8: Graph of T against lambda where, T is the quenching time (Implicit scheme).

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