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On the Solution of Chance-Constrained Multiobjective Integer Quadratic Programming Problem with Some Stability Notions

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Abstract

In this study we present an algorithm for solving multiobjective integer quadratic programming problems having random parameters in the objective functions and in the constraints. Some basic stability notions are characterized for the problem of concern and the stability concept of this problem is introduced. An illustrative numerical example of bicriterion integer quadratic test problem under randomness is given.

Keywords: *Efficiency, Multiple criteria, Chance-constrained programming, Integer quadratic programming, Stability.*

1 Introduction

Decision problems of stochastic or probabilistic optimization arise when certain coefficients of an optimization model are random quantities. Stochastic multiobjective integer programs are challenging from both computational and theoretical points of view since they combine three different types of models into one. Until now algorithmic results have been limited to special instances.

In recent years methods of multiobjective stochastic optimization have become increasingly important in scientifically based decision-making involved in practical problems arising in practical problems in transportation, scheduling, agriculture, military purposes and technology [1-3]. We should point the reader to an extensive list of papers maintained by Maarten van der Vlerk at the Web Site: http://mally.eco.rug.nl/biblio/SPlist.html.

In literature there are many papers that deal with stability of solutions for stochastic multiobjective optimization problems. Among the many suggested approaches for treating stability of these problems [4-9].

In [4], a qualitative analysis of some basic notions of stochastic vector optimization problem with random parameters in the right-hand side of the constraints has been presented. These notions were the set of feasible parameters and the stability set of the first kind.

Also, the determination of the stability set of the first kind has been suggested when the right-hand side of the constraints are normally distributed. Theories and applications of stochastic multiobjective optimization problems have been introduced in [5]. The solution of chance-constrained multiobjective linear programming problems has been discussed in [6] together with a parametric study on the problem of concern. In [7], the author assumed a deterministic multiobjective programming problem which is approximated by surrogate problems based on estimations for the objective functions and the constraints. Making use of a large deviations approach, the behaviour was investigated for the constraint sets, the sets of efficient points and the solution sets if the size of the underlying sample tends to infinity. The results were illustrated by applying them to stochastic programming with chance constraints, where (i) the distribution function of the random variables is estimated by the empirical distribution function, (ii) certain parameters have to be estimated. A parametric study on stochastic multiobjective integer linear programming problems was presented in [8] and the stability of efficient solutions for such problems has been investigated. A comparison study has been given in [9] between fuzzy and stochastic approaches for solving multiobjective integer nonlinear programming problems. Moreover, the study of stability of efficient solutions for such problems in the decision space has been investigated.

In this study, we will extend the work in the sited papers [10, 11 and 12] for the author and with others to cover the case of multiobjective integer quadratic programming problem (MOIQP) and a stochastic approach is presented to treat these problems. The problem under consideration involves random parameters in the objective functions and in the constraints. In Section 2, the mathematical formulation of (MOIQP) is introduced. Some basic stability notions such as the set of feasible parameters; the solvability set and the stability set of the first kind are characterized for (MOIQP) in Section 3. A solution algorithm for solving such problems is described in Section 4. Section 5 contains an illustrative example to explain and clarify the proposed solution algorithm. Finally, the paper is concluded in Section 6.

2 Problem Formulation and the Solution Concept

The problem to be considered in this study is the multiobjective integer quadratic programming problem having random parameters. These random parameters are involved in the objective functions and in the constraints. The problem of concern is formulated mathematically as follows:

$$(MOIQP)_1: \quad \min \ F(x,y), \quad F: R^n \to R^K$$

where $F(x,y)=(f_1(x,y^1), f_2(x,y^2),..., f_K(x,y^k))$ is a vector valued criterion with $f_k(x,y^k)$, $k=1,2,...,K$ are real valued convex objective functions. Furthermore, X is the feasible set and might be, for example, of the form:

$$X=\{x\in R^n \,/\, P(g_r(x)=\sum_{j=1}^{n} a_{rj}x_j \le b_r) \ge \alpha_r, \qquad r=1,2,...,m,$$
$$x_j \ge 0 \text{ and integers}, \ j=1,2,...,n\},$$

In the above problem $(MOIQP)_1$, the objective function $f_k(x,y^k)$ has the following quadratic form:

$$f_k(x,y^k)=\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}^k \, x_i x_j + \sum_{j=1}^{n} y_j^k x_j,$$

where, we suppose that $[C_{ij}^k]$, $i,j=1,2,...,n$ are symmetric positive semi-definite matrices, $[y_j^k]$, $k=1,2,...,K$ are n- vectors of random parameters normally distributed and independently with each other with known means μ_j^k and variances $\sigma_j^{2\,k}$, respectively. Moreover, P means probability and α_r, $r=1,2,...,m$ is a specified probability. This means that the linear constraints $g_r(x)$ may be violated some of the time, and at most for $100(1-\alpha_r)\%$ of the time. For the sake of simplicity, we assume that the random parameters b_r, $r=1,2,...,m$ are also distributed normally with known means $E\{b_r\}=\mu_r$ and variances $Var\{b_r\}=\sigma_r^2$, respectively and independently of each other.

The concept of the efficient solution of problem $(MOIQP)_1$ is introduced in the following definition.

Definition 1: *A point* $x^* \in X$ *is said to be an efficient solution of problem* $(MOIQP)_1$ *if there is no other* $x \in X$ *such that*

$$P(y^k / f_k(x, y^k) \le f_k(x^*, y^k) = 1, \quad k = 1, 2, ..., K,$$

With strict inequality holds for at least one k and

$$P(g_r(x^*) = \sum_{j=1}^{n} a_{rj} x^* \le b_r) \ge \alpha_r,\ \alpha_r \in [0,1], \quad r = 1, 2, ..., m.$$

Now, going back to problem $(MOIQP)_1$ and for the purpose of optimization, new objective functions $f_k(x)$, $k=1,2,...,K$ can be constructed using the chance-constrained programming technique [13, 14] as:

$$f_k(x) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}^k x_i x_j + \beta_1^k \sum_{j=1}^{n} \mu_j^k x_j + \beta_2^k \sqrt{\sum_{j=1}^{n} \overset{2}{\sigma}{}_j^{\,k} x_j^2} \qquad k = 1, 2, ..., K,$$

where μ_j^k = mean of $\{y_j^k\}$ and $\overset{2}{\sigma}{}_j^{\,k}$ = variance of $\{y_j^k\}$. In addition, β_1^k and β_2^k are nonnegative constants whose values indicate the relative importance of μ_j^k and the standard deviation $\overset{2}{\sigma}{}_j^{\,k}$ of the minimization, respectively. Therefore, problem $(MOIQP)_1$ will take the following equivalent form:

$(MOIQP)_2$:

$$\min \quad f_k(x) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}^k x_i x_j + \beta_1^k \sum_{j=1}^{n} \mu_j^k x_j + \beta_2^k \sqrt{\sum_{j=1}^{n} \overset{2}{\sigma}{}_j^{\,k} x_j^2} \qquad k = 1, 2, ..., K,$$

where

$$X = \{ x \in R^n \; / \; P(g_r(x) = \sum_{j=1}^{n} a_{rj} x_j \le b_r) \ge \alpha_r, \qquad r = 1, 2, ..., m,$$
$$x_j \ge 0 \text{ and integers}, \; j = 1, 2, ..., n \},$$

Taking $h_r(x) = g_r(x) - b_r$, then $h_r(x)$, $r = 1, 2, ..., m$ are random parameters normally distributed with characteristics $\eta_r = g_r(x) - \mu_r$ and $\theta_r^2 = \sigma_r^2$.

The inequalities

$$P(g_r(x) = \sum_{j=1}^{n} a_{rj} x_j \le b_r) \ge \alpha_r, \qquad r = 1, 2, ..., m,$$

are equivalent to

$$P(h_r(x) \le 0) \ge \alpha_r, \quad r = 1, 2, ..., m$$

This leads to

$$\frac{1}{\sqrt{2\pi}\,\sigma_r} \int_{-\infty}^{0} e^{-\frac{1}{2}\left(\frac{v - \eta_r(x)}{\theta_r(x)}\right)^2} dv \ge \alpha_r, \quad r = 1, 2, ..., m.$$

Putting

$$\phi(v) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} e^{-v^2/2} \, dv$$

Then, the above inequality can be rewritten as:

$$\phi\left(\frac{-\eta_r(x)}{\sigma_r^2}\right) \ge \alpha_r, \quad r = 1, 2, ..., m$$

i.e. $$\eta_r(x) + \phi^{-1}(\alpha_r)\,\sigma_r^2 \le 0 \; ,$$

This gives

$$(g_r(x) - \mu_r) + \phi^{-1}(\alpha_r)\,\sigma_r^2 \le 0$$

and consequently, problem $(MOIQP)_2$ can be reduced to the following multiobjective integer nonlinear programming problem [10, 12]:

$$(MOINLP): \qquad \min \ f_k(x), \quad k=1,2...,K,$$
$$\text{subject to}$$
$$X(\mu,\sigma)=\{x\in R^n \,/\, g_r(x)=\sum_{j=1}^{n} a_{rj}x_j-\mu_r+\phi^{-1}(\alpha_r)\sigma_r^2\le 0, \quad r=1,2,...,m,$$
$$x_j\ge 0 \ \text{ and integers}, \quad j=1,2,...,n\}.$$

Problem $(MOINLP)$ can be treated using the nonnegative weighted sum approach [15] i.e. by considering the following integer nonlinear programming problem with single-objective function:

$$P(w): \qquad \min \ \sum_{k=1}^{K} w_k f_k(x),$$
$$\text{subjec to}$$
$$x\in X(\mu,\sigma),$$

where $w_k \succ 0$, $k=1,2,...,K$ and $\sum_{k=1}^{K} w_k=1.$

Clearly, problem $P(w)$ above can be solved using any available nonlinear programming package, for example, GINO [16] coupled with the branch- and-bound method [17].

It should be noted from the scalarization theorem [18] that if x^* is a unique optimal solution of problem $P(w)$ at certain $w^*\in R^k$, $w_k^*\succ 0$ for all $k=1,2,...,K$ then x^* becomes an efficient solution for the original problem $(MOIQP)_1$.

3 Some Basic Stability Notions for Problem $(MOIQP)_1$.

The Set of Feasible Parameters

Definition 2: *The set of feasible parameters of problem* $(MOIQP)_1$ *is defined via problem* $(MOINLP)$ *as:*

$$A=\{w\in R_+^k-\{0\}, b_r\in N(\mu,\sigma)\,/\,X(\mu,\sigma)\ne\varphi\}.$$

where $N(\mu,\sigma)$ denotes the normal distribution. Equivalently, the set A can be redefined as:

$$A=\{w\in R_+^k-\{0\}, (\mu,\sigma)/X(\mu,\sigma)\ne\varphi\}.$$

The Solvability Set

Definition 3: *The solvability set of problem* $(MOIQP)_1$ *is defined via problem* $(MOINLP)$ *as:*

$$B=\{(w_{k,}\, b_r)\in A \;/\; \text{Problem}\,(MOINLP)\ \text{has an efficient solution}\ x^*\}.$$

The Stability Set of the First Kind $S(x^*)$

Definition 4: *Let* x^* *be an efficient solution of problem* $(MOIQP)_1$ *corresponding to* $(w_k^*, b_r^*)\in B$ *then the stability set of the first kind of problem* $(MOIQP)_1$, *denoted by* $S(x^*)$, *is defined as:*

$$S(x^*)=\{(w_k, b_r)\in B \;/\; x^*\ \text{is an efficient solution of problem}\ (MOINLP)_1\}.$$

As mentioned before, the random parameters b_r, $r=1,2,...,m$ can be defined exactly if its characteristics $E\{b_r\}=\mu_r$ and $Var\{b_r\}=\sigma_r^2$ are known earlier.

Before we go any further, a nonlinear programming relaxation problem equivalent and corresponding to problem $P(w)$ can be stated in the following form:

$$\tilde{P}(w): \qquad \min \sum_{k=1}^{K} w_k f_k(x),$$

$$\text{subject to}$$

$$g_r(x)=\sum_{j=1}^{n} a_{rj}x_j-\mu_r+\phi^{-1}(\alpha_r)\sigma_r^2\le 0,\quad r=1,2,...,m,$$

$$x_j\ge\delta_j,\qquad j\in I\subseteq N=\{1,2,...,n\},$$

$$x_j\le\gamma_j,\qquad j\in J\subseteq N=\{1,2,...,n\}.$$

where $I\cup J\subseteq N=\{1,2,...,n\}$, $I\cap J=\varphi$ and the constraints $x_j\ge\delta_j$, $x_j\le\gamma_j$ are additional and have been added to the feasible solution set of problem $P(w)$ through the use of the branch-and-bound process to obtain its integer optimal solution x^*.

Determination of Stability Set of the First Kind Set $S(x^*)$

In what follows, we suppose that the functions $f_k(x), k=1,2,...,K$; $g_r(x)$ and $\eta_r(x), r=1,2,...,m$ are convex, then the Kuhn-Tucker necessary optimality conditions corresponding to problem $\tilde{P}(w)$ will take the following form:

$$\nabla \sum_{k=1}^{K} w_k f_k(x) + \lambda_r [\nabla g_r(x) + \nabla(\frac{-\eta_r(x)}{\sigma_r})\sigma_r] - u_j + v_j = 0,$$

$$\lambda_r [g_r(x) - \mu_r - \eta_r(x)] = 0,$$

$$g_r(x) - \mu_r - \eta_r(x) \le 0,$$

$$x_j \ge \delta_j,$$

$$x_j \le \gamma_j,$$

$$u_j x_j = 0, \quad j \in I \subseteq N = \{1,2,...,n\},$$

$$v_j x_j = 0, \quad j \in J \subseteq N = \{1,2,...,n\},$$

$$\lambda_r \ge 0, \quad r = 1,2,...,m,$$

$$u_j \ge 0, \quad j \in I,$$

$$v_j \ge 0, \quad j \in J,$$

where $w_k \phi\, 0$ and $\sum_{k=1}^{K} w_k = 1$.

It should be noted that all the relations of the above system are evaluated at the optimal integer solution x^* and λ_r, u_j, v_j are the Lagrange multipliers. The first and the last three relations of the above system represent a Polytope in $\lambda u v-$space for which its vertices can be determined using any algorithm which is based upon the Simplex Method, for example, Balinski [19].

According to whether any of the variables $\lambda_r, (r = 1, 2, ..., m)$, $u_j, (j \in I \subset N)$ and $v_j, (j \in J \subseteq N)$ are zero or positive, then the set of parameters $w_k, (k = 1,2,...,K)$ and $\lambda_r, (r = 1,2,...,m)$ for which the Kuhn-Tucker necessary optimality conditions are utilized will be determined and is denoted by $T(x^*)$ .Clearly, we can write that $T(x^*) \subseteq S(x^*)$.

4 The Solution Algorithm

Now, we describe an algorithm to solve problem $(MOIQP)_1$ which has been formulated earlier in Section 2. This suggested algorithm terminates in a series of finite number of steps and can be summarized in the following manner.

Step 1: Determine μ_j^k = mean of $\{y_j^k\}$ and $\sigma_j^{2\,k}$ = variance of $\{y_j^k\}$ .Also, determine $E\{b_r\}=\mu_r$ and variances $Var\{b_r\}=\sigma_r^2$.

Step 2: Formulate the deterministic objective functions $f_k(x),\ k=1,2,...,K$ and convert the set of constraints X of problem $(MOIQP)_1$ into the set of constraints $X(\mu,\sigma)$ of problem $(MOINLP)$.

Step 3: Choose $w = w_k^* \in R^k$ such that $w_k^* \succ 0$ and $\sum_{k=1}^{K} w_k^* = 1$ and then solve the resulting single-objective integer nonlinear problem $P(w^*)$ using GINO software package [16] together with the Branch-and-Bound method [17]. Let x^* be a unique optimal integer solution of problem $P(w^*)$.

Step 4: Determine the stability set $T(x^*)$ by utilizing the Kuhn-Tucker necessary optimality conditions corresponding to the parametric integer nonlinear problem $\tilde{P}(w)$ at the optimal integer solution x^* .

Step 5: Choose another vector of nonnegative weights $w = w^{**} \in R^k$ and then go to Step 4 again.

A systematic variation of the nonnegative weights will lead to a set of optimal integer solutions of problem $\tilde{P}(w)$.

5 An Illustrative Example

In this section, an illustrative numerical example is provided to clarify the suggested solution algorithm. The problem under consideration is a bicriterion integer quadratic programming problem $(BIQP)$ and having random parameters y_1, y_2 and b_1, b_2 in the objectives and in the constraints, respectively. For the purpose of illustration, let us consider the $(BIQP)$ as follows:

$$(BIQP): \qquad \min\ F(x)=[\ f_1(x,y);\ f_2(x,y)],$$
$$\text{subject to}$$
$$x \in X,$$

where the feasible solution set X is defined by:

$$X = \left\{ x \in R^2 \left| \begin{array}{l} P(x_1 + x_2 \leq b_1) \geq 0.95, \\ P(x_1 + 2x_2 \leq 3b_2) \geq 090, \\ x_1, x_2 \geq 0 \text{ and integers.} \end{array} \right. \right\}$$

and

$$f_1(x, y) = 2x_1^2 + x_2^2 - y_1 x_1, \qquad f_2(x, y) = x_1^2 + 3x_2^2 - y_2 x_2.$$

Step 1: Assume that the random variables y_1, y_2, b_1 and b_2 are normally distributed with the following means and variances, respectively.

$\mu_1 = \text{mean}(y_1) = 3,$ $\mu_2 = \text{mean}(y_2) = 4,$

$\sigma_1^2 = \text{variance}(y_1) = 4,$ $\sigma_2^2 = \text{variance}(y_2) = 16,$

and

$E(b_1) = 9$, $E(b_2) = 1,$ $Var(b_1) = 4, Var(b_2) = 25.$

Step 2: The deterministic bicriterion integer quadratic programming problem equivalent to the above problem $(BIQP)$ can be written in the following form:

$$\begin{array}{l} \min \quad F(x) = [\ f_1(x);\ f_2(x)], \\ \text{subject to} \\ \qquad x \in \tilde{X}, \end{array}$$

Where

$$\tilde{X} = \left\{ x \in R^2 \left| \begin{array}{l} x_1 + x_2 \leq 12.29, \\ x_1 + 2x_2 \leq 22.275, \\ x_1, x_2 \geq 0 \text{ and integers} \end{array} \right. \right\}$$

and

$$f_1(x) = 2x_1^2 + x_2^2 - 3\beta_1 x_1 - 2\beta_2 x_1; \quad f_2(x) = x_1^2 + 3x_2^2 - 4\beta_1 x_2 - 4\beta_2 x_2,$$

Provided that $\beta_1, \beta_2 \geq 0$ and are supposed to be $0 \leq \beta_1 \leq 5$ and $0 \leq \beta_2 \leq 6.$

Step 3: Using the nonnegative weighted sum approach [15], then we have:

$$P(w): \qquad \min\,[\,w_1(2x_1^2 + x_2^2 - 3\beta_1 x_1 - 2\beta_2 x_1) + w_2(x_1^2 + 3x_2^2 - 4\beta_1 x_2 - 4\beta_2 x_2)\,],$$

subject to

$$x \in \tilde{X},$$

where $w_1, w_2 \phi 0$ and $w_1 + w_2 = 1.$

Step 4: Choose $w_1^* = w_2^* = 0.5,$ therefore we get:

$$P(w^*): \qquad \min\ (\frac{3}{2}x_1^2 + 2x_2^2 - \frac{3}{2}\beta_1 x_1 - \beta_2 x_1 - 2\beta_1 x_2 - 2\beta_2 x_2,$$

subjet to

$$x \in \tilde{X}.$$

The optimal integer solution of problem $P(w^*)$, using the Branch-and Bound method [17], has been found as $(x_1^*, x_2^*) = (5, 5).$

Step 5: The nonlinear programming problem equivalent to problem $P(w)$, in its parametric form, can be written as follows:

$$\tilde{P}(w): \qquad \min\,[\,w_1(2x_1^2 + x_2^2 - 3\beta_1 x_1 - 2\beta_2 x_1) + w_2(x_1^2 + 3x_2^2 - 4\beta_1 x_2 - 4\beta_2 x_2)\,],$$

subject to

$$\begin{aligned} x_1 + x_2 &\le 12.29, \\ x_1 + 2x_2 &\le 22.275, \\ x_1 &\ge 5, \\ x_2 &= 5. \end{aligned}$$

It should be noted that the constraints $x_1 \ge 5$ and $x_2 = 5$ are additional and that have been added to the constraint set $\tilde{X}$ of problem $P(w^*)$ to find its optimal integer solution through the Branch-and- Bound process.

Step 6: The Kuhn-Tucker necessary optimality conditions corresponding to problem $\tilde{P}(w)$ will take the following form:

$$
\begin{aligned}
4x_1w_1 - 3\beta_1 w_1 - 2\beta_2 w_1 + 2w_2 x_1 + \lambda_1 + \lambda_2 - \nu_1 &= 0,\\
2x_2 w_1 - 4\beta_1 w_2 - 4\beta_2 w_2 + +6w_2 x_2 + \lambda_1 + 2\lambda_2 + \nu_2 &= 0,\\
\lambda_1(x_1 + x_2 - 12.29) &= 0,\\
\lambda_2(x_1 + 2x_2 - 22.275) &= 0,\\
\nu_1(-x_1 + 5) &= 0,\\
\nu_2(x_2 - 5) &= 0,\\
x_1 + x_2 &\le 12.29,\\
x_1 + 2x_2 &\le 22.275,\\
x_1 &\ge 5,\\
x_2 &= 5,\\
\lambda_1, \lambda_2, \nu_1, \nu_2 &\ge 0.
\end{aligned}
$$

where the relations of the above system are evaluated at the optimal integer solution $(x_1^*, x_2^*) = (5, 5)$.

Also, it can be shown that $\lambda_1 = \lambda_2 = 0$ and $\nu_1, \nu_2 \ge 0$ and therefore, the stability set $T(5,5)$ is given by:

$$T(5,5) = \left\{ w \in R^2 \middle| -7w_1 + 10w_2 \ge 0,\ -10w_1 + 14w_2 \ge 0 \text{ and } w_1, w_2 \ge 0,\ \text{where } w_1 + w_2 = 1 \right\}$$

Clearly,

$$T(5,5) \subseteq S(5,5).$$

On the other hand, choosing $(\overline{w}_1, \overline{w}_2) = (\frac{2}{3}, \frac{1}{3}) \notin T(5,5)$ gives the optimal integer solution $(\bar{x}_1, \bar{x}_2) = (3, 6)$. This will yield $\lambda_1 = \lambda_2 = 0$ and $\nu_1, \nu_2 \ge 0$ with the stability set $\overline{T}(3, 6)$, which is given by:

$$\overline{T}(3,6) = \left\{ w \in R^2 \middle| 5w_1 - 2w_2 \ge 0,\ 3w_1 + 2w_2 \ge 0 \text{ and } w_1, w_2 \ge 0,\ \text{where } w_1 + w_2 = 1 \right\}.$$

and clearly $\overline{T}(3,6) \subseteq S(3,6).$

6 Conclusions

The main objective of this study was to present a solution algorithm for solving multiobjective integer quadratic programming problems having random parameters in the objective functions and in the right-hand side of the constraints. Some basic stability notions of the problem of concern have been defined and characterized.

Certainly, there are many other aspects and questions should be explained in the field of stochastic multiobjective integer quadratic programming problems. This study was an attempt to establish underlying results which hopefully will help other researchers to discuss such problems from different directions.

However, there remain several open points for discussion and should be solved in future. Some of these points are the following:

(i) A procedure is needed to enlarge the set $T(x^*)$ such that $T(x^*)$ becomes $S(x^*)$.
(ii) An algorithm is required for solving large-scale stochastic multiobjective integer quadratic programming problems.
(iii) Computer codes should be introduced for the implementation of the solution for these problems and the computational complexity must be discussed.

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