



Gen. Math. Notes, Vol. 20, No. 1, January 2014, pp.77-92

ISSN 2219-7184; Copyright ©ICSRS Publication, 2014

www.i-csrs.org

Available free online at <http://www.geman.in>

Almost Slightly νg –Continuous Functions

S. Balasubramanian

Department of Mathematics
Government Arts College (Autonomous)
Karur-639 005, Tamilnadu, India
E-mail: mani55682@rediffmail.com

(Received: 22-10-13 / Accepted: 3-12-13)

Abstract

In this paper we discuss a new type of continuous functions called almost slightly νg –continuous functions; its properties and interrelation with other continuous functions are studied.

Keywords: *slightly continuous functions; slightly semi-continuous functions; slightly β –continuous functions; slightly γ –continuous functions and slightly ν –continuous functions.*

1. Introduction

T.M.Nour introduced slightly semi-continuous functions during the year 1995. After him T.Noiri and G.I.Ghae further studied slightly semi-continuous functions on 2000. During 2001 T.Noiri individually studied slightly β –continuous functions. C.W.Baker introduced slightly precontinuous functions. Erdal Ekici and M. Caldas studied slightly γ –continuous functions. Arse Nagli Uresin and others studied slightly δ –continuous functions. Recently the Author of the present paper studied slightly νg –continuous functions. Inspired with these developments the author introduce in this paper a new variety of slightly continuous functions called almost slightly νg –continuous function and study its basic properties; interrelation with other type of such functions available in the literature. Throughout the paper a space X means a topological space (X, τ) .

2. Preliminaries

Definition 2.1: $A \subset X$ is called

(i) closed[resp: Semi-closed; ν -closed] if its complement is open[resp:semi-open; ν -open].

(ii) $r\alpha$ -closed if $\exists U \in \alpha O(X) \ni U \subset A \subset \overline{\alpha(U)}$.

(iii) semi- θ -open if it is the union of semi-regular sets and its complement is semi- θ -closed.

(iv) Regular closed[resp: α -closed; pre-closed; β -closed] if $A = \overline{A^o}$ [resp: $(\overline{A^o})^o \subseteq A$; $\overline{A^o} \subseteq A$; $\overline{A^o} \subseteq A$].

(v) g -closed[resp: rg -closed] if $\overline{A} \subseteq U$ whenever $A \subseteq U$ and U is open in X .

(vi) sg -closed[resp: gs -closed] if $s(\overline{A}) \subseteq U$ whenever $A \subseteq U$ and U is semi-open{open} in X .

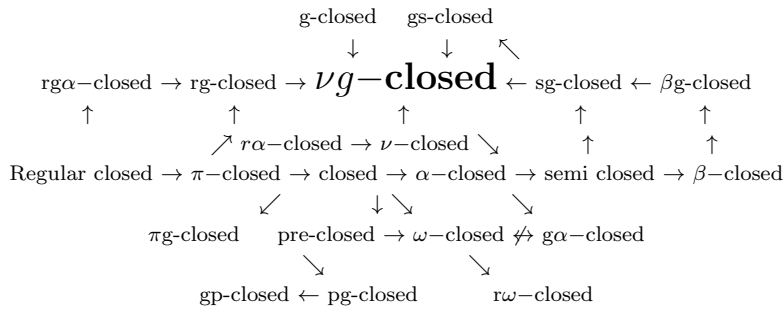
(vii) pg -closed[resp: gp -closed; gpr -closed] if $p(\overline{A}) \subseteq U$ whenever $A \subseteq U$ and U is pre-open{open; regular-open} in X .

(viii) αg -closed[resp: $g\alpha$ -closed; $rg\alpha$ -closed] if $\alpha(\overline{A}) \subseteq U$ whenever $A \subseteq U$ and U is $\{\alpha$ -open; $r\alpha$ -open}open in X .

(ix) νg -closed if $\nu(\overline{A}) \subseteq U$ whenever $A \subseteq U$ and U is ν -open in X .

(x) clopen[resp: r -clopen] if it is both open and closed[resp: regular-open and regular-closed]

Note 1: From the above definitions we have the following interrelations among the closed sets.



Definition 2.2: A function $f: X \rightarrow Y$ is said to be

(i) continuous[resp: nearly-continuous; $r\alpha$ -continuous; ν -continuous; α -continuous; semi-continuous; β -continuous; pre-continuous] if inverse image of each open set is open[resp: regular-open; $r\alpha$ -open; ν -open; α -open; semi-open; β -open; preopen].

(ii) nearly-irresolute[resp: $r\alpha$ -irresolute; ν -irresolute; α -irresolute; irresolute; β -irresolute; pre-irresolute] if inverse image of each regular-open[resp: $r\alpha$ -open; ν -open; α -open; semi-open; β -open; preopen] set is regular-open[resp: $r\alpha$ -open; ν -open; α -open; semi-open; β -open; preopen].

(iii) almost continuous[resp: almost $r\alpha$ -continuous; almost ν -continuous; almost α -continuous; almost semi-continuous; almost β -continuous; almost

pre-continuous] if for each $x \in X$ and each open set $(V, f(x))$, there exists an open[resp: α -open; ν -open; α -open; semi-open; β -open; preopen] set $(U, x) \ni f(U) \subset (\overline{V})^o$.

(iv) weakly continuous[resp: weakly nearly-continuous; weakly α -continuous; weakly ν -continuous; weakly α -continuous; weakly semi-continuous; weakly β -continuous; weakly pre-continuous] if for each $x \in X$ and each open set $(V, f(x))$, there exists an open[resp: regular-open; α -open; ν -open; α -open; semi-open; β -open; preopen] set $(U, x) \ni f(U) \subset \overline{V}$.

(v) slightly continuous[resp: slightly semi-continuous; slightly pre-continuous; slightly β -continuous; slightly γ -continuous; slightly α -continuous; slightly r -continuous; slightly ν -continuous] at $x \in X$ if for each clopen subset V in Y containing $f(x)$, $\exists U \in \tau(X)$ [$\exists U \in SO(X)$; $\exists U \in PO(X)$; $\exists U \in \beta O(X)$; $\exists U \in \gamma O(X)$; $\exists U \in \alpha O(X)$; $\exists U \in RO(X)$; $\exists U \in \nu O(X)$] containing x such that $f(U) \subseteq V$.

(vi) slightly continuous[resp: slightly semi-continuous; slightly pre-continuous; slightly β -continuous; slightly γ -continuous; slightly α -continuous; slightly r -continuous; slightly ν -continuous] if it is slightly-continuous[resp: slightly semi-continuous; slightly pre-continuous; slightly β -continuous; slightly γ -continuous; slightly α -continuous; slightly r -continuous; slightly ν -continuous] at each $x \in X$.

(vii) almost strongly θ -semi-continuous[resp: strongly θ -semi-continuous] if for each $x \in X$ and for each $V \in \sigma(Y, f(x))$, $\exists U \in SO(X, x) \ni f(s(\overline{U})) \subset s(\overline{V})$ [resp: $f(s(\overline{U})) \subset V$].

Definition 2.3: A function $f: X \rightarrow Y$ is said to be [almost-] slightly g -continuous [resp: [almost-] slightly sg -continuous; [almost-] slightly pg -continuous; [almost-] slightly βg -continuous; [almost-] slightly γg -continuous; [almost-] slightly αg -continuous; [almost-] slightly rg -continuous] at $x \in X$ if for each $V \in CO(Vf(x))$, [resp: $V \in RCO(Vf(x))$], $\exists U \in GO(X, x)$ [$\exists U \in SGO(X, x)$; $\exists U \in PGO(X, x)$; $\exists U \in \beta GO(X, x)$; $\exists U \in \gamma GO(X, x)$; $\exists U \in \alpha GO(X, x)$; $\exists U \in RGO(X, x)$] $\ni f(U) \subseteq V$, and [almost-] slightly g -continuous [resp: [almost-] slightly sg -continuous; [almost-] slightly pg -continuous; [almost-] slightly βg -continuous; [almost-] slightly γg -continuous; [almost-] slightly αg -continuous; [almost-] slightly rg -continuous] if it is [almost-] slightly g -continuous [resp: [almost-] slightly sg -continuous; [almost-] slightly pg -continuous; [almost-] slightly βg -continuous; [almost-] slightly γg -continuous; [almost-] slightly αg -continuous; [almost-] slightly rg -continuous] at each $x \in X$.

Definition 2.4: X is said to be a

(i) compact[resp: nearly-compact; α -compact; ν -compact; α -compact; semi-compact; β -compact; pre-compact; mildly-compact] space if every open [resp: regular-open; α -open; ν -open; α -open; semi-open; β -open; preopen;

clopen] cover has a finite subcover.

(ii) countably-compact[resp: countably-nearly-compact; countably - α - compact; countably - ν - compact; countably- α - compact; countably - semi - compact; countably - β - compact; countably-pre-compact; mildly-countably compact] space if every countable open[resp: regular-open; α - oover.

(iii) closed-compact[resp: closed-nearly-compact; closed-r α - compact; closed- ν - compact; closed- α - compact; closed-semi-compact; closed- β -compact; closed-pre-compact] space if every closed[resp: regular-closed; α -closed; ν - closed; α -closed; semi-closed; β -closed; preclosed] cover has a finite subcover.

(iv) Lindeloff [resp: nearly-Lindeloff; α - Lindeloff; ν - Lindeloff; α -Lindeloff; semi-Lindeloff; β - Lindeloff; pre-Lindeloff; mildly-Lindeloff] space if every open[resp: regular-open; α -open; ν -open; α -open; semi-open; β -open; pre-open; clopen] cover has a countable subcover.

(v) Extremely disconnected[briefly e.d] if the closure of each open set is open.

Definition 2.5: X is said to be a

(i) T_0 [resp: r- T_0 ; α - T_0 ; ν - T_0 ; α - T_0 ; semi- T_0 ; β - T_0 ; pre- T_0 ; Ultra T_0] space if for each $x \neq y \in X \exists U \in \tau(X)$ [resp: rO(X); α O(X); ν O(X); α O(X); SO(X); β O(X); PO(X); CO(X)] containing either x or y.

(ii) T_1 [resp: r- T_1 ; α - T_1 ; ν - T_1 ; α - T_1 ; semi- T_1 ; β - T_1 ; pre- T_1 ; Ultra T_1] space if for each $x \neq y \in X \exists U, V \in \tau(X)$ [resp: rO(X); α O(X); ν O(X); α O(X); SO(X); β O(X); PO(X); CO(X)] such that $x \in U - V$ and $y \in V - U$.

(iii) T_2 [resp: r- T_2 ; α - T_2 ; ν - T_2 ; α - T_2 ; semi- T_2 ; β - T_2 ; pre- T_2 ; Ultra T_2] space if for each $x \neq y \in X \exists U, V \in \tau(X)$ [resp: rO(X); α O(X); ν O(X); $\alpha\beta$ O(X); PO(X); CO(X)] such that $x \in U$; $y \in V$ and $U \cap V = \phi$.

(iv) C_0 [resp: r- C_0 ; α - C_0 ; ν - C_0 ; α - C_0 ; semi- C_0 ; β - C_0 ; pre- C_0 ; Ultra C_0] space if for each $x \neq y \in X \exists U \in \tau(X)$ [resp: rO(X); α O(X); $\nu\alpha\beta$ O(X); PO(X); CO(X)]whose closure contains either x or y

(v) C_1 [resp: r- C_1 ; α - C_1 ; ν - C_1 ; α - C_1 ; semi- C_1 ; β - C_1 ; pre- C_1 ; Ultra C_1] space if for each $x \neq y \in X \exists U, V \in \tau(X)$ [resp: rO(X); $\alpha\nu\alpha\beta$

(vi) C_2 [resp: r- C_2 ; α - C_2 ; ν - C_2 ; α - C_2 ; semi- C_2 ; β - C_2 ; pre- C_2 ; Ultra C_2] space if for each $x \neq y \in X \exists U, V \in \tau(X)$ [resp: rO(X); $\alpha\nu\alpha\betaU \cap V = \phi$.

(vii) D_0 [resp: r- D_0 ; α - D_0 ; ν - D_0 ; α - D_0 ; semi- D_0 ; β - D_0 ; pre- D_0 ; Ultra D_0] space if for each $x \neq y \in X \exists U \in D(X)$ [resp: rD(X); α D(X); ν D(X); α D(X); SD(X); β D(X); PD(X); COD(X)] containing either x or y.

(viii) D_1 [resp: r- D_1 ; α - D_1 ; ν - D_1 ; α - D_1 ; semi- D_1 ; β - D_1 ; pre- D_1 ; Ultra D_1] space if for each $x \neq y \in X \exists U, V \in D(X)$ [resp: rD(X); α D(X); ν D(X); α D(X); SD(X); β D(X); PD(X); COD(X)] $\ni x \in U - V$ and $y \in V - U$.

(ix) D_2 [resp: r- D_2 ; α - D_2 ; ν - D_2 ; α - D_2 ; semi- D_2 ; β - D_2 ; pre- D_2 ; Ultra

D_2] space if for each $x \neq y \in X \exists U, V \in D(X)$ [resp: $rD(X)$; $r\alpha D(X)$; $\nu D(X)$; $\alpha D(X)$; $SD(X)$; $\beta D(X)$; $PD(X)$; $CD(X)$] such that $x \in U$; $y \in V$ and $U \cap V = \phi$.
(x) R_0 [resp: $r-R_0$; $r\alpha - R_0$; $\nu - R_0$; $\alpha - R_0$; semi- R_0 ; $\beta - R_0$; pre- R_0 ; Ultra R_0] space if for each $x \in X \exists U \in \tau(X)$ [resp: $RO(X)$; $r\alpha O(X)$; $\nu O(X)$; $\alpha O(X)$; $SO(X)$; $\beta O(X)$; $PO(X)$; $CO(X)$] $\{x\} \subseteq U$ [resp: $r\{x\} \subseteq U$; $\nu\{x\} \subseteq U$; $\alpha\{x\} \subseteq U$; $s\{x\} \subseteq U$] whenever $x \in U \in \tau(X)$ [resp: $x \in U \in RO(X)$; $x \in U \in \nu O(X)$; $x \in U \in \alpha O(X)$; $x \in U \in SO(X)$]
(xi) R_1 [resp: $r-R_1$; $r\alpha - R_1$; $\nu - R_1$; $\alpha - R_1$; semi- R_1 ; $\beta - R_1$; pre- R_1 ; Ultra R_1] space if for $x, y \in X \ni \overline{\{x\}} \neq \overline{\{y\}}$ [resp: $\ni r\overline{\{x\}} \neq r\overline{\{y\}}$; $\ni r\alpha\overline{\{x\}} \neq r\alpha\overline{\{y\}}$; $\ni \nu\overline{\{x\}} \neq \nu\overline{\{y\}}$; $\ni \alpha\overline{\{x\}} \neq \alpha\overline{\{y\}}$; $\ni s\overline{\{x\}} \neq s\overline{\{y\}}$; $\ni \beta\overline{\{x\}} \neq \beta\overline{\{y\}}$; $\ni p\overline{\{x\}} \neq p\overline{\{y\}}$; $\ni CO\overline{\{x\}} \neq CO\overline{\{y\}}$]; $V \in \tau(X) \ni$ disjoint $U, V \in \tau(X) \ni \{x\} \subseteq U$ [resp: $RO(X) \ni r\{x\} \subseteq U$; $R\alpha O(X) \ni r\alpha\{x\} \subseteq U$; $\nu O(X) \ni \nu\{x\} \subseteq U$; $RO(X) \ni \alpha\{x\} \subseteq U$; $SO(X) \ni s\{x\} \subseteq U$; $\beta O(X) \ni \beta\{x\} \subseteq U$; $PO(X) \ni p\{x\} \subseteq U$; $CO(X) \ni co\{x\} \subseteq U$] and $\{y\} \subseteq V$ [resp: $RO(X) \ni r\{y\} \subseteq V$; $R\alpha O(X) \ni r\alpha\{y\} \subseteq V$; $\nu O(X) \ni \nu\{y\} \subseteq V$; $RO(X) \ni \alpha\{y\} \subseteq V$; $SO(X) \ni s\{y\} \subseteq V$; $\beta O(X) \ni \beta\{y\} \subseteq V$; $PO(X) \ni p\{y\} \subseteq V$; $CO(X) \ni co\{y\} \subseteq V$].

Lemma 2.1:

- (i) Let A and B be subsets of a space X , if $A \in \nu O(X)$ and $B \in RO(X)$, then $A \cap B \in \nu O(B)$.
- (ii) Let $A \subset B \subset X$, if $A \in \nu O(B)$ and $B \in RO(X)$, then $A \in \nu O(X)$.

Remark 1: $\nu GO(X, x)$ [resp: $RCO(X, x)$] represents νg -open set containing x [resp: r -clopen set containing x].

Theorem 2.1:

- (i) If f is $\nu g.c.$, then f is al. $\nu g.c.$
- (i) If f is $c.\nu g.c.$, then f is al. $c.\nu g.c.$

3. Almost Slightly νg -Continuous Functions:

Definition 3.1: A function $f: X \rightarrow Y$ is said to be almost slightly νg -continuous function at $x \in X$ if for each $V \in RCO(Y, f(x))$, $\exists U \in \nu GO(X, x)$ such that $f(U) \subseteq V$ and almost slightly νg -continuous function if it is almost slightly νg -continuous at each $x \in X$.

Note 2: Here after we call almost slightly νg -continuous function as al.sl. $\nu g.c$ function shortly.

Example 3.1: $X = Y = \{a, b, c\}$; $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$. Let f is identity function, then f is al.sl. $\nu g.c.$

Example 3.2: $X = Y = \{a, b, c, d\}; \tau = \sigma = \{\phi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, X\}$. Let f be defined by $f(a) = b; f(b) = c; f(c) = d$ and $f(d) = a$, then f is not $sl.\nu g.c.$, and not $al.sl.\nu g.c.$ Since it is not satisfying the condition at the points c and d .

Theorem 3.1: The following are equivalent.

- (i) f is $al.sl.\nu g.c.$
- (ii) $f^{-1}(V)$ is νg -open for every r -clopen set V in Y .
- (iii) $f^{-1}(V)$ is νg -closed for every r -clopen set V in Y .
- (iv) $f(\nu g(A)) \subseteq \nu g(f(A))$.

Corollary 3.1: The following are equivalent.

- (i) f is $al.sl.\nu g.c.$
- (ii) For each $x \in X$ and each r -clopen subset $V \in (Y, f(x)) \exists U \in \nu GO(X, x) \ni f(U) \subseteq V$.

Proof: Straightforward from definition 3.1.

Theorem 3.2: Let $\Sigma = \{U_i : i \in I\}$ be any cover of X by regular open sets in X . A function f is $al.sl.\nu g.c.$ iff $f_{/U_i}$ is $al.sl.\nu g.c.$, for each $i \in I$.

Proof: Let $i \in I$ be an arbitrary index and $U_i \in RO(X)$. Let $x \in U_i$ and $V \in RCO(Y, f_{/U_i}(x))$. For f is $al.sl.\nu g.c.$, $\exists U \in \nu GO(X, x) \ni f(U) \subset V$. Since $U_i \in RO(X)$, by lemma 2.1 $x \in U \cap U_i \in \nu GO(U_i)$ and $(f_{/U_i})U \cap U_i = f(U \cap U_i) \subset f(U) \subset V$. Hence $f_{/U_i}$ is $al.sl.\nu g.c.$

Conversely Let $x \in X$ and $V \in RCO(Y, f(x))$, $\exists i \in I \ni x \in U_i$. Since $f_{/U_i}$ is $al.sl.\nu g.c.$, $\exists U \in \nu GO(U_i, x) \ni f_{/U_i}(U) \subset V$. By lemma 2.1, $U \in \nu GO(X)$ and $f(U) \subset V$. Hence f is $al.sl.\nu g.c.$

Theorem 3.3:

- (i) If f is νg -irresolute and g is $al.sl.\nu g.c.$ [$al.sl.c.$], then $g \circ f$ is $al.sl.\nu g.c.$
- (ii) If f is νg -irresolute and g is $al.\nu g.c.$, then $g \circ f$ is $al.sl.\nu g.c.$
- (iii) If f is νg -continuous and g is $al.sl.c.$, then $g \circ f$ is $al.sl.\nu g.c.$
- (iv) If f is rg -continuous and g is $al.sl.\nu g.c.$ [$al.sl.c.$], then $g \circ f$ is $al.sl.\nu g.c.$

Theorem 3.4: If f is νg -irresolute, νg -open and $\nu GO(X) = \tau$ and g be any function, then $g \circ f$ is $al.sl.\nu g.c$ iff g is $al.sl.\nu g.c.$

Proof: If part: Theorem 3.3(i)

Only if part: Let A be r -clopen subset of Z . Then $(g \circ f)^{-1}(A)$ is a νg -open subset of X and hence open in X [by assumption]. Since f is νg -open $f(g \circ f)^{-1}(A) = g^{-1}(A)$ is νg -open in Y . Thus g is $al.sl.\nu g.c.$

Corollary 3.2: If f is νg -irresolute, νg -open and $\nu GO(X) = RO(X)$ and g be any function, then $g \circ f$ is al.sl. νg .c iff g is al.sl. νg .c.

Corollary 3.3: If f is νg -irresolute, νg -open and bijective, g is a function. Then g is al.sl. νg .c. iff $g \circ f$ is al.sl. νg .c.

Theorem 3.5: If $g : X \rightarrow X \times Y$, defined by $g(x) = (x, f(x)) \forall x \in X$ be the graph function of $f : X \rightarrow Y$. Then g is al.sl. νg .c iff f is al.sl. νg .c.

Proof: Let $V \in RCO(Y)$, then $X \times V \in RCO(X \times Y)$. Since g is al.sl. νg .c., $f^{-1}(V) = f^{-1}(X \times V) \in \nu GO(X)$. Thus f is al.sl. νg .c.

Conversely, let $x \in X$ and $F \in RCO(X \times Y, g(x))$. Then $F \cap (\{x\} \times Y) \in RCO(\{x\} \times Y, g(x))$. Also $\{x\} \times Y$ is homeomorphic to Y . Hence $\{y \in Y : (x, y) \in F\} \in RCO(Y)$. Since f is al.sl. νg .c. $\bigcup \{f^{-1}(y) : (x, y) \in F\} \in \nu GO(X)$. Further $x \in \bigcup \{f^{-1}(y) : (x, y) \in F\} \subseteq g^{-1}(F)$. Hence $g^{-1}(F)$ is νg -open. Thus g is al.sl. νg .c.

Theorem 3.6:

(i) If $f : X \rightarrow \prod Y_\lambda$ is al.sl. νg .c, then $P_\lambda \circ f : X \rightarrow Y_\lambda$ is al.sl. νg .c for each $\lambda \in \Lambda$, where P_λ is the projection of $\prod Y_\lambda$ onto Y_λ .

(ii) $f : \prod X_\lambda \rightarrow \prod Y_\lambda$ is al.sl. νg .c, iff $f_\lambda : X_\lambda \rightarrow Y_\lambda$ is al.sl. νg .c for each $\lambda \in \Lambda$.

Remark 2:

(i) Composition, Algebraic sum and product of al.sl. νg .c functions is not in general al.sl. νg .c.

(iii) The pointwise limit of a sequence of al.sl. νg .c functions is not in general al.sl. νg .c.

Example 3.3: Let $X = Y = [0, 1]$. Let $f_n : X \rightarrow Y$ is defined as follows $f_n(x) = x_n$ for $n = 1, 2, 3, \dots$, then f defined by $f(x) = 0$ if $0 \leq x < 1$ and $f(x) = 1$ if $x = 1$. Therefore each f_n is al.sl. νg .c but f is not al.sl. νg .c. For $(\frac{1}{2}, 1]$ is r-clopen in Y , but $f^{-1}((\frac{1}{2}, 1]) = \{1\}$ is not νg -open in X .

However we can prove the following:

Theorem 3.7: The uniform limit of a sequence of al.sl. νg .c functions is al.sl. νg .c.

Note 3: Pasting lemma is not true for al.sl. νg .c functions. However we have the following weaker versions.

Theorem 3.8: Let X and Y be topological spaces such that $X = A \cup B$ and let f/A and g/B are al.sl.r.c maps such that $f(x) = g(x) \forall x \in A \cap B$. If

$A, B \in RO(X)$ and $RO(X)$ is closed under finite unions, then the combination $\alpha : X \rightarrow Y$ is al.sl.vg.c continuous.

Theorem 3.9: Pasting lemma Let X and Y be spaces such that $X = A \cup B$ and let $f|_A$ and $g|_B$ are al.sl.vg.c maps such that $f(x) = g(x) \forall x \in A \cap B$. If $A, B \in RO(X)$ and $\nu GO(X)$ is closed under finite unions, then the combination $\alpha : X \rightarrow Y$ is al.sl.vg.c.

Proof: Let $F \in RCO(Y)$, then $\alpha^{-1}(F) = f^{-1}(F) \cup g^{-1}(F)$, where $f^{-1}(F) \in \nu GO(A)$ and $g^{-1}(F) \in \nu GO(B) \Rightarrow f^{-1}(F); g^{-1}(F) \in \nu GO(X) \Rightarrow f^{-1}(F) \cup g^{-1}(F) = \alpha^{-1}(F) \in \nu GO(X)$ [by assumption]. Hence $\alpha : X \rightarrow Y$ is al.sl.vg.c.

4. Comparisons:

Theorem 4.1: If f is sl.vg.c., then f is al.sl.vg.c.

Proof: Let $x \in X$ and $V \in RCO(Y, f(x))$, then $x \in X$ and $V \in CO(Y, f(x))$. Since f is sl.vg.c., \exists an $U \in \nu GO(X, x) \ni f(U) \subset V$. Hence f is al.sl.vg.c.

Theorem 4.2: If f is v.g.c., then f is sl.vg.c.

Proof: Let $x \in X$ and $V \in CO(Y, f(x))$, then $x \in X$ and $V \in \sigma(Y, f(x))$. Since f is v.g.c., $f^{-1}(V) \in \nu GO(X, x)$ i.e., \exists an $U_x \in \nu GO(X, x) \ni U_x \subset f^{-1}(V) \Rightarrow f(U_x) \subset V$. Hence f is sl.vg.c.

Theorem 4.3: If f is c.vg.c., then f is sl.vg.c.

Proof: Let $x \in X$ and $V \in CO(Y, f(x))$, then $x \in X$ and V is closed in Y containing $f(x)$. Since f is c.vg.c., $f^{-1}(V) \in \nu GO(X, x)$ i.e., \exists an $U_x \in \nu GO(X, x) \ni U_x \subset f^{-1}(V) \Rightarrow f(U_x) \subset V$. Hence f is sl.vg.c.

Theorem 4.4: If f is al.vg.c., then f is al.sl.vg.c.

Proof: Let $x \in X$ and $V \in RCO(Y, f(x))$, then $x \in X$ and $V \in \sigma(Y, f(x))$. Since f is al.vg.c., $f^{-1}(V) \in \nu GO(X, x)$ i.e., \exists an $U_x \in \nu GO(X, x) \ni U_x \subset f^{-1}(V) \Rightarrow f(U_x) \subset V$. Hence f is sl.vg.c.

Theorem 4.5: If f is al.c.vg.c., then f is al.sl.vg.c.

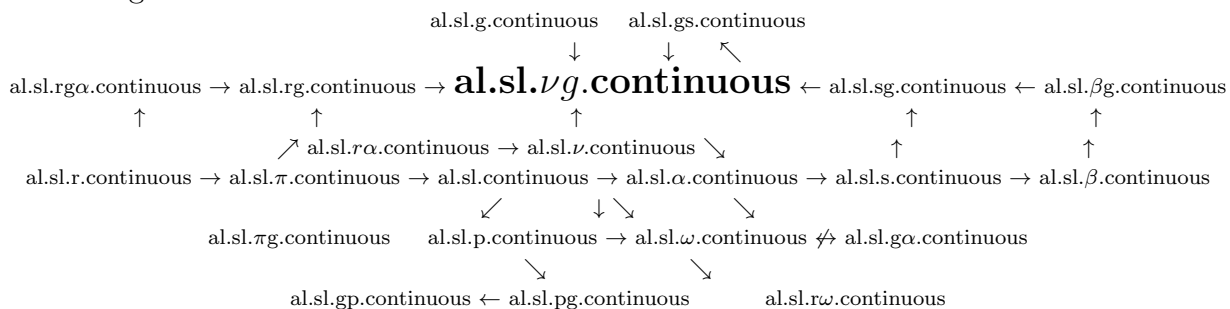
Proof: Let $x \in X$ and $V \in RCO(Y, f(x))$, then $x \in X$ and V is closed in Y containing $f(x)$. Since f is al.c.vg.c., $f^{-1}(V) \in \nu GO(X, x)$ i.e., \exists an $U_x \in \nu GO(X, x) \ni U_x \subset f^{-1}(V) \Rightarrow f(U_x) \subset V$. Hence f is sl.vg.c.

Theorem 4.6:

- (i) If f is al.sl.rg.c, then f is al.sl.vg.c.
- (ii) If f is al.sl.sg.c, then f is al.sl.vg.c.
- (iii) If f is al.sl.g.c, then f is al.sl.vg.c.
- (iv) If f is al.sl.s.c, then f is al.sl.vg.c.

- (v) If f is al.sl. ν .c, then f is al.sl. νg .c.
- (vi) If f is al.sl.r.c, then f is al.sl. νg .c.
- (vii) If f is al.sl.c, then f is al.sl. νg .c.
- (viii) If f is al.sl. ω .c, then f is al.sl. νg .c.
- (ix) If f is al.sl.rg α .c, then f is al.sl.rg.c.
- (x) If f is al.sl. ω -irresolute, then f is al.sl. νg .c.
- (xi) If f is al.sl.r. ω .c, then f is al.sl. νg .c.
- (xii) If f is al.sl. π .c, then f is al.sl. νg .c.
- (xiii) If f is al.sl. α .c, then f is al.sl. νg .c.
- (xiv) If f is al.sl.g α .c, then f is al.sl. νg .c.

Note 4: By note 1 and from the above theorem we have the following implication diagram.



Theorem 4.7:

- (i) If $\text{R}\alpha\text{O}(X) = \text{RO}(X)$ then f is al.sl.r α .c. iff f is al.sl.r.c.
- (ii) If $\nu\text{GO}(X) = \text{R}\alpha\text{O}(X)$ then f is al.sl.r α .c. iff f is al.sl. νg .c.
- (iii) If $\nu\text{GO}(X) = \text{RO}(X)$ then f is al.sl.r α .c. iff f is al.sl. νg .c.
- (iv) If $\nu\text{GO}(X) = \alpha\text{O}(X)$ then f is al.sl. α .c. iff f is al.sl. νg .c.
- (v) If $\nu\text{GO}(X) = \text{SO}(X)$ then f is al.sl.s.c. iff f is al.sl. νg .c.
- (vi) If $\nu\text{GO}(X) = \beta\text{O}(X)$ then f is al.sl. β .c. iff f is al.sl. νg .c.

Theorem 4.8: If f is al.sl. νg .c., from a discrete space X into a e.d space Y , then f is w.s.c.

Proof: Follows from note 3 above and theorem 3[41] of T.M.Nour.

Corollary 4.1: If f is al.sl. νg .c., from a discrete space X into a e.d space Y , then:

- (i) f is w.s.c.
- (ii) f is w. β .c.
- (iii) f is w.p.c.

Proof: Follows from note 3 above and theorem 4.8.

Theorem 4.9: If f is al.sl. νg .c., and X is discrete and e.d, then f is al.sl.c.

Proof: Let $x \in X$ and $V \in RCO(Y, f(x))$. Since f is al.sl. ν g.c., $\exists U \in \nu GO(X, x) \ni f(U) \subset V \Rightarrow U \in SR(X, x) \ni f(U) \subset V$. Since X is discrete and e.d. $U \in CO(X)$. Hence f is al.sl.c.

Corollary 4.2: If f is al.sl. ν g.c., and X is $\nu T_{\frac{1}{2}}$, discrete and e.d, then:

- (i) f is al.sl.c.
- (ii) f is al.sl. α .c.
- (iii) f is al.sl.s.c.
- (iv) f is al.sl. β .c.
- (v) f is al.sl.p.c.

Proof: Follows from note 3 above and theorem 4.9.

Theorem 4.10: If f is al.sl. ν g.c., from a discrete space X into a e.d space Y , then f st. θ .s.c.

Proof: Let $x \in X$ and $V \in \sigma(Y, f(x))$, then $\overline{s(V)} \subset (\overline{V})^o \in RO(Y)$. Since Y is e.d, $\overline{s(V)} \in CO(Y)$. Since f is al.sl. ν g.c, f is al.sl.s.c, $\exists U \in SO(X, x) \ni f(\overline{s(U)}) \subset \overline{s(V)}$, so f is a.st. θ .s.c.

Theorem 4.11: If f is al.sl. ν g.c from a discrete space X into a T_3 space Y , then f st. θ .s.c.

Proof: Let $x \in X$ and $V \in \sigma(Y, f(x))$. Since Y is Ultra regular, $\exists W \in CO(Y) \ni f(x) \in W \subset V$. Since f is al.sl. ν g.c, $\exists U \in SO(X, x) \ni f(\overline{s(U)}) \subset W$ and $f(\overline{s(U)}) \subset V$. Thus f is st. θ .s.c.

Example 4.1: Example 3.1 above f is al.sl. ν g.c; al.sl.sg.c; al.sl.gs.c; al.sl.r α .c; al.sl. ν .c; al.sl.s.c. and al.sl. β .c; but not al.sl.g.c; al.sl.rg.c; al.sl.gr.c; al.sl.pg.c; al.sl.gp.c; al.sl.gpr.c; al.sl.g α .c; al.sl. α g.c; al.sl.rg α .c; al.sl.r.c; al.sl.p.c; al.sl. α .c; and al.sl.c;

Example 4.2: Example 3.2 above f is al.sl.r α .c; and al.sl.gpr.c; but not al.sl. ν g.c; al.sl.sg.c; al.sl.gs.c; al.sl. ν .c; al.sl.s.c; al.sl. β .c; al.sl.g.c; al.sl.rg.c; al.sl.gr.c; al.sl.pg.c; al.sl.gp.c; al.sl.g α .c; al.sl. α g.c; al.sl.rg α .c; al.sl.r.c; al.sl.p.c; al.sl. α .c; and al.sl.c;

Remark 4.1: al.sl.r α .c; al.sl.gpr.c; and al.sl. ν g.c. are independent to each other.

Example 4.3: Example 3.1 above f is al.sl. ν g.c and al.sl.r α .c; but not al.sl.gpr.c

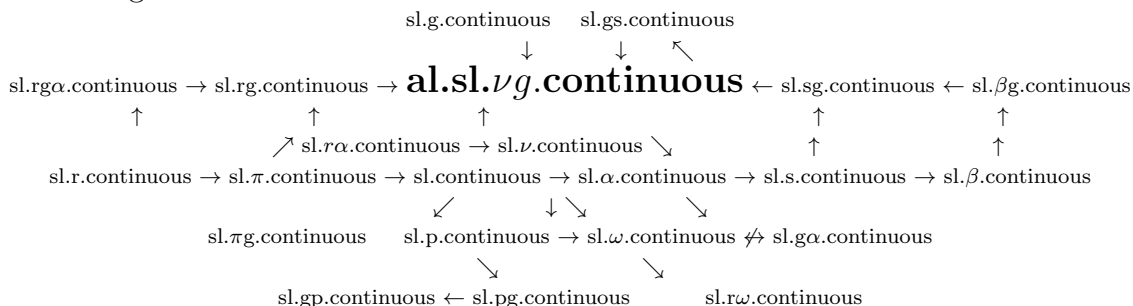
Example 4.4: Example 3.2 above f is al.sl.r α .c; and al.sl.gpr.c; but not al.sl. ν g.c

Theorem 4.12:

- (i) If f is sl.rg.c, then f is al.sl. νg .c.
- (ii) If f is sl.sg.c, then f is al.sl. νg .c.
- (iii) If f is sl.g.c, then f is al.sl. νg .c.
- (iv) If f is sl.s.c, then f is al.sl. νg .c.
- (v) If f is sl. ν .c, then f is al.sl. νg .c.
- (vi) If f is sl.r.c, then f is al.sl. νg .c.
- (vii) If f is sl.c, then f is al.sl. νg .c.
- (viii) If f is sl. ω .c, then f is al.sl. νg .c.
- (ix) If f is sl.rg α .c, then f is al.sl.rg.c.
- (x) If f is sl. ω -irresolute, then f is al.sl. νg .c.
- (xi) If f is sl.r. ω .c, then f is al.sl. νg .c.
- (xii) If f is sl. π .c, then f is al.sl. νg .c.
- (xiii) If f is sl. α .c, then f is al.sl. νg .c.
- (xiv) If f is sl.g α .c, then f is al.sl. νg .c.

Proof: Follows from Note 3[12] and above theorem.

Note 5: By note 1 and from the above theorem we have the following implication diagram.



5. Covering and Separation Properties:

Theorem 5.1: If f is al.sl. νg .c.[resp: al.sl.rg.c] surjection and X is νg -compact, then Y is compact.

Proof: Let $\{G_i : i \in I\}$ be any open cover for Y . Then each G_i is open in Y and hence each G_i is r -clopen in Y . Since f is al.sl. νg .c., $f^{-1}(G_i)$ is νg -open in X . Thus $\{f^{-1}(G_i)\}$ forms a νg -open cover for X and hence have a finite subcover, since X is νg -compact. Since f is surjection, $Y = f(X) = \bigcup_{i=1}^n G_i$. Therefore Y is compact.

Corollary 5.1: If f is al.sl. ν .c.[resp: al.sl.r.c] surjection and X is νg -compact, then Y is compact.

Theorem 5.2: If f is al.sl. νg .c., surjection and X is νg -compact [νg -lindeloff] then Y is mildly compact [mildly lindeloff].

Proof: Let $\{U_i : i \in I\}$ be r-clopen cover for Y . For each $x \in X$, $\exists \alpha_x \in I \ni f(x) \in U_{\alpha_x}$ and $\exists V_x \in \nu GO(X, x) \ni f(V_x) \subset U_{\alpha_x}$. Since the family $\{V_i : i \in I\}$ is a cover of X by νg -open sets of X , there exists a finite subset I_0 of $I \ni X \subset \{V_x : x \in I_0\}$. Therefore $Y \subset \bigcup \{f(V_x) : x \in I_0\} \subset \bigcup \{U_{\alpha_x} : x \in I_0\}$. Hence Y is mildly compact.

Corollary 5.2:

(i) If f is al.sl.rg.c.[resp: al.sl. ν .c.; al.sl.r.c.] surjection and X is νg -compact [νg -lindeloff] then Y is mildly compact [mildly lindeloff].

(ii) If f is al.sl. νg .c.[resp: al.sl.rg.c.; al.sl. ν .c.; al.sl.r.c.] surjection and X is locally νg -compact [resp: νg -Lindeloff; locally νg -lindeloff], then Y is locally compact [resp: Lindeloff; locally lindeloff].

(iii) If f is al.sl. νg .c.[al.sl.r.c.], surjection and X is locally νg -compact [resp: νg -lindeloff; locally νg -lindeloff] then Y is locally mildly compact {resp: locally mildly lindeloff}.

Theorem 5.3: If f is al.sl. νg .c., surjection and X is s-closed then Y is mildly compact [mildly lindeloff].

Proof: Let $\{V_i : V_i \in RCO(Y); i \in I\}$ be a cover of Y , then $\{f^{-1}(V_i) : i \in I\}$ is νg -open cover of X [by Thm 3.1] and so there is finite subset I_0 of I , such that $\{f^{-1}(V_i) : i \in I_0\}$ covers X . Therefore $\{(V_i) : i \in I_0\}$ covers Y since f is surjection. Hence Y is mildly compact.

Corollary 5.3: If f is al.sl.rg.c.[resp: al.sl. ν .c.; al.sl.r.c.] surjection and X is s-closed then Y is mildly compact [mildly lindeloff].

Theorem 5.4: If f is al.sl. νg .c., [resp: al.sl.rg.c.; al.sl. ν .c.; al.sl.r.c.] surjection and X is νg -connected, then Y is connected.

Proof: If Y is disconnected, then $Y = A \cup B$ where A and B are disjoint r-clopen sets in Y . Since f is al.sl. νg .c. surjection, $X = f^{-1}(Y) = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A) f^{-1}(B)$ are disjoint νg -open sets in X , which is a contradiction for X is νg -connected. Hence Y is connected.

Corollary 5.4: The inverse image of a disconnected space under a al.sl. νg .c., [resp: al.sl.rg.c.; al.sl. ν .c.; al.sl.r.c.] surjection is νg -disconnected.

Theorem 5.5: If f is al.sl. νg .c. [resp: al.sl.rg.c.; al.sl. ν .c.], injection and Y is UT_i , then X is νg_i $i = 0, 1, 2$.

Proof: Let $x_1 \neq x_2 \in X$. Then $f(x_1) \neq f(x_2) \in Y$ since f is injective. For Y is $UT_2 \exists V_j \in RCO(Y) \ni f(x_j) \in V_j$ and $\cap V_j = \phi$ for $j = 1, 2$. By Theorem 3.1,

$x_j \in f^{-1}(V_j) \in \nu GO(X)$ for $j = 1, 2$ and $\cap f^{-1}(V_j) = \phi$ for $j = 1, 2$. Thus X is νg_2 .

Theorem 5.6: If f is al.sl. νg .c.[resp: al.sl.rg.c.; al.sl. ν .c.], injection; closed and Y is UT_i , then X is νg_i $i = 3, 4$.

Proof:(i) Let $x \in X$ and F be disjoint closed subset of X not containing x , then $f(x)$ and $f(F)$ are disjoint closed subset of Y , since f is closed and injection. Since Y is ultraregular, $f(x)$ and $f(F)$ are separated by disjoint r-clopen sets U and V respectively. Hence $x \in f^{-1}(U)$; $F \subseteq f^{-1}(V)$, $f^{-1}(U)$; $f^{-1}(V) \in \nu GO(X)$ and $f^{-1}(U) \cap f^{-1}(V) = \phi$. Thus X is νg_3 .

(ii) Let F_j and $f(F_j)$ are disjoint closed subsets of X and Y respectively for $j = 1, 2$, since f is closed and injection. For Y is ultranormal, $f(F_j)$ are separated by disjoint r-clopen sets V_j respectively for $j = 1, 2$. Hence $F_j \subseteq f^{-1}(V_j)$ and $f^{-1}(V_j) \in \nu GO(X)$ and $\cap f^{-1}(V_j) = \phi$ for $j = 1, 2$. Thus X is νg_4 .

Theorem 5.7: If f is al.sl. νg .c.[resp: al.sl.rg.c.; al.sl. ν .c.], injection and

(i) Y is UC_i [resp: UD_i] then X is νgC_i [resp: νgD_i] $i = 0, 1, 2$.

(ii) Y is UR_i , then X is νgR_i $i = 0, 1$.

Theorem 5.8: If f is al.sl. νg .c.[resp: al.sl. ν .c.; al.sl.rg.c; al.sl.r.c] and Y is UT_2 , then the graph $G(f)$ of f is νg -closed in the product space $X \times Y$.

Proof: Let $(x_1, x_2) \notin G(f) \Rightarrow y \neq f(x) \Rightarrow \exists$ disjoint r-clopen sets V and $W \ni f(x) \in V$ and $y \in W$. Since f is al.sl. νg .c., $\exists U \in \nu GO(X) \ni x \in U$ and $f(U) \subset W$. Therefore $(x, y) \in U \times V \subset X \times Y - G(f)$. Hence $G(f)$ is νg -closed in $X \times Y$.

Theorem 5.9: If f is al.sl. νg .c.[resp: al.sl. ν .c.; al.sl.rg.c; al.sl.r.c] and Y is UT_2 , then $A = \{(x_1, x_2) | f(x_1) = f(x_2)\}$ is νg -closed in the product space $X \times X$.

Proof: If $(x_1, x_2) \in X \times X - A$, then $f(x_1) \neq f(x_2) \Rightarrow \exists$ disjoint $V_j \in RCO(Y) \ni f(x_j) \in V_j$, and since f is al.sl. νg .c., $f^{-1}(V_j) \in \nu GO(X, x_j)$ for each $j = 1, 2$. Thus $(x_1, x_2) \in f^{-1}(V_1) \times f^{-1}(V_2) \in \nu GO(X \times X)$ and $f^{-1}(V_1) \times f^{-1}(V_2) \subset X \times X - A$. Hence A is νg -closed.

Theorem 5.10: If f is al.sl.r.c.[resp: al.sl.c.]; g is al.sl. νg .c[resp: al.sl.rg.c; al.sl. ν .c]; and Y is UT_2 , then $E = \{x \in X : f(x) = g(x)\}$ is νg -closed in X .

Conclusion: In this paper we defined almost slightly- νg -continuous functions, studied its properties and their interrelations with other types of almost slightly-continuous functions.

References

- [1] M.E. Abd El-Monsef, S.N. Eldeeb and R.A. Mahmoud, β -open sets and β -continuous mappings, *Bull. Fac. Sci. Assiut. Univ.*, A 12(1) (1983), 77-90.
- [2] Andreivic, β -open sets, *Math. Vestnick.*, 38(1986), 24-32.
- [3] A.N. Uresin, A. Kerkin and T. Noiri, Slightly δ -precontinuous funtions, *Commen. Fac. Sci. Univ. Ark.*, Series 41, 56(2) (2007), 1-9.
- [4] S.P. Arya and M.P. Bhamini, Some weaker forms of semi-continuous functions, *Ganita*, 33(1-2) (1982), 124-134.
- [5] C.W. Baker, Slightly precontinuous funtions, *Acta Math Hung.*, 94(1-6) (2002), 45-52.
- [6] S. Balasubramanian, C. Sandhya and P.A.S. Vyjayanthi, On $\nu - T_i, \nu - R_i$ and $\nu - C_i$ axioms, *Scientia Magna*, 4(4) (2008), 86-103.
- [7] S. Balasubramanian, νg -closed sets, *Bull. Kerala Math. Association*, 5(2) (2009), 81-92.
- [8] S. Balasubramanian, C. Sandhya and P.A.S. Vyjayanthi, On νD -sets and separation axioms, *Int. J. Math. Anal.*, 4(19) (2010), 909-919.
- [9] S. Balasubramanian, νg -continuity, *Proc. International Seminar on New Trends in Applied Mathematics*, Bharatha Matha College, Ernakulam, Kerala, (2011).
- [10] S. Balasubramanian, Slightly νg -continuity, *Inter. J. Math. Archive*, 2(8) (2011), 1455-1463.
- [11] S. Balasubramanian, On νg -closed sets, *Inter. J. Math. Archive*, 2(10) (2011), 1909-1915.
- [12] S. Balasubramanian, νg -open mappings, *Inter. J. Comp. Math. Sci. and Application*, 5(2) (2011), 7-14.
- [13] S. Balasubramanian, νg -boundary and νg -exterior operators, *Acta Ciencia Indica*, 37(M)(1) (2011), 11-18.
- [14] S. Balasubramanian, Almost νg -Continuity, *Scientia Magna*, 7(3) (2011), 1-11.
- [15] S. Balasubramanian, Somewhat almost νg -continuity and somewhat almost νg -open map, *Proc. ICMANW*, (2011).

- [16] S. Balasubramanian, Contra νg -Continuity, *General Mathematical Notes*, 10(1) (2012), 1-18.
- [17] S. Balasubramanian, Somewhat νg -continuity, *Bull. Kerala Math. Soc.*, 9(1) (2012), 185-197.
- [18] S. Balasubramanian, Almost contra νg -Continuity, *International Journal of Mathematical Engineering and Science*, 1(8) (2012), 51-65.
- [19] S. Balasubramanian, Somewhat M- νg -open map, *Aryabhatta J. Math and Informatics*, 4(2) (2012), 315-320.
- [20] S. Balasubramanian, Somewhat almost νg -open map, *Ref. Des. Era, J. Math. Sci.*, 7(4) (November) (2012), 289-296.
- [21] S. Balasubramanian, Somewhat almost νg -continuity, *Ref. Des. Era, J. Math. Sci.*, 7(4) (November) (2012), 335-342.
- [22] S. Balasubramanian, νg -separation axioms, *Scientia Magna*, 9(2) (2013), 57-75.
- [23] S. Balasubramanian, More on νg -separation axioms, *Scientia Magna*, 9(2) (2013), 76-92.
- [24] Y. Beceron, S. Yukseh and E. Hatir, On almost strongly θ -semi continuous functions, *Bull. Cal. Math. Soc.*, 87(2013), 329.
- [25] A. Davis, Indexed system of neighbourhoods for general topological spaces, *Amer. Math. Monthly*, 68(1961), 886-893.
- [26] G. Di. Maio, A separation axiom weaker than R_0 , *IJPAM*, 16(1983), 373-375.
- [27] G. Di. Maio and T. Noiri, On s-closed spaces, *Indian J. Pure and Appl. Math*, 18(3) (1987), 226-233.
- [28] W. Dunham, $T_{\frac{1}{2}}$ spaces, *Kyungpook Math. J.*, 17(1977), 161-169.
- [29] E. Ekici and M. Caldas, Slightly γ -continuous functions, *Bol. Sac. Paran. Mat*, (38) (V.22.2) (2004), 63-74.
- [30] S.N. Maheswari and R. Prasad, On R_0 spaces, *Portugal Math.*, 34(1975), 213-217.
- [31] S.N. Maheswari and R. Prasad, Some new separation axioms, *Ann. Soc. Sci, Bruxelle*, 89(1975), 395.

- [32] S.N. Maheswari and R. Prasad, On s-normal spaces, *Bull. Math. Soc. Sci. R.S. Roumania*, 22(70) (1978), 27.
- [33] S.N. Maheswari and S.S. Thakur, On α -iresolute mappings, *Tamkang J. Math.*, 11(1980), 201-214.
- [34] R.A. Mahmoud and M.E. Abd El-Monsef, β -irresolute and β -topological invariant, *Proc. Pak. Acad. Sci*, 27(3) (1990), 285-296.
- [35] A.S. Mashhour, M.E. Abd El-Monsef and S.N. El-Deep, On precontinuous and weak precontinuous functions, *Proc. Math. Phy. Soc. Egypt*, 3(1982), 47-53.
- [36] A.S. Mashhour, M.E. Abd El-Monsef and S.N. El-Deep, α -continuous and α -open mappings, *Acta Math Hung.*, 41(3-4) (1983), 231-218.
- [37] G.B. Navalagi, Further propertis on pre- T_0 , pre- T_1 , pre- T_2 spaces, (Preprint).
- [38] T. Noiri and G.I. Chae, A note on slightly semi continuous functions, *Bull. Cal. Math. Soc*, 92(2) (2000), 87-92.
- [39] T. Noiri, Slightly β -continuous functions, *I.J.M.&M.S.*, 28(8) (2001), 469-478.
- [40] T.M. Nour, Slightly semi continuous functions, *Bull. Cal. Math. Soc*, 87(1995), 187-190.
- [41] Singhal and Singhal, Almost continuous mappings, *Yoko. J. Math.*, 16(1968), 63-73.