## Research Article

# Improved Robust Stability Criteria of Uncertain Neutral Systems with Mixed Delays 

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#### Abstract

The problem of robust stability for a class of neutral control systems with mixed delays is investigated. Based on Lyapunov stable theory, by constructing a new Lyapunov-Krasovskii function, some new stable criteria are obtained. These criteria are formulated in the forms of linear matrix inequalities (LMIs). Compared with some previous publications, our results are less conservative. Simulation examples are presented to illustrate the improvement of the main results.


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## 1. Introduction

As one of important dynamical systems, neutral system has been received considerable attention in past years. Large numbers of monographs and papers on the stability of neutral type system with or without time delays have been published. A wide variety of methods disposing the stability problems of neutral system have been proposed [1-6]. It is well known that, because of the finite switching speed, memory effect, and so on, time delays are unavoidable in nature and technology. They can make important effects on the stability of concerned dynamical systems. Thus, the studies on stability of time-delayed neutral system are of great significance. In recent years, all kinds of delays such as time-varying delay [7-10], distributed delay [11-13], and mixed delay [14-16] were considered, and corresponding stable criteria have been derived. In practical, during the design of control system and its hardware implementation, the convergence of a control system may often be destroyed by its unavoidable uncertainty due to the existence of modeling error, the deviation of vital data, and so on. Therefore, the studies on robust convergence of delayed control
system have been a hot research topic, and many sufficient conditions have been derived to guarantee the robust asymptotic or exponential stability for different class of delayed systems (see $[2,7,8,10,12,13,17-20]$ ). General speaking, these criteria can be divided into two categories [21]: that is, delay-independent criteria and delay-dependent criteria. As pointed out in [12], when the size of time delay is small, delay-dependent criteria may be less conservative than those of delay-independent criteria, and the more free-weighting matrices are introduced in criterion, the less conservative it may be. On the other hand, compared with traditional matrix measure, matrix norm, and Riccati matrix criteria, linear matrix inequality (LMI) technique can be easily checked by LMI toolbox in MATLAB software and can make free weighting matrices easy to select. Thus, it becomes one of the most extensively used techniques in control system. In addition, the admissible allowed upper bound on the delay is usually regarded as the performance index for measuring the conservatism of the conditions obtained.

Motivated by the afore-mentioned analysis, in this paper, based on the equivalent equation of the zero which is similar to [2] in the derivative of a Lyapunov-Krasovskii functional, we will focus on deriving some improved robust stable criteria for a class of neutral control systems with mixed delays. By constructing a new Lyapunov function, some new delay-dependent stable criteria are derived via sufficiently employing NewtonLeibniz formula to introduce large numbers of free weighting matrices. These free weighting matrices express the influence of the relationship among terms $x(t), \dot{x}(t), x(t-$ $\left.\tau_{1}\right), x\left(t-\tau_{2}\right), \dot{x}\left(t-\tau_{2}\right), \int_{t-\tau_{1}}^{t} \dot{x}(s) d s, \int_{t-\tau_{2}}^{t} \dot{x}(s) d s$. Since these criteria are both discrete delay-dependent and distributed delay-dependent, they are less conservative than some previous methods for the concerned systems. When norm-bounded parameter uncertainties appear in the concerned system, delay-dependent robust asymptotic stability criteria are also presented. All of these criteria are expressed in the forms of linear matrix inequalities (LMIs), which can be easily solved. Finally, numerical examples are given to illustrate the improvement of the main results. Simulations show that our results are valid.

## 2. Preliminaries

Consider uncertain neutral system with mixed delays [4] as follows:

$$
\Sigma:\left\{\begin{array}{l}
\dot{x}(t)-C \dot{x}\left(t-\tau_{2}\right)=(A+\Delta A(t)) x(t)+(B+\Delta B(t)) x\left(t-\tau_{1}\right)  \tag{2.1}\\
\quad+(D+\Delta D(t)) \int_{t-h}^{t} x(s) d s, \quad t>0, \\
x(t)=\phi(t), t \in[-\tau, 0],
\end{array}\right.
$$

where $x(t) \in \mathcal{R}^{n}$ is the state vector; $A, B, C \in \mathcal{R}^{n \times n}$ represent the weighting matrices; $\tau_{1}, \tau_{2}$ are discrete delays; $h$ is distributed delay; $\phi(t)$ is initial condition which is continuous on interval $[-\tau, 0]$, where $\tau=\max \left\{\tau_{1}, \tau_{2}, h\right\} ; \Delta A(t), \Delta B(t), \Delta D(t)$ denote the time-varying structured uncertainties which are of the following form:

$$
\begin{equation*}
[\Delta A(t), \Delta B(t), \Delta D(t)]=K F(t)\left[E_{a}, E_{b}, E_{d}\right] \tag{2.2}
\end{equation*}
$$

where $K, E_{a}, E_{b}, E_{d}$ are the known constant matrices with appropriate dimensions; $F(t)$ is unknown continuous time-varying matrix function satisfying $F^{T}(t) F(t) \leq I$, for all $t \geq 0$.

The nominal $\Sigma_{0}$ of $\Sigma$ can be defined as

$$
\Sigma_{0}:\left\{\begin{array}{l}
\dot{x}(t)-C \dot{x}\left(t-\tau_{2}\right)=A x(t)+B x\left(t-\tau_{1}\right)+D \int_{t-h}^{t} x(s) d s, \quad t>0  \tag{2.3}\\
x(t)=\phi(t), t \in[-\tau, 0]
\end{array}\right.
$$

For further discussion, we first introduce the following lemmas.
Lemma 2.1 (see [22]). Given constant symmetric matrices $\Sigma_{1}, \Sigma_{2}, \Sigma_{3}$ where $\Sigma_{1}^{T}=\Sigma_{1}$ and $0<\Sigma_{2}=$ $\Sigma_{2}^{T}$, then $\Sigma_{1}+\Sigma_{3}^{T} \Sigma_{2}^{-1} \Sigma_{3}<0$ if and only if

$$
\left(\begin{array}{cc}
\Sigma_{1} & \Sigma_{3}^{T}  \tag{2.4}\\
\Sigma_{3} & -\Sigma_{2}
\end{array}\right)<0, \quad \text { or } \quad\left(\begin{array}{cc}
-\Sigma_{2} & \Sigma_{3} \\
\Sigma_{3}^{T} & \Sigma_{1}
\end{array}\right)<0
$$

Lemma 2.2 (see [23]). For given matrices $Q=Q^{T}, H, E$, and $R=R^{T}>0$ with appropriate dimensions, then

$$
\begin{equation*}
Q+H F E+E^{T} F^{T} H^{T}<0, \tag{2.5}
\end{equation*}
$$

for all $F$ satisfying $F^{T} F \leq R$ if and only if there exists a positive number $\varepsilon>0$, such that

$$
\begin{equation*}
Q+\varepsilon^{-1} H H^{T}+\varepsilon E^{T} R E<0 \tag{2.6}
\end{equation*}
$$

Lemma 2.3. For any real vector $X, Y$ and positive definite matrix $\Sigma>0$ with appropriate dimensions, it follows that

$$
\begin{equation*}
2 X^{T} Y \leq X^{T} \Sigma X+Y^{T} \Sigma^{-1} Y \tag{2.7}
\end{equation*}
$$

## 3. Main Results

In this section, we will analyse the stability problem of uncertain neutral systems with mixed delays described by (2.1). First, we consider the stability problem for the nominal system
(2.3) with $\Delta A(t)=0, \Delta B(t)=0, \Delta D(t)=0$. In order to introduce free-weighting matrix, we can use the following fact:

$$
\begin{equation*}
M\left[x(t)-x\left(t-\tau_{1}\right)-\int_{t-\tau_{1}}^{t} \dot{x}(s) d s\right]=0 \tag{3.1}
\end{equation*}
$$

where $M$ is an arbitrary matrix with appropriate dimensions. Substituting zero equation (3.1) into system (2.3), the original system can be transformed into the following form:

$$
\begin{gather*}
\dot{x}(t)-C \dot{x}\left(t-\tau_{2}\right)=(A-M) x(t)+(B+M) x\left(t-\tau_{1}\right)+ \\
D \int_{t-h}^{t} x(s) d s+M \int_{t-\tau_{1}}^{t} \dot{x}(s) d s, \quad t>0,  \tag{3.2}\\
x(t)=\phi(t), \quad t \in[-\tau, 0] .
\end{gather*}
$$

For the asymptotic stability of system (3.2), we can obtain the following results.
Theorem 3.1. For any given matrix $M$, scalars $\tau_{1}>0, \tau_{2}>0, h>0$, the nominal system $\Sigma_{0}$ is asymptotically stable if $\|C\|<1$, and there exist positive definite matrices $P_{1}, Q_{1}, Q_{2}, Q_{3}, Q_{4}, Q_{5}$, arbitrary matrices $P_{i}(i=2,3, \ldots, 25), F_{i}(i=1,2, \ldots, 8)$ with appropriate dimensions such that the following linear matrix inequality is feasible:

$$
\Xi_{1}=\left(\begin{array}{ccccccccc}
\Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} & \Xi_{17} & \Xi_{18} & \Xi_{19}  \tag{3.3}\\
* & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} & \Xi_{26} & \Xi_{27} & \Xi_{28} & \Xi_{29} \\
* & * & \Xi_{33} & \Xi_{34} & \Xi_{35} & \Xi_{36} & \Xi_{37} & \Xi_{38} & \Xi_{39} \\
* & * & * & \Xi_{44} & \Xi_{45} & \Xi_{46} & \Xi_{47} & \Xi_{48} & \Xi_{49} \\
* & * & * & * & \Xi_{55} & \Xi_{56} & \Xi_{57} & \Xi_{58} & \Xi_{59} \\
* & * & * & * & * & \Xi_{66} & \Xi_{67} & \Xi_{68} & \Xi_{69} \\
* & * & * & * & * & * & \Xi_{77} & \Xi_{78} & \Xi_{79} \\
* & * & * & * & * & * & * & \Xi_{88} & \Xi_{89} \\
* & * & * & * & * & * & * & * & \Xi_{99}
\end{array}\right)<0,
$$

where

$$
\begin{aligned}
& \Xi_{11}=P_{2}^{T}(A-M)+P_{10}^{T}+P_{18}^{T}+(A-M)^{T} P_{2}+P_{10}+P_{18}+h Q_{3}+Q_{1}, \\
& \Xi_{12}=P_{1}^{T}-P_{2}^{T}+(A-M)^{T} P_{3}+P_{11}+P_{19}, \\
& \Xi_{13}=P_{2}^{T}(B+M)-P_{18}^{T}+(A-M)^{T} P_{4}+P_{12}+P_{20}, \\
& \Xi_{14}=-P_{10}^{T}+(A-M)^{T} P_{5}+P_{13}+P_{21}, \\
& \Xi_{15}=P_{2}^{T} C+(A-M)^{T} P_{6}+P_{14}+P_{22}, \\
& \Xi_{16}=P_{2}^{T} M-P_{18}^{T}+(A-M)^{T} P_{7}+P_{15}+P_{23}-F_{1}^{T}, \\
& \Xi_{17}=-P_{10}^{T}+(A-M)^{T} P_{8}+P_{16}+P_{24}-F_{1}^{T}, \\
& \Xi_{18}=P_{2}^{T} D+(A-M)^{T} P_{9}+P_{17}+P_{25}-F_{1}^{T}, \\
& \Xi_{19}=F_{1}^{T}, \\
& \Xi_{22}=-P_{3}^{T}+P_{3}^{T}+\tau_{1} Q_{4}+\tau_{2} Q_{5}+Q_{2}, \\
& \Xi_{23}=P_{3}^{T}(B+M)-P_{19}^{T}-P_{4}, \Xi_{24}=-P_{11}^{T}-P_{5}^{T}, \\
& \Xi_{25}=P_{3}^{T} C-P_{6}^{T}, \Xi_{26}=P_{3}^{T} M-P_{19}^{T}-P_{7}-F_{2}^{T}, \\
& \Xi_{27}=-P_{11}^{T}-P_{8}-F_{2}^{T}, \Xi_{28}=P_{3}^{T} D-P_{9}-F_{2}^{T}, \\
& \Xi_{29}=F_{2}^{T}, \\
& \Xi_{33}=P_{4}^{T}(B+M)-P_{20}^{T}+(B+M)^{T} P_{4}-P_{20}-Q_{1}, \\
& \Xi_{34}=-P_{12}^{T}+(B+M)^{T} P_{5}-P_{21}, \\
& \Xi_{35}=P_{4}^{T} C+(B+M)^{T} P_{6}-P_{22}, \\
& \Xi_{36}=P_{4}^{T} M-P_{20}^{T}+(B+M)^{T} P_{7}-P_{23}-F_{3}^{T}, \\
& \Xi_{37}=-P_{12}^{T}+(B+M)^{T} P_{8}-P_{24}-F_{3}^{T}, \\
& \Xi_{38}=P_{4}^{T} D+(B+M)^{T} P_{9}-P_{25}-F_{3}^{T}, \\
& \Xi_{39}=F_{3}^{T}, \\
& \Xi_{44}=-P_{13}^{T}-P_{13}, \\
&
\end{aligned}
$$

$$
\begin{align*}
& \Xi_{45}=P_{5}^{T} C-P_{14}, \\
& \Xi_{46}=P_{5}^{T} M-P_{21}^{T}-P_{15}-F_{4}^{T}, \\
& \Xi_{47}=-P_{13}^{T}-P_{16}-F_{4}^{T}, \\
& \Xi_{48}=P_{5}^{T} D-P_{17}-F_{4}^{T}, \\
& \Xi_{49}=F_{4}^{T}, \\
& \Xi_{55}=P_{6}^{T} C+C^{T} P_{6}-Q_{2}, \\
& \Xi_{56}=P_{6}^{T} M-P_{22}^{T}+C^{T} P_{7}-F_{5}^{T}, \\
& \Xi_{57}=-P_{14}^{T}+C^{T} P_{8}-F_{5}^{T}, \\
& \Xi_{58}=P_{6}^{T} D+C^{T} P_{9}-F_{5}^{T}, \\
& \Xi_{59}=F_{5}^{T}, \\
& \Xi_{66}=P_{7}^{T} M-P_{23}^{T}+M^{T} P_{7}-P_{23}-F_{6}^{T}-F_{6}, \\
& \Xi_{67}=-P_{15}^{T}+M^{T} P_{8}-P_{24}-F_{6}^{T}-F_{7}^{T}, \\
& \Xi_{68}=P_{7}^{T} D+M^{T} P_{9}-P_{25}-F_{6}^{T}-F_{8}^{T}, \\
& \Xi_{69}=F_{6}^{T} \\
& \Xi_{89}=F_{8}^{T}, \\
& \Xi_{88}=P_{9}^{T} D+D^{T} P_{9}-F_{8}^{T}-F_{8}^{T}, \\
& \Xi_{77}=-P_{16}^{T}-P_{16}-F_{7}^{T}-F_{7}, \\
& \tau_{2} \\
& \Xi_{79}=P_{8}^{T} D-P_{17}-F_{8}^{T}-F_{7}^{T}, \\
& \Xi_{78}  \tag{3.4}\\
& \Xi_{7} \\
& \hline
\end{align*},
$$

Proof. Constructing a new Lyapunov functional candidate for system (3.2) as follows:

$$
\begin{equation*}
V(t)=V_{1}(t)+V_{2}(t)+V_{3}(t)+V_{4}(t)+V_{5}(t)+V_{6}(t) \tag{3.5}
\end{equation*}
$$

where

$$
\begin{align*}
V_{1}(t)= & Y^{T}(t) P Y(t), \\
Y(t)= & {\left[x^{T}(t), \dot{x}^{T}(t), x^{T}\left(t-\tau_{1}\right), x^{T}\left(t-\tau_{2}\right), \dot{x}^{T}\left(t-\tau_{2}\right),\left(\int_{t-\tau_{1}}^{t} \dot{x}(s) d s\right)^{T},\right.} \\
& \left.\left(\int_{t-\tau_{2}}^{t} \dot{x}(s) d s\right)^{T},\left(\int_{t-h}^{t} x(s) d s\right)^{T}\right]^{T}, \\
V_{2}(t)= & \int_{t-\tau_{1}}^{t} x^{T}(s) Q_{1} x(s) d s, \quad V_{3}(t)=\int_{t-\tau_{2}}^{t} \dot{x}^{T}(s) Q_{2} \dot{x}(s) d s, \quad V_{4}(t)=\int_{t-h}^{t} \int_{s}^{t} x^{T}(\xi) Q_{3} x(\xi) d \xi d s, \\
V_{5}(t)= & \int_{t-\tau_{1}}^{t} \int_{s}^{t} \dot{x}^{T}(\xi) Q_{4} \dot{x}(\xi) d \xi d s, \quad V_{6}(t)=\int_{t-\tau_{2}}^{t} \int_{s}^{t} \dot{x}^{T}(\xi) Q_{5} \dot{x}(\xi) d \xi d s, \tag{3.6}
\end{align*}
$$

$$
P=\left(\begin{array}{llllllll}
P_{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0  \tag{3.7}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right) .
$$

Set

$$
\tilde{P}=\left(\begin{array}{cccccccc}
P_{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0  \tag{3.8}\\
P_{2} & P_{3} & P_{4} & P_{5} & P_{6} & P_{7} & P_{8} & P_{9} \\
P_{10} & P_{11} & P_{12} & P_{13} & P_{14} & P_{15} & P_{16} & P_{17} \\
P_{18} & P_{19} & P_{20} & P_{21} & P_{22} & P_{23} & P_{24} & P_{25}
\end{array}\right)
$$

along the trajectories of system (3.2), the derivative of $V(t)$ is given by

$$
\begin{equation*}
\dot{V}(t)=\dot{V}_{1}(t)+\dot{V}_{2}(t)+\dot{V}_{3}(t)+\dot{V}_{4}(t)+\dot{V}_{5}(t)+\dot{V}_{6}(t) \tag{3.9}
\end{equation*}
$$

where

$$
\begin{align*}
& \dot{V}_{1}(t)=2 Y^{T}(t) P \dot{Y}(t) \\
& =2 Y^{T}(t) \tilde{P}^{T} \\
& \times\left(\begin{array}{c}
\dot{x}(t) \\
-\dot{x}(t)+C \dot{x}\left(t-\tau_{2}\right)+(A-M) x(t) \\
+(B+M) x\left(t-\tau_{1}\right)+D \int_{t-h}^{t} x(s) d s+M \int_{t-\tau_{1}}^{t} \dot{x}(s) d s \\
x(t)-x\left(t-\tau_{2}\right)-\int_{t-\tau_{2}}^{t} \dot{x}(s) d s \\
x(t)-x\left(t-\tau_{1}\right)-\int_{t-\tau_{1}}^{t} \dot{x}(s) d s
\end{array}\right)  \tag{3.10}\\
& =2 Y^{T}(t) \widetilde{P}^{T}\left(\begin{array}{cccccccc}
0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\
A-M & -I & B+M & 0 & C & M & 0 & D \\
I & 0 & 0 & -I & 0 & 0 & -I & 0 \\
I & 0 & -I & 0 & 0 & -I & 0 & 0
\end{array}\right) Y(t), \\
& \dot{V}_{2}(t)=x^{T}(t) Q_{1} x(t)-x^{T}\left(t-\tau_{1}\right) Q_{1} x\left(t-\tau_{1}\right), \\
& \dot{V}_{3}(t)=\dot{x}^{T}(t) Q_{2} \dot{x}(t)-\dot{x}^{T}\left(t-\tau_{2}\right) Q_{2} \dot{x}\left(t-\tau_{2}\right), \\
& \dot{V}_{2}(t)+\dot{V}_{3}(t)=Y^{T}(t)\left(\begin{array}{cccccccc}
Q_{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & Q_{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & -Q_{1} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -Q_{2} & 0 & 0 & 0 \\
* & * & * & * & * & 0 & 0 & 0 \\
* & * & * & * & * & * & 0 & 0 \\
* & * & * & * & * & * & * & 0
\end{array}\right) Y(t) . \tag{3.11}
\end{align*}
$$

By Lemma 2.3, similar to the disposal route in [34], we have

$$
\begin{align*}
\dot{V}_{4}(t) & =h x^{T}(t) Q_{3} x(t)-\int_{t-h}^{t} x^{T}(s) Q_{3} x(s) d s \\
& \leq h x^{T}(t) Q_{3} x(t)+\int_{t-h}^{t}\left[-2 x^{T}(s) F Y(t)+Y^{T}(t) F^{T} Q_{3}^{-1} F Y(t)\right] d s  \tag{3.12}\\
& =h x^{T}(t) Q_{3} x(t)-2\left(\int_{t-h}^{t} x(s) d s\right)^{T} F Y(t)+h Y^{T}(t) F^{T} Q_{3}^{-1} F Y(t),
\end{align*}
$$

where $F=\left[F_{1}, F_{2}, F_{3}, F_{4}, F_{5}, F_{6}, F_{7}, F_{8}\right]$. Similarly, we have

$$
\begin{align*}
\dot{V}_{5}(t) & =\tau_{1} \dot{x}^{T}(t) Q_{4} \dot{x}(t)-\int_{t-\tau_{1}}^{t} \dot{x}^{T}(s) Q_{4} \dot{x}(s) d s \\
& \leq \tau_{1} \dot{x}^{T}(t) Q_{4} \dot{x}(t)+\int_{t-\tau_{1}}^{t}\left[-2 \dot{x}^{T}(s) F Y(t)+Y^{T}(t) F^{T} Q_{4}^{-1} F Y(t)\right] d s  \tag{3.13}\\
& =\tau_{1} \dot{x}^{T}(t) Q_{4} \dot{x}(t)-2\left(\int_{t-\tau_{1}}^{t} \dot{x}(s) d s\right)^{T} F Y(t)+\tau_{1} Y^{T}(t) F^{T} Q_{4}^{-1} F Y(t), \\
\dot{V}_{6}(t) & =\tau_{2} \dot{x}^{T}(t) Q_{5} \dot{x}(t)-\int_{t-\tau_{2}}^{t} \dot{x}^{T}(s) Q_{5} \dot{x}(s) d s \\
& \leq \tau_{2} \dot{x}^{T}(t) Q_{5} \dot{x}(t)+\int_{t-\tau_{2}}^{t}\left[-2 \dot{x}^{T}(s) F Y(t)+Y^{T}(t) F^{T} Q_{5}^{-1} F Y(t)\right] d s  \tag{3.14}\\
& =\tau_{2} \dot{x}^{T}(t) Q_{5} \dot{x}(t)-2\left(\int_{t-\tau_{2}}^{t} \dot{x}(s) d s\right)^{T} F Y(t)+\tau_{2} Y^{T}(t) F^{T} Q_{5}^{-1} F Y(t) .
\end{align*}
$$

From (3.12)-(3.14), we get

$$
\begin{align*}
& \dot{V}_{4}(t)+\dot{V}_{5}(t)+\dot{V}_{6}(t) \\
& \leq\left(\begin{array}{cccccccc}
h Q_{3} & 0 & 0 & 0 & 0 & -F_{1}^{T} & -F_{1}^{T} & -F_{1}^{T} \\
* & \tau_{1} Q_{4}+\tau_{2} Q_{5} & 0 & 0 & 0 & -F_{2}^{T} & -F_{2}^{T} & F_{2}^{T} \\
* & * & 0 & 0 & 0 & -F_{3}^{T} & -F_{3}^{T} & -F_{3}^{T} \\
* & * & * & 0 & 0 & -F_{4}^{T} & -F_{4}^{T} & -F_{4}^{T} \\
* & * & * & * & 0 & -F_{5}^{T} & -F_{5}^{T} & -F_{5}^{T} \\
* & * & * & * & * & -F_{6}^{T}-F_{6} & -F_{7}^{T}-F_{6}^{T} & -F_{8}^{T}-F_{6}^{T} \\
* & * & * & * & * & * & -F_{7}^{T}-F_{7}^{T} & -F_{8}^{T}-F_{7}^{T} \\
* & * & * & * & * & * & * & -F_{8}^{T}-F_{8}^{T}
\end{array}\right) \Upsilon(t)  \tag{3.15}\\
& \quad+Y^{T}(t) F^{T}\left(h Q_{3}^{-1}+\tau_{1} Q_{4}^{-1}+\tau_{2} Q_{5}^{-1}\right) F Y(t) .
\end{align*}
$$

Hence,

$$
\begin{equation*}
\dot{V}(t) \leq Y^{T}(t)\left[\Xi^{\prime}+F^{T}\left(h Q_{3}^{-1}+\tau_{1} Q_{4}^{-1}+\tau_{2} Q_{5}^{-1}\right) F\right] Y(t) \tag{3.16}
\end{equation*}
$$

where

$$
\Xi^{\prime}=\left(\begin{array}{cccccccc}
\Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} & \Xi_{17} & \Xi_{18}  \tag{3.17}\\
* & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} & \Xi_{26} & \Xi_{27} & \Xi_{28} \\
* & * & \Xi_{33} & \Xi_{34} & \Xi_{35} & \Xi_{36} & \Xi_{37} & \Xi_{38} \\
* & * & * & \Xi_{44} & \Xi_{45} & \Xi_{46} & \Xi_{47} & \Xi_{48} \\
* & * & * & * & \Xi_{55} & \Xi_{56} & \Xi_{57} & \Xi_{58} \\
* & * & * & * & * & \Xi_{66} & \Xi_{67} & \Xi_{68} \\
* & * & * & * & * & * & \Xi_{77} & \Xi_{78} \\
* & * & * & * & * & * & * & \Xi_{88}
\end{array}\right) .
$$

In views of Lemma 2.1 and (3.3), we have $\dot{V}(t)<0$, namely, there exists a positive scalar $\lambda>0$ such that $\dot{V}(t) \leq-\lambda\|Y(t)\|^{2}$. According to [35], system (2.3) is asymptotically stable, this completes the proof.

Remark 3.2. Motivated by the results obtained in [34], free-weighting matrices $F_{i}(i=$ $1,2, \ldots, 8)$ are introduced in Theorem 3.1 so as to reduce the conservatism of the delaydependent result. Moreover, more free-weighting matrices are introduced by the construction and disposal of $V_{1}(t)$, which may make the conservatism reduce further.

Remark 3.3. The transformation from system (2.3) to (3.2) enables us to utilize the information of the relationship among terms $x(t), x\left(t-\tau_{1}\right), \int_{t-\tau_{1}}^{t} \dot{x}(s) d s$. Combined with the arbitrariness of matrix $M$, the conservatism of stability criterion is reduced further.

Remark 3.4. When $M=I$, we can obtain the following simplified corollary.
Corollary 3.5. For given positive scalars $\tau_{1}>0, \tau_{2}>0, h>0$, the nominal system $\Sigma_{0}$ is asymptotically stable if $\|C\|<1$, and there exist positive definite matrices $P_{1}, Q_{1}, Q_{2}, Q_{3}, Q_{4}, Q_{5}$ and arbitrary matrices $P_{i}(i=2,3, \ldots, 25), F_{i}(i=1,2, \ldots, 8)$ with appropriate dimensions such that the following linear matrix inequality is feasible:

$$
\Xi_{2}=\left(\begin{array}{ccccccccc}
\Xi_{11}^{\prime} & \Xi_{12}^{\prime} & \Xi_{13}^{\prime} & \Xi_{14}^{\prime} & \Xi_{15}^{\prime} & \Xi_{16}^{\prime} & \Xi_{17}^{\prime} & \Xi_{18}^{\prime} & \Xi_{19}  \tag{3.18}\\
* & \Xi_{22} & \Xi_{23}^{\prime} & \Xi_{24} & \Xi_{25} & \Xi_{26}^{\prime} & \Xi_{27} & \Xi_{28} & \Xi_{29} \\
* & * & \Xi_{33}^{\prime} & \Xi_{34}^{\prime} & \Xi_{35}^{\prime} & \Xi_{36}^{\prime} \Xi_{37}^{\prime} & \Xi_{38}^{\prime} & \Xi_{39} \\
* & * & * & \Xi_{44} & \Xi_{45} & \Xi_{46}^{\prime} & \Xi_{47} & \Xi_{48} & \Xi_{49} \\
* & * & * & * & \Xi_{55} & \Xi_{56}^{\prime} & \Xi_{57} & \Xi_{58} & \Xi_{59} \\
* & * & * & * & * & \Xi_{66}^{\prime} & \Xi_{67}^{\prime} & \Xi_{68}^{\prime} & \Xi_{69} \\
* & * & * & * & * & * & \Xi_{77} & \Xi_{78} & \Xi_{79} \\
* & * & * & * & * & * & * & \Xi_{88} & \Xi_{89} \\
* & * & * & * & * & * & * & * & \Xi_{99}
\end{array}\right)<0,
$$

where

$$
\begin{align*}
& \Xi_{11}^{\prime}=P_{2}^{T}(A-I)+P_{10}^{T}+P_{18}^{T}+(A-I)^{T} P_{2}+P_{10}+P_{18}+h Q_{3}+Q_{1}, \\
& \Xi_{12}^{\prime}=P_{1}^{T}-P_{2}^{T}+(A-I)^{T} P_{3}+P_{11}+P_{19}, \\
& \Xi_{13}^{\prime}=P_{2}^{T}(B+I)-P_{18}^{T}+(A-I)^{T} P_{4}+P_{12}+P_{20}, \\
& \Xi_{14}^{\prime}=-P_{10}^{T}+(A-I)^{T} P_{5}+P_{13}+P_{21}, \\
& \Xi_{15}^{\prime}=P_{2}^{T} C+(A-I)^{T} P_{6}+P_{14}+P_{22}, \\
& \Xi_{16}^{\prime}=P_{2}^{T}-P_{18}^{T}+(A-I)^{T} P_{7}+P_{15}+P_{23}-F_{1}^{T}, \\
& \Xi_{18}^{\prime}=P_{2}^{T} D+(A-I)^{T} P_{9}+P_{17}+P_{25}-F_{1}^{T}, \\
& \Xi_{23}^{\prime}=P_{3}^{T}(B+I)-P_{19}^{T}-P_{4}, \\
& \Xi_{26}^{\prime}=P_{3}^{T}-P_{19}^{T}-P_{7}-F_{2}^{T}, \\
& \Xi_{33}^{\prime}=P_{4}^{T}(B+I)-P_{20}^{T}+(B+I)^{T} P_{4}-P_{20}-Q_{1},  \tag{3.19}\\
& \Xi_{34}^{\prime}=-P_{12}^{T}+(B+I)^{T} P_{5}-P_{21}, \\
& \Xi_{35}^{\prime}=P_{4}^{T} C+(B+I)^{T} P_{6}-P_{22}, \\
& \Xi_{36}^{\prime}=P_{4}^{T}-P_{20}^{T}+(B+I)^{T} P_{7}-P_{23}-F_{3}^{T}, \\
& \Xi_{37}^{\prime}=-P_{12}^{T}+(B+I)^{T} P_{8}-P_{24}-F_{3}^{T}, \\
& \Xi_{38}^{\prime}=P_{4}^{T} D+(B+I)^{T} P_{9}-P_{25}-F_{3}^{T}, \\
& \Xi_{46}^{\prime}=P_{5}^{T}-P_{21}^{T}-P_{15}-F_{4}^{T}, \\
& \Xi_{56}^{\prime}=P_{6}^{T}-P_{22}^{T}+C^{T} P_{7}-F_{5}^{T}, \\
& \Xi_{66}^{\prime}=P_{7}^{T}-P_{23}^{T}+P_{7}-P_{23}-F_{6}^{T}-F_{6}, \\
& \Xi_{67}^{\prime}=-P_{15}^{T}+P_{8}-P_{24}-F_{6}^{T}-F_{7}^{T}, \\
& \Xi_{68}^{\prime}=P_{7}^{T} D+P_{9}-P_{25}-F_{6}^{T}-F_{8}^{T}, \\
&
\end{align*},
$$

Remark 3.6. Based on Theorem 3.1 and Corollary 3.5, by using Lemmas 2.1 and 2.2, we can perform the robust asymptotic stability analysis for system (2.1) with uncertainty $\Delta A(t), \Delta B(t), \Delta D(t)$ as follows.

Theorem 3.7. For any given matrix $M$, scalars $\tau_{1}>0, \tau_{2}>0, h>0$, the original system $\Sigma$ is robustly and asymptotically stable if $\|C\|<1$, and there exist positive definite matrices $P_{1}, Q_{1}, Q_{2}, Q_{3}, Q_{4}, Q_{5}$, positive scalar $\delta$, and arbitrary matrices $P_{i}(i=2,3, \ldots, 25), F_{i}(i=1,2, \ldots, 8)$ with appropriate dimensions such that the following linear matrix inequality is feasible:

$$
\Xi_{3}=\left(\begin{array}{ccccccccc}
\Xi_{11}+\delta E_{a}^{T} E_{a} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} & \Xi_{17} & \Xi_{18} & \Xi_{19}  \tag{3.20}\\
* & \Xi_{22}^{T} K & \Xi_{23} & \Xi_{24} & \Xi_{25} & \Xi_{26} & \Xi_{27} & \Xi_{28} & \Xi_{29} \\
P_{3}^{T} K \\
* & * & \Xi_{33}+\delta E_{b}^{T} E_{b} & \Xi_{34} & \Xi_{35} & \Xi_{36} & \Xi_{37} & \Xi_{38} & \Xi_{39} \\
P_{4}^{T} K \\
* & * & * & \Xi_{44} & \Xi_{45} & \Xi_{46} & \Xi_{47} & \Xi_{48} & \Xi_{49} \\
P_{5}^{T} K \\
* & * & * & * & \Xi_{55} & \Xi_{56} & \Xi_{57} & \Xi_{58} & \Xi_{59} \\
P_{6}^{T} K \\
* & * & * & * & * & \Xi_{66}+\delta E_{c}^{T} E_{c} & \Xi_{67} & \Xi_{68} & \Xi_{69} \\
P_{7}^{T} K \\
* & * & * & * & * & * & \Xi_{77} & \Xi_{78} & \Xi_{79} \\
P_{8}^{T} K \\
* & * & * & * & * & * & * & \Xi_{88} & \Xi_{89} \\
P_{9}^{T} K \\
* & * & * & * & * & * & * & * & \Xi_{99} \\
* \\
* & * & * & * & * & * & * & * & *
\end{array}\right)<0 .
$$

Proof. Replacing $A, B, D$ in (3.3) with $A+K F(t) E_{a}, B+K F(t) E_{b}$, and $D+K F(t) E_{d}$, respectively, (3.3) for system (2.1) is equivalent to the following form:

$$
\begin{equation*}
\Xi+\Pi_{1}^{T} F^{T}(t) \Pi_{2}+\Pi_{2}^{T} F(t) \Pi_{1}<0, \tag{3.21}
\end{equation*}
$$

where $\Pi_{1}=\left[K^{T} P_{2}, K^{T} P_{3}, K^{T} P_{4}, K^{T} P_{5}, K^{T} P_{6}, K^{T} P_{7}, K^{T} P_{8}, K^{T} P_{9}\right], \Pi_{2}=\left[E_{a}, 0, E_{b}, 0,0, E_{c}\right.$, $0,0,0$ ]. From Lemma 2.2, a sufficient condition for (3.20) is that there exists a positive scalar $\delta>0$ such that

$$
\begin{equation*}
\Xi+\delta^{-1} \Pi_{1}^{T} \Pi_{1}+\delta \Pi_{2}^{T} \Pi_{2}<0 . \tag{3.22}
\end{equation*}
$$

In views of Lemma 2.1, we can easily obtain this conclusion, this completes the proof.
Corollary 3.8. For given positive scalars $\tau_{1}>0, \tau_{2}>0, h>0$, the original system $\Sigma$ is robustly and asymptotically stable if $\|C\|<1$, and there exist positive definite matrices $P_{1}, Q_{1}, Q_{2}, Q_{3}, Q_{4}, Q_{5}$,

Table 1: Stability bounds of time delays (Example 4.1).

|  | $[24]$ | $[25]$ | $[26]$ | $[27]$ | $[28]$ | $[12]$ | Ours |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\tau_{1}$ | $0<\tau_{1}<0.4991$ | $0<\tau_{1}<0.7602$ | $0<\tau_{1}<0.4991$ | $0<\tau_{1}<1.6965$ | $\tau_{1}>0$ | $\tau_{1}>0$ | $\tau_{1}>0$ |

Table 2: Some comparison for allowable upper bounds on $\tau_{1}$ (Example 4.2).

|  | $[29]$ | $[30]$ | $[31]$ | $[24]$ | $[28]$ | $[32]$ | $[13]$ | $[12]$ | $[33]$ | Ours |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\tau_{1}, \tau_{2}$ | 0.3 | 0.71 | 0.74 | 0.8844 | 1.3718 | 1.6525 | 1.6525 | 1.6525 | 2.2254 | 2.2254 |

positive scalar $\delta$, and arbitrary matrices $P_{i}(i=2,3, \ldots, 25), F_{i}(i=1,2, \ldots, 8)$ with appropriate dimensions such that the following linear matrix inequality is feasible:

$$
\Xi_{4}=\left(\begin{array}{cccccccccc}
\Xi_{11}^{\prime}+\delta E_{a}^{T} E_{a} & \Xi_{12}^{\prime} & \Xi_{13}^{\prime} & \Xi_{14}^{\prime} & \Xi_{15}^{\prime} & \Xi_{16}^{\prime} & \Xi_{17}^{\prime} & \Xi_{18}^{\prime} & \Xi_{19} & P_{2}^{T} K  \tag{3.23}\\
* & \Xi_{22} & \Xi_{23}^{\prime} & \Xi_{24} & \Xi_{25} & \Xi_{26}^{\prime} & \Xi_{27} & \Xi_{28} & \Xi_{29} & P_{3}^{T} K \\
* & * & \Xi_{33}^{\prime}+\delta E_{b}^{T} E_{b} & \Xi_{34}^{\prime} & \Xi_{35}^{\prime} & \Xi_{36}^{\prime} & \Xi_{37}^{\prime} & \Xi_{38}^{\prime} & \Xi_{39} & P_{4}^{T} K \\
* & * & * & \Xi_{44} & \Xi_{45} & \Xi_{46}^{\prime} & \Xi_{47} & \Xi_{48} & \Xi_{49} & P_{5}^{T} K \\
* & * & * & * & \Xi_{55} & \Xi_{56}^{\prime} & \Xi_{57} & \Xi_{58} & \Xi_{59} & P_{6}^{T} K \\
* & * & * & * & * & \Xi_{66}^{\prime}+\delta E_{c}^{T} E_{c} & \Xi_{67}^{\prime} & \Xi_{68}^{\prime} & \Xi_{69} & P_{7}^{T} K \\
* & * & * & * & * & * & \Xi_{77} & \Xi_{78} & \Xi_{79} & P_{8}^{T} K \\
* & * & * & * & * & * & * & \Xi_{88} & \Xi_{89} & P_{9}^{T} K \\
* & * & * & * & * & * & * & * & \Xi_{99} & 0 \\
* & * & * & * & * & * & * & * & * & -\delta I
\end{array}\right)<0 .
$$

## 4. Numerical Examples

In this section, some numerical examples will be presented to show the validity and improvement of the main results derived earlier.

Example 4.1. Consider the following neutral system presented in Park and Kwon [28]:

$$
\begin{equation*}
\dot{x}(t)-C \dot{x}\left(t-\tau_{2}\right)=A x(t)+B x\left(t-\tau_{1}\right)+D \int_{t-h}^{t} x(s) d s, \tag{4.1}
\end{equation*}
$$

with

$$
A=\left(\begin{array}{cc}
-3 & -2  \tag{4.2}\\
1 & 0
\end{array}\right), \quad B=\left(\begin{array}{cc}
-0.5 & 0.1 \\
0.3 & 0
\end{array}\right), \quad C=0, \quad D=0 .
$$



Figure 1: State trajectories of system (12) with $c c=0.4, h=\tau_{1}=\tau_{2}=1.60, E_{a}=E_{b}=E_{d}=0.2 I, K=I$.


Figure 2: State trajectories of system (12) with $c c=0.4, \tau_{1}=\tau_{2}=0.3, h=1.94, E_{a}=E_{b}=E_{d}=0.2 I, K=I$.


Figure 3: State trajectories of system (12) with $c c=0.4, \tau_{1}=\tau_{2}=0.3, h=2.61, E_{a}=E_{b}=E_{d}=0$.

Table 3: Calculated allowable size of distributed delay $h(\Delta A(t)=\Delta B(t)=\Delta D(t)=0)$ (Example 4.3).

| Liu [12] | cc | 0 | 0.05 | 0.1 | 0.15 | 0.2 | 0.25 | 0.3 | 0.35 | 0.4 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\tau_{1}=\tau_{2}=h$ | 0.67 | 0.62 | 0.56 | 0.51 | 0.46 | 0.41 | 0.36 | 0.32 | 0.28 |
|  | $\tau_{1}=\tau_{2}=0.3$ | 0.79 | 0.78 | 0.73 | 0.66 | 0.58 | 0.50 | 0.41 | 0.37 | 0.21 |
| Ours |  |  |  |  |  |  |  |  |  |  |
|  | $\tau_{1}=\tau_{2}=h$ | 2.15 | 2.13 | 2.11 | 2.09 | 2.07 | 2.04 | 2.02 | 1.99 | 1.97 |
|  | $\tau_{1}=\tau_{2}=0.3$ | 2.61 | 2.61 | 2.61 | 2.61 | 2.61 | 2.61 | 2.61 | 2.61 | 2.61 |

Table 4: Calculated allowable size of distributed delay $h\left(K=I, E_{a}=E_{b}=E_{d}=0.2 I\right)$ (Example 4.3).

| Liu [12] | cc | 0 | 0.05 | 0.1 | 0.15 | 0.2 | 0.25 | 0.3 | 0.35 | 0.4 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\tau_{1}=\tau_{2}=h$ | 0.67 | 0.62 | 0.56 | 0.51 | 0.46 | 0.41 | 0.36 | 0.32 | 0.28 |
|  | $\tau_{1}=\tau_{2}=0.3$ | 0.79 | 0.78 | 0.73 | 0.66 | 0.58 | 0.50 | 0.41 | 0.37 | 0.21 |
| Ours |  |  |  |  |  |  |  | 0.3 |  |  |
|  | $\tau_{1}=\tau_{2}=h$ | 1.82 | 1.79 | 1.77 | 1.75 | 1.72 | 1.70 | 1.67 | 1.63 | 1.60 |
|  | $\tau_{1}=\tau_{2}=0.3$ | 1.94 | 1.94 | 1.94 | 1.94 | 1.94 | 1.94 | 1.94 | 1.94 | 1.94 |

Set $M=\left(\begin{array}{cc}-1 & 0 \\ 0 & -2\end{array}\right)$. For comparisons, we calculate the allowable upper bound of $\tau_{1}$ for which the asymptotic stability is guaranteed. For this example, Table 1 shows that Theorem 3.1 obtained in this paper is less conservative than the related results obtained in [12,24-28].

Example 4.2. Consider the following neutral system studied in He et al. [32]:

$$
\begin{equation*}
\dot{x}(t)-C \dot{x}\left(t-\tau_{2}\right)=A x(t)+B x\left(t-\tau_{1}\right)+D \int_{t-h}^{t} x(s) d s, \tag{4.3}
\end{equation*}
$$

with

$$
A=\left(\begin{array}{cc}
-0.9 & 0.2  \tag{4.4}\\
0.1 & -0.9
\end{array}\right), \quad B=\left(\begin{array}{cc}
-1.1 & -0.2 \\
-0.1 & -1.1
\end{array}\right), \quad C=\left(\begin{array}{cc}
-0.2 & 0 \\
0.2 & -0.1
\end{array}\right), \quad D=0
$$

Set $M=\left(\begin{array}{cc}-1 & 0 \\ 0 & -2\end{array}\right)$.
For this example, we calculate the allowable upper bound of $\tau_{1}$ for which the asymptotic stability is guaranteed. Table 2 shows that Theorem 3.1 obtained in this paper is less conservative than the related results obtained in [12, 13, 24, 28-33].

Example 4.3. Consider the neutral system studied in Liu et al. [12]:

$$
\begin{equation*}
\dot{x}(t)-C \dot{x}\left(t-\tau_{2}\right)=(A+\Delta A(t)) x(t)+(B+\Delta B(t)) x\left(t-\tau_{1}\right)+(D+\Delta D(t)) \int_{t-h}^{t} x(s) d s \tag{4.5}
\end{equation*}
$$

with

$$
A=\left(\begin{array}{cc}
-2 & 0  \tag{4.6}\\
0 & -15
\end{array}\right), \quad B=\left(\begin{array}{cc}
1 & 3 \\
-3 & 1
\end{array}\right), \quad C=\left(\begin{array}{cc}
c c & 0 \\
0 & c c
\end{array}\right), \quad D=\left(\begin{array}{cc}
1 & 0.5 \\
-0.5 & 1
\end{array}\right), \quad 0 \leq c c<1
$$



Figure 4: State trajectories of system (12) with $c c=0.4, \tau_{1}=\tau_{2}=h=1.97, E_{a}=E_{b}=E_{d}=0$.

Table 5: Comparison of $\boldsymbol{\tau}_{\max }$ using different methods (Example 4.4).

| cc | 0 | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 |
| :--- | :---: | :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| Han [33] | 3.13 | 2.98 | 2.83 | 2.66 | 2.49 | 2.31 | 2.12 | 1.93 |
| Han [36] | 1.77 | 1.63 | 1.48 | 1.33 | 1.16 | 0.98 | 0.79 | 0.59 |
| He et al. [32] | 2.39 | 2.05 | 1.75 | 1.49 | 1.27 | 1.08 | 0.91 | 0.76 |
| Theorem 3.7 | 3.45 | 3.21 | 3.02 | 2.88 | 2.62 | 2.54 | 2.51 | 2.23 |

For the convenience of comparison, let $\Delta A(t)=\Delta B(t)=\Delta D(t)=0$. Set $M=\left(\begin{array}{cc}-1 & 0 \\ 0 & -2\end{array}\right)$. The comparative results between Theorem 3.1 with the result obtained in [12] are given in Table 3. When

$$
\begin{equation*}
[\Delta A(t), \Delta B(t), \Delta D(t)]=K F(t)\left[E_{a}, E_{b}, E_{d}\right] \tag{4.7}
\end{equation*}
$$

where $K=I, E_{a}=E_{b}=E_{d}=0.2 I$, Table 4 shows that the robust stability criterion obtained in this paper is also less conservative than the related result obtained in [12]. From the simulation figures (see Figures 1-4), one can see that the results derived in this paper are valid.

Example 4.4. Consider the following uncertain neutral system $[32,33,36]$ :

$$
\begin{equation*}
\dot{x}(t)-C \dot{x}(t-\tau)=\left[A+K F(t) E_{a}\right] x(t)+\left[B+K F(t) E_{b}\right] x(t-\tau) \tag{4.8}
\end{equation*}
$$

where $A=\left(\begin{array}{cc}-2 & 0 \\ 0 & -0.9\end{array}\right), B=\left(\begin{array}{cc}-1 & 0 \\ -1 & -1\end{array}\right), C=\left(\begin{array}{cc}c & 0 \\ 0 & c\end{array}\right), 0 \leq c<1, L=\left(\begin{array}{cc}0.2 & 0 \\ 0 & 0.2\end{array}\right), E_{a}=E_{b}=\left(\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}\right)$. Table 5 gives out the related comparative results, from which one can see that Theorem 3.7 obtained in this paper is less conservative than those established in [32, 33, 36].

## 5. Conclusion

By constructing a new Lyapunov function, some new robust stable criteria for a class of neutral control systems with mixed delays are obtained. These criteria are formulated in the forms of linear matrix inequalities. Compared with some previous publications, our results are less conservative. Numerical examples and simulations show that our results are valid.

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