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Research Article

Some Identities of the Twisted q-Genocchi Numbers and Polynomials with Weight α and q-Bernstein Polynomials with Weight α

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Recently mathematicians have studied some interesting relations between q-Genocchi numbers, q-Euler numbers, polynomials, Bernstein polynomials, and q-Bernstein polynomials. In this paper, we give some interesting identities of the twisted q-Genocchi numbers, polynomials, and q-Bernstein polynomials with weighted α.

1. Introduction

Throughout this paper, let p be a fixed odd prime number. The symbols $\mathbb{Z}_p$, $\mathbb{Q}_p$, and $\mathbb{C}_p$ denote the ring of p-adic integers, the field of p-adic rational numbers, and the completion of algebraic closure of $\mathbb{Q}_p$. Let $\mathbb{N}$ be the set of natural numbers and let $\mathbb{Z}_+ = \mathbb{N} \cup \{0\}$. As a well-known definition, the p-adic absolute value is given by $|x|_p = p^{-r}$, where $x = p^r t/s$ with $(t,p) = (s,p) = (t,s) = 1$. When one talks of q-extension, q is variously considered as an indeterminate, a complex number $q \in \mathbb{C}$, or a p-adic number $q \in \mathbb{C}_p$. In this paper we assume that $q \in \mathbb{C}_p$ with $|1 - q|_p < 1$.

We assume that $\mathrm{UD}(\mathbb{Z}_p)$ is the space of the uniformly differentiable function on $\mathbb{Z}_p$. For $f \in \mathrm{UD}(\mathbb{Z}_p)$, Kim defined the fermionic p-adic q-integral on $\mathbb{Z}_p$ as follows:

$$I_{-q}(f) = \int_{\mathbb{Z}_p} f(x)\,d\mu_{-q}(x) = \lim_{N\to\infty} \frac{1}{[p^N]_{-q}} \sum_{x=0}^{p^N-1} f(x)(-q)^x. \tag{1.1}$$

For $n \in \mathbb{N}$, let $f_n(x) = f(x+n)$ be translation. As a well known equation, by (1.1), we have

$$q^n \int_{\mathbb{Z}_p} f(x+n)\,d\mu_{-q}(x) = (-1)^n I_{-q}(f) + [2]_q \sum_{l=0}^{n-1} (-1)^{n-1-l} q^l f(l), \tag{1.2}$$

compared [1–4]. Throughout this paper we use the notation:

$$[x]_q = \frac{1-q^x}{1-q}, \quad [x]_{-q} = \frac{1-(-q)^x}{1+q}, \tag{1.3}$$

(cf. [1–16]). $\lim_{q\to 1}[x]_q = x$ for any x with $|x|_p \le 1$ in the present p-adic case. To investigate relation of the twisted q-Genocchi numbers and polynomials with weight α and the Bernstein polynomials with weight α, we will use useful property for $[x]_{q^\alpha}$ as follows;

$$\begin{aligned} [x]_{q^\alpha} &= 1 - [1-x]_{q^{-\alpha}}, \\ [1-x]_{q^{-\alpha}} &= 1 - [x]_{q^\alpha}. \end{aligned} \tag{1.4}$$

The twisted q-Genocchi numbers and polynomials with weight α are defined by the generating function as follows, respectively:

$$G_{n,q,w}^{(\alpha)} = n\int_{\mathbb{Z}_p} \phi_w(x)[x]_{q^\alpha}^{n-1} d\mu_{-q}(x), \tag{1.5}$$

$$G_{n,q,w}^{(\alpha)}(x) = n\int_{\mathbb{Z}_p} \phi_w(y)[y+x]_{q^\alpha}^{n-1} d\mu_{-q}(y). \tag{1.6}$$

In the special case, $x = 0$, $G_{n,q,w}^{(\alpha)}(0) = G_{n,q,w}^{(\alpha)}$ are called the nth twisted q-Genocchi numbers with weight α (see [9]).

Let $C_{p^n} = \{w \mid w^{p^n} = 1\}$ be the cyclic group of order p^n and let

$$T_p = \lim_{n\to\infty} C_{p^n} = \bigcup_{n\ge 1} C_{p^n}, \tag{1.7}$$

see [9, 12–15].

Kim defined the q-Bernstein polynomials with weight α of degree n as follows:

$$B_{n,k}^{(\alpha)}(x) = \binom{n}{k}[x]_{q^\alpha}^{k}[1-x]_{q^{-\alpha}}^{n-k}, \quad \text{where } x\in[0,1],\ n,k\in\mathbb{Z}_+, \tag{1.8}$$

compare [4, 7].

In this paper, we investigate some properties for the twisted q-Genocchi numbers and polynomials with weight α. By using these properties, we give some interesting identities on the twisted q-Genocchi polynomials with weight α and q-Bernstein polynomials with weight α.

2. Some Identities on the Twisted q-Genocchi Polynomials with Weight α and q-Bernstein Polynomials with Weight α

From (1.8), we can derive the following recurrence formula for the twisted q-Genocchi numbers with weight α:

$$G_{0,q,w}^{(\alpha)} = 0, \qquad qwG_{n,q,w}^{(\alpha)}(1) + G_{n,q,w}^{(\alpha)} = \begin{cases} [2]_q, & \text{if } n = 1, \\ 0, & \text{if } n > 1, \end{cases} \tag{2.1}$$

$$G_{0,q,w}^{(\alpha)} = 0, \qquad qw\left(1 + q^{\alpha}G_{q,w}^{(\alpha)}\right)^n + q^{\alpha}G_{n,q,w}^{(\alpha)} = \begin{cases} q^{\alpha}[2]_q, & \text{if } n = 1, \\ 0, & \text{if } n > 1, \end{cases} \tag{2.2}$$

$$q^{\alpha x}G_{n+1,q,w}^{(\alpha)}(x) = \left([x]_q^{\alpha} + q^{\alpha x}G_{q,w}^{(\alpha)}\right)^{n+1} \tag{2.3}$$

with usual convention about replacing $(G_{q,w}^{(\alpha)})^n$ by $G_{n,q,w}^{(\alpha)}$.

By (1.5), we easily get

$$G_{n,q,w}^{(\alpha)}(x) = n[2]_q\left(\frac{1}{1-q^{\alpha}}\right)^{n-1}\sum_{l=0}^{n-1}\binom{n-1}{l}(-1)^l q^{\alpha x l}\frac{1}{1+wq^{\alpha l+1}}. \tag{2.4}$$

By (2.4), we obtain the theorem below.

Theorem 2.1. *Let $n \in \mathbb{Z}_+$. For $w \in T_p$, one has*

$$G_{n,q,w}^{(\alpha)}(x) = (-1)^{n-1}w^{-1}q^{\alpha(1-n)}G_{n,q^{-1},w^{-1}}^{(\alpha)}(1-x). \tag{2.5}$$

By (2.1), (2.2), and (2.3) we note that

$$\begin{aligned} G_{n,q,w}^{(\alpha)} &= -qwG_{n,q,w}^{(\alpha)}(1) \\ &= -nwqG_{1,q,w}^{(\alpha)} + w^2q^{2-\alpha}\sum_{l=2}^{n}\binom{n}{l}q^{\alpha l}G_{l,q,w}^{(\alpha)}(1) \\ &= -nwqG_{1,q,w}^{(\alpha)} + w^2q^{2-2\alpha}\sum_{l=2}^{n}\binom{n}{l}q^{\alpha l}\left(1 + q^{\alpha}G_{q,w}^{(\alpha)}\right)^l \\ &= -nwqG_{1,q,w}^{(\alpha)} + w^2q^{2-2\alpha}\left([2]_{q^{\alpha}} + q^{2\alpha}G_{q,w}^{(\alpha)}\right)^n - nw^2q^2G_{1,q,w}^{(\alpha)} \\ &= -nwqG_{1,q,w}^{(\alpha)} + w^2q^2G_{n,q,w}^{(\alpha)}(2) - nw^2q^2G_{1,q,w}^{(\alpha)}. \end{aligned} \tag{2.6}$$

Therefore, by (2.6), we obtain the theorem below.

Theorem 2.2. *For $n \in \mathbb{N}$ with $n > 1$, one has*

$$G_{n,q,w}^{(\alpha)}(2) = w^{-2}q^{-2}G_{n,q,w}^{(\alpha)} + w^{-1}q^{-1}\frac{n[2]_q}{1+qw} + \frac{n[2]_q}{1+qw}. \tag{2.7}$$

By (1.6) and Theorem 2.2,

$$\begin{aligned} \frac{G_{n+1,q,w}^{(\alpha)}(2)}{n+1} &= \int_{\mathbb{Z}_p} \phi_w(y)\,[y+2]_{q^\alpha}^n d\mu_{-q}(y) \\ &= \frac{1}{n+1}\left(\frac{(n+1)[2]_q}{1+qw} + \frac{(n+1)w^{-1}q^{-1}[2]_q}{1+qw}\right) + w^{-2}q^{-2}\frac{G_{n+1,q,w}^{(\alpha)}}{n+1} \\ &= \frac{[2]_q}{1+qw} + w^{-1}q^{-1}\frac{[2]_q}{1+qw} + w^{-2}q^{-2}\frac{G_{n+1,q,w}^{(\alpha)}}{n+1}. \end{aligned} \tag{2.8}$$

Hence, we obtain the corollary below.

Corollary 2.3. *For $n \in \mathbb{N}$, one has*

$$\int_{\mathbb{Z}_p} \phi_w(y)\,[y+2]_{q^\alpha}^n d\mu_{-q}(y) = \frac{[2]_q}{1+qw} + w^{-1}q^{-1}\frac{[2]_q}{1+qw} + w^{-2}q^{-2}\frac{G_{n+1,q,w}^{(\alpha)}}{n+1}. \tag{2.9}$$

By fermionic integral on $\mathbb{Z}_p$,Theorems 2.1 and 2.2, we note that

$$\begin{aligned} \int_{\mathbb{Z}_p} \phi_w(x)[1-x]_{q^{-\alpha}}^n d\mu_{-q}(x) &= (-1)^n q^{\alpha n}\int_{\mathbb{Z}_p} \phi_w(x)[x-1]_{q^\alpha}^n d\mu_{-q}(x) \\ &= (-1)^n q^{\alpha n}\frac{G_{n+1,q,w}^{(\alpha)}(-1)}{n+1} \\ &= w^{-1}\frac{G_{n+1,q^{-1},w^{-1}}^{(\alpha)}(2)}{n+1} \\ &= w^{-1}\left(\frac{[2]_{q^{-1}}}{1+q^{-1}w^{-1}} + wq\frac{[2]_{q^{-1}}}{1+q^{-1}w^{-1}} + w^2q^2\frac{G_{n+1,q^{-1},w^{-1}}^{(\alpha)}}{n+1}\right) \\ &= \frac{[2]_q}{1+qw} + wq\frac{[2]_q}{1+qw} + wq^2\frac{G_{n+1,q^{-1},w^{-1}}^{(\alpha)}}{n+1}. \end{aligned} \tag{2.10}$$

Therefore, we have the theorem below.

Theorem 2.4. *For $n \in \mathbb{N}$ with $n > 1$, one has*

$$\int_{\mathbb{Z}_p} \phi_w(x)[1-x]_{q^{-\alpha}}^n d\mu_{-q}(x) = \frac{[2]_q}{1+qw} + wq\frac{[2]_q}{1+qw} + wq^2\frac{G_{n+1,q^{-1},w^{-1}}^{(\alpha)}}{n+1}. \tag{2.11}$$

By (1.4), Theorem 2.4, we take the fermionic p-adic invariant integral on $\mathbb{Z}_p$ for one q-Bernstein polynomials as follows:

$$\begin{aligned}\int_{\mathbb{Z}_p} \phi_w(x) B_{n,k}(x,q) d\mu_{-q}(x) &= \int_{\mathbb{Z}_p} \phi_w(x)\binom{n}{k}[x]_{q^\alpha}^k[1-x]_{q^{-\alpha}}^{n-k} d\mu_{-q}(x) \\ &= \binom{n}{k}\int_{\mathbb{Z}_p} \phi_w(x)[x]_{q^\alpha}^k\left(1-[x]_{q^\alpha}\right)^{n-k} d\mu_{-q}(x) \\ &= \binom{n}{k}\sum_{l=0}^{n-k}\binom{n-k}{l}(-1)^l\frac{G_{k+l+1,q,w}^{(\alpha)}}{k+l+1}.\end{aligned} \tag{2.12}$$

By symmetry of q-Bernstein polynomials with weight α of degree n, we get the following formula;

$$\begin{aligned}&\int_{\mathbb{Z}_p} \phi_w(x) B_{n,k}(x,q) d\mu_{-q}(x) \\ &\quad= \int_{\mathbb{Z}_p} \phi_w(x)\binom{n}{k}[x]_{q^\alpha}^{n-k}[1-x]_{q^{-\alpha}}^{k} d\mu_{-q}(x) \\ &\quad= \int_{\mathbb{Z}_p} \phi_w(x)\binom{n}{k}[1-x]_{q^{-\alpha}}^{k}\left(1-[1-x]_{q^{-\alpha}}\right)^{n-k} d\mu_{-q}(x) \\ &\quad= \binom{n}{k}\sum_{l=0}^{n-k}\binom{n-k}{l}(-1)^{n-k-l}\int_{\mathbb{Z}_p} \phi_w(x)[1-x]_{q^{-\alpha}}^{n-l} d\mu_{-q}(x) \\ &\quad= \binom{n}{k}\sum_{l=0}^{n-k}\binom{n-k}{l}(-1)^{n-k-l}\left(\frac{[2]_q}{1+qw} + wq\frac{[2]_q}{1+qw} + wq^2\frac{G_{n-l+1,q^{-1},w^{-1}}^{(\alpha)}}{n-l+1}\right).\end{aligned} \tag{2.13}$$

Therefore, by (2.12) and (2.13), we have the theorem below.

Theorem 2.5. *For $n \in \mathbb{N}$ with $n > 1$, one has*

$$\begin{aligned}&\sum_{l=0}^{n-k}\binom{n-k}{l}(-1)^l\frac{G_{k+l+1,q,w}^{(\alpha)}}{k+l+1} \\ &\quad= \sum_{l=0}^{n-k}\binom{n-k}{l}(-1)^{n-k-l}\left(\frac{[2]_q}{1+qw} + wq\frac{[2]_q}{1+qw} + wq^2\frac{G_{n-l+1,q^{-1},w^{-1}}^{(\alpha)}}{n-l+1}\right).\end{aligned} \tag{2.14}$$

Also, we note that

$$\begin{aligned}
&\int_{\mathbb{Z}_p} \phi_w(x) B_{n,k}(x,q)\, d\mu_{-q}(x) \\
&\quad = \binom{n}{k} \sum_{l=0}^{n-k} \binom{n-k}{l} (-1)^l \frac{G^{(\alpha)}_{k+l+1,q,w}}{k+l+1} \\
&\quad = \binom{n}{k} \int_{\mathbb{Z}_p} \phi_w(x) [1-x]^{n-k}_{q^{-\alpha}} [x]^k_{q^\alpha}\, d\mu_{-q}(x) \\
&\quad = \binom{n}{k} \int_{\mathbb{Z}_p} \phi_w(x) [1-x]^{n-k}_{q^{-\alpha}} \left(1 - [1-x]_{q^{-\alpha}}\right)^k d\mu_{-q}(x) \\
&\quad = \binom{n}{k} \sum_{l=0}^{k} \binom{k}{l} (-1)^{k-l} \int_{\mathbb{Z}_p} \phi_w(x) [1-x]^{n-l}_{q^{-\alpha}}\, d\mu_{-q}(x) \\
&\quad = \binom{n}{k} \sum_{l=0}^{k} \binom{k}{l} (-1)^{k-l} \left(\frac{[2]_q}{1+qw} + wq \frac{[2]_q}{1+qw} + wq^2 \frac{G^{(\alpha)}_{n-l+1,q^{-1},w^{-1}}}{n-l+1} \right).
\end{aligned} \tag{2.15}$$

Therefore, we have the theorem below.

Theorem 2.6. *For $n, k \in \mathbb{Z}_+$ with $n > k+1$, one has*

$$\begin{aligned}
&\int_{\mathbb{Z}_p} \phi_w(x) B_{k,n}(x,q)\, d\mu_{-q}(x) \\
&\quad = \binom{n}{k} \sum_{l=0}^{k} \binom{k}{l} (-1)^{k-l} \left(\frac{[2]_q}{1+qw} + wq \frac{[2]_q}{1+qw} + wq^2 \frac{G^{(\alpha)}_{n-l+1,q^{-1},w^{-1}}}{n-l+1} \right).
\end{aligned} \tag{2.16}$$

By (2.11) and Theorem 2.6, we have the theorem below.

Theorem 2.7. *Let $n, k \in \mathbb{Z}_+$ with $n > k+1$. Then one has*

$$\begin{aligned}
&\sum_{l=0}^{n-k} \binom{n-k}{l} (-1)^l \frac{G^{(\alpha)}_{k+l+1,q,w}}{k+l+1} \\
&\quad = \sum_{l=0}^{k} \binom{k}{l} (-1)^{k-l} \left(\frac{[2]_q}{1+qw} + wq \frac{[2]_q}{1+qw} + wq^2 \frac{G^{(\alpha)}_{n-l+1,q^{-1},w^{-1}}}{n-l+1} \right).
\end{aligned} \tag{2.17}$$

Let $n_1, n_2, k \in \mathbb{Z}_+$ with $n_1 + n_2 > 2k + 1$. Then we get

$$\int_{\mathbb{Z}_p} \phi_w(x) B_{n_1,k}^{(\alpha)}(x,q) B_{n_2,k}^{(\alpha)}(x,q) d\mu_{-q}(x)$$
$$= \binom{n_1}{k}\binom{n_2}{k}\sum_{l=0}^{2k}\binom{2k}{l}(-1)^{2k-l}\int_{\mathbb{Z}_p}\phi_w(x)[1-x]_{q^{-\alpha}}^{n_1+n_2-l} d\mu_{-q}(x)$$
$$= \binom{n_1}{k}\binom{n_2}{k}\sum_{l=0}^{2k}\binom{2k}{l}(-1)^{2k-l}\left(\frac{[2]_q}{1+qw} + wq\frac{[2]_q}{1+qw} + wq^2\frac{G_{n_1+n_2-l+1,q^{-1},w^{-1}}^{(\alpha)}}{n_1+n_2-l+1}\right). \tag{2.18}$$

Therefore, we obtain the theorem below.

Theorem 2.8. *For $n_1, n_2, k \in \mathbb{Z}_+$, one has*

$$\int_{\mathbb{Z}_p} \phi_w(x) B_{n_1,k}^{(\alpha)}(x,q) B_{n_2,k}^{(\alpha)}(x,q) d\mu_{-q}(x)$$
$$= \binom{n_1}{k}\binom{n_2}{k}\sum_{l=0}^{2k}\binom{2k}{l}(-1)^{2k-l}\left(\frac{[2]_q}{1+qw} + wq\frac{[2]_q}{1+qw} + wq^2\frac{G_{n_1+n_2-l+1,q^{-1},w^{-1}}^{(\alpha)}}{n_1+n_2-l+1}\right)$$
$$= \begin{cases} \dfrac{[2]_q}{1+qw} + wq\dfrac{[2]_q}{1+qw} + wq^2\dfrac{G_{n_1+n_2-l+1,q^{-1},w^{-1}}^{(\alpha)}}{n_1+n_2-l+1}, & \text{if } k=0, \\ wq^2\dbinom{n_1}{k}\dbinom{n_2}{k}\displaystyle\sum_{l=0}^{2k}\dbinom{2k}{l}(-1)^{2k-l}\dfrac{G_{n_1+n_2-l+1,q^{-1},w^{-1}}}{n_1+n_2-l+1}, & \text{if } k>0, \end{cases} \tag{2.19}$$

By simple calculation, we easily see that

$$\int_{\mathbb{Z}_p} \phi_w(x) B_{n_1,k}^{(\alpha)}(x,q) B_{n_2,k}^{(\alpha)}(x,q) d\mu_{-q}(x)$$
$$= \binom{n_1}{k}\binom{n_2}{k}\sum_{l=0}^{n_1+n_2-2k}(-1)^l\binom{n_1+n_2-2k}{l}\int_{\mathbb{Z}_p}\phi_w(x)[x]_{q^{\alpha}}^{2k+l} d\mu_{-q}(x)$$
$$= \binom{n_1}{k}\binom{n_2}{k}\sum_{l=0}^{n_1+n_2-2k}(-1)^l\binom{n_1+n_2-2k}{l}\frac{G_{2k+l+1,q,w}^{(\alpha)}}{2k+l+1}, \quad \text{where } n_1, n_2, k \in \mathbb{Z}_+. \tag{2.20}$$

Therefore, by (2.20) and Theorem 2.8, we obtain the theorem below.

Theorem 2.9. *Let $n_1, n_2, k \in \mathbb{Z}_+$ with $n_1 + n_2 > 2k + 1$. Then one has*

$$\sum_{l=0}^{2k} \binom{2k}{l} (-1)^{2k-l} \left(\frac{[2]_q}{1+qw} + wq \frac{[2]_q}{1+qw} + wq^2 \frac{G^{(\alpha)}_{n_1+n_2-l+1,q^{-1},w^{-1}}}{n_1+n_2-l+1} \right) = \sum_{l=0}^{n_1+n_2-2k} (-1)^l \binom{n_1+n_2-2k}{l} \frac{G^{(\alpha)}_{2k+l+1,q,w}}{2k+l+1}. \tag{2.21}$$

For $n_1, n_2, \ldots, n_s, k \in \mathbb{Z}_+$, $n_1 + n_2 + \cdots + n_s > sk + 1$, and let $\sum_{i=1}^{s} n_i = m$, then by the symmetry of q-Bernstein polynomials with weight α, we see that

$$\begin{aligned} &\int_{\mathbb{Z}_p} \phi_w(x) \prod_{i=1}^{s} B^{(\alpha)}_{k,n_i}(x,q)\, d\mu_{-q}(x) \\ &= \prod_{i=1}^{s} \binom{n_i}{k} \sum_{l=0}^{sk} \binom{sk}{l} (-1)^{sk-l} \int_{\mathbb{Z}_p} \phi_w(x) [1-x]^{m-l}_{q^{-\alpha}}\, d\mu_{-q}(x) \\ &= \prod_{i=1}^{s} \binom{n_i}{k} \sum_{l=0}^{sk} \binom{sk}{l} (-1)^{sk-l} \left(\frac{[2]_q}{1+qw} + wq \frac{[2]_q}{1+qw} + wq^2 \frac{G^{(\alpha)}_{m-l+1,q^{-1},w^{-1}}}{m-l+1} \right). \end{aligned} \tag{2.22}$$

Therefore, we have the theorem below.

Theorem 2.10. *For $n_1, n_2, \ldots, n_s, k \in \mathbb{Z}_+$ with $n_1 + n_2 + \cdots + n_s > sk + 1$, one has*

$$\begin{aligned} &\int_{\mathbb{Z}_p} \phi_w(x) \prod_{i=1}^{s} B^{(\alpha)}_{k,n_i}(x,q)\, d\mu_{-q}(x) \\ &= \prod_{i=1}^{s} \binom{n_i}{k} \sum_{l=0}^{sk} \binom{sk}{l} (-1)^{sk-l} \left(\frac{[2]_q}{1+qw} + wq \frac{[2]_q}{1+qw} + wq^2 \frac{G^{(\alpha)}_{m-l+1,q^{-1},w^{-1}}}{m-l+1} \right), \end{aligned} \tag{2.23}$$

where $n_1 + \cdots + n_s = m$.

In the same manner as in (2.15), we can get the following relation:

$$\begin{aligned} &\int_{\mathbb{Z}_p} \phi_w(x) \prod_{i=1}^{s} B^{(\alpha)}_{k,n_i}(x,q)\, d\mu_{-q}(x) \\ &= \prod_{i=1}^{s} \binom{n_i}{k} \int_{\mathbb{Z}_p} \phi_w(x) [x]^{sk}_{q^{\alpha}} \sum_{l=0}^{m-sk} (-1)^l \binom{m-sk}{l} (-1)^l [x]^l_{q^{\alpha}}\, d\mu_{-q}(x) \\ &= \prod_{i=1}^{s} \binom{n_i}{k} \sum_{l=0}^{m-sk} (-1)^l \binom{m-sk}{l} \frac{G^{(\alpha)}_{sk+l+1,q,w}}{sk+l+1}, \end{aligned} \tag{2.24}$$

where $n_1, n_2, \ldots, n_s, k \in \mathbb{Z}_+$ with $m = n_1 + n_2 + \cdots + n_s > sk + 1$.

By Theorem 2.10 and (2.13), we have the following corollary.

Corollary 2.11. *Let* $m \in \mathbb{N}$*. For* $n_1, n_2, \ldots, n_s, k \in \mathbb{Z}_+$ *with* $n_1 + \cdots + n_s > mk + 1$*, one has*

$$\sum_{l=0}^{sk} \binom{sk}{l} (-1)^{sk-l} \left(\frac{[2]_q}{1+qw} + wq \frac{[2]_q}{1+qw} + wq^2 \frac{G^{(\alpha)}_{m-l+1,q^{-1},w^{-1}}}{m-l+1} \right) = \sum_{l=0}^{m-sk} (-1)^l \binom{m-sk}{l} \frac{G^{(\alpha)}_{sk+l+1,q,w}}{sk+l+1}, \tag{2.25}$$

where $n_1 + \cdots + n_s = m$*.*

References

[1] T. Kim, "Some identities on the q-Euler polynomials of higher order and q-Stirling numbers by the fermionic p-adic integral on $\mathbb{Z}_p$," *Russian Journal of Mathematical Physics*, vol. 16, no. 4, pp. 484–491, 2009.

[2] T. Kim, "Note on the Euler numbers and polynomials," *Advanced Studies in Contemporary Mathematics*, vol. 17, no. 2, pp. 131–136, 2008.

[3] T. Kim, "q-Volkenborn integration," *Russian Journal of Mathematical Physics*, vol. 9, no. 3, pp. 288–299, 2002.

[4] T. Kim, J. Choi, and Y.-H. Kim, "Some identities on the q-Bernstein polynomials, q-Stirling numbers and q-Bernoulli numbers," *Advanced Studies in Contemporary Mathematics*, vol. 20, no. 3, pp. 335–341, 2010.

[5] I. N. Cangul, H. Ozden, and Y. Simsek, "A new approach to q-Genocchi numbers and their interpolation functions," *Nonlinear Analysis*, vol. 71, no. 12, pp. e793–e799, 2009.

[6] I. N. Cangul, H. Ozden, V. Kurt, and Y. Simsek, "On the higher-order w-q-Genocchi numbers," *Advanced Studies in Contemporary Mathematics*, vol. 19, no. 1, pp. 39–57, 2009.

[7] T. Kim, L. C. Jang, and H. Yi, "A note on the modified q-Bernstein polynomials," *Discrete Dynamics in Nature and Society*, vol. 2010, Article ID 706483, 12 pages, 2010.

[8] T. Kim, J. Choi, Y. H. Kim, and C. S. Ryoo, "On the fermionic p-adic integral representation of Bernstein polynomials associated with Euler numbers and polynomials," *Journal of Inequalities and Applications*, vol. 2010, Article ID 864247, 12 pages, 2010.

[9] H. Y. Lee, "A note on the twisted q-Genocchi numbers and polynomials with weight α," *Journal Of Applied Mathematics and Informatics*. In press.

[10] H. Ozden, I. N. Cangul, and Y. Simsek, "Hurwitz type multiple genocchi Zeta function," in *Numerical Analysis and Applied Math*, AIP Conference Proceedings, pp. 1148–1781, 2009.

[11] H. Ozden, Y. Simsek, and H. M. Srivastava, "A unified presentation of the generating functions of the generalized Bernoulli, Euler and Genocchi polynomials," *Computers and Mathematics with Applications*, vol. 60, no. 10, pp. 2779–2787, 2010.

[12] S. H. Rim, J. H. Jin, E. J. Moon, and S. J. Lee, "Some identities on the q-Genocchi polynomials of higher-order and q-Stirling numbers by the fermionic p-adic integral on $\mathbb{Z}_p$," *International Journal of Mathematics and Mathematical Sciences*, vol. 2010, Article ID 860280, 14 pages, 2010.

[13] C. S. Ryoo, "Some identities of the twisted q-Euler numbers and polynomials associated with q-Bernstein polynomials," vol. 14, no. 2, pp. 239–248, 2011.

[14] C. S. Ryoo, "Some relations between twisted q-Euler numbers and Bernstein polynomials," *Advanced Studies in Contemporary Mathematics*, vol. 21, no. 2, pp. 217–223, 2011.

[15] Y. Simsek, V. Kurt, and D. Kim, "New approach to the complete sum of products of the twisted (h, q)-Bernoulli numbers and polynomials," *Journal of Nonlinear Mathematical Physics*, vol. 14, no. 1, pp. 44–56, 2007.

[16] Y. Simsek and M. Acikgoz, "A new generating function of q-Bernstein-type polynomials and their interpolation function," *Abstract and Applied Analysis*, vol. 2010, Article ID 769095, 12 pages, 2010.