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Research Article

Fractional Models for Thermal Modeling and Temperature Estimation of a Transistor Junction

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The thermal behavior of a power transistor mounted on a dissipator is considered in order to estimate the transistor temperature junction using a measure of the dissipator temperature only. The thermal transfers between the electric power applied to the transistor, the junction temperature, and the dissipator temperature are characterized by two fractional transfer functions. These models are then used in a Control Output Observer (COO) to estimate the transistor junction temperature.

1. Introduction

Fractional differentiation has been widely used in the modelling of many physical and chemical processes and engineering systems such as electrochemistry and diffusion waves, electromagnetic waves, fractal electrical networks, electrical machines, nanotechnology, viscoelastic materials and systems, quantum evolution of complex systems [1], and heat conduction [2].

Automatic control is also a field in which many applications of fractional differentiations has been proposed. Recently, the authors have demonstrated that the real state of a fractional order system is not exactly observable [3]. However, they also have demonstrated that the pseudostate vector of the pseudostate space description (also defined in this paper) can be estimated using a Luenberger like observer.

In this paper, this theory is applied to the estimation of the junction temperature of a power transistor. Temperature management and control are among the most critical functions in power electronic devices, as operating temperature and thermal cycling can affect device performance and reliability. Transistor junction temperature estimation is a problem that was several times addressed in the literature. However, some of the proposed methods are open loop estimations [4], or cannot be implemented on-line due to the complex model used [5, 6] or also require additional devices [7].

In this paper, a simple fractional model is used to evaluate on-line the transistor junction temperature. The considered transistors are fitted with protection diodes that are used in a preliminary study and after their characterisations as a junction temperature sensor. This substitution sensor is used to characterise the transistor mounted on a dissipator by two transfer functions. The first one links the electric power applied to the transistor to its junction temperature and the second one links the junction temperature to the sensor temperature. Given the link existing between diffusion-based systems and fractional systems, the transfer functions obtained are fractional with orders multiples of 0.5 [2]. These transfer functions are then used to derive a pseudostate space description in which the junction temperature and the dissipator temperature are pseudostate variables. An observer based on a dynamic feedback of the real dissipator temperature is used to estimate the junction temperature.

2. Preliminaries

A multiinput, multioutput fractional system is described by the differential equation system involving fractional derivatives of the system input $u(t) \in \mathbb{R}^m$ and of the system output $y(t) \in \mathbb{R}^p$:

$$\sum_{k=0}^{N_a} S_k \left(\frac{d}{dt}\right)^{\gamma_{a_k}} y(t) = \sum_{k=0}^{N_b} T_k \left(\frac{d}{dt}\right)^{\gamma_{b_k}} u(t) \tag{2.1}$$

in which $S_k \in \mathbb{R}^{p\times p}$ and $T_k \in \mathbb{R}^{p\times m}$. $(d/dt)^{\gamma_{a_k}}$ and $(d/dt)^{\gamma_{b_k}}$ denote fractional differentiation operators of orders $\gamma_{a_k} \in \mathbb{R}$ and $\gamma_{b_k} \in \mathbb{R}$, respectively. Such operators are defined in [8–11] and a detailed survey of the properties linked to these definitions can be found in [8].

If orders γ_{a_k} and γ_{b_k} verify relations $\gamma_{a_k} = k/q$, $\gamma_{b_k} = k/q$, $q \in \mathrm{N}^*$, then differentiation orders γ_{a_k} and γ_{b_k} are commensurate [12] (multiple of the same rational number $1/q$).

Using order commensurability condition and for zero initial conditions, differential equation (2.1) admits a pseudostate space representation of the form:

$$\begin{aligned} \frac{d}{dt}^{\gamma} x(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) + Du(t), \end{aligned} \tag{2.2}$$

where $x \in \mathbb{R}^n$ is the pseudostate vector, $\gamma = 1/q$ is the fractional order of the system, and $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, and $D \in \mathbb{R}^{p\times m}$ are constant matrices.

As explained in [3], representation (2.2) is not strictly a state space representation and this is why it is denoted in the sequel *pseudostate space representation*. In the usual integer order system theory, the state of the system, $x(t)$, known at any given time point, along with the

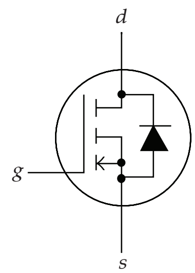


Figure 1: BUK 552 transistor internal description.

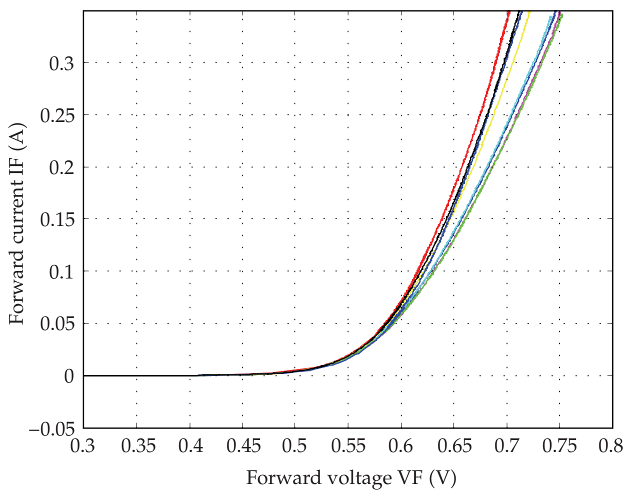


Figure 2: Ten transistors protection diode forward characteristics.

system equations and system inputs, is sufficient to predict the response of the system. That comment can be found in [13].

As demonstrated in [3], and whatever the fractional derivative definition used (excepted Caputo's definition but this last one is not physically acceptable [14]), the value of vector $x(t)$ at initial time t_0 in (2.2) is not enough to predict the future behavior of the system. Vector $x(t)$ in (2.2) is thus not a state vector of the system. However, as also shown in [3], a Luenberger type observer can be used to estimate pseudostate vector $x(t)$.

3. Experimental System Description

Ten BUK 552 transistors have been used in this study. As shown on the component internal description of Figure 1, a protection diode is associated to the MOSFET transistor.

A preliminary study of the forward characteristic protection diode of these transistors has demonstrated that the diodes can be used as temperature sensors. As shown in Figure 2, for the same temperature, the forward voltage for a 25 mA current is quasiconstant. But as shown in Figure 3, this voltage varies with temperature.

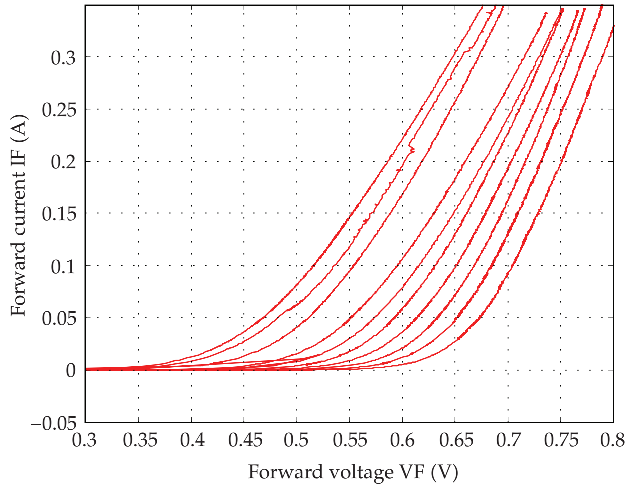


Figure 3: Protection diode forward characteristics for ten temperatures from 20°C (curve on the right) to 120°C (curve on the left) with an increment between 10 and 20°C.

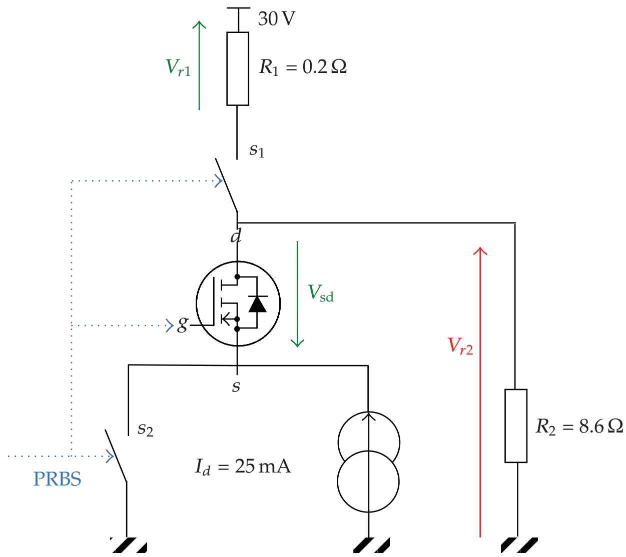


Figure 4: Electrical system used.

One can thus conclude that the protection diode in the considered transistors can be used as a temperature sensor through the measure of its forward drop when a forward current of 25 mA is applied.

To exploit this property the electrical system of Figure 4 was built.

This electrical system permits to characterize the protection diode forward characteristic and to apply a current to the transistor to heat it. The two switches s_1 and s_2 and the transistor grid are controlled by a laptop, a National Instruments I/O card and a Virtual Instrument designed for the experiment. With such a system, the following operations are performed.

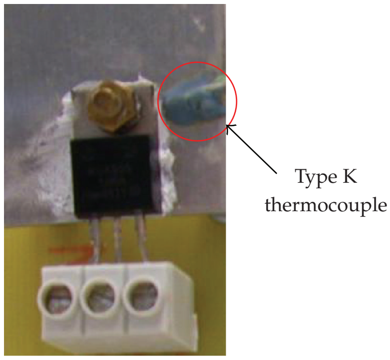


Figure 5: Transistor mounted on a dissipator.

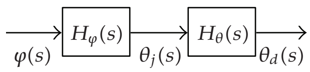


Figure 6: System model.

(i) Protection diode forward characterization is obtained by switching off s_1 and s_2. Forward current is measured through the shunt resistor R_2 and forward voltage—V_{sd} is measured at the transistor terminals.

(ii) Transistor on-state operation is obtained by switching on s_1 and s_2; transistor on-state current is measured through the difference of the voltage at the shunt resistors R_1 and R_2 terminals. V_{sd} voltage is also measured.

As shown in Figure 5 and during the tests, the ten transistors are mounted on a dissipator (aluminum plate). The dissipator temperature is measured by a type K thermocouple glued on the plate.

4. System Modeling

Two transfer functions have been used to characterize the system constituted by the transistor mounted on a dissipator. As shown on Figure 6, the first one links the electrical power applied to the transistor $\varphi(t)$ to the junction temperature $\theta_j(t)$, and the second one links the junction temperature $\theta_j(t)$ to the dissipator temperature $\theta_d(t)$. These two transfer functions are denoted $H_\varphi(s)$ and $H_\theta(s)$, respectively.

It was shown in [2] that fractional system with orders equal to 0.5 are well adapted to model thermal-diffusion-based systems. $H_\varphi(s)$ and $H_\theta(s)$ transfer functions are supposed of the form:

$$H_\varphi(s) = \frac{\sum_{k=0}^{N_b} b_k s^{0.5k}}{\sum_{k=1}^{N_a} a_k s^{0.5k}} \quad H_\theta(s) = \frac{\sum_{k=0}^{N_d} d_k s^{0.5k}}{\sum_{k=1}^{N_c} c_k s^{0.5k}} \quad N_b < N_a, \ N_d < N_c. \tag{4.1}$$

To obtain the numerical values associated to these transfer functions a Pseudo-Random Binary Sequence (PRBS) is applied at the transistor grid. The sampling period of

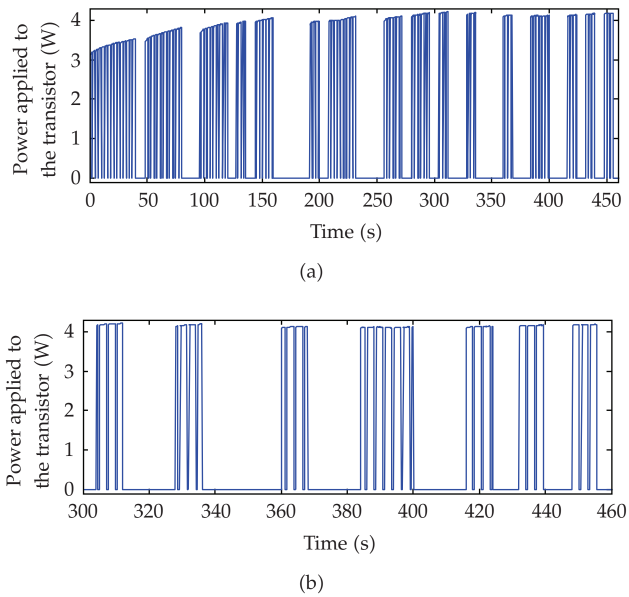


Figure 7: Electrical power applied to the transistor and a zoom on this signal.

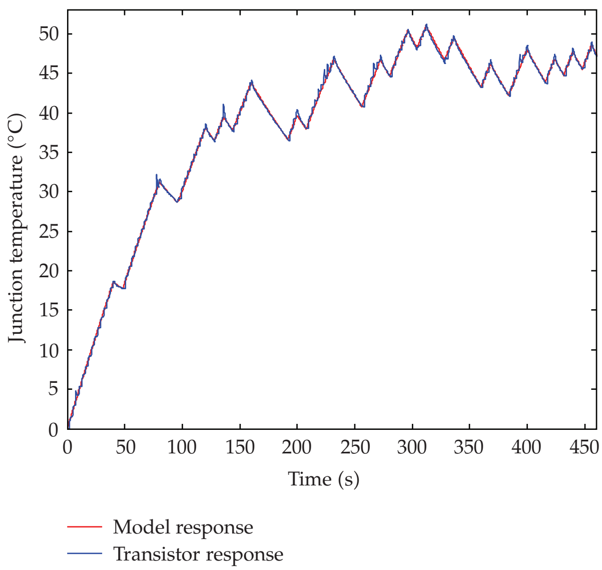


Figure 8: Transistor junction temperature evolution.

this PRBS is $T_e = 100\,\text{ms}$. This signal is interrupted each 2 s during 10 ms in order to measure the junction and the dissipator temperatures. This PRBS, represented on Figure 7, generates an electric power signal applied to the transistor.

The PRBS generates the junction and dissipator temperatures evolutions, respectively represented on Figures 8 and 9.

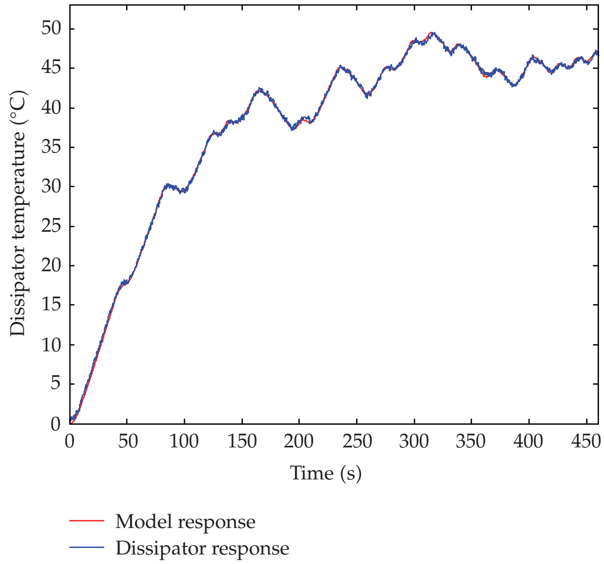


Figure 9: Dissipator temperature evolution.

Table 1: Coefficients of transfer functions $H_\varphi(s)$ and $H_\theta(s)$.

$H_\varphi(s)$	
$a_0 = 1$	
$a_1 = 0$	$b_0 = 34.51$
$a_2 = 173.55$	$b_1 = 0$
$a_3 = 11.70$	$b_2 = 0$
$H_\theta(s)$	
$c_0 = 1$	
$c_1 = 25.26$	$d_0 = 1$
$c_2 = 0$	$d_1 = 22.07$
$c_3 = 220.33$	$d_2 = 14.46$

Coefficients of transfer functions $H_\varphi(s)$ and $H_\theta(s)$ are identified using the identification module of the CRONE Matlab toolbox [15, 16]. Assuming that the fractional orders are imposed, a least square-based method can here be used. The obtained parameters are gathered in Table 1.

5. Temperature Estimation

The Controller Output Observer (COO), used in this section has been developed by MARGOLIS at Davis University in California [17]. This observer has been mainly applied in the mechanical domain (estimator of torque, speed, etc.). Here, it is used to estimate the temperature of a MosFet junction using the temperature measurement of the dissipator only.

Given relation (4.1), one can write

$$\theta_d(s) = H_\theta(s)\theta_j(s). \tag{5.1}$$

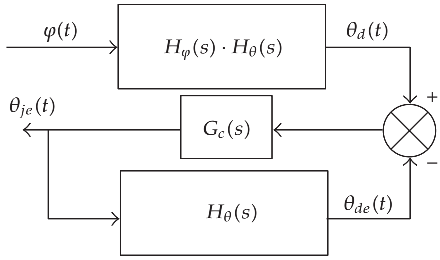


Figure 10: Dynamic output feedback-based observer.

A physical model of the system is supposed described by the relation

$$\theta_{de}(s) = H_\theta(s)\theta_{je}(s), \tag{5.2}$$

where $\theta_{de}(s)$ and $\theta_{je}(s)$ represent, respectively, an estimation of the dissipator and junction temperatures.

The COO provides an estimation of the system input by minimizing the error between model and system outputs through a controller of transfer functions $G_c(s)$ (see Figure 10). Such a minimization is mathematically described by

$$\theta_{je}(s) = G_c(s)(\theta_d(s) - \theta_{de}(s)). \tag{5.3}$$

Using (5.1) and (5.2), into (5.3) leads to:

$$\theta_{je}(s) = G_c(s)\left(H_\theta(s)\theta_j(s) - H_\theta(s)\theta_{je}(s)\right), \tag{5.4}$$

and thus

$$\theta_{je}(s) = \frac{G_c(s)H_\theta(s)}{1 + G_c(s)H_\theta(s)}\theta_j(s). \tag{5.5}$$

Relation (5.5) highlights that the estimation dynamics can be imposed by a convenient choice of the controller $G_c(s)$. Here, a 1st-order dynamics is imposed for θ_j estimation, namely

$$\theta_{je}(s) = \frac{1}{\tau s + 1}\theta_j(s). \tag{5.6}$$

τ is the time-constant associated to the estimation dynamics. If relation (5.6) is overridden into (5.5) it is found that

$$G_c(s) = \frac{1}{H_\theta(s)}\frac{1}{\tau s}. \tag{5.7}$$

Relation (5.7) shows that the estimation of $\theta_j(s)$ is based on the inversion of $H_\theta(s)$ transfer function. This estimator can thus be used only with a stable system with all its zeros in the left half complex plane (with the inversion the zeros of the system become the

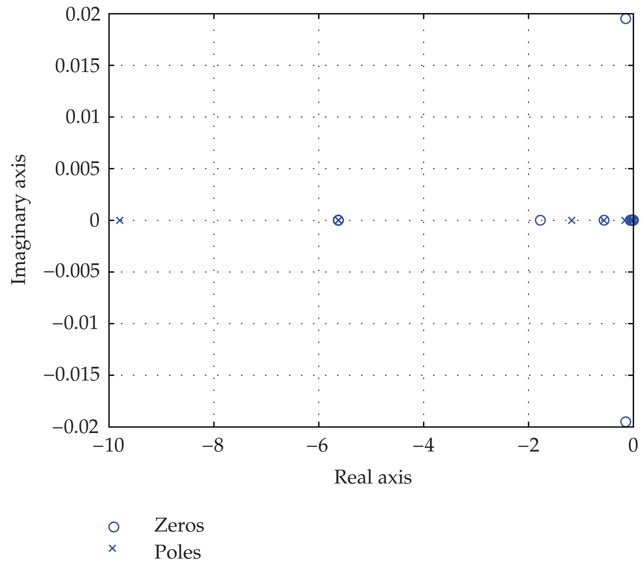


Figure 11: Poles, zeros map of $H_\theta(s)$.

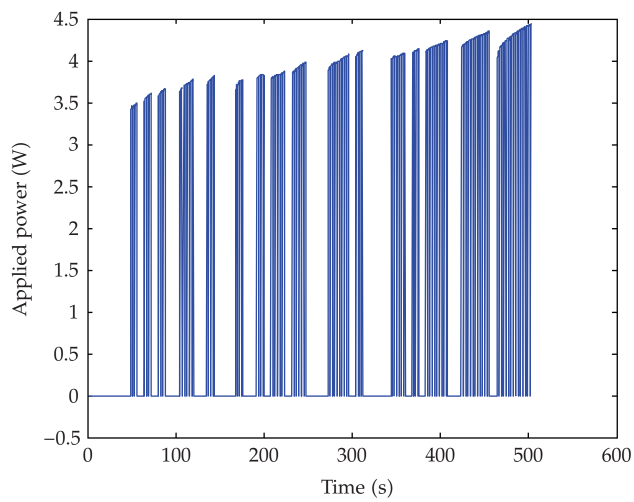
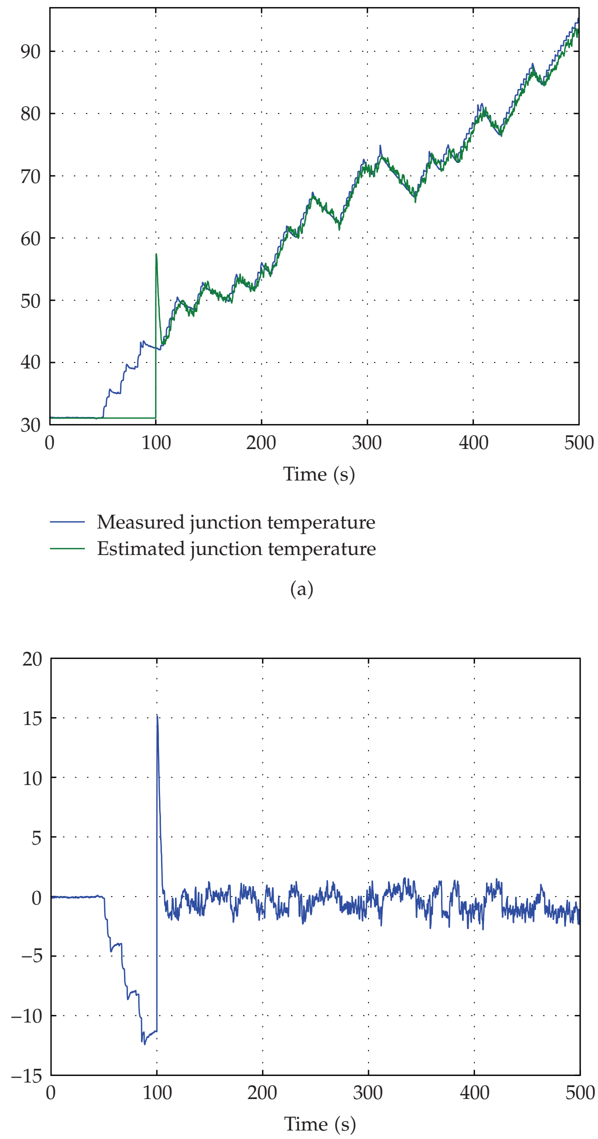


Figure 12: Electrical power applied to the thermal system.

poles of the estimator and conversely). For its implementation in the scheme of Figure 10, transfer function $H_\theta(s)$ must be approximated by an integer transfer function. Here, an approximation based on a recursive distribution of poles and zero is used [18]. Figure 11 shows the poles and zeros of the approximation obtained and proves that all the poles and the zeros are in left half complex plane.

6. Implementation and Results

To evaluate the efficiency of the observer, the electrical power represented in Figure 12 is used as the input $\varphi(t)$ in the real system of Figure 10.



(a)

(b)

Figure 13: Comparison of the real temperature $\theta_j(t)$ with the estimated temperature and corresponding error.

In Figure 13, the COO is started at time $t = 100\,\text{s}$ in order to observe its convergence. The observer converges very quickly because the parameter τ is chosen equal to 2s. On the other hand, due to the amplification of the measurement noises, the error oscillates within the interval $[-1 \quad 1]$°C and never converges towards 0.

An advantage of this estimator with respect to the traditional ones (Luenberger like for instance) is that it only requires one information to estimate the junction temperature $\theta_j(t)$: the dissipator temperature $\theta_d(t)$ (no measure of the input).

7. Conclusion

In this paper, the efficiency of fractional models for thermal systems modeling is demonstrated through an application to transistor junction temperature estimation. Two fractional models are indeed used to represent the dynamic evolution of the junction temperature and of the dissipator temperature on which the transistor is mounted. These two models are then used to estimate a transistor junction temperature. Such a temperature diagnostic is really important on power electronic devices to optimize the heat management of IGBTs (Insulated Gate Bipolar Transistor). This estimator provides information about junction temperature using a dynamic output feedback-based observer, with only a measure of the dissipator temperature. This limited number of required measures and the simple heat transfer fractional models obtained permit a simple and online implementation of the estimator. The estimator has been implemented and produces very accurate junction temperature estimation. Such an estimator should thus be used to monitor electronic systems such as motor controller, welding, DC/DC and AC/DC converters and to prevent damages in electronic systems on which the considered transistors (or similar components) are used.

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