Hindawi Publishing Corporation
Advances in Mathematical Physics
Volume 2011, Article ID 138358, 13 pages
doi:10.1155/2011/138358

Research Article

Dissipative Effect and Tunneling Time

Samyadeb Bhattacharya and Sisir Roy

Physics and Applied Mathematics Unit, Indian Statistical Institute, 203 Barrackpore Trunk Road, Kolkata 700 108, India

Correspondence should be addressed to Sisir Roy, sisir@isical.ac.in

Received 15 March 2011; Revised 7 June 2011; Accepted 10 June 2011

Academic Editor: Yao-Zhong Zhang

The quantum Langevin equation has been studied for dissipative system using the approach of Ford et al. Here, we have considered the inverted harmonic oscillator potential and calculated the effect of dissipation on tunneling time, group delay, and the self-interference term. A critical value of the friction coefficient has been determined for which the self-interference term vanishes. This approach sheds new light on understanding the ion transport at nanoscale.

1. Introduction

Caldeira and Laggett [1] started a systematic investigation of the quantum dissipative system and quantum tunneling in dissipative media. After that influential work, many authors have discussed the dissipative tunneling in numerous papers but not with profound illustration in the aspect of tunneling time. In fact, the proper definition of tunneling time has been debated for decades and it is yet to have definite answer [2]. Hauge and Støveng [3] mentioned seven different definitions of tunneling time of which the dwell time and the phase time or group delay are well accepted by the community. Winful [4] studied a general relation between the group delay and the dwell time. In case of quantum dissipative system Caldeira and Laggett used the path integral technique to study the dissipative quantum tunneling. Brouard et al. [5] made an important clarification of the existence of many tunneling times and the relations among them in a comprehensive framework. Ford et al. [6] investigated the dissipative quantum tunneling using quantum Langevin equation. The quantum Langevin equation is nothing but the Heisenberg equation of motion for the coordinate operator of a particle with certain mass, under a particular potential. This is the macroscopic description of a quantum particle interacting with a bath. The interaction with the bath corresponds to energy loss; in other words, it is the signature of dissipation. The memory function present in the equation describes the interaction with the bath. The nature of the dissipation is contained

in the memory function. In a recent work [7] one of the present authors (S. Roy) investigated the transport of ions in biological system and constructed a nonlinear Schrödinger equation where the transport of ion occurs at nanoscale. Here, the authors proposed a particular type of memory kernel associated to the non-Markovian behaviour of ions so as to understand the observational findings. The mechanism of ion transport at nanoscale is becoming an important area of research. Considering the memory kernel for ion transport at nanoscale, we will discuss the tunneling phenomena within quantum Langevin framework. Here we consider a parabolic potential barrier of the form $V(x) = (1/2)m\Omega^2(d^2 - x^2)$. The transmittance is calculated using the method devised by Ford et al. [6]. The misty aspect of tunneling time and various approaches towards it are critically analyzed. We will also try to figure out a process to calculate the tunneling time for a dissipative medium. In Section 2 we will briefly review quantum Langevin equation for convenience. Then, we will discuss various concepts related to tunneling time, that is, phase time, dwell time, and so forth, and the effect of dissipation in Section 3. The self-interference effect is discussed within this framework for dissipative systems. This method has been applied to understand the transport of K^+ ion in the biological domain which is much relevant at the nanoscale in Section 4. Possible implications are indicated in Section 5.

2. Quantum Langevin Equation

We begin with the discussion on the tunneling of the ions through the dissipative potential barrier. The theory of dissipative quantum tunneling is pioneered by Caldeira and Laggett [1], where they treated the problem through the technique of path integral. But we will address how quantum Langevin equation can be used to discuss dissipative quantum tunneling using the approach developed by Ford et al. [6, 8, 9]. Here it is easy to incorporate non-Markovian and strong coupling effects using suitable memory function. The memory function present in the quantum Langevin equation describes the interaction with the bath. At first we will briefly discuss the general theory with a general memory function. Then, we will deal with a specific memory function which we have at our hands [7], in our problem of potassium ion transfer through ion channels. We consider an inverted harmonic oscillator potential barrier and see how the transmission coefficient is modified for inclusion of the memory function. The quantum Langevin equation has the form

$$m\ddot{x} + \int_{-\infty}^{t} dt_1 \mu(t - t_1)\ddot{x}(t_1) + U'(x) = F(t), \tag{2.1}$$

where the dot and the prime, respectively, describ the derivative with respect to t and x. This is nothing but the Heisenberg equation of motion for the coordinate operator x of a particle of mass m, under a potential $U(x)$. Here the coupling with the bath is described by two terms, the random force $F(t)$ with mean zero and a mean force characterized by the memory function $\mu(t - t_1)$. The autocorrelation of $F(t)$ is given by

$$\frac{1}{2}\langle F(t)F(t_1) + F(t_1)F(t)\rangle = \frac{1}{\pi}\int_0^\infty d\omega\ \mathrm{Re}\left[\tilde{\mu}(\omega + i0^+)\right]\hbar\omega\coth\left(\frac{\hbar\omega}{2KT}\right)\cos[\omega(t - t_1)]. \tag{2.2}$$

In this expression

$$\tilde{\mu}(z) = \int_0^\infty dt e^{izt} \mu(t); \quad \mathrm{Im}(z) > 0 \tag{2.3}$$

is nothing but the Fourier transformation of $\mu(t)$. The coupling to the bath is given by the function $\tilde{\mu}(z)$.

Now this function has three important mathematical properties corresponding to some very important physical principles [5, 7].

(1) The $\mathrm{Im}(z) > 0$ condition states that the function is analytic in the upper half plane. This is a consequence of causality.

(2) The second condition is the "positivity condition" stated as

$$\mathrm{Re}\left[\tilde{\mu}(\omega + i0^+)\right] \geq 0; \quad -\infty < \omega < \infty. \tag{2.4}$$

This is a consequence of the 2nd law of thermodynamics.

(3) The third condition is the reality condition stated as

$$\tilde{\mu}(-\omega + i0^+)^* = \tilde{\mu}(\omega + i0^+). \tag{2.5}$$

This follows from the fact that x is a Hermitian operator. These properties are very elaborately explained by Ford and his coauthors. Based on these three properties, the function can be specified to belong in a restricted class of functions, having a general representation in the upper half plane

$$\tilde{\mu}(z) = -icz + \frac{2iz}{\pi} \int_0^\infty d\omega \frac{\mathrm{Re}\left[\tilde{\mu}(\omega + i0^+)\right]}{z^2 - \omega^2}, \tag{2.6}$$

where c is a positive constant, which can be absorbed in the particle mass.

Ford and his collaborators have considered harmonic oscillator potential $U(x) = (1/2)m\omega^2 x^2$ as a simple example. Under this potential, the quantum Langevin equation takes the form

$$m\ddot{x} + \int_0^\infty dt' \mu(t - t')\dot{x} + m\omega_0^2 x = F(t). \tag{2.7}$$

This equation can be solved by the method of Fourier transformation. We get the Fourier transformation of the coordinate operator x as

$$\tilde{x}(\omega) = \eta(\omega)\tilde{F}(\omega), \tag{2.8}$$

where $\eta(\omega)$ is called the susceptibility and expressed as

$$\eta(\omega) = \left[-m\omega^2 + m\omega_0^2 - i\omega\tilde{\mu}(\omega)\right]^{-1}. \tag{2.9}$$

We consider an inverted harmonic oscillator potential of the form $U(x) = (1/2)m\Omega_0^2(d^2 - x^2)$, where $2d$ is the width of the barrier. In the absence of dissipation we have an exact expression for the transmittance by the WKB approximation method

$$D_0 = \exp\left[-\frac{\pi m\Omega_0}{2\hbar}\left(d^2 - \frac{2E}{m\Omega_0^2}\right)\right]. \tag{2.10}$$

If dissipation is included, the tunneling frequency will be changed. Then the expression will be modified by replacing the frequency in the nondissipative case by that in the dissipative case.

In case of the inverted harmonic oscillator potential the susceptibility takes the form

$$\eta(\omega) = \left[-m\omega^2 - m\Omega_0^2 - i\omega\tilde{\mu}(\omega)\right]^{-1}. \tag{2.11}$$

The normal mode frequencies of this coupled system are the poles of the susceptibility. However, there is an isolated imaginary normal mode frequency corresponding to a pole of the susceptibility, which is classically forbidden and can be interpreted as the tunneling frequency $\omega = i\Omega(\Omega)$. The determining equation for Ω is

$$\left[\eta(i\Omega)\right]^{-1} = m\Omega^2 - m\Omega_0^2 + \Omega\tilde{\mu}(i\Omega) = 0. \tag{2.12}$$

Putting the expression of $\tilde{\mu}(i\Omega)$ from (2.6), we get

$$\Omega^2 + \frac{2\Omega^2}{m\pi}\int_0^\infty d\omega \frac{\mathrm{Re}\left[\tilde{\mu}(\omega + i0^+)\right]}{\Omega^2 + \omega^2} = \Omega_0^2. \tag{2.13}$$

Since the left-hand side of (2.13) is a monotonically increasing function, the value of Ω will always be less than Ω_0.

Let us consider a simple frictional coefficient "γ". Under which the frequency determining equation becomes

$$\Omega^2 + \gamma\Omega - \Omega_0^2 = 0. \tag{2.14}$$

Since Ω must be real and positive, we take the positive solution of this quadratic equation

$$\Omega = -\frac{\gamma}{2} + \Omega_0\left(1 + \frac{\gamma^2}{4\Omega_0^2}\right)^{-1}. \tag{2.15}$$

Considering $\gamma \ll \Omega_0$, we get $\Omega \approx \Omega_0 - (\gamma/2)$.

Then by replacing Ω_0 by Ω in the expression of transmittance, we get the transmittance for dissipative medium:

$$D = \exp\left[-\frac{\pi m\Omega}{2\hbar}\left(d^2 - \frac{2E}{m\Omega^2}\right)\right], \tag{2.16}$$

$$\text{that is, } D = D_0 \exp\left[\frac{\pi m\gamma d^2}{2\hbar} + \frac{\pi E\gamma}{\hbar\Omega_0^2}\right], \tag{2.17}$$

where D and D_0 are the transmittance with and without dissipation, respectively.

Now the presence of dissipation can be incorporated in the potential function. The potential barrier without dissipation is $V_0(x) = (1/2)m\Omega_0^2(d^2 - x^2)$.

The potential barrier with dissipation can be expressed as $V(x) = (1/2)m\Omega^2(d^2 - x^2)$. Relating these two, we get

$$V(x) \approx V_0(x)\left(1 - \frac{\gamma}{\Omega_0}\right). \tag{2.18}$$

So we can say that dissipation reduces the potential function. The dissipative contribution is included in the tunneling frequency Ω. We will use this fact to calculate the tunneling time and incorporate effect of dissipation in it.

3. Tunneling Time

In case of quantum mechanical tunneling through a barrier, it is well known how to calculate the probability of tunneling, the escape rate, and the lifetime in initial well. But the question is if there is a time analogous to classical time spent in the barrier region how long does it take a particle to tunnell through a barrier? The subject of this so-called tunneling time or traversal time has been covered by many authors in numerous independent approaches [3, 10–15]. Brouard et al. [5] have discussed various aspects of tunneling time in a very systematic approach. The very elegant review of Hauge and Støveng [3] lists at least seven different types of tunneling time of which the phase time (group delay) and dwell time are considered well established.

3.1. Relation between Phase Time and Dwell Time

The group delay or phase time measures the delay between appearance of a wave packet at the beginning and the end of the barrier. By the method of stationary phase, it is given by the energy derivative of the transmission phase shift:

$$t_{pT} = \hbar\frac{d\phi_T}{dE}. \tag{3.1}$$

Here $\phi_T = \phi_t + kL$, where L is the length of the barrier. Similarly the group delay for reflection is given by

$$t_{pR} = \hbar \frac{d\phi_R}{dE}, \tag{3.2}$$

where ϕ_R is the reflection phase shift. The total group delay is defined as the total group delay:

$$t_p = |T|^2 t_{pT} + |R|^2 t_{pR}, \tag{3.3}$$

where T and R are the transmission and reflection coefficients, respectively. In case of symmetric barriers $t_p = t_{pT} = t_{pR}$.

Regardless of transmission or reflection, the dwell time is a measure of the time spent by a particle in the barrier region $A < x < B$. It is given by the expression

$$t_d = \frac{\int_A^B |\psi(x)|^2 dx}{j_{\text{in}}}, \tag{3.4}$$

where $\psi(x)$ is the wave function corresponding to energy E and $j_{\text{in}} = \hbar k/m$ is the flux of the incident particles. This equation gives us the time that the incident flux has to be turned on, to provide the accumulated particle storage in the barrier. Winful [4] has discussed that delay time and dwell time are related by a linear relation:

$$t_p = t_d + t_i, \tag{3.5}$$

where t_i is called the self-interference term given by the expression

$$t_i = -\frac{\text{Im}(R)}{k} \hbar \frac{\partial k}{\partial E}. \tag{3.6}$$

The self-interference term comes from the overlap of incident and reflected waves in front of the barrier. This term is of considerable importance at low energies, when the particle spends most of its time dwelling in front of the barrier, interfering with itself. In the relation given by (3.5), the self-interference term is disentangled and given by a separate expression in (3.6).

3.2. Calculation of Dwell Time and Self-Interference Term in Dissipative Case

The delay time can be calculated considering the effect of dissipation. In order to do that, first we will calculate the dwell time for the parabolic barrier we have taken as a model potential. Following Er-Juan and Qi-Qing [16] we begin with the parabolic potential barrier and then subdivide the potential into infinitesimal rectangular barrier elements and then summing up the individual dwell times spent by the particles inside the barrier elements, and the dwell time of the parabolic barrier is calculated.

Let us take a rectangular potential

$$\begin{aligned} V(x) &= V_i \quad \text{for } x_{i-1} < x < x_i, \\ V(x) &= 0 \quad \text{for } x > x_{i-1} \text{ or } x < x_i. \end{aligned} \tag{3.7}$$

The solutions for the three regions can be written as

$$\begin{aligned} \psi_1(x) &= e^{ikx} + \mathrm{Re}^{-ikx} \quad \text{for } x < x_i, \\ \psi_2(x) &= Ce^{\kappa x} + De^{-\kappa x} \quad \text{for } x_{i-1} < x < x_i, \\ \psi_3(x) &= Te^{ikx} \quad \text{for } x > x_i. \end{aligned} \tag{3.8}$$

After some manipulations

$$\begin{aligned} C &= T\left(\frac{1}{2}\right)\left(1 + i\frac{k}{\kappa}\right)e^{ik\Delta - \kappa\Delta}, \\ D &= T\left(\frac{1}{2}\right)\left(1 - i\frac{k}{\kappa}\right)e^{ik\Delta + \kappa\Delta}. \end{aligned} \tag{3.9}$$

Now the transmission coefficient can be found as

$$T = \frac{-4ik\kappa e^{-\kappa\Delta}e^{-ik\Delta}}{(\kappa - ik)^2 - (\kappa + ik)^2 e^{-2\kappa\Delta}}, \tag{3.10}$$

and the probability of transmission is

$$P = |T|^2 = \frac{16k^2\kappa^2}{\left|(\kappa - ik)^2 - (\kappa + ik)^2 e^{-2\kappa\Delta}\right|^2}. \tag{3.11}$$

For thin barrier we have $e^{-2\kappa\Delta} \approx 1$. Then the transmission coefficient and probability of transmission, respectively, become $T \approx e^{-\kappa\Delta}e^{-ik\Delta}$ and $P \approx e^{-2\kappa\Delta}$. The dwell time t_d is defined as $t_d = (1/\mathrm{j}) \int_B |\psi_2|^2 dx$. Substituting these values, we get

$$t_d = \frac{1}{j}\int_B e^{-2\kappa\Delta}dx = \frac{1}{j}\int_B |T|^2 dx = \frac{1}{j}\int_B P\,dx. \tag{3.12}$$

Considering such barrier as a succession of the adjacent thin rectangular barrier elements in width $\Delta_i = x_i - x_{i-1}$,

The corresponding dwell time for the ith barrier

$$t_d^i = \frac{1}{j}\int_{\Delta_i} P_i dx = \frac{e^{-2\kappa_i\Delta_i}}{j}\Delta_i = \frac{P_i\Delta_i}{j}. \tag{3.13}$$

The dwell time for the entire barrier becomes

$$\begin{aligned} t_d &= \frac{P_1\Delta_1}{j} + \frac{P_1P_2\Delta_2}{j} + \frac{P_1P_2P_3\Delta_3}{j} + \cdots + \frac{\left(\prod_{n=1}^{i} P_n\right)\Delta_i}{j} + \cdots \\ \Longrightarrow t_d &= \sum_{i=1}^{\infty} \frac{\left(\prod_{n=1}^{i} P_n\right)\Delta_i}{j} = \frac{1}{j}\sum_{i=1}^{\infty}\left(e^{-2\sum_i \tilde{\kappa}_i(x)\Delta_i}\right)\Delta_i. \end{aligned} \tag{3.14}$$

Since the length of each barrier is very small, the summation can be replaced by integral:

$$t_d = \frac{1}{j}\int_{x_1}^{x_2}\left(e^{-2\int_{x_1}^{x}\tilde{\kappa}(x)dx}\right)dx, \tag{3.15}$$

where $\tilde{\kappa}(x) = \sqrt{2m(\overline{V}-E)/\hbar^2}$, $\overline{V} = (1/(x_2-x_1))\int_{x_1}^{x_2} V(x)dx$ is a kind of average height approximation for the potential barrier $V(x)$. Our model potential is the inverse harmonic oscillator potential of the form $V(x) = (1/2)m\Omega^2(d^2-x^2)$. Therefore $\overline{V} = (1/3)V_0^D + (2/3)E$, where $V_0^D = (1/2)m\Omega^2d^2$.

Therefore, the integrant becomes

$$\begin{aligned} P_x &= e^{-2\int_{x_1}^{x}\left[\sqrt{2m(\overline{V}-E)/\hbar^2}\right]dx}, \\ \text{that is, } P_x &= e^{-2\left[\sqrt{2m(\overline{V}-E)/\hbar^2}\right]\left[x-\sqrt{d^2-(2E/m\Omega^2)}\right]}. \end{aligned} \tag{3.16}$$

Now $t_d = (1/j)\int_{x_1}^{x_2} P_x dx$. Finally putting the value of P_x, we get

$$t_d = \sqrt{\frac{m}{2E}}\frac{e^{-2\tilde{\kappa}A}}{\tilde{\kappa}}\sinh 2\tilde{\kappa}A, \tag{3.17}$$

where $\tilde{\kappa}A = \sqrt{2m(\overline{V}-E)/\hbar^2}\sqrt{d^2-(2E/m\Omega^2)} = d\sqrt{2m/3\hbar^2V_0^D}(V_0^D-E)$.

The effect of dissipation is included in V_0^D, where $V_0^D = (1/2)m\Omega^2d^2 = (1/2)m\Omega_0^2d^2(1-(\gamma/2\Omega_0))^2 \approx V_0(1-(\gamma/\Omega_0))$, γ is the coefficient of friction.

Now we have to calculate the self-interference term $t_i = -(\mathrm{Im}(R)/k)\hbar(\partial k/\partial E)$. To calculate this term, we have to know the reflection coefficient R for the concerned potential.

By the method of WKB approximation the reflection coefficient R can be easily shown as

$$R = \frac{((\theta/4)-(1/\theta))}{((\theta/4)+(1/\theta))}e^{i(\pi/2)}. \tag{3.18}$$

Therefore,

$$\mathrm{Im}(R) = \frac{((\theta/4) - (1/\theta))}{((\theta/4) + (1/\theta))}, \tag{3.19}$$

where

$$\theta = \exp\left(-\int_{x_1}^{x_2} \kappa(x)dx\right) = \exp\left[-\frac{\pi m\Omega d^2}{4\hbar}\left(1 - \frac{2E}{m\Omega^2 d^2}\right)\right] = e^{-\alpha}. \tag{3.20}$$

For small θ,

$$\mathrm{Im}(R) \approx -\frac{(3+5\alpha)}{(5+3\alpha)}. \tag{3.21}$$

Therefore, the self-interference term is found to be

$$t_i = \frac{\hbar}{2E}\frac{(3+5\alpha)}{(5+3\alpha)}. \tag{3.22}$$

So from (3.17) and (3.22), we find the complete expression of the group delay for the case of a particle tunneling through a barrier of inverse harmonic oscillator potential:

$$t_p = \sqrt{\frac{m}{2E}}\frac{e^{-2\tilde{\kappa}A}}{\tilde{\kappa}}\sinh 2\tilde{\kappa}A + \frac{\hbar}{2E}\frac{(3+5\alpha)}{(5+3\alpha)}. \tag{3.23}$$

3.3. *Effect of Dissipation in Self-Interference Term*

Now we will estimate the effect of dissipation in the self-interference term. The effect of dissipation is included in the frequency term $\Omega = (\Omega_0 - (\gamma/2))$. From (3.20) we can write

$$\begin{aligned}\alpha &= \frac{\pi m\Omega d^2}{4\hbar} - \frac{\pi E}{2\hbar\Omega} \\ &\approx \left(\frac{\pi m\Omega_0 d^2}{4\hbar} - \frac{\pi E}{2\hbar\Omega_0}\right) - \left(\frac{m\Omega_0\pi d^2}{8\hbar} + \frac{\pi E}{4\hbar\Omega_0}\right)\frac{\gamma}{\Omega_0}.\end{aligned} \tag{3.24}$$

Hence, we can write

$$\alpha = \alpha_0 - \alpha'\frac{\gamma}{\Omega_0}, \tag{3.25}$$

where $\alpha_0 = ((\pi m\Omega_0 d^2/4\hbar) - (\pi E/2\hbar\Omega_0))$ and $\alpha' = ((m\Omega_0\pi d^2/8\hbar) + (\pi E/4\hbar\Omega_0))$.

Therefore,

$$t_i = \frac{5\hbar}{6E}\left(1 - \frac{1.067}{(1.67+\alpha_0) - \alpha'(\gamma/\Omega_0)}\right), \tag{3.26}$$

α' and γ are both positive terms. Therefore, the denominator of the 2nd term on the right-hand side will reduce due to them and the 2nd term will increase. It will reduce the whole term. So the presence of dissipative term reduces the self-interference effect.

Now if we take $\mathrm{Im}(R) = 0$, then

$$\gamma = \left(\frac{\alpha_0 + 0.603}{\alpha'}\right)\Omega_0. \tag{3.27}$$

Therefore, it is evident that, for a critical value of dissipation coefficient given by (3.27), the self-interference term vanishes. Let us now apply the approach to a biological phenomena, that is, transport of potassium ion through ion channels.

4. Potassium Ion Transfer through Ion Channel

Here we will consider a special case for potassium ion transport through ion channel. From the above discussion we can emphasize on the fact that the memory function μ is the all important function in this theory. It signifies the nature of the dissipative medium. By choosing this memory function properly, we can determine the tunneling coefficients and tunneling times for various dissipative media. Now at this very moment, our memory function representing the potassium ion transfer through ion channel comes into play. Ion channels are transmembrane protein structures that selectively allow given ion species to travel across the cell membrane. Zhou et al. [17] demonstrate that the channel protein transiently stabilizes three K^+ states, two within the selectivity filter and one within the water basket towards the intracellular side of the selectivity filter. Experimental evidence indicates that the selectivity filter is devoid of water molecules other than single water molecule between K ions [18]. The memory kernel of our specific problem [7] can be written as

$$\mu(t-t') = a_0\delta(t-t') + \frac{a_1}{\tau_1}e^{-|t-t'|/\tau_1} - \frac{a_2}{\tau_2}e^{-|t-t'|/\tau_2}. \tag{4.1}$$

The oscillatory ionic dynamics in K^+ ion channels is proposed to occur at the limit of the weak non-Markovian approximation associated with a time reversible Markov process, at the selectivity filter. This reversible stochastic process belongs to a different time scale to that governing diffusion across the rest of the channel, which is determined by the glue-like properties of water at the water basket. The framework of stochastic mechanics provides a model for such dissipative force in terms of quantum theory. That channel ionic permeation can be associated with nonlinear Schrödinger equation which addresses the issue of decoherence and time scale considerations. Now the memory kernel contains both Markovian and non-Markovian contributions that allows a continuous change from Markovian to non-Markovian dynamics and enables identification of both the terms. The non-Markovian process has two time scales τ_1 and τ_2 whose contributions are dominated by the parameters

a_1 and a_2. The first term contains the Markovian contribution. It is also clear that $a_1, a_2 \ll a_0$ is the weak non-Markovian limit.

Averaging over t' and taking the Fourier transformation over the memory kernel, we get

$$\mathrm{Re}\left[\tilde{\mu}(\omega)\right] = (a_0 + a_1 - a_2) - \frac{a_1\tau_1}{1+\omega^2\tau_1^2} + \frac{a_2\tau_2}{1+\omega^2\tau_2^2}. \tag{4.2}$$

The determining equation of the tunneling frequency (Ω) is given by (2.13). Under the present circumstances, the equation becomes

$$\Omega^2 + \frac{1}{m}\left[(a_0 + a_1 - a_2) - \left(\frac{a_1\tau_1}{1+\Omega\tau_1} - \frac{a_2\tau_2}{1+\Omega\tau_2}\right)\right]\Omega - \Omega_0^2 = 0. \tag{4.3}$$

Here we consider an approximation $\Omega\tau_1, \Omega\tau_2 \ll 1$. That is, $\Omega \ll 1/\tau_1, 1/\tau_2$. That is, the time scales τ_1 and τ_2 are very small. Since ω is a finite positive quantity, it is very small compared to the inverse of τ_1 and τ_2.

Taking up to the first order of the binomial terms, we get

$$\Omega^2 + \frac{1}{m}[(a_0 + a_1 - a_2) - a_1\tau_1(1-\Omega\tau_1) + a_2\tau_2(1-\Omega\tau_2)]\Omega - \Omega_0^2 = 0. \tag{4.4}$$

Neglecting the second-order terms of τ_1 and τ_2, we get

$$\Omega^2 + \frac{1}{m}[(a_0 + a_1 - a_2) - (a_1\tau_1 - a_2\tau_2)]\Omega - \Omega_0^2 = 0. \tag{4.5}$$

Let $\gamma = (1/m)[(a_0 + a_1 - a_2) - (a_1\tau_1 - a_2\tau_2)]$. So (4.5) may be written in the same form of (2.14),

$$\Omega^2 + \gamma\Omega - \Omega_0^2 = 0. \tag{4.6}$$

The tunneling coefficient is found to be

$$D = D_0 \exp\left[\left(\frac{\pi m d^2}{2\hbar} + \frac{\pi E}{\hbar\Omega_0^2}\right)\cdot\frac{1}{m}[(a_0 + a_1 - a_2) - (a_1\tau_1 - a_2\tau_2)]\right]. \tag{4.7}$$

In case of weak non-Markovian limit ($a_0 \gg a_1, a_2$), we neglect the a_1, a_2 part, and

$$D_{wnm} = D_0 \exp\left[\left(\frac{\pi m d^2}{2\hbar} + \frac{\pi E}{\hbar\Omega_0^2}\right)\cdot\frac{a_0}{m}\right]. \tag{4.8}$$

This is similar to (2.17).

For the strong non-Markovian case, we get

$$D_{snm} = D_{wnm} \exp\left[\left(\frac{\pi m d^2}{2\hbar} + \frac{\pi E}{\hbar\Omega_0^2}\right) \cdot \frac{1}{m}[(a_1 - a_2) - (a_1\tau_1 - a_2\tau_2)]\right]. \tag{4.9}$$

If we put this γ in the tunneling time expression, we get the group delay for this specific case of potassium ion transfer too.

The expression of delay time is given by (3.23), where $\tilde{\kappa}A = d\sqrt{2m/3\hbar^2 V_0^D}(V_0^D - E)$. The effect of dissipation is included in $V_0^D \approx V_0(1 - (\gamma/\Omega_0))$. So we get

$$\begin{aligned}\tilde{\kappa}A &= d\sqrt{\frac{2m}{3\hbar^2 V_0}}\left(1 - \frac{\gamma}{2\Omega_0}\right)^{-1}\left(V_0 - E - \frac{V_0\gamma}{\Omega_0}\right) \\ &\approx d\sqrt{\frac{2m}{3\hbar^2 V_0}}(V_0 - E) - d\sqrt{\frac{2m}{3\hbar^2 V_0}}\frac{\gamma}{2\Omega_0}(V_0 + E).\end{aligned} \tag{4.10}$$

In this case of potassium ion channel with the memory kernel as given in (4.1), the expression of the delay time will be

$$t_p = \sqrt{\frac{m}{2E}}\frac{e^{-2\tilde{\kappa}A}}{\tilde{\kappa}}\sinh 2\tilde{\kappa}A + \frac{\hbar}{2E}\frac{(3 + 5\alpha)}{(5 + 3\alpha)}, \tag{4.11}$$

with

$$\begin{aligned}\tilde{\kappa}A &= d\sqrt{\frac{2m}{3\hbar^2 V_0}}(V_0 - E) - d\sqrt{\frac{2m}{3\hbar^2 V_0}}\frac{(1/m)[(a_0 + a_1 - a_2) - (a_1\tau_1 - a_2\tau_2)]}{2\Omega_0}(V_0 + E), \\ \alpha &= \alpha_0 - \alpha'\frac{1}{m\Omega_0}[(a_0 + a_1 - a_2) - (a_1\tau_1 - a_2\tau_2)].\end{aligned} \tag{4.12}$$

So the delay time or phase time will depend on the parameter values a_0, a_1, a_2, τ_1, and τ_2.

5. Possible Implications

It is evident from the above analysis that the effect of dissipation on group delay can be estimated directly in terms of the frictional coefficient. It is also possible to express the self-interference term in terms of the friction coefficient (γ), and we can estimate the critical value of γ for which the interference term vanishes. The chosen biological example indicates that the present approach may play an important role in understanding the ion transport at nanoscale which will be considered in subsequent papers. We are also interested in the numerical estimation of tunneling time and the effect of dissipation on it. For that purpose, currently we have the required data for electron tunneling through water. But when one considers electron tunneling through water, the electron-phonon interaction must also play

an important role. This interaction will contribute in the potential in a considerable manner. But at nanoscale water behaves more like frozen ice [19]. In that frozen water configuration, the electron transfer through water is only weakly affected by electron-phonon interaction [20]. But currently we do not have sufficient data for that numerical calculation. We hope to present this thorough numerical estimation of tunneling times in subsequent papers.

References

[1] A. O. Caldeira and A. J. Laggett, "Quantum tunneling in a dissipative system," *Annals of Physics*, vol. 149, pp. 374–456, 1983.

[2] R. Y. Chiao and A. M. Steinberg, "Tunneling times and superluminality," in *Progress in Optics*, E. Wolf, Ed., vol. 37, p. 345, Elsevier, New York, NY, USA, 1997.

[3] E. H. Hauge and J. A. Støveng, "Tunneling times: a critical review," *Reviews of Modern Physics*, vol. 61, no. 4, pp. 917–936, 1989.

[4] H. G. Winful, "Delay time and the hartman effect in quantum tunneling," *Physical Review Letters*, vol. 91, no. 26, Article ID 260401, 4 pages, 2003.

[5] S. Brouard, R. Sala, and J. G. Muga, "Systematic approach to define and classify quantum transmission and reflection times," *Physical Review. A*, vol. 49, no. 6, pp. 4312–4325, 1994.

[6] G. W. Ford, J. T. Lewis, and R. F. O'Connell, "Dissipative quantum tunneling: quantum Langevin equation approach," *Physics Letters. A*, vol. 128, no. 1-2, pp. 29–34, 1988.

[7] S. Roy and R. Llinás, "Relevance of quantum mechanics on some aspects of ion channel function," *Comptes Rendus Biologies*, vol. 332, no. 6, pp. 517–522, 2009.

[8] G. W. Ford, J. T. Lewis, and R. F. O'Connell, "Quantum Langevin equation," *Physical Review. A*, vol. 37, no. 11, pp. 4419–4428, 1988.

[9] G. W. Ford, J. T. Lewis, and R. F. O'Connell, "Quantum oscillator in a blackbody radiation field," *Physical Review Letters*, vol. 55, no. 21, pp. 2273–2276, 1985.

[10] R. Landauer, "Barrier traversal time," *Nature*, vol. 341, pp. 567–568, 1989.

[11] M. B. Büttiker, "Larmor precession and the traversal time for tunneling," *Physical Review. B*, vol. 27, no. 10, pp. 6178–6188, 1983.

[12] M. Büttiker and R. Landauer, "Traversal time for tunneling," *Physical Review Letters*, vol. 49, no. 23, pp. 1739–1742, 1982.

[13] R. Landauer and T. Mritin, "Barrier interaction time in tunneling," *Reviews of Modern Physics*, vol. 66, no. 1, pp. 217–228, 1994.

[14] C. R. Leavens and G. C. Aers, "Dwell time and phase times for transmission and reflection," *Physical Review. B*, vol. 39, no. 2, pp. 1202–1206, 1989.

[15] D. Sokolovski, "Path integrals and equations of motion for the traversal-time distributions in classical diffusion and quantum mechanics," *Physical Review. A*, vol. 52, no. 1, pp. R5–R8, 1995.

[16] B. Er-Juan and S. Qi-Qing, "Dwell time of particles in tunneling barriers of arbitrary shape," *Chinese Physics Letters*, vol. 15, no. 12, p. 865, 1998.

[17] Y. Zhou, J. H. Morais-Cabral, A. Kaufman, and R. Mackinnon, "Chemistry of ion coordination and hydration revealed by a K^+ channel–fab complex at 2.0 Å resolution," *Nature*, vol. 414, pp. 43–48, 2001.

[18] D. A. Doyle, J. M. Cabral, R. A. Pfuetzner et al., "The structure of the potassium channel: molecular basis of K^+ conduction and selectivity," *Science*, vol. 280, pp. 69–77, 1998.

[19] T.-D. Li, J. Gao, R. Szoszkiewicz, U. Landman, and E. Riedo, "Structured and viscous water in subnanometer gaps," *Physical Review. B*, vol. 75, Article ID 115415, 6 pages, 2007.

[20] M. Galperin and A. Nitzan, "Inelastic effects in electron tunneling through water layers," *Journal of Chemical Physics*, vol. 115, Article ID 2681, 14 pages, 2001.