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Transposed Markov matrix as a new decision tool of how to choose among competing investment options in academic medicine

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Medical institutions face the challenge of promoting excellence in a variety of competing focus areas, such as grants, publications, income, research, faculty, variety, patient care and teaching. A transposed Markov chain is used analyse the interactions between the various focus areas and their transition towards steady-state. In contradistinction with a regular Markov chain, in the transposed chain used for the present analysis, the sum of *inputs* (rather than *outputs*) of each individual state is 100%, whereas the outputs are left to assume any possible value. The mathematics of calculating the steady state conditions of a transposed Markov matrix are similar to those of a regular Markov matrix. The analysis shows that a focus area more dependent on other areas is also more likely to lose its investment, whereas largely self-reliant areas will generate the largest return. Full strength of all academic focus areas can be achieved only by investments in all areas. In academic systems with one or several exclusively self-reliant focus areas, only investment in these particular areas will invigorate the system, as all other investments are bound to dissipate over time. The newly developed decision tool of a transposed Markov matrix could be helpful in stochastic modelling of medical phenomena.

Keywords: decision analysis; decision tree; Markov chain; stochastic modelling

1. Introduction

The medical departments or divisions of academic centres face the challenge of promoting excellence in a variety of competing areas [1]. For instance, a division of gastroenterology provides care to patients with gastrointestinal disease and consultation to physicians from other medical specialties. Such an academic division needs to teach students, housestaff, and fellows the theory and practice of gastroenterology. At the same time, members of the division are expected to carry out research and publish in scholarly journals. The clinical and research activities should generate an income that supports a clinical and laboratory infrastructure and allows the division to be engaged in the most modern diagnostic and interventional procedures, as well as in state-of-art research techniques. Ideally, a self-supporting medical division involved in clinical and research activities would also aid financially the medical school as a whole.

On one hand, a division chief could decide to focus on one or a few areas and aim at excellence in these few at the expense of other areas. Which area would be most promising and would yield the highest returns? Is it possible that by concentrating on one particular area alone, excellence achieved here would eventually promote excellence in other areas

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as well? On the other hand, one could also try to invest in all divisional activities equally and hope to achieve excellence in all areas in the long run. Obviously, the outcome of such strategies depends on the type and magnitude of interactions between the various academic endeavours. The aim of the present article is to analyse such interactions and develop a decision model to answer these questions.

Besides the problem of decision making and medical administration, the present analysis is also inspired by a mathematical problem. In a decision tree, as well as a Markov state model, the probabilities of all transitions leaving a given state add up to 100%. In several recent studies, this general concept of decision trees was modified to have all transitions that enter (rather than leave) any given state add up to 100%. This modified decision tool proved useful in modelling medical conditions beyond the realm of conventional decision trees [2–4]. In the present decision analysis, it is tested whether such a concept could also be applied to a medical decision analysis involving Markov chain models.

2. Methods

The circles of Figure 1 symbolize the various areas which medical departments or specialty divisions strive to excel in or focus on. The eight focus areas represent grants, publications, income, research, faculty, division size and variety, clinical care, and teaching. These terms are only meant to serve as abbreviations for a multitude of activities

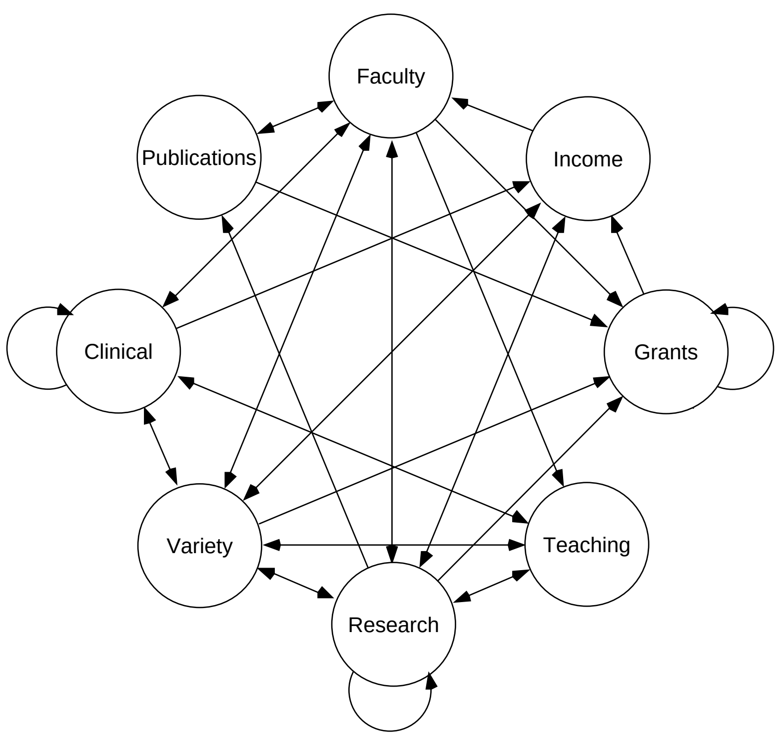

Figure 1. State diagram of an academic division focusing on eight different areas of activity. The arrows symbolize interactions between the various states. The curved arrows indicate states that, in addition to other states, depend on their own activity.

that are in reality associated with each individual focus area. For instance, 'grants' includes funding through governmental agencies, industry and medical societies. It involves the expertise in grantmanship, seeking approval of projects by the institutional review boards, and administration of funds, just to name a few of the many issues involved. Similarly, the term 'faculty' stands for a working environment that can attract knowledgeable, skilful and dedicated academicians who enjoy their work and pursue their personal and academic goals in a fashion that is gratifying to themselves and contributes to the overall well-being of the division. Lastly, the term 'variety' relates to a division that is sufficiently big to support the whole spectrum of a given specialty or subspecialty. In gastroenterology, for instance, the division would not be focused on hepatology or oesophageal disease alone, and it would be able to support a large variety of diagnostic and interventional endoscopic procedures.

The arrows in Figure 1 symbolize interactions between the different focus areas. A double-headed arrow is used as short-form for the presence of two independent interactions. The double-headed arrow, for instance, between 'variety' and 'teaching' indicates that a large variety would contribute to teaching, and that a thriving teaching environment in itself would also promote divisional variety. Theoretically, every focus area could interact with every other focus area. In the baseline model, however, it is assumed that only parts of all potential interactions are realized. Although in some environments, for instance, publications could stem from a large volume or variety of clinical activity, in general the demands of clinical activity may mostly interfere with the spare time necessary to analyse, write-up, and submit clinical data for scholarly publications. In a sensitivity analysis, other types of scenarios have also been investigated.

The interactions between the different focus areas are depicted by the transition matrix of Table 1. Each interaction can potentially assume a value between 0 and 100%. Each matrix element m_{ij} represents the contribution of the *i* focus area on the *j* focus area. The sum of interactions affecting a single focus area (and shown in each single column of the matrix) add up to 100%, that is, $100\% = \sum_i \mathrm{m}_{ij}$. The transition matrix **M** of Table 1 represents the transposition of a Markov matrix, and the same mathematical laws apply to it as to a regular Markov chain [5–6]. Bold capital letters are used here and throughout the text to indicate a matrix. Different from a regular Markov chain, where the sum of *row* elements is 100%, in the present transition matrix the *column* elements add up to 100%. Similar to a regular Markov chain, the steady-state behaviour of the system can also be predicted by the k-step transition matrix $\mathbf{M}^k$, where $\mathbf{M}^k = \mathbf{M} \times \mathbf{M} \times \mathbf{M} \ldots$ *k*-times and k is assumed to be large, for instance, $k > 16$.

The parameters of interest are the strengths associated with each focus area, the performance of a division being described by a row matrix $\mathbf{S} = \{\mathrm{s}_1, \mathrm{s}_2, \mathrm{s}_3, \ldots, \mathrm{s}_8\}$, where s_1 through s_8 represent the strengths or levels of excellence achieved by each focus area from grants to teaching. The overall divisional strength S corresponds to the sum of the individual strengths, that is, $S = \sum_i \mathrm{s}_i$. An initial investment in grants, for instance, as indicated by a row matrix $\mathbf{I} = \{1, 0, 0, \ldots, 0\}$, would raise the strength associated with grants to $\mathrm{s}_1 = 1$. Using the same notation, joint investment in grants, publications and teaching or even all areas simultaneously would be indicated by the row matrices $\mathbf{I} = \{1, 1, 0, \ldots, 0, 1\}$ or $\mathbf{I} = \{1, 1, 1, \ldots, 1, 1\}$, respectively. Until now, the term 'investment' has been used as an abbreviation to indicate time, effort and money spent in order to support a particular type of divisional activity. It could also entail the assignment of office or laboratory space, equipment, and the distribution of clinical or administrative workload among the division members. Monetary units, such as US$, represent the most commonly used unit of measurement to describe these entities. In the present analysis, we measure investments and strengths either in dollars or as arbitrary dimensionless units.

Table 1. Transition and steady-state matrices of three scenarios.

Transition Matrix

First Scenario

	Grants	Publications	Income	Research	Faculty	Variety	Clinical	Teaching	**Sum**
Grants	0.10	0.00	0.50	0.00	0.00	0.00	0.00	0.00	**0.60**
Publications	0.20	0.00	0.00	0.00	0.20	0.00	0.00	0.00	**0.40**
Income	0.00	0.00	0.00	0.40	0.20	0.15	0.00	0.00	**0.75**
Research	0.40	0.50	0.25	0.20	0.20	0.15	0.00	0.10	**1.80**
Faculty	0.20	0.50	0.00	0.25	0.00	0.20	0.20	0.30	**1.65**
Variety	0.10	0.00	0.05	0.10	0.20	0.00	0.25	0.20	**0.90**
Clinical	0.00	0.00	0.20	0.00	0.20	0.30	0.45	0.40	**1.55**
Teaching	0.00	0.00	0.00	0.05	0.00	0.20	0.10	0.00	**0.35**
Sum	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**8.00**

Second Scenario

	Grants	Publications	Income	Research	Faculty	Variety	Clinical	Teaching	**Sum**
Grants	0.10	0.00	0.50	0.00	0.00	0.00	0.00	0.00	**0.60**
Publications	0.20	0.00	0.00	0.00	0.20	0.00	0.00	0.00	**0.40**
Income	0.00	0.00	0.00	0.40	0.20	0.15	0.00	0.00	**0.75**
Research	0.40	0.50	0.25	0.20	0.20	0.15	0.00	0.10	**1.80**
Faculty	0.20	0.50	0.00	0.25	0.00	0.20	0.05	0.30	**1.50**
Variety	0.10	0.00	0.05	0.10	0.20	0.00	0.05	0.20	**0.70**
Clinical	0.00	0.00	0.20	0.00	0.20	0.30	0.85	0.40	**1.95**
Teaching	0.00	0.00	0.00	0.05	0.00	0.20	0.05	0.00	**0.30**
Sum	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**8.00**

Third Scenario

	Grants	Publications	Income	Research	Faculty	Variety	Clinical	Teaching	**Sum**
Grants	1.00	0.00	0.50	0.00	0.00	0.00	0.00	0.00	**1.50**
Publications	0.00	0.00	0.00	0.00	0.20	0.00	0.00	0.00	**0.20**
Income	0.00	0.00	0.00	0.00	0.20	0.15	0.00	0.00	**0.35**
Research	0.00	0.50	0.25	1.00	0.20	0.15	0.00	0.10	**2.20**
Faculty	0.00	0.50	0.00	0.00	0.00	0.20	0.00	0.30	**1.00**
Variety	0.00	0.00	0.05	0.00	0.20	0.00	0.00	0.20	**0.45**
Clinical	0.00	0.00	0.20	0.00	0.20	0.30	1.00	0.40	**2.10**
Teaching	0.00	0.00	0.00	0.00	0.00	0.20	0.00	0.00	**0.20**
Sum	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**8.00**

Steady-State Matrix

First Scenario

	Grants	Publications	Income	Research	Faculty	Variety	Clinical	Teaching	**Sum**
Grants	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	**0.56**
Publications	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	**0.38**
Income	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	**1.00**
Research	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	**1.42**
Faculty	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	**1.36**
Variety	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	**1.05**
Clinical	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22	**1.77**
Teaching	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	**0.46**
Sum	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**8.00**

Second Scenario

	Grants	Publications	Income	Research	Faculty	Variety	Clinical	Teaching	**Sum**
Grants	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	**0.34**
Publications	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	**0.24**
Income	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	**0.61**
Research	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	**0.88**
Faculty	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	**0.84**
Variety	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	**0.60**
Clinical	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	**4.13**
Teaching	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	**0.37**
Sum	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**8.00**

Third Scenario

	Grants	Publications	Income	Research	Faculty	Variety	Clinical	Teaching	**Sum**
Grants	1.00	0.07	0.51	0.00	0.14	0.12	0.00	0.06	**1.89**
Publications	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.00**
Income	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.00**
Research	0.00	0.73	0.27	1.00	0.47	0.35	0.00	0.31	**3.13**
Faculty	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.00**
Variety	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.00**
Clinical	0.00	0.20	0.23	0.00	0.39	0.54	1.00	0.63	**2.98**
Teaching	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.00**
Sum	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**8.00**

The initial strength of the division corresponds with the initial investment, that is, $\mathbf{S}(0) = \mathbf{I}$. The long-term or steady-state outcome of the investment is given by the matrix product $\mathbf{S}(k) = \mathbf{I} \times \mathbf{M}^k$. Such calculations are easily carried out on an Excel® spreadsheet (from Microsoft, Redmond, WA) [7]. The built-in MMULT function is used for matrix multiplications to calculate $\mathbf{M}^k = \mathbf{M} \times \mathbf{M} \times \mathbf{M} \ldots$ or $\mathbf{S}(k) = \mathbf{I} \times \mathbf{M}^k$. Similar input–output matrices have been used by economists to model long-term behaviour of companies with multiple subdivisions or the US economy as a whole [8]. One can also simulate the transition process of the system towards a steady-state directly on a spreadsheet [9]. Each column represents a different focus area and each row represents another cycle of the system moving towards its steady-state. The first row represents the initial strengths of the eight focus areas corresponding with the initial investment in these areas. The data of each row (n), multiplied by the interactions shown in the transition matrix of Table 1, are used to calculate the data of its subsequent row ($n + 1$). For instance, $\text{income}_{n+1} = 50\% \times \text{grants}_n + 25\% \times \text{research}_n + 5\% \times \text{variety}_n + 20\% \times \text{clinic}_n$. Eventually, the simulation reaches the same steady-state strengths as given by $\mathbf{S}(k) = \mathbf{I} \times \mathbf{M}^k$.

3. Results

If one decides to invest only in grants, each focus area eventually reaches an identical strength of 0.07 (Table 1). For instance, for each \$1.00 spent initially only on grants, each focus area gains strength by \$0.07. The overall strength of a division comprised of eight focus areas is raised by 8 × \$0.07 = \$0.56 per \$1.00 invested. In other words, \$0.44 of the original investment has dissipated. As a second example, if one decides to invest only in clinical care, all eight focus areas gain in strength by 0.22. Thus, with each \$1.00 invested initially only in clinical care, eventually each focus area gains strength by \$0.22, and the overall divisional strength rises by 8 × \$0.22 = \$1.76, that is, \$0.76 more than the initial investment. Lastly, if more resources were available and one could afford to invest simultaneously \$1.00 in grants and \$1.00 in clinical care, each area would eventually attain a level of \$0.29 = \$0.07 + \$0.22. For the sake of simplicity, the \$-sign has been omitted from the subsequent presentation and the dimensionless numbers are used to describe changes per unit of investment. As demonstrated above, one can easily revert from dimensionless numbers to statements about changes in dollars per dollars invested.

The examples from above illustrate the following obvious pattern or principle. The strengths of the divisional focus areas depend on the area of initial investment and the appearance of the steady-state matrix $\mathbf{M}^k$, as given by the formula $\mathbf{S}(k) = \mathbf{I} \times \mathbf{M}^k$ from above. Each row of $\mathbf{M}^k$ represent the long-term strengths in all individual areas, even if one decided to invest initially only in one focus area associated with a particular row.

A second pattern that emerges from the examples of above or from inspection of the steady-state matrix is that some investments result in higher overall strengths of the division than others. For instance, investment in grants yields an overall strength of $S = 8 \times 0.07 = 0.56$ as compared to $S = 8 \times 0.22 = 1.76$ associated with investing in clinical care. Different from a pure Markov chain, the strength or overall performance of a system as described by Table 1 changes during consecutive cycles. Over time investment in grants leads to a net loss, whereas investment in the clinical arena leads to a net gain. The steady-state matrix also reveals that the division could never assume its full strength of $S = 8$ unless one invested equally in all eight areas.

Why are some focus areas more productive than others? Table 1 shows that excellence in clinical care was modelled with a relatively strong self-supportive interaction, that is, the interaction of clinical excellence with itself being $m_{77} = 0.45$. This interaction was chosen to reflect the fact that excellence in the clinical focus area would be less dependent on the input from other areas, such as grants, publication, or research, and that it would depend primarily on its own level of performance. In the second scenario of Table 1, raising this interaction even further from $m_{77} = 0.45$ to $m_{77} = 0.85$ (at the expense of reducing all other contributions to 0.05) increases the steady-state strength of each individual divisional area from 0.22 to 0.52. As an extreme scenario, one could envisage a clinical enterprise that is completely self-reliant and does not need any input from the other focus areas. This type of transition matrix would result in a steady-state matrix filled with 0, except for the clinical row filled with 1. Under such circumstances only investment in the clinical area would result in steady-state strength with an eight-fold return. Investment in any other or even all other areas simultaneously would eventually dissipate and not benefit the overall system. In contradistinction with a *sink* or *absorptive* state in a regular Markov chain, the clinical area of this last example functions like a *source* or *productive* state that can sustain the entire system and yield an eight-fold return to each unit of investment.

The third scenario from Table 1 contains the example of a division with three productive, that is, self-supportive states, namely, grants, research, and clinical care. The steady-state matrix is different from previous matrices in that the numbers in each row vary. An investment in grants, for instance, leads to different strengths for each of the seven other focus areas. Only a simultaneous investment in all three self-supportive states would bring the division to full strength. On the other hand, any investment in any of the other areas is lost as it dissipates over time.

4. Discussion

The present analysis strives to model the interactions among various competing influences that determine the level of performance of a medical division or department at an academic institution. A division chief constrained by limited resources is frequently facing the question of where to invest the resources in trying to strengthen a division. A decision model is developed to predict which type of investment would provide the biggest return in the long run and what type of rules govern the response of a system to efforts to strengthen its performance. At the onset, it may seem that there could be few fixed rules that govern academic divisions alike and that each division would function with its own set of constraints. For instance, bench research has a lesser impact on training of fellows in some divisions compared to others. While many medical divisions make money from procedural activities, under certain constraints clinical activities may be associated with income loss, and grants provide the main source for generating revenues. Other examples abound. In spite of a seemingly large heterogeneity in the function of various academic medical divisions, however, the present analysis may still be insightful in establishing a set of rules that apply similarly to many divisions.

These are the general principles that can be extracted from the present model: (1) An initial investment to strengthen a particular focus area can result in a net gain or loss, depending on the types of interactions of this given area with other areas. A focus area more dependent on other areas is more likely to spoil its investment. (2) In case of limited resources, therefore, investment in largely self-reliant areas will generate in the long run most ‘bang for the buck.’ (3) However, maximum strength in all divisional areas will be achieved ultimately only by investments in all divisional focus areas. (4) In academic systems with one or several exclusively self-reliant areas, only investment in these particular areas will invigorate the system, as all other investments are bound to dissipate over time. These four principles reflect the mathematics underlying the present model and must apply similarly to real academic divisions.

One essential prerequisite of a regular Markov chain is that all *outputs* of a given state add up to 100%. A regular Markov chain is used to model the probable outcomes of all individual states. Each state can evolve into or provide output to a predefined set of multiple other states. A Markov chain lends itself to model the movement of patients among various health conditions [10,11]. The model is focused on the fate of patients in each state and their transitions out of this state into other states of the Markov chain. In contradistinction with a regular Markov chain, in the modified chain, the sum of *inputs* into each individual state is set to be 100%, while the outputs are left to assume any possible value. Mathematically, this leads to a transposition of the regular Markov matrix. The mathematics of calculating the steady state conditions remain unchanged. In the modified Markov chain, one is concerned with the entirety of influences or inputs that change the fate of each state over time. The modified chain is thus focused on the input into individual states and how each state becomes transformed by the sum of its outside influences. This changed perspective of the transposed matrix opens up the possibility to model medical and social phenomena that are different from those of the regular Markov state model.

In conclusion, a modified input-output model is developed to analyse the interactions among various research and clinical activities that characterize an academic medical division. The model allows one to derive a set of general principles that apply to the management of real academic divisions. The newly developed decision tool of a transposed Markov matrix could be helpful in stochastic modelling of other medical phenomena as well.

References

[1] W.W. Souba, *Reinventing the academic medical center*, J. Surg. Res. 81 (1999), pp. 113–122.

[2] A. Sonnenberg and M.W. Gavin, *Timing of surgery for enterovesical fistula in Crohn's disease: Decision analysis using a time-dependent compartment model*, Inflamm. Bowel Dis. 6 (2000), pp. 280–285.

[3] A. Sonnenberg and M.W. Gavin, *General principles to enhance practice patterns in gastrointestinal endoscopy*, Aliment. Pharmacol. Ther. 16 (2002), pp. 1003–1009.

[4] A. Sonnenberg, *What to do about* Helicobacter pylori? *A decision analysis of its implication on public health*, Helicobacter J. 7 (2002), pp. 60–67.

[5] J.J. Higgins and S. Keller-McNulty, *Concepts in Probability and Stochastic Modeling*. Duxbury Press, Belmont, CA, 1995, pp. 136–188.

[6] H.M. Taylor and S. Karlin, *An Introduction to Stochastic Modeling*, 3rd ed., Academic Press, San Diego, 1998, pp. 95–266.

[7] W.J. Orvis, *Excel for Scientists and Engineers*, 2nd ed., Sybex, San Francisco, 26 and 416, 1996.

[8] D. Gale, *Linear Economic Models*, McGraw-Hill, New York, 1960.

[9] F.J. Clauss, *Applied Management Science and Spreadsheet Modeling*. Duxbury Press, Belmont, CA, 1996, pp. 454–496.

[10] F.A. Sonnenberg and J.R. Beck, *Markov models in medical decision making: a practical guide*, Med. Decis. Making 13 (1993), pp. 322–338.

[11] D.B. Petitti, *Meta-Analysis, Decision Analysis and Cost-Effectiveness Analysis: Methods for Quantitative Synthesis in Medicine*. Oxford University Press, New York, 2000, pp. 152–158.