

A NEW CHAOTIC ATTRACTOR FROM 2D DISCRETE MAPPING VIA BORDER-COLLISION PERIOD-DOUBLING SCENARIO

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The following map is studied: $(x, y) \rightarrow (1 + a(|x| - y^2) + y, bx)$. It is proved numerically that this model can display two different chaotic attractors, one is new and the other is a Lozi-type attractor. The new chaotic attractor is allowed via a border-collision period-doubling scenario, which is different from the classical period-doubling bifurcation.

1. Introduction

The discrete mathematical models are gotten directly via scientific experiences, or by the use of the Poincaré section for the study of a continuous model. One of these models is the Henon map. Many papers have described chaotic systems, one of the most famous being a two-dimensional discrete map which models the original Henon map [3, 4, 5, 7, 8]. Moreover, it is possible to change the form of the Henon map for obtaining others chaotic attractors [2, 6], this type of applications is used in secure communications using the notions of chaos.

The Lozi map is 2D noninvertible iterated map proposed by Lozi [6] as follow:

$$(x, y) \longrightarrow (1 - a|x| + y, bx). \quad (1.1)$$

This model gives a chaotic attractor called *Lozi attractor* and its shape is resembled to the one shown in Figure 2.1.

2. The proposed model

In this article, we essentially study the following modified Lozi map:

$$(x, y) \longrightarrow (1 + a(|x| - y^2) + y, bx). \quad (2.1)$$

We show numerically that the new model (2.1) can display two different chaotic attractors; including the classical Lozi-type attractor as shown in Figure 2.1 to Figure 4.2.

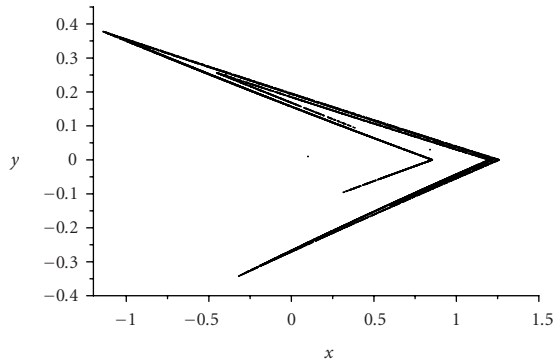


Figure 2.1. A Lozi-type chaotic attractor obtained from system (2.1) for $a = -1.8, b = 0.3$.

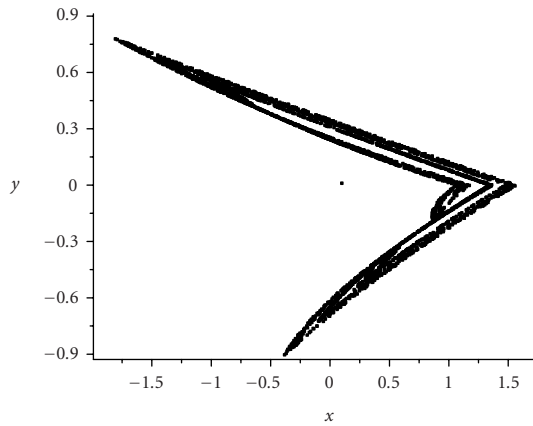


Figure 2.2. A Lozi-type chaotic attractor obtained from system (2.1) for $a = -1.8, b = 0.5$.

3. Comparison with the Lozi system

The system (2.1) has the same complexity as the Lozi system (1.1), they are both two-dimensional discrete noninvertible maps. However, the two models are topologically not equivalent because the Lozi system (1.1) is a piecewise-linear map, but model (2.1) is a nonlinear system. In addition; it can be rigorously proved that a non-singular homeomorphism that transforms each system to other does not exist. The proof needed some straightforward but tedious algebra which leads to a system of algebraic equations without solutions.

4. Route to chaos

It is well known that while varying the parameter a , the Hénon attractor is obtained via a period-doubling bifurcation route to chaos as a typical future [3], unless for Lozi map, no

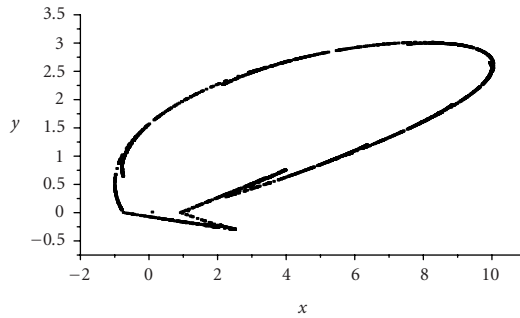


Figure 4.1. A typical orbit of system (2.1) obtained for $a = 1.35$, $b = 0.3$.

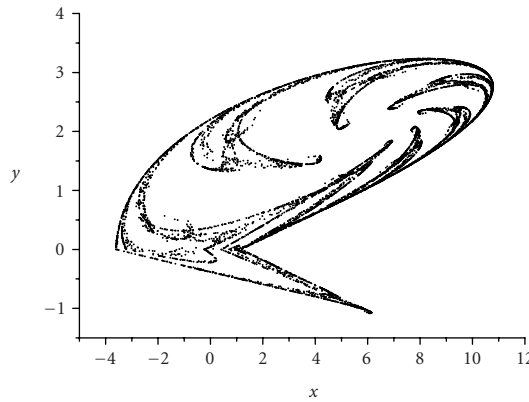


Figure 4.2. The new chaotic attractor obtained for $a = 1.4$, $b = 0.3$.

period doubling route to chaos is allowed, and the attractor goes directly from a border-collision bifurcation developed from a stable periodic orbit [4], and for the same case the new chaotic attractor given by system (2.1) is obtained from a border-collision period-doubling bifurcation scenario [1]; this scenario (see Figure 4.3) is formed by a sequence of pairs of bifurcations, whereby each pair consists of a border-collision bifurcation and a pitchfork bifurcation. Thus, the three chaotic attractors go via different and distinguishable route to chaos as a typical future.

5. Conclusion

This paper reports the finding of a new two-dimensional discrete chaotic attractor obtained via direct modification of the Lozi mapping. The new chaotic attractor is allowed via a border-collision period-doubling scenario. More detailed analysis about the structure, dynamics and bifurcations of the new system (2.1) will be provided in near future.

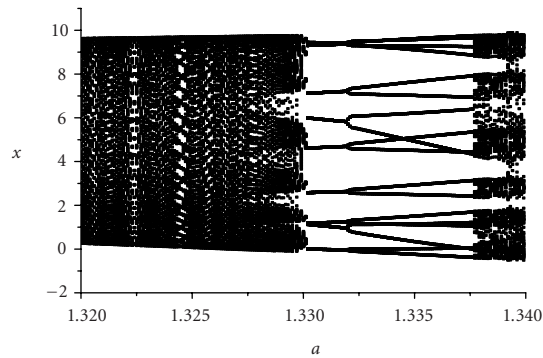


Figure 4.3. Border-collision period-doubling scenario route to chaos observed for system (2.1).

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