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Research Article

Positive Solutions for Third-Order Nonlinear p-Laplacian m-Point Boundary Value Problems on Time Scales

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We study the following third-order p-Laplacian m-point boundary value problems on time scales: $(\phi_p(u^{\Delta\nabla}))^{\nabla} + a(t)f(t,u(t)) = 0$, $t \in [0,T]_{\mathbf{T}}$, $\beta u(0) - \gamma u^{\Delta}(0) = 0$, $u(T) = \sum_{i=1}^{m-2} a_i u(\xi_i)$, $\phi_p(u^{\Delta\nabla}(0)) = \sum_{i=1}^{m-2} b_i \phi_p(u^{\Delta\nabla}(\xi_i))$, where $\phi_p(s)$ is p-Laplacian operator, that is, $\phi_p(s) = |s|^{p-2}s$, $p > 1$, $\phi_p^{-1} = \phi_q$, $1/p + 1/q = 1$, $0 < \xi_1 < \cdots < \xi_{m-2} < \rho(T)$. We obtain the existence of positive solutions by using fixed-point theorem in cones. The conclusions in this paper essentially extend and improve the known results.

1. Introduction

The theory of time scales was initiated by Hilger [1] as a means of unifying and extending theories from differential and difference equations. The study of time scales has lead to several important applications in the study of insect population models, neural networks, heat transfer, and epidemic models, see, for example [2–6]. Recently, the boundary value problems with p-Laplacian operator have also been discussed extensively in the literature, for example, see [7–13].

A time scale $\mathbf{T}$ is a nonempty closed subset of $\mathbb{R}$. We make the blanket assumption that $0, T$ are points in $\mathbf{T}$. By an interval $(0, T)$, we always mean the intersection of the real interval $(0, T)$ with the given time scale; that is $(0, T)\cap\mathbf{T}$.

In [14], Anderson considered the the following third-order nonlinear boundary value problem (BVP):

$$\begin{gathered} x'''(t) = f(t, x(t)), \quad t_1 \le t \le t_3, \\ x(t_1) = x'(t_2) = 0, \qquad \gamma x(t_3) + \delta x''(t_3) = 0. \end{gathered} \tag{1.1}$$

Author studied the existence of solutions for the nonlinear boundary value problem by using the Krasnoselskii's fixed point theorem and Leggett and Williams fixed point theorem, respectively.

In [8, 9], He considered the existence of positive solutions of the p-Laplacian dynamic equations on time scales

$$\left(\phi_p\left(u^{\Delta}\right)\right)^{\nabla}+a(t)f\left(u(t)\right)=0, \quad t\in[0,T]_{\mathbf{T}}, \tag{1.2}$$

satisfying the boundary conditions

$$u(0)-B_0\left(u^{\Delta}(\eta)\right)=0, \qquad u^{\Delta}(T)=0, \tag{1.3}$$

or

$$u^{\Delta}(0)=0, \qquad u(T)-B_1\left(u^{\Delta}(\eta)\right)=0, \tag{1.4}$$

where $\eta\in(0,\rho(T))$. He obtained the existence of at least double and triple positive solutions of the boundary value problems by using a new double fixed point theorem and triple fixed point theorem, respectively.

In [13], Zhou and Ma firstly studied the existence and iteration of positive solutions for the following third-order generalized right-focal boundary value problem with p-Laplacian operator:

$$\begin{gathered}\left(\phi_p\left(u''\right)\right)'(t)=q(t)f\left(t,u(t)\right), \quad 0\le t\le 1,\\ u(0)=\sum_{i=1}^{m}\alpha_i u\left(\xi_i\right), \qquad u'(\eta)=0, \qquad u''(1)=\sum_{i=1}^{n}\beta_i u''\left(\theta_i\right).\end{gathered} \tag{1.5}$$

They established a corresponding iterative scheme for the problem by using the monotone iterative technique.

However, to the best of our knowledge, little work has been done on the existence of positive solutions for third-order p-Laplacian m-point boundary value problems on time scales. This paper attempts to fill this gap in the literature.

In this paper, by using different method, we are concerned with the existence of positive solutions for the following third-order p-Laplacian m-point boundary value problems on time scales:

$$\begin{gathered}\left(\phi_p\left(u^{\Delta\nabla}\right)\right)^{\nabla}+a(t)f\left(t,u(t)\right)=0, \quad t\in[0,T]_{\mathbf{T}},\\ \beta u(0)-\gamma u^{\Delta}(0)=0, \qquad u(T)=\sum_{i=1}^{m-2}a_i u\left(\xi_i\right), \qquad \phi_p\left(u^{\Delta\nabla}(0)\right)=\sum_{i=1}^{m-2}b_i\phi_p\left(u^{\Delta\nabla}\left(\xi_i\right)\right),\end{gathered} \tag{1.6}$$

where $\phi_p(s)$ is p-Laplacian operator, that is, $\phi_p(s)=|s|^{p-2}s,\ p>1,\ \phi_p^{-1}=\phi_q,\ (1/p)+(1/q)=1,$

and a_i, b_i, a, f satisfy

(H_1) $\beta, \gamma \geq 0$, $\beta + \gamma > 0$, $a_i \in [0, +\infty)$, $i = 1, 2, \ldots, m-3$, $a_{m-2} > 0$, $0 < \xi_1 < \cdots < \xi_{m-2} < \rho(T)$, $0 < \sum_{i=1}^{m-2} b_i < 1$, $0 < \sum_{i=1}^{m-2} a_i \xi_i < T$, $d = \beta(T - \sum_{i=1}^{m-2} a_i \xi_i) + \gamma(1 - \sum_{i=1}^{m-2} a_i) > 0$;

(H_2) $f : [0, T]_{\mathbf{T}} \times [0, +\infty) \to R^+$ is continuous, $a \in C_{\mathrm{ld}}((0, T)_{\mathbf{T}}, \mathbb{R}^+)$ and there exists $t_0 \in [\xi_{m-2}, T)_{\mathbf{T}}$ such that $a(t_0) > 0$, where $\mathbb{R}^+ = [0, +\infty)$.

2. Preliminaries and lemmas

For convenience, we list the following definitions which can be found in [1–5].

Definition 2.1. A time scale $\mathbf{T}$ is a nonempty closed subset of real numbers $\mathbb{R}$. For $t < \sup \mathbf{T}$ and $r > \inf \mathbf{T}$, define the forward jump operator σ and backward jump operator ρ, respectively, by

$$\sigma(t) = \inf\{\tau \in \mathbf{T} \mid \tau > t\} \in \mathbf{T}, \qquad \rho(r) = \sup\{\tau \in \mathbf{T} \mid \tau < r\} \in \mathbf{T}, \tag{2.1}$$

for all $t, r \in \mathbf{T}$. If $\sigma(t) > t$, t is said to be right scattered; if $\rho(r) < r$, r is said to be left scattered; if $\sigma(t) = t$, t is said to be right dense; if $\rho(r) = r$, r is said to be left dense. If $\mathbf{T}$ has a right scattered minimum m, define $\mathbf{T}_k = \mathbf{T} - \{m\}$, otherwise set $\mathbf{T}_k = \mathbf{T}$. If $\mathbf{T}$ has a left scattered maximum M, define $\mathbf{T}^k = \mathbf{T} - \{M\}$, otherwise set $\mathbf{T}^k = \mathbf{T}$.

Definition 2.2. For $f : \mathbf{T} \to \mathbb{R}$ and $t \in \mathbf{T}^k$, the delta derivative of f at the point t is defined to be the number $f^{\Delta}(t)$, (provided it exists), with the property that for each $\epsilon > 0$, there is a neighborhood U of t such that

$$\left| f(\sigma(t)) - f(s) - f^{\Delta}(t)(\sigma(t) - s) \right| \leq \epsilon \left| \sigma(t) - s \right|, \tag{2.2}$$

for all $s \in U$.

For $f : \mathbf{T} \to \mathbb{R}$ and $t \in \mathbf{T}_k$, the nabla derivative of f at t, denoted by $f^{\nabla}(t)$ (provided it exists), with the property that for each $\epsilon > 0$, there is a neighborhood U of t such that

$$\left| f(\rho(t)) - f(s) - f^{\nabla}(t)(\rho(t) - s) \right| \leq \epsilon \left| \rho(t) - s \right|, \tag{2.3}$$

for all $s \in U$.

Definition 2.3. A function f is left-dense continuous (i.e., ld-continuous), if f is continuous at each left-dense point in $\mathbf{T}$ and its right-sided limit exists at each right-dense point in $\mathbf{T}$.

Definition 2.4. If $\phi^{\Delta}(t) = f(t)$, then one defines the delta integral by

$$\int_a^b f(t)\Delta t = \phi(b) - \phi(a). \tag{2.4}$$

If $F^{\nabla}(t) = f(t)$, then one defines the nabla integral by

$$\int_a^b f(t)\nabla t = F(b) - F(a). \tag{2.5}$$

Lemma 2.5. *If $d = \beta(T - \sum_{i=1}^{m-2} a_i\xi_i) + \gamma(1 - \sum_{i=1}^{m-2} a_i) > 0$, then for $h \in C_{ld}[0,T]_{\mathbf{T}}$, the boundary value problem (BVP)*

$$\begin{gathered} u^{\Delta\nabla} + h(t) = 0, \quad t \in [0,T]_{\mathbf{T}}, \\ \beta u(0) - \gamma u^{\Delta}(0) = 0, \qquad u(T) = \sum_{i=1}^{m-2} a_i u(\xi_i) \end{gathered} \tag{2.6}$$

has the unique solution

$$\begin{aligned} u(t) = &-\int_0^t (t-s)h(s)\nabla s + \frac{\beta t + \gamma}{d}\int_0^T (T-s)h(s)\nabla s \\ &- \frac{\beta t + \gamma}{d}\sum_{i=1}^{m-2} a_i \int_0^{\xi_i} (\xi_i - s)h(s)\nabla s. \end{aligned} \tag{2.7}$$

Proof. By direct computation, we can easily get (2.7). So, we omit it. □

Lemma 2.6. *If $0 < \sum_{i=1}^{m-2} b_i < 1$, $0 < \sum_{i=1}^{m-2} a_i\xi_i < T$, $d = \beta(T - \sum_{i=1}^{m-2} a_i\xi_i) + \gamma(1 - \sum_{i=1}^{m-2} a_i) > 0$, then for $h \in C_{ld}[0,T]_{\mathbf{T}}$, the boundary value problem (BVP)*

$$\begin{gathered} \left(\phi_p(u^{\Delta\nabla})\right)^{\nabla} + h(t) = 0, \quad t \in [0,T]_{\mathbf{T}}, \\ \beta u(0) - \gamma u^{\Delta}(0) = 0, \qquad u(T) = \sum_{i=1}^{m-2} a_i u(\xi_i), \qquad \phi_p(u^{\Delta\nabla}(0)) = \sum_{i=1}^{m-2} b_i \phi_p(u^{\Delta\nabla}(\xi_i)) \end{gathered} \tag{2.8}$$

has the unique solution

$$\begin{aligned} u(t) = &-\int_0^t (t-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s + \frac{\beta t + \gamma}{d}\int_0^T (T-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s \\ &- \frac{\beta t + \gamma}{d}\sum_{i=1}^{m-2} a_i \int_0^{\xi_i} (\xi_i - s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s, \end{aligned} \tag{2.9}$$

where $B = \sum_{i=1}^{m-2} b_i \int_0^{\xi_i} h(r)\nabla r / (1 - \sum_{i=1}^{m-2} b_i)$.

Proof. Integrating both sides of (1.6) on $[0,t]$, we have

$$\phi_p(u^{\Delta\nabla}(t)) = \phi_p(u^{\Delta\nabla}(0)) - \int_0^t h(r)\nabla r. \tag{2.10}$$

So

$$\phi_p(u^{\Delta\nabla}(\xi_i)) = \phi_p(u^{\Delta\nabla}(0)) - \int_0^{\xi_i} h(r)\nabla r. \tag{2.11}$$

By boundary value condition $\phi_p(u^{\Delta\nabla}(0)) = \sum_{i=1}^{m-2} b_i \phi_p(u^{\Delta\nabla}(\xi_i))$, we have

$$\phi_p(u^{\Delta\nabla}(0)) = -\frac{\sum_{i=1}^{m-2} b_i \int_0^{\xi_i} h(r)\nabla r}{1-\sum_{i=1}^{m-2} b_i}. \tag{2.12}$$

By (2.10) and (2.12), we know

$$u^{\Delta\nabla}(t) = -\phi_q\left(\frac{\sum_{i=1}^{m-2} b_i \int_0^{\xi_i} h(r)\nabla r}{1-\sum_{i=1}^{m-2} b_i} + \int_0^t h(r)\nabla r\right). \tag{2.13}$$

This together with Lemma 2.5 implies that

$$\begin{aligned} u(t) = &-\int_0^t (t-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s + \frac{\beta t+\gamma}{d}\int_0^T (T-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s \\ &- \frac{\beta t+\gamma}{d}\sum_{i=1}^{m-2} a_i \int_0^{\xi_i} (\xi_i - s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s, \end{aligned} \tag{2.14}$$

where $B = \sum_{i=1}^{m-2} b_i \int_0^{\xi_i} h(r)\nabla r / (1-\sum_{i=1}^{m-2} b_i)$. The proof is complete. □

Lemma 2.7. *Let* $0 < \sum_{i=1}^{m-2} a_i\xi_i < 1$, $d > 0$. *If* $h \in C_{ld}[0,T]_{\mathbf{T}}$ *and* $h(t) \geq 0$, *then the unique solution* u *of* (2.8) *satisfies*

$$u(t) \geq 0. \tag{2.15}$$

Proof. By $u^{\Delta\nabla}(t) = -\phi_q(\sum_{i=1}^{m-2} b_i \int_0^{\xi_i} h(r)\nabla r/(1-\sum_{i=1}^{m-2} b_i)) + \int_0^t h(r)\nabla r) \leq 0$, we can know that the graph of $u(t)$ is concave down on $(0,T)_{\mathbf{T}}$. So we only prove $u(0) \geq 0$, $u(T) \geq 0$.

Firstly, we will prove $u(0) \geq 0$ by the following two perspectives.

(i) If $0 < \sum_{i=1}^{m-2} a_i \leq 1$, we have

$$\begin{aligned} u(0) &= \frac{\gamma}{d}\left[\int_0^T (T-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s - \sum_{i=1}^{m-2} a_i \int_0^{\xi_i} (\xi_i - s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s\right] \\ &\geq \frac{\gamma}{d}\left[\int_0^T (T-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s - \sum_{i=1}^{m-2} a_i \int_0^{T} (T - s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s\right] \\ &= \frac{\gamma}{d}\left(1-\sum_{i=1}^{m-2} a_i\right)\int_0^T (T-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s \geq 0. \end{aligned} \tag{2.16}$$

(ii) If $\sum_{i=1}^{m-2} a_i > 1$, by (2.8), we have

$$u(0) = \frac{\gamma}{d}\left[\int_0^T (T-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s - \sum_{i=1}^{m-2} a_i \int_0^{\xi_i} (\xi_i - s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s\right]$$
$$\geq \frac{\gamma}{d}\left[\int_0^T (T-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s - \sum_{i=1}^{m-2} a_i \int_0^{T} (\xi_i - s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s\right]$$
$$= \frac{\gamma}{d}\int_0^T \left[\left(T - \sum_{i=1}^{m-2} a_i\xi_i\right) + \left(\sum_{i=1}^{m-2} a_i - 1\right)s\right]\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s \geq 0. \tag{2.17}$$

On the other hand, we have

$$u(T) = -\int_0^T (T-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s + \frac{\beta+\gamma}{d}\int_0^T (T-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s$$
$$- \frac{\beta+\gamma}{d}\sum_{i=1}^{m-2} a_i \int_0^{\xi_i} (\xi_i - s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s$$
$$\geq \frac{\beta}{d}\left[\sum_{i=1}^{m-2} a_i \int_0^{\xi_i} (\xi_i(T-s) - T(\xi_i - s))\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s\right.$$
$$\left. + \sum_{i=1}^{m-2} a_i\xi_i \int_{\xi_i}^T (T-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s\right]$$
$$+ \frac{\gamma}{d}\sum_{i=1}^{m-2} a_i\left[\int_0^T (T-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s - \int_0^T (\xi_i - s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s\right]$$
$$= \frac{\beta}{d}\sum_{i=1}^{m-2} a_i\left[\int_0^{\xi_i} (T-\xi_i)s\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s + \xi_i\int_{\xi_i}^T (T-s)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s\right]$$
$$+ \frac{\gamma}{d}\sum_{i=1}^{m-2} a_i\left[\int_0^T (T-\xi_i)\phi_q\left(\int_0^s h(r)\nabla r + B\right)\nabla s\right] \geq 0. \tag{2.18}$$

The proof is completed. □

Lemma 2.8. *Let $a_i \geq 0$, $i = 1, \ldots, m-2$, $0 < \sum_{i=1}^{m-2} a_i\xi_i < T$, $d > 0$. If $h \in C_{ld}[0,T]_{\mathbf{T}}$ and $h(t) \geq 0$, then the unique positive solution $u(t)$ of (BVP)* (2.8) *satisfies*

$$\inf_{t\in[\xi_{m-2},T]_{\mathbf{T}}} u(t) \geq \sigma\|u\|, \tag{2.19}$$

where $\sigma = \min\{a_{m-2}(T-\xi_{m-2})/(T - a_{m-2}\xi_{m-2}), a_{m-2}\xi_{m-2}/T, \xi_{m-2}/T\}$, $\|u\| = \sup_{t\in[0,T]_{\mathbf{T}}}|u(t)|$.

Proof. Let $u(\bar{t}) = \max_{t\in[0,T]_{\mathbf{T}}} u(t) = \|u\|$, we shall discuss it from the following two perspectives.

Case 1. If $0 < \sum_{i=1}^{m-2} a_i < 1$.

Firstly, assume $\bar{t} < \xi_{m-2} < T$, then $\min_{t\in[\xi_{m-2},T]_{\mathbf{T}}} u(t) = u(T)$. By $u(T) = \sum_{i=1}^{m-2} a_i u(\xi_i) \geq a_{m-2}u(\xi_{m-2})$, we have

$$\begin{aligned} u(\bar{t}) &\leq u(T) + \frac{u(T) - u(\xi_{m-2})}{T - \xi_{m-2}}(0 - T) = u(T) - \frac{T}{T - \xi_{m-2}}u(T) + \frac{T}{T - \xi_{m-2}}u(\xi_{m-2}) \\ &\leq u(T)\left(1 - \frac{T}{T - \xi_{m-2}} + \frac{T}{a_{m-2}(T - \xi_{m-2})}\right) = u(T)\frac{T - a_{m-2}\xi_{m-2}}{a_{m-2}(T - \xi_{m-2})}. \end{aligned} \tag{2.20}$$

So

$$\min_{t\in[\xi_{m-2},T]_{\mathbf{T}}} u(t) \geq \frac{a_{m-2}(T - \xi_{m-2})}{T - a_{m-2}\xi_{m-2}}\|u\|. \tag{2.21}$$

Secondly, assume $\xi_{m-2} < \bar{t} < T$, then $\min_{t\in[\xi_{m-2},T]_{\mathbf{T}}} u(t) = u(T)$. Otherwise, we have $\min_{t\in[\xi_{m-2},T]_{\mathbf{T}}} u(t) = u(\xi_{m-2})$, then $\bar{t} \in [\xi_{m-2}, T]_{\mathbf{T}}$, $u(\xi_{m-2}) \geq u(\xi_{m-1}) \geq \cdots \geq u(\xi_2) \geq u(\xi_1)$. By $0 < \sum_{i=1}^{m-2} a_i < 1$, we have

$$u(T) = \sum_{i=1}^{m-2} a_i u(\xi_i) \leq \sum_{i=1}^{m-2} a_i u(\xi_{m-2}) < u(\xi_{m-2}) \leq u(T), \tag{2.22}$$

a contradiction.

By concave of $u(t)$, we get $u(\xi_{m-2})/\xi_{m-2} \geq u(\bar{t})/\bar{t} \geq u(\bar{t})/T$. In fact, since $u(T) \geq a_{m-2}u(\xi_{m-2})$, then $u(T)/a_{m-2}\xi_{m-2} \geq u(\bar{t})/T$, which implies

$$\min_{t\in[\xi_{m-2},T]_{\mathbf{T}}} u(t) \geq \frac{a_{m-2}\xi_{m-2}}{T}\|u\|. \tag{2.23}$$

Case 2. If $\sum_{i=1}^{m-2} a_i > 1$.

Firstly, assume $u(\xi_{m-2}) \leq u(T)$, then $\min_{t\in[\xi_{m-2},T]_{\mathbf{T}}} u(t) = u(\xi_{m-2})$. By concave of $u(t)$, we have $\bar{t} \in [\xi_{m-2}, t]_{\mathbf{T}}$, which implies $u(\xi_{m-2})/\xi_{m-2} \geq u(\bar{t})/\bar{t} \geq u(\bar{t})/T$, then

$$\min_{t\in[\xi_{m-2},T]_{\mathbf{T}}} u(t) \geq \frac{\xi_{m-2}}{T}\|u\|. \tag{2.24}$$

Secondly, assume $u(\xi_{m-2}) > u(t)$, then $\min_{t\in[\xi_{m-2},T]_{\mathbf{T}}} u(t) = u(T)$, and $\bar{t} \in [\xi_1, T]_{\mathbf{T}}$. If not, $\bar{t} \in [0, \xi_1)$, then $u(\xi_1) \geq \cdots \geq u(\xi_{m-2}) > u(T)$. So, we have

$$u(T) = \sum_{i=1}^{m-2} a_i u(\xi_i) > u(T)\sum_{i=1}^{m-2} a_i \geq u(T), \tag{2.25}$$

a contradiction. By $\sum_{i=1}^{m-2} a_i > 1$, there exists $\bar{\xi} \in \{\xi_1, \xi_2, \ldots, \xi_{m-2}\}$ such that $u(\bar{\xi}) \le u(T)$, then $u(\xi_1) \le u(\xi_2) \le \cdots \le u(\xi_{m-2}) \le u(1)$. By concave of $u(t)$, we have $u(1)/\xi_1 \ge u(\xi_1)/\xi_1 \ge u(\bar{t})/\bar{t} \ge u(\bar{t})/T$, then

$$\min_{t \in [\xi_{m-2}, T]_{\mathbf{T}}} u(t) \ge \xi_1 \|u\|. \tag{2.26}$$

Therefore, by (2.21)–(2.26), we have

$$\inf_{t \in [\xi_{m-2}, T]_{\mathbf{T}}} u(t) \ge \sigma \|u\|, \tag{2.27}$$

where $\sigma = \min\{a_{m-2}(T - \xi_{m-2})/(T - a_{m-2}\xi_{m-2}), a_{m-2}\xi_{m-2}/T, \xi_{m-2}/T\}$. The proof is complete. □

Let $E = C_{\mathrm{ld}}[0, T]_{\mathbf{T}}$ be endowed with the ordering $x \le y$ if $x(t) \le y(t)$, for all $t \in [0, T]_{\mathbf{T}}$, and $\|u\| = \max_{t \in [0,T]_{\mathbf{T}}} |u(t)|$ is defined as usual by maximum norm. Clearly, it follows that $(E, \|u\|)$ is a Banach space.

We define a cone by

$$K = \left\{ u : u \in E,\ u(t) \text{ is concave, nonnegative on } [0, T]_{\mathbf{T}},\ \inf_{t \in [\xi_{m-2}, T]_{\mathbf{T}}} u(t) \ge \sigma \|u\| \right\}. \tag{2.28}$$

Define an operator $S : K \to E$ by setting

$$\begin{aligned} Su(t) = &- \int_0^t (t - s)\phi_q\left(\int_0^s a(r) f(r, u(r)) \nabla r + A\right) \nabla s \\ &+ \frac{\beta t + \gamma}{d} \int_0^T (T - s)\phi_q\left(\int_0^s a(r) f(r, u(r)) \nabla r + A\right) \nabla s \\ &- \frac{\beta t + \gamma}{d} \sum_{i=1}^{m-2} a_i \int_0^{\xi_i} (\xi_i - s)\phi_q\left(\int_0^s a(r) f(r, u(r)) \nabla r + A\right) \nabla s, \end{aligned} \tag{2.29}$$

where $A = \sum_{i=1}^{m-2} b_i \int_0^{\xi_i} a(r) f(r, u(r)) \nabla r / (1 - \sum_{i=1}^{m-2} b_i)$. Obviously, u is a solution of boundary value problem (1.6) if and only if u is a fixed point of operator S.

Lemma 2.9. *$S : K \to K$ is completely continuous.*

Proof. By (H_2) and Lemmas 2.7-2.8, we easily get $SK \subset K$. By Arzela-Ascoli theorem and Lebesgue dominated convergence theorem, we can easily prove S is completely continuous. □

Lemma 2.10 (see [15]). *Let K be a cone in a Banach space X. Let D be an open bounded subset of X with $D_K = D \cap K \neq \varnothing$ and $\overline{D}_K \neq K$. Assume that $A : \overline{D}_K \to K$ is a compact map such that $x \neq Ax$ for $x \in \partial D_K$. Then the following results hold.*

(1) *If* $\|Ax\| \leq \|x\|$, $x \in \partial D_K$, *then* $i_K(A, D_K) = 1$.

(2) *If there exists* $x_0 \in K \setminus \{0\}$ *such that* $x \neq Ax + \lambda x_0$, *for all* $x \in \partial D_K$ *and all* $\lambda > 0$, *then* $i_K(A, D_K) = 0$.

(3) *Let* U *be open in* X *such that* $\overline{U} \subset D_K$. *If* $i_K(A, D_K) = 1$ *and* $i_K(A, U_K) = 0$, *then* A *has a fixed point in* $D_K \setminus \overline{U}_K$. *The same result holds if* $i_K(A, D_K) = 0$ *and* $i_K(A, U_K) = 1$, *where* $i_K(A, D_K)$ *denotes fixed point index.*

One defines

$$K_\rho = \{u(t) \in K : \|u\| < \rho\}, \qquad \Omega_\rho = \left\{u(t) \in K : \min_{\xi_{m-2} \leq t \leq T} x(t) < \sigma\rho\right\}. \tag{2.30}$$

Lemma 2.11 (see [15]). Ω_ρ *defined above has the following properties:*

(a) $K_{\sigma\rho} \subset \Omega_\rho \subset K_\rho$;

(b) Ω_ρ *is open relative to* K;

(c) $x \in \partial\Omega_\rho$ *if and only if* $\min_{\xi_{m-2} \leq t \leq T} x(t) = \sigma\rho$;

(d) *if* $x \in \partial\Omega_\rho$, *then* $\sigma\rho \leq x(t) \leq \rho$, *for* $t \in [\xi_{m-2}, T]_{\mathbf{T}}$.

For the convenience, we introduce the following notations:

$$f_{\sigma\rho}^{\rho} = \min\left\{\min_{\xi_{m-2} \leq t \leq T} \frac{f(t,u)}{\phi_p(\rho)} : u \in [\sigma\rho, \rho]\right\}, \qquad f_0^{\rho} = \max\left\{\max_{0 \leq t \leq T} \frac{f(t,u)}{\phi_p(\rho)} : u \in [0, \rho]\right\},$$

$$f^{\alpha} = \limsup_{u \to \alpha} \max_{0 \leq t \leq T} \frac{f(t,u)}{\phi_p(u)}, \qquad f_{\alpha} = \liminf_{u \to \alpha} \max_{\xi_{m-2} \leq t \leq T} \frac{f(t,u)}{\phi_p(u)} \qquad (\alpha := \infty \text{ or } 0^+),$$

$$\frac{1}{m} = \frac{(\beta T + \gamma)}{d} \int_0^T (T - s)\nabla s \phi_q\left(\int_0^T a(r)\nabla r + \frac{\sum_{i=1}^{m-2} b_i \int_0^{\xi_i} a(r)\nabla r}{1 - \sum_{i=1}^{m-2} b_i}\right),$$

$$\frac{1}{M} = \frac{1}{d}\int_{\xi_{m-2}}^{T} (T-s)\phi_q\left(\int_{\xi_{m-2}}^{s} a(r)\nabla r\right)\nabla s \min\left\{\beta\xi_{m-2}+\gamma, \beta \max\left\{\sum_{i=1}^{m-2} a_i\xi_1, a_{m-2}\xi_{m-2}\right\}+\gamma\sum_{i=1}^{m-2} a_i\right\}. \tag{2.31}$$

Lemma 2.12. *If* f *satisfies the following condition:*

$$f_0^{\rho} \leq \phi_p(m), \quad u \neq Su, \; u \in \partial K_\rho, \tag{2.32}$$

then

$$i_K(S, K_\rho) = 1. \tag{2.33}$$

Proof. For $u \in \partial K_\rho$, then from (2.32), we have

$$\begin{aligned}\int_0^s a(r) f(r, u(r)) \nabla r + A &= \int_0^s a(r) f(r, u(r)) \nabla r + \frac{\sum_{i=1}^{m-2} b_i \int_0^{\xi_i} a(r) f(r, u(r)) \nabla r}{1 - \sum_{i=1}^{m-2} b_i} \\ &\leq \int_0^T a(r) f(r, u(r)) \nabla r + \frac{\sum_{i=1}^{m-2} b_i \int_0^{\xi_i} a(r) f(r, u(r)) \nabla r}{1 - \sum_{i=1}^{m-2} b_i} \\ &\leq \phi_p(m\rho) \left(\int_0^T a(r) \nabla r + \frac{\sum_{i=1}^{m-2} b_i \int_0^{\xi_i} a(r) \nabla r}{1 - \sum_{i=1}^{m-2} b_i} \right).\end{aligned} \tag{2.34}$$

So that

$$\phi_q\left(\int_0^s a(r) f(r, u(r)) \nabla r + A \right) \leq m\rho \phi_q\left(\int_0^T a(r) \nabla r + \frac{\sum_{i=1}^{m-2} b_i \int_0^{\xi_i} a(r) \nabla r}{1 - \sum_{i=1}^{m-2} b_i} \right). \tag{2.35}$$

Therefore,

$$\begin{aligned}Su(t) &\leq \frac{\beta t + \gamma}{d} \int_0^T (T - s) \phi_q\left(\int_0^s a(r) f(r, u(r)) \nabla r + A \right) \nabla s \\ &\leq \frac{(\beta T + \gamma) m\rho}{d} \int_0^T (T - s) \nabla s \phi_q\left(\int_0^T a(r) \nabla r + \frac{\sum_{i=1}^{m-2} b_i \int_0^{\xi_i} a(r) \nabla r}{1 - \sum_{i=1}^{m-2} b_i} \right) \\ &= \rho.\end{aligned} \tag{2.36}$$

This implies that $\|Su\| \leq \|u\|$ for $u \in \partial K_\rho$. Hence, by Lemma 2.10(1) it follows that $i_K(S, K_\rho) = 1$. □

Lemma 2.13. *If f satisfies the following condition:*

$$f_{\sigma\rho}^{\rho} \geq \phi_p(M\sigma), \quad u \neq Su, \ u \in \partial\Omega_\rho, \tag{2.37}$$

then

$$i_K(S, \Omega_\rho) = 0. \tag{2.38}$$

Proof. Let $e(t) \equiv 1$ for $t \in [0, T]_{\mathbf{T}}$. Then $e \in \partial K_1$. We claim that

$$u \neq Su + \lambda e, \quad u \in \partial\Omega_\rho, \ \lambda > 0. \tag{2.39}$$

In fact, if not, there exist $u_0 \in \partial\Omega_\rho$ and $\lambda_0 > 0$ such that $u_0 = Su_0 + \lambda_0 e$. By $f_{\sigma\rho}^{\rho} \geq \phi_p(M\sigma)$, we have

$$\begin{aligned}\int_0^s a(r)f(r,u_0(r))\nabla r + A &= \int_0^s a(r)f(r,u_0(r))\nabla r + \frac{\sum_{i=1}^{m-2} b_i \int_0^{\xi_i} a(r)f(r,u_0(r))\nabla r}{1-\sum_{i=1}^{m-2} b_i} \\ &\geq \int_{\xi_{m-2}}^s a(r)f^+(r,u(r))\nabla r \\ &\geq \phi_p(M\sigma\rho)\left(\int_{\xi_{m-2}}^s a(r)\nabla r\right).\end{aligned} \tag{2.40}$$

So that

$$\phi_q\left(\int_0^s a(r)f(r,u(r))\nabla r + A\right) \geq M\sigma\rho\phi_q\left(\int_{\xi_{m-2}}^s a(r)\nabla r\right). \tag{2.41}$$

By [16, Theorem 2.2(iv)], for $t > 0$, we have

$$\left(\frac{\int_0^t (t-s)\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r + A\right)\nabla s}{t}\right)^{\Delta} = \frac{\int_0^t s\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r + A\right)\nabla s}{t\sigma(t)} \geq 0. \tag{2.42}$$

So, for $i = 1, 2, \ldots, m-2$, we have

$$\frac{\int_0^{\xi_{m-2}} (\xi_{m-2}-s)\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r + A\right)\nabla s}{\xi_{m-2}} \geq \frac{\int_0^{\xi_i} (\xi_i - s)\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r + A\right)\nabla s}{\xi_i}. \tag{2.43}$$

Therefore,

$$\begin{aligned}Su_0(\xi_{m-2}) \geq \frac{\beta}{d}\Bigg[&\xi_{m-2}\int_0^T (t-s)\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r + A\right)\nabla s \\ &- T\int_0^{\xi_{m-2}} (\xi_{m-2}-s)\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r + A\right)\nabla s\Bigg] \\ &+ \frac{\gamma}{d}\Bigg[\int_0^T (t-s)\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r + A\right)\nabla s \\ &- \int_0^{\xi_{m-2}} (\xi_{m-2}-s)\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r + A\right)\nabla s\Bigg]\end{aligned}$$

$$\geq \frac{\beta\xi_{m-2}+\gamma}{d}\int_{\xi_{m-2}}^{T}(T-s)\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r+A\right)\nabla s$$
$$\geq \frac{(\beta\xi_{m-2}+\gamma)M\sigma\rho}{d}\int_{\xi_{m-2}}^{T}(T-s)\phi_q\left(\int_{\xi_{m-2}}^s a(r)\nabla r\right)\nabla s, \tag{2.44}$$

$$\begin{aligned} Su_0(T) \geq{}& \frac{\beta}{d}\sum_{i=1}^{m-2}a_i\left[\int_0^{\xi_i}(t-\xi_i)s\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r+A\right)\nabla s\right.\\ &\left.+\xi_i\int_{\xi_i}^{T}(T-s)\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r+A\right)\nabla s\right]\\ &+\frac{\gamma}{d}\sum_{i=1}^{m-2}\left[\int_0^{T}(t-s)\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r+A\right)\nabla s\right.\\ &\left.-\int_0^{\xi_i}(T-s)\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r+A\right)\nabla s\right]\\ \geq{}& \frac{\beta}{d}\sum_{i=1}^{m-2}a_i\xi_i\int_{\xi_i}^{T}(t-s)\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r+A\right)\nabla s\\ &+\frac{\gamma}{d}\sum_{i=1}^{m-2}\int_{\xi_i}^{T}(t-s)\phi_q\left(\int_0^s a(r)f(r,u_0(r))\nabla r+A\right)\nabla s\\ \geq{}& \frac{M\sigma\rho}{d}\left(\beta\max\left\{\sum_{i=1}^{m-2}a_i\xi_1, a_{m-2}\xi_{m-2}\right\}+\gamma\sum_{i=1}^{m-2}\right)\int_{\xi_{m-2}}^{T}(T-s)\phi_q\left(\int_{\xi_{m-2}}^s a(r)\nabla r\right)\nabla s. \end{aligned} \tag{2.45}$$

Obviously, we can know

$$\begin{aligned} \min_{t\in[\xi_{m-2},T]_{\mathbf{T}}} Su_0(t) &= \min\{Su_0(\xi_{m-2}), Su_0(T)\}\\ &\geq \frac{M\sigma\rho}{d}\int_{\xi_{m-2}}^{T}(T-s)\phi_q\left(\int_{\xi_{m-2}}^s a(r)\nabla r\right)\nabla s\\ &\quad\times\min\left\{\beta\xi_{m-2}+\gamma, \beta\max\left\{\sum_{i=1}^{m-2}a_i\xi_1, a_{m-2}\xi_{m-2}\right\}+\gamma\sum_{i=1}^{m-2}\right\}\\ &\geq \sigma\rho. \end{aligned} \tag{2.46}$$

For $t\in[\xi_{m-2},T]_{\mathbf{T}}$, then

$$\begin{aligned} u_0(t) = Su_0(t)+\lambda_0 e(t) &\geq \min_{t\in[\xi_{m-2},T]} Su_0(t)+\lambda_0\\ &= \min\{Su_0(\xi_{m-2}), Su_0(T)\}+\lambda_0 \geq \sigma\rho+\lambda_0. \end{aligned} \tag{2.47}$$

This together with Lemma 2.11(c) implies that

$$\sigma\rho \geq \sigma\rho + \lambda_0, \tag{2.48}$$

a contradiction. Hence, by Lemma 2.10(2), it follows that $i_K(S, \Omega_\rho) = 0$. □

3. Main results

We now give our results on the existence of positive solutions of BVP (1.6).

Theorem 3.1. *Suppose conditions* (H_1) *and* (H_2) *hold, and assume that one of the following conditions holds.*

(H_3) *There exist* $\rho_1, \rho_2 \in (0, +\infty)$ *with* $\rho_1 < \sigma\rho_2$ *such that* $f_0^{\rho_1} \leq \phi_p(m)$, $f_{\sigma\rho_2}^{\rho_2} \geq \phi_p(M\sigma)$.

(H_4) *There exist* $\rho_1, \rho_2 \in (0, +\infty)$ *with* $\rho_1 < \rho_2$ *such that* $f_0^{\rho_2} \leq \phi_p(m)$, $f_{\sigma\rho_1}^{\rho_1} \geq \phi_p(M\sigma)$.

Then, the boundary value problem (1.6) *has at least one positive solution.*

Proof. Assume that (H_3) holds, we show that S has a fixed point u_1 in $\Omega_{\rho_2} \backslash \overline{K}_{\rho_1}$. By $f_0^{\rho_1} \leq \phi_p(m)$ and Lemma 2.12, we have that

$$i_K(S, K_{\rho_1}) = 1. \tag{3.1}$$

By $f_{\sigma\rho_2}^{\rho_2} \geq \phi_p(M\sigma)$ and Lemma 2.13, we have that

$$i_K(S, K_{\rho_2}) = 0. \tag{3.2}$$

By Lemma 2.11(a) and $\rho_1 < \sigma\rho_2$, we have $\overline{K}_{\rho_1} \subset K_{\sigma\rho_2} \subset \Omega_{\rho_2}$. It follows from Lemma 2.10(3) that S has a fixed point u_1 in $\Omega_{\rho_2} \setminus \overline{K}_{\rho_1}$. When condition (H_4) holds, the proof is similar to the above, so we omit it here.

As a special case of Theorem 3.1, we obtain the following result. □

Corollary 3.2. *Suppose conditions* (H_1) *and* (H_2) *hold, and assume that one of the following conditions holds.*

(H_5) $0 \leq f^0 < \phi_p(m)$ *and* $\phi_p(M) < f_\infty \leq \infty$.

(H_6) $0 \leq f^\infty < \phi_p(m)$ *and* $\phi_p(M) < f_0 \leq \infty$.

Then, the boundary value problem (1.6) *has at least one positive solution.*

Theorem 3.3. *Assume conditions* (H_1) *and* (H_2) *hold, and suppose that one of the following conditions holds.*

(H_7) *There exist* ρ_1, ρ_2*, and* $\rho_3 \in (0, +\infty)$ *with* $\rho_1 < \sigma\rho_2$ *and* $\rho_2 < \rho_3$ *such that*

$$f_0^{\rho_1} \leq \phi_p(m), \quad f_{\sigma\rho_2}^{\rho_2} \geq \phi_p(M\sigma), \quad u \neq Su, \quad \forall u \in \partial\Omega_{\rho_2}, \quad f_0^{\rho_3} \leq \phi_p(m). \tag{3.3}$$

(H_8) *There exist* ρ_1, ρ_2, *and* $\rho_3 \in (0, +\infty)$ *with* $\rho_1 < \rho_2 < \sigma\rho_3$ *such that*

$$f_0^{\rho_2} \leq \phi_p(m), \quad f_{\sigma\rho_1}^{\rho_1} \geq \phi_p(M\sigma), \quad u \neq Su, \quad \forall\, u \in \partial K_{\rho_2}, \quad f_{\sigma\rho_3}^{\rho_3} \geq \phi_p(M\sigma). \tag{3.4}$$

Then, the boundary value problem (1.6) *has at least two positive solutions. Moreover, if in* (H_7) $f_0^{\rho_1} \leq \phi_p(m)$ *is replaced by* $f_0^{\rho_1} < \phi_p(m)$, *then the BVP* (1.6) *has a third positive solution* $u_3 \in K_{\rho_1}$.

Proof. Assume that condition (H_7) holds, we show that either S has a fixed point u_1 in ∂K_{ρ_1} or $\Omega_{\rho_2} \setminus \overline{K}_{\rho_1}$. If $u \neq Su$ for $u \in \partial K_{\rho_1} \cup \partial K_{\rho_3}$. By Lemma 2.12 and Lemma 2.13, we have that

$$\begin{aligned} i_K(S, K_{\rho_1}) &= 1, \\ i_K(S, K_{\rho_3}) &= 1, \\ i_K(S, K_{\rho_2}) &= 0. \end{aligned} \tag{3.5}$$

By Lemma 2.11(a) and $\rho_1 < \sigma\rho_2$, we have $\overline{K}_{\rho_1} \subset K_{\sigma\rho_2} \subset \Omega_{\rho_2}$. It follows from Lemma 2.10(3) that S has a fixed point u_1 in $\Omega_{\rho_2} \setminus \overline{K}_{\rho_1}$. Similarly, S has a fixed point u_2 in $K_{\rho_3} \setminus \overline{\Omega}_{\rho_2}$. When condition ($H_8$) holds, the proof is similar to the above, so we omit it here.

As a special case of Theorem 3.3, we obtain the following result. □

Corollary 3.4. *Assume conditions* (H_1) *and* (H_2) *hold, if there exists* $\rho > 0$ *such that one of the following conditions holds.*

(H_9) $0 \leq f^0 < \phi_p(m)$, $f_{\sigma\rho}^{\rho} \geq \phi_p(M\sigma)$, $u \neq Su$, $\forall u \in \partial\Omega_\rho$ *and* $0 \leq f^\infty < \phi_p(m)$.

(H_{10}) $\phi_p(M) < f_0 \leq \infty$, $f_0^{\rho} \leq \phi_p(m)$, $u \neq Su$, $\forall u \in \partial K_\rho$ *and* $\phi_p(M) < f_\infty \leq \infty$.

Then, the boundary value problem (1.6) *has at least two positive solutions.*

4. Some examples

In this section, we present some simple examples to explain our results. We only study the case $\mathbf{T} = \mathbb{R}$, $(0, T)_{\mathbf{T}} = (0, 1)$.

Example 4.1. Consider the following three-point boundary value problem with p-Laplacian:

$$\begin{gathered} (\phi_p(u''))' + a(t) f(t, u) = 0, \quad 0 < t < 1, \\ u'(0) = 0, \qquad u(1) = \frac{1}{2} u\left(\frac{1}{3}\right), \qquad (\phi_p(u'')(0)) = \frac{1}{4}\left(\phi_p(u'')\left(\frac{1}{3}\right)\right), \end{gathered} \tag{4.1}$$

where $\beta = 0$, $\gamma = 1$, $a_1 = 1/2$, $b_1 = 1/4$, $\xi_1 = 1/3$, $a(t) = 1$, $p = q = 2$. By computing, we can know $\sigma = 1/6$, $M = 819/16$, $m = 9/10$. Let $\rho_1 = 1$, $\rho_2 = 208$, then $\sigma\rho_1 < \rho_1 < \sigma\rho_2 < \rho_2$. We define a nonlinearity f as follows:

$$f(t,u)=\begin{cases}\dfrac{9t^3}{10}\left(\dfrac{1}{6}-u\right)^3, & 0<t<1,\ u\in\left[0,\dfrac{1}{6}\right],\\[2ex] \dfrac{9t^3}{10}\sin\left(\dfrac{6}{5}\dfrac{\pi}{2}u-\dfrac{1}{5}\dfrac{\pi}{2}\right), & 0<t<1,\ u\in\left[\dfrac{1}{6},1\right],\\[2ex] \dfrac{9t^3}{10}\left(\dfrac{208}{202}-\dfrac{6}{202}u\right)+\dfrac{819}{96}\left(\dfrac{6}{202}u-\dfrac{6}{202}\right), & 0<t<1,\ u\in\left[1,\dfrac{208}{6}\right],\\[2ex] \dfrac{819}{96}+t^3\left(u-\dfrac{208}{6}\right)^2, & 0<t<1,\ u\in\left[\dfrac{208}{6},208\right],\\[2ex] \dfrac{819}{96}+t^3\left(208-\dfrac{208}{6}\right)^2\left[1+(u-208)\right], & 0<t<1,\ u\in\left[208,+\infty\right].\end{cases}\tag{4.2}$$

Then, by the definition of f, we have

(i) $f_0^{\rho_1}\le\phi_p(m)=9/10$;

(ii) $f_{\sigma\rho_2}^{\rho_2}\ge\phi_p(M\sigma)=819/19968$.

So condition (H_3) holds, by Theorem 3.1, boundary value problem (4.1) has at least one positive solution.

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