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Research Article

On the Solutions of the System of Difference Equations $x_{n+1} = \max\{A/x_n, y_n/x_n\}$, $y_{n+1} = \max\{A/y_n, x_n/y_n\}$

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We study the behavior of the solutions of the following system of difference equations $x_{n+1} = \max\{A/x_n, y_n/x_n\}$, $y_{n+1} = \max\{A/y_n, x_n/y_n\}$ where the constant A and the initial conditions are positive real numbers.

1. Introduction

Recently, there has been a great interest in studying the periodic nature of nonlinear difference equations. Although difference equations are relatively simple in form, it is, unfortunately, extremely difficult to understand thoroughly the periodic behavior of their solutions. The periodic nature of nonlinear difference equations of the max type has been investigated by many authors. See, for example [1–24].

In this paper we study the behavior of the solutions of the following system of difference equations:

$$x_{n+1} = \max\left\{\frac{A}{x_n}, \frac{y_n}{x_n}\right\}, \qquad y_{n+1} = \max\left\{\frac{A}{y_n}, \frac{x_n}{y_n}\right\}, \tag{1.1}$$

where the constant A and the initial conditions are positive real numbers.

2. Main Result

Definition 2.1. Fibonacci sequence is $f_1 = 1$, $f_2 = 1$ and for $n \geq 3$, $f_n = f_{n-1} + f_{n-2}$.

Definition 2.2. The symbol [[ ]] symbolizes the greatest integer function.

Definition 2.3. The sequence of $a(n) \equiv n (\mathrm{mod}\ 2)$.

Definition 2.4. The sequence of

$$k(n) = \begin{cases} n, & n = 0, 2, 4, \ldots, \\ n+1, & n = 1, 3, 5, \ldots. \end{cases} \tag{2.1}$$

Theorem 2.5. *Let (x_n, y_n) be the solution of the system of difference equations* (1.1) *for $A < x_0 < y_0$ and $y_0/x_0 > A$.*

If $n \geq 1$, then

$$x_n = \left(\frac{A^{f_{k(n)-1} - a(n)} x_0^{f_{k(n)}}}{y_0^{f_{k(n)}}} \right)^{(-1)^n}, \tag{2.2}$$

$y_1 = x_0/y_0$, and if $n \geq 2$

$$y_n = \left(\frac{y_0^{f_{k(n-1)}+1}}{A^{f_{k(n-1)} + a(n) - 1} x_0^{f_{k(n-1)}+1}} \right)^{(-1)^n}. \tag{2.3}$$

Proof. Let $y_0 > A$, then

$$\begin{aligned}
x_1 &= \max\left\{ \frac{A}{x_0}, \frac{y_0}{x_0} \right\} = \frac{y_0}{x_0} > A, \\
y_1 &= \max\left\{ \frac{A}{y_0}, \frac{x_0}{y_0} \right\} = \frac{x_0}{y_0} < A, \\
x_2 &= \max\left\{ \frac{A}{x_1}, \frac{y_1}{x_1} \right\} = \frac{Ax_0}{y_0} < A, \\
y_2 &= \max\left\{ \frac{A}{y_1}, \frac{x_1}{y_1} \right\} = \frac{x_1}{y_1} = \frac{y_0^2}{x_0^2} > A, \\
x_3 &= \max\left\{ \frac{A}{x_2}, \frac{y_2}{x_2} \right\} = \frac{y_2}{x_2} = \frac{y_0^3}{Ax_0^3} > A, \\
y_3 &= \max\left\{ \frac{A}{y_2}, \frac{x_2}{y_2} \right\} = \frac{A}{y_2} = \frac{Ax_0^2}{y_0^2} < A,
\end{aligned}$$

$$x_4 = \max\left\{\frac{A}{x_3}, \frac{y_3}{x_3}\right\} = \frac{A^2 x_0^3}{y_0^3} < A,$$
$$y_4 = \max\left\{\frac{A}{y_3}, \frac{x_3}{y_3}\right\} = \frac{x_3}{y_3} = \frac{y_0^5}{A^2 x_0^5} > A,$$
$$x_5 = \max\left\{\frac{A}{x_4}, \frac{y_4}{x_4}\right\} = \frac{y_4}{x_4} = \frac{y_0^8}{A^3 x_0^8} > A,$$
$$y_5 = \max\left\{\frac{A}{y_4}, \frac{x_4}{y_4}\right\} = \frac{A}{y_4} = \frac{A^3 x_0^5}{y_0^5} < A,$$
$$\vdots \tag{2.4}$$

$n \geq 1$ then

$$x_n = \left(\frac{A^{f_{k(n)-1}-a(n)} x_0^{f_{k(n)}}}{y_0^{f_{k(n)}}}\right)^{(-1)^n}, \tag{2.5}$$

$y_1 = x_0/y_0$ then $n \geq 2$,

$$y_n = \left(\frac{y_0^{f_{k(n-1)}+1}}{A^{f_{k(n-1)}+a(n)-1} x_0^{f_{k(n-1)}+1}}\right)^{(-1)^n}. \tag{2.6}$$

□

Theorem 2.6. *Let (x_n, y_n) be the solution of the system of difference equations* (1.1) *for $A < y_0 < x_0$. $x_1 = x_0/y_0$ and if $n \geq 2$,*

$$x_n = \left(\frac{A^{f_{k(n-1)}+a(n)-1} x_0^{f_{k(n-1)}+1}}{y_0^{f_{k(n-1)}+1}}\right)^{(-1)^n}. \tag{2.7}$$

If $n \geq 1$, then

$$y_n = \left(\frac{y_0^{f_{k(n)}}}{A^{f_{k(n)-1}-a(n)} x_0^{f_{k(n)}}}\right)^{(-1)^n}. \tag{2.8}$$

Proof. Similarly we can obtain the proof as the proof of Theorem 2.5. □

Theorem 2.7. *Let (x_n, y_n) be the solution of the system of difference equations* (1.1) *for $A < x_0 < y_0$ and $(y_0/x_0) > A$.*

$$\begin{aligned}
&\text{(a)} \quad \lim_{n\to\infty} x_{2n} = 0, \\
&\qquad \lim_{n\to\infty} y_{2n} = \infty. \\
&\text{(b)} \quad \lim_{n\to\infty} x_{2n+1} = \infty, \\
&\qquad \lim_{n\to\infty} y_{2n+1} = 0.
\end{aligned} \tag{2.9}$$

Proof. (a) We obtain that

$$\begin{aligned}
\lim_{n\to\infty} x_{2n} &= \lim_{n\to\infty} \left(\frac{A^{f_{k(2n)-1}-a(2n)} x_0^{f_{k(2n)}}}{y_0^{f_{k(2n)}}} \right)^{(-1)^{2n}} \\
&= \lim_{n\to\infty} \left(\frac{A^{f_{k(2n)-1}-a(2n)} x_0^{f_{k(2n)}}}{y_0^{f_{k(2n)}}} \right)^{(-1)^{2k}} \\
&= \lim_{n\to\infty} \left(\frac{A^{f_{2n-1}-a(2n)} x_0^{f_{2n}}}{y_0^{f_{2n}}} \right) \\
&= 0, \\
\lim_{n\to\infty} y_{2n} &= \lim_{n\to\infty} \left(\frac{y_0^{f_{k(2n-1)}+1}}{A^{f_{k(2n-1)}+a(2n)-1} x_0^{f_{k(2n-1)}+1}} \right)^{(-1)^{2n}} \\
&= \lim_{n\to\infty} \left(\frac{y_0^{f_{k(2n-1)}+1}}{A^{f_{k(2n-1)}+a(2n)-1} x_0^{f_{k(2n-1)}+1}} \right)^{(-1)^{2k}} \\
&= \lim_{k\to\infty} \left(\frac{y_0^{f_{2n}+1}}{A^{f_{2n}-1} x_0^{f_{2n}+1}} \right) \\
&= \infty.
\end{aligned} \tag{2.10}$$

(b) Similarly we can obtain the proof of (b) as the proof of (a). □

Theorem 2.8. *Let* (x_n, y_n) *be the solution of the system of difference equations* (1.1) *for* $A < y_0 < x_0$ *and* $(x_0/y_0) > A$.

$$\begin{aligned} \text{(a)} \quad & \lim_{n\to\infty} x_{2n} = \infty, \\ & \lim_{n\to\infty} y_{2n} = 0. \\ \text{(b)} \quad & \lim_{n\to\infty} x_{2n+1} = 0, \\ & \lim_{n\to\infty} y_{2n+1} = \infty. \end{aligned} \tag{2.11}$$

Proof. Similarly we can obtain the proof as the proof of Theorem 2.7. □

Theorem 2.9. *Let* (x_n, y_n) *be the solution of the system of difference equations* (1.1) *for* $1 < x_0 < y_0 < A$.

If $n \geq 1$, *then*

$$x_n = \left(\frac{x_0}{A^{a(n)}}\right)^{(-1)^n}. \tag{2.12}$$

If $n \geq 1$, *then*

$$y_n = \left(\frac{y_0}{A^{a(n)}}\right)^{(-1)^n}. \tag{2.13}$$

Proof. Let

$$\begin{aligned}
x_1 &= \max\left\{\frac{A}{x_0}, \frac{y_0}{x_0}\right\} = \frac{A}{x_0} < A, \\
y_1 &= \max\left\{\frac{A}{y_0}, \frac{x_0}{y_0}\right\} = \frac{A}{y_0} < A, \\
x_2 &= \max\left\{\frac{A}{x_1}, \frac{y_1}{x_1}\right\} = \frac{Ax_0}{A} = x_0 < A, \\
y_2 &= \max\left\{\frac{A}{y_1}, \frac{x_1}{y_1}\right\} = \frac{A}{y_1} = y_0 < A, \\
x_3 &= \max\left\{\frac{A}{x_2}, \frac{y_2}{x_2}\right\} = \frac{A}{x_2} = \frac{A}{x_0} < A, \\
y_3 &= \max\left\{\frac{A}{y_2}, \frac{x_2}{y_2}\right\} = \frac{A}{y_2} = \frac{A}{y_0} < A, \\
x_4 &= \max\left\{\frac{A}{x_3}, \frac{y_3}{x_3}\right\} = \frac{A}{x_3} = x_0 < A,
\end{aligned}$$

$$
\begin{aligned}
y_4 &= \max\left\{\frac{A}{y_3}, \frac{x_3}{y_3}\right\} = \frac{A}{y_3} = y_0 < A, \\
x_5 &= \max\left\{\frac{A}{x_4}, \frac{y_4}{x_4}\right\} = \frac{A}{x_4} = \frac{A}{x_0} < A, \\
y_5 &= \max\left\{\frac{A}{y_4}, \frac{x_4}{y_4}\right\} = \frac{A}{y_4} = \frac{A}{y_0} < A, \\
&\vdots
\end{aligned}
\tag{2.14}
$$

$n \geq 1$, then

$$
\begin{aligned}
x_n &= \left(\frac{x_0}{A^{a(n)}}\right)^{(-1)^n}, \\
y_n &= \left(\frac{y_0}{A^{a(n)}}\right)^{(-1)^n}.
\end{aligned}
\tag{2.15}
$$

□

Theorem 2.10. *Let* (x_n, y_n) *be the solution of the system of difference equations* (1.1) *for* $1 < x_0 < y_0 < A$.

$$
\begin{aligned}
\text{(a)} \quad & \lim_{n\to\infty} x_{2n} = x_0, \\
& \lim_{n\to\infty} y_{2n} = y_0. \\
\text{(b)} \quad & \lim_{n\to\infty} x_{2n+1} = \frac{A}{x_0}, \\
& \lim_{n\to\infty} y_{2n+1} = \frac{A}{y_0}.
\end{aligned}
\tag{2.16}
$$

Proof. (a) We obtain that

$$
\begin{aligned}
\lim_{n\to\infty} x_{2n} &= \lim_{n\to\infty}\left(\frac{x_0}{A^{a(2n)}}\right)^{(-1)^{2n}} = \lim_{n\to\infty}\left(\frac{x_0}{A^0}\right)^{(-1)^{2n}} = \lim_{n\to\infty}\left(\frac{x_0}{A^0}\right) = x_0, \\
\lim_{n\to\infty} y_{2n} &= \lim_{n\to\infty}\left(\frac{y_0}{A^{a(2n)}}\right)^{(-1)^{2n}} = \lim_{n\to\infty}\left(\frac{y_0}{A^0}\right)^{(-1)^{2n}} = \lim_{n\to\infty}\left(\frac{y_0}{A^0}\right) = y_0.
\end{aligned}
\tag{2.17}
$$

(b) Similarly we can obtain the proof of (b) as the proof of (a). □

Lemma 2.11. *Let* (x_0, y_0) *be the initial condition of* (1.1) *for* $0 < x_0 < 1 < y_0 < A$*; there is at least an* $i_0 \in N$ *such that every* $n \in N$ *for* $n > i_0$, $y_0/x_0^n > A$.

Proof. We consider that $x_0 < 1$ hence $\lim_{n\to\infty}(y_0/x_0^n) = \infty$ and that proofs the existing of i_0 defined in hypothesis. □

Theorem 2.12. *Let* (x_n, y_n) *be the solution of the system of difference equations* (1.1) *for* $0 < x_0 < 1 < y_0 < A$*, and* i_0 *is the number, defined by Lemma 2.11.*

$1 \leq n \leq i_0$

$$x_n = A^{a(n)}(x_0)^{(-1)^n}, \tag{2.18}$$

$1 \leq n \leq i_0$

$$y_n = A^{a(n)}\left(\frac{y_0}{x_0^{[[n/2]]}}\right)^{(-1)^n}, \tag{2.19}$$

and when $n > i_0$*, the solutions will be different for every different constant* A*.*

Proof. Let $y_0 < A$, then

$$\begin{gathered}
x_1 = \max\left\{\frac{A}{x_0}, \frac{y_0}{x_0}\right\} = \frac{A}{x_0} > 1, \\
y_1 = \max\left\{\frac{A}{y_0}, \frac{x_0}{y_0}\right\} = \frac{A}{y_0} > 1, \\
x_2 = \max\left\{\frac{A}{x_1}, \frac{y_1}{x_1}\right\} = \frac{A}{x_1} = x_0 < 1, \\
y_2 = \max\left\{\frac{A}{y_1}, \frac{x_1}{y_1}\right\} = \frac{x_1}{y_1} = \frac{y_0}{x_0} > 1, \\
x_3 = \max\left\{\frac{A}{x_2}, \frac{y_2}{x_2}\right\} = \frac{A}{x_2} = \frac{A}{x_0} > 1, \\
y_3 = \max\left\{\frac{A}{y_2}, \frac{x_2}{y_2}\right\} = \frac{A}{y_2} = \frac{Ax_0}{y_0}, \\
x_4 = \max\left\{\frac{A}{x_3}, \frac{y_3}{x_3}\right\} = \frac{A}{x_3} = x_0 < 1, \\
y_4 = \max\left\{\frac{A}{y_3}, \frac{x_3}{y_3}\right\} = \frac{x_3}{y_3} = \frac{y_0}{x_0^2}, \\
x_5 = \max\left\{\frac{A}{x_4}, \frac{y_4}{x_4}\right\} = \frac{A}{x_4} = \frac{A}{x_0} > 1, \\
y_5 = \max\left\{\frac{A}{y_4}, \frac{x_4}{y_4}\right\} = \frac{A}{y_4} = \frac{Ax_0^2}{y_0},
\end{gathered} \tag{2.20}$$

$1 \le n \le i_0$,

$$x_n = A^{a(n)} (x_0)^{(-1)^n},$$
$$y_n = A^{a(n)} \left(\frac{y_0}{x_0^{[[n/2]]}} \right)^{(-1)^n}. \tag{2.21}$$

□

Lemma 2.13. *Let* (x_0, y_0) *be the initial condition of* (1.1) *for* $0 < y_0 < 1 < x_0 < A$*; there is at least an* $i_0 \in N$ *such that every* $n \in N$ *for* $n > i_0$*,* $x_0/y_0^n > A$*.*

Proof. Similarly we can obtain the proof as the proof of Lemma 2.11. □

Theorem 2.14. *Let* (x_n, y_n) *be the solution of the system of difference equations* (1.1) *for* $0 < y_0 < 1 < x_0 < A$*, and* i_0 *is the number, defined by Lemma 2.13.*

$1 \le n \le i_0$

$$x_n = A^{a(n)} \left(\frac{x_0}{y_0^{[[n/2]]}} \right)^{(-1)^n}, \tag{2.22}$$

$1 \le n \le i_0$

$$y_n = A^{a(n)} (y_0)^{(-1)^n}, \tag{2.23}$$

and when $n > i_0$*, the solutions will be different for every different constant* A*.*

Proof. Similarly we can obtain the proof of be as the proof of Theorem 2.12. □

Lemma 2.15. *Let* (x_0, y_0) *be the initial condition of* (1.1) *for* $0 < y_0 < x_0 < 1$*; there is at least an* $i_0 \in N$ *such that every* $n \in N$ *for* $n > i_0$*,* $(x_0/y_0)^n > A$*.*

Proof. We consider that $y_0 < x_0$ hence $\lim_{n \to \infty} (x_0/y_0)^n = \infty$ and that proofs the existing of i_0 defined in hypothesis. □

Theorem 2.16. *Let* (x_n, y_n) *be the solution of the system of difference equations* (1.1) *for* $0 < y_0 < x_0 < 1$*,* $A > 1$*, and* i_0 *is the number, defined by Lemma 2.15.*

$x_1 = A/x_0$ *and if* $1 < n \le i_0$*,*

$$x_n = A^{a(n)} \left(\frac{x_0}{y_0} \right)^{[[n/2]](-1)^n}, \tag{2.24}$$

$y_1 = A/y_0$*, and if* $1 < n \le i_0$*,*

$$y_n = A^{a(n)} \left(\frac{y_0}{x_0} \right)^{(-1)^n}, \tag{2.25}$$

and when $n > i_0$*, the solutions will be different for every different constant* A*.*

Proof. Let $y_0 < A$, then

$$
\begin{aligned}
x_1 &= \max\left\{\frac{A}{x_0}, \frac{y_0}{x_0}\right\} = \frac{A}{x_0} > 1, \\
y_1 &= \max\left\{\frac{A}{y_0}, \frac{x_0}{y_0}\right\} = \frac{A}{y_0} > 1, \\
x_2 &= \max\left\{\frac{A}{x_1}, \frac{y_1}{x_1}\right\} = \frac{y_1}{x_1} = \frac{x_0}{y_0} > 1, \\
y_2 &= \max\left\{\frac{A}{y_1}, \frac{x_1}{y_1}\right\} = \frac{x_1}{y_1} = \frac{y_0}{x_0} < 1, \\
x_3 &= \max\left\{\frac{A}{x_2}, \frac{y_2}{x_2}\right\} = \frac{A}{x_2} = \frac{Ay_0}{x_0}, \\
y_3 &= \max\left\{\frac{A}{y_2}, \frac{x_2}{y_2}\right\} = \frac{A}{y_2} = \frac{Ax_0}{y_0} > 1, \\
x_4 &= \max\left\{\frac{A}{x_3}, \frac{y_3}{x_3}\right\} = \frac{y_3}{x_3} = \frac{x_0^2}{y_0^2} > 1, \\
y_4 &= \max\left\{\frac{A}{y_3}, \frac{x_3}{y_3}\right\} = \frac{A}{y_3} = \frac{y_0}{x_0} < 1,
\end{aligned}
\tag{2.26}
$$

$n > 1$ then

$$
x_n = A^{a(n)}\left(\frac{x_0}{y_0}\right)^{[[n/2]](-1)^n}, \tag{2.27}
$$

$n \geq 1$ then

$$
y_n = A^{a(n)}\left(\frac{y_0}{x_0}\right)^{(-1)^n}. \tag{2.28}
$$

□

Lemma 2.17. *Let (x_0, y_0) be the initial condition of* (1.1) *for $0 < y_0 < x_0 < 1$; there is at least an $i_0 \in N$ such that every $n \in N$ for $n > i_0$, $(y_0/x_0)^n > A$.*

Proof. Similarly we can obtain the proof as the proof of Lemma 2.15. □

Theorem 2.18. *Let (x_n, y_n) be the solution of the system of difference equations* (1.1) *for $0 < x_0 < y_0 < 1$, $A > 1$, and i_0 is the number, defined by Lemma 2.17.*

$x_1 = A/x_0$, and if $1 < n \leq i_0$,

$$
x_n = A^{a(n)}\left(\frac{x_0}{y_0}\right)^{(-1)^n}, \tag{2.29}
$$

$y_1 = A/y_0$, and if $1 < n \leq i_0$,

$$y_n = A^{a(n)} \left(\frac{y_0}{x_0}\right)^{[[n/2]](-1)^n}, \tag{2.30}$$

and when $n > i_0$, the solutions will be different for every different constant A.

Proof. Similarly we can obtain the proof as the proof of Theorem 2.16, which completes the proofs of theorems. □

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