Research Article

Global Dynamics Behaviors of Viral Infection Model for Pest Management

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According to biological strategy for pest control, a mathematical model with periodic releasing virus particles for insect viruses attacking pests is considered. By using Floquet's theorem, small-amplitude perturbation skills and comparison theorem, we prove that all solutions of the system are uniformly ultimately bounded and there exists a globally asymptotically stable pest-eradication periodic solution when the amount of virus particles released is larger than some critical value. When the amount of virus particles released is less than some critical value, the system is shown to be permanent, which implies that the trivial pest-eradication solution loses its stability. Further, the mathematical results are also confirmed by means of numerical simulation.

1. Introduction and Model Formulation

Currently, applications of chemical pesticides to combat pests are still one of the main measures to improve crop yields. Though chemical crop protection plays an important role in modern agricultural practices, it is still viewed as a profit-induced poisoning of the environment. The nondegradable chemical residues, which accumulate to harmful levels, are the root cause of health and environmental hazards and deserve most of the present hostility toward them. Moreover, synthetic pesticides often disrupt the balanced insect communities. This leads to the interest in Biological control methods for insects and plant pests [1, 2].

Biological control is, generally, man's use of a suitably chosen living organism, referred as the biocontrol agent, to control another. Biocontrol agents can be predators, pathogens, or parasites of the organism to be controlled that either kill the harmful organism or interfere with its biological processes [3]. In a large number of biopesticides, the insect virus pesticide because of its high pathogenicity, specificity, and ease production plays an important role in pest biological control. The control of rabbit pests in Australia by the virus disease called

"myxomatosis" provides a spectacular example of a virus controlling pest [4]. The insect viruses for the biological control of pests are mainly baculoviruses. Baculoviruses comprise a family of double-stranded DNA viruses which are pathogenic for arthropods, mainly insects. The polyhedral occlusion body (OB) is the characteristic phenotypic appearance of baculoviruses and in case of a nucleopolyhedrovirus (NPV) typically comprised of a proteinaceous matrix with a large number of embedded virus particles. Baculoviruses have a long history as effective and environmentally benign insect control agents in field crops, vegetables, forests, and pastures [5].

Transmission is also key to the persistence of baculoviruses in the environment [6, 7]. Transmission occurs primarily when an NPV-infected larva dies and lyses, releasing a massive number of OBs onto foliage and soil. Susceptible hosts become infected when they ingest OBs while feeding. Defecation and regurgitation by infected larvae have been reported as additional routes of contamination of host plants with viruses [7–9]. Moreover, some studies suggest that cannibalism and predation may also be routes of virus transmission [10]. Environmental factors such as rainfall, wind transport, and contaminated ovipositors of parasitic hymenopterans could contribute to NPV transmission as well [11, 12].

Insight in the epidemiological dynamics, it is necessary to predict optimal timing, frequency, and dosage of virus application and to assess the short and longer term persistence of NPV in insect populations and the environment. Modeling studies can help to obtain preliminary assessments of expected ecological dynamics at the short and longer term. There is a vast amount of literature on the applications of microbial disease to suppress pests [13–15], but there are only a few papers on mathematical models of the dynamics of viral infection in pest control [16, 17]. System with impulsive effects describing evolution processes is characterized by the fact that at certain moments of time they abruptly experience a change of state. Processes of such type are studied in almost every domain of applied science. Impulsive differential equations have been recently used in population dynamics in relation to impulsive vaccination, population ecology, the chemotherapeutic treatment of disease, and the theory of the chemostat [18–22].

In this paper, according to the above description, we should construct a more realistic model by introducing additional virus particles (i.e., using viral pesticide) to investigate the dynamical behavior of viruses attacking pests, which is described as follows:

$$
\begin{aligned}
S'(t) &= rS(t)\left(1 - \frac{S(t) + I(t)}{K}\right) - \beta S(t)I(t) - \theta S(t)V(t), \\
I'(t) &= \beta S(t)I(t) + \theta S(t)V(t) - \lambda I(t), \qquad\qquad t \neq nT, \\
V'(t) &= -\theta S(t)V(t) + b\lambda I(t) - \mu V(t), \\
\Delta S(t) &= 0, \\
\Delta I(t) &= 0, \quad t = nT, \ n = 1, 2, \ldots, \\
\Delta V(t) &= p,
\end{aligned}
\tag{1.1}
$$

where $S(t)$, $I(t)$, and $V(t)$ denote the density of susceptible pests, infected pests, and virus particles at time t, respectively. T is the impulsive period, $n = \{1, 2, \ldots\}$, p is the release amount

of virus particles, $\Delta S(t) = S(t^+) - S(t), \Delta I(t) = I(t^+) - I(t)$, and $\Delta V(t) = V(t^+) - V(t)$. The assumptions in the model are as follows.

(*A*1) We assume that only susceptible pests S are capable of reproducing with logistic law; that is, the infected pests I are removed by lysis before having the possibility of reproducing. However, they still contribute with S to population growth toward the carrying capacity. r is intrinsic birth rate and $K(> 0)$ is carrying capacity.

(*A*2) The term θVS denotes that the susceptible pests S become infected I as they ingest foods contaminated with virus particles, in which θ is positive constant and represents the "effective per pest contact rate with viruses." And βSI denotes that the susceptible pests S become infected I by the transmission of infective pests I according to other ways; perhaps β is very small and close to zero.

(*A*3) An infective pest I has a latent period, which is the period between the instant of infection and that of lysis, during which the virus reproduces inside the pest. The lysis death rate constant λ. λ gives a measure of such a latency period T being $\lambda = 1/T$. The lysis of infected pests, on the average, produces b virus particles $(b > 1)$. b is the virus replication factor.

(*A*4) The virus particles V have a natural death rate μ due to all kinds of possible mortality of viruses such as enzymatic attack, pH dependence, temperature changes, UV radiation, and photooxidation.

The paper is organized as follows: in Section 2, some auxiliary results which establish the a priori boundedness of the solutions, together with the asymptotic properties of certain reduced systems which are used throughout the paper as a basis of several comparison arguments, are stated. In Section 3, by using Floquet's theory for impulsive differential equations, small-amplitude perturbation methods, and comparison techniques, we provide the sufficient conditions for the local and global stability of the pest-eradication periodic solution and the conditions for the permanence of the system. Finally, a brief discussion and numerical examples are given. We also point out some future research directions.

2. Preliminary

We give some definitions, notations, and lemmas which will be useful for stating and proving our main results. Let $R_+ = [0, \infty), R_+^3 = \{(x_1, x_2, x_3) \mid x_i > 0, \ i = 1, 2, 3\}$. Denote by $f = (f_1, f_2, f_3)^T$ the map defined by the right hand of the first three equations in system (1.1). Let $V : R_+ \times R_+^3 \to R_+$, then $V \in V_0$ if

(i) V is continuous in $(nT, (n + 1)T] \times R_+^3$ and for each $z \in R_+^3, \ n \in N$

$$\lim_{(t,z) \to (nT^+, z)} V(t, z) = V(nT^+, z) \tag{2.1}$$

exists;

(ii) V is locally Lipschitzian in z.

Definition 2.1. $V \in V_0$, then for $(t, z) \in (nT, (n+1)T] \times R^3$, the upper right derivative of $V(t, z)$ with respect to system (1.1) is defined as

$$D^+V(t, z) = \lim_{h \to 0^+} \sup \frac{1}{h}[V(t + h, z + hf(t, z)) - V(t, z)]. \tag{2.2}$$

The solution of (1.1), denoted by $z(t) = (S(t), I(t), V(t))$, is a piecewise continuous function $z(t): R_+ \to R_+^3$, $z(t)$ is continuous on $(nT, (n+1)T]$, $n \in N$, and $z(nT^+) = \lim_{t \to nT^+} z(t)$ exists. Obviously, the existence and uniqueness of the solution of (1.1) is guaranteed by the smoothness properties of f (for more details see [18]).

Lemma 2.2. *Suppose that $z(t)$ is a solution of (1.1) with $z(0^+) \geq 0$, then $z(t) \geq 0$ for all $t \geq 0$. Moreover, if $z(0^+) > 0$, then $z(t) > 0$ for all $t \geq 0$.*

Lemma 2.3. *Let $V : R_+ \times R_+^n \to R_+$ and $V \in V_0$. Assume that*

$$D^+V(t, z(t)) \leq (\geq)g(t, V(t, z)), \quad t \neq \tau_k,$$

$$V(t, z(t^+)) \leq (\geq)\Psi_n(V(t, z(t))), \quad t = \tau_k, \ k \in N, \tag{2.3}$$

$$z(0^+) = z_0,$$

where $g : R_+ \times R_+^n \to R^n$ is continuous in $(\tau_k, \tau_{k+1}] \times R_+^n$ and for each $v \in R_+^n, n \in N$

$$\lim_{(t, \iota) \to (\tau_k^+, v)} g(t, \iota) = g(\tau_k^+, v) \tag{2.4}$$

exists, and $\Psi_n : R_+^n \to R_+^n$ is nondecreasing. Let $\mathfrak{R}(t) = \mathfrak{R}(t, 0, U_0)$ be the maximal(minimal) solution of the scalar impulsive differential equation

$$U'(t) = g(t, U), \quad t \neq \tau_k,$$

$$U(t^+) = \Psi_n(U(t)), \quad t = \tau_k, \ k \in N, \tag{2.5}$$

$$U(0^+) = U_0$$

existing on $[0, \infty)$. Then $V(0^+, z_0) \leq (\geq)U_0$ implies that

$$V(t, z(t)) \leq (\geq)\mathfrak{R}(t), \quad t \geq 0, \tag{2.6}$$

where $z(t)$ is any solution of (1.1) existing on $[0, \infty]$.

Note that if one has some smoothness conditions of g to guarantee the existence and uniqueness of solutions for (2.5), then $\mathfrak{R}(t)$ is exactly the unique solution of (2.5).

Lemma 2.4. *There exists a constant $M_1 > 0, M_2 > 0$ such that $S(t) \leq M_1, I(t) \leq M_1, V(t) \leq M_2$ for each positive solution $(S(t), I(t), V(t))$ of (1.1) with t being large enough.*

Proof. Define a function L such that $L(t) = S(t) + I(t)$. Then we have

$$D^+L(t)|_{(1.1)} + \lambda L(t) = -\frac{r}{K}S^2(t) + (\lambda + r)S(t) - \frac{rS(t)I(t)}{K}$$

$$\leq -\frac{r}{K}S^2(t) + (\lambda + r)S(t), \quad t \in (nT, (n+1)T]. \tag{2.7}$$

Obviously, the right hand of the above equality is bounded; thus, there exists $M_0 = K(\lambda + r)^2/4r > 0$ such that $D^+L(t) \leq -\lambda L(t) + M_0$. It follows that $\liminf_{t\to\infty} L(t) \leq \limsup_{t\to\infty} L(t) \leq M_0/\lambda$. Therefore, by the definition of $L(t)$ we obtain that there exists a constant $M_1 = K(\lambda + r)^2/4r\lambda > 0$ such that $S(t) \leq M_1, I(t) \leq M_1$. From the third and sixth equations of system (1.1), we have

$$V'(t) = -\theta S(t)V(t) + b\lambda I(t) - \mu V(t) \leq b\lambda M_1 - \mu V(t), \quad t \neq nT,$$

$$V(nT^+) = V(nT) + p, \quad t = nT. \tag{2.8}$$

According to Lemma 2.3 in [18] we derive

$$V(t) \leq V(0)e^{-\mu t} + \int_0^t b\lambda M_1 e^{-\mu(t-s)}ds + \sum_{0<kT<t} pe^{-\mu(t-kT)}$$

$$\longrightarrow \frac{b\lambda M_1}{\mu} + \frac{pe^{\mu T}}{e^{\mu T} - 1} \quad \text{as } t \longrightarrow \infty. \tag{2.9}$$

Therefore, there exists a constant $M_2 > 0$ such that $V(t) \leq M_2$. The proof is complete. □

Next, we give some basic property of the following subsystem:

$$y'(t) = -dy(t), \quad t \neq nT,$$

$$\Delta y(t) = p, \quad t = nT, \tag{2.10}$$

$$v'(t) = a - bv(t), \quad t \neq nT,$$

$$\Delta v(t) = \theta, \quad t = nT. \tag{2.11}$$

Lemma 2.5. *System* (2.10) *has a positive periodic solution* $y^*(t)$ *and for every positive solution* $y(t)$ *of system* (2.10), $|y(t) - y^*(t)| \to 0$ *as* $t \to \infty$, *where* $y^*(t) = pe^{-d(t-nT)}/(1 - e^{-dT})$ *and* $y^*(0^+) = p/(1 - e^{-dT})$.

Lemma 2.6. *System* (2.11) *has a positive periodic solution* $v^*(t)$ *and for every positive solution* $v(t)$ *of system* (2.11), $|v(t) - v^*(t)| \to 0$ *as* $t \to \infty$, *where* $v^*(t) = a/b + \theta e^{-b(t-nT)}/(1 - e^{-bT})$ *and* $v^*(0^+) = a/b + \theta/(1 - e^{-bT})$.

3. Main Results

When $S(t) = 0$, from the second and fifth equations of system (1.1), we have $\lim_{t \to \infty} I(t) = 0$. Further, from the third and sixth equations of system (1.1), we have

$$
\begin{aligned}
V'(t) &= -\mu V(t), \quad t \neq nT, \\
\Delta V(t) &= p, \quad t = nT,
\end{aligned}
\tag{3.1}
$$

By Lemma (2.10), we can obtain the unique positive periodic solution of system (3.1): $V^*(t) = pe^{-\mu(t-nT)}/(1-e^{-\mu T})$, $nT < t \leq (n+1)T$, with initial value $V^*(0^+) = p/(1-e^{-\mu T})$. Thus the pest-eradication solution is explicitly shown. That is, system (1.1) has a so-called pest-eradication periodic solution $(0, 0, V^*(t))$. Next, we shall give the condition to assure its global asymptotic stability.

Theorem 3.1. *Let $(S(t), I(t), V(t))$ be any solution of system (1.1) with positive initial values. Then the pest-eradication periodic solution $(0, 0, V^*(t))$ is locally asymptotically stable provided that*

$$
rT < \theta \int_0^T V^*(t)dt.
\tag{3.2}
$$

Proof. The local stability of periodic solution $(0, 0, V^*(t))$ may be determined by considering the behavior of small amplitude perturbation of the solution. Let $S(t) = u(t)$, $I(t) = v(t)$, $V(t) = w(t) + V^*(t)$. The corresponding linearized system of (1.1) at $(0, 0, V^*)$ is

$$
\begin{aligned}
u'(t) &= (r - \theta V^*(t))u(t), \\
v'(t) &= \theta V^*(t)u(t) - \lambda v(t), \qquad\qquad t \neq nT, \\
w'(t) &= -\theta V^*(t)u(t) + b\lambda v(t) - \mu w(t), \\[4pt]
u(t^+) &= u(t), \\
v(t^+) &= v(t), \qquad t = nT, \ n = 1, 2, \ldots, \\
w(t^+) &= w(t),
\end{aligned}
\tag{3.3}
$$

Let $\Phi(t)$ be the fundamental matrix of (3.3), then $\Phi(t)$ satisfies

$$
\frac{d\Phi(t)}{dt} = \begin{pmatrix} r - \theta V^*(t) & 0 & 0 \\ \theta V^*(t) & -\lambda & 0 \\ -\theta V^*(t) & b\lambda & -\mu \end{pmatrix} \Phi(t),
\tag{3.4}
$$

and $\Phi(0) = E_3$ (unit 3×3 matrix). Hence, the fundamental solution matrix is

$$
\Phi(t) = \begin{pmatrix} e^{\int_0^t (r - \theta V^*(t))dt} & 0 & 0 \\ \Delta & e^{-\lambda t} & 0 \\ \Delta & \Delta & e^{-\mu t} \end{pmatrix},
\tag{3.5}
$$

where the exact expressions of Δ are omitted, since they are not used subsequently. The resetting impulsive condition of (3.3) becomes

$$
\begin{pmatrix} u(nT^+) \\ v(nT^+) \\ w(nT^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u(nT) \\ v(nT) \\ w(nT) \end{pmatrix}. \tag{3.6}
$$

Hence, if all the eigenvalues of

$$
M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Phi(T) \tag{3.7}
$$

have absolute values less than one, then the periodic solution $(0,0,V^*(t))$ is locally stable. Since the eigenvalues of M are

$$
\lambda_1 = e^{-\lambda T} < 1, \qquad \lambda_2 = e^{-\mu T} < 1, \qquad \lambda_3 = e^{\int_0^T (r - \theta V^*(t))dt} \tag{3.8}
$$

and $|\lambda_3| < 1$ if and only if (3.2) holds, according to Floquet's theory of impulsive differential equation, the pest-eradication periodic solution $(0,0,V^*(t))$ is locally stable.

In fact, for condition (3.2), rT represents the normalized gain of the pest in a period, while $\theta \int_0^T V^*(t)dt$ represents the normalized loss of the pest in a period due to viral disease. That is, this condition is a balance condition for the pest near the pest-eradication periodic solution, which asserts the fact that in a vicinity of this solution $(0,0,V^*(t))$ the pest is depleted faster than they can recover and consequently the pest is condemned to extinction. □

Theorem 3.2. *Let $(S(t),I(t),V(t))$ be any solution of system* (1.1) *with positive initial values. Then the pest-eradication periodic solution $(0,0,V^*(t))$ is globally asymptotically stable provided that*

$$
rT < \frac{\theta p}{\mu + \theta K}. \tag{3.9}
$$

Proof. From (3.9), we know that (3.2) also holds. By Theorem 3.1, we know that $(0,0,V^*(t))$ is locally stable. Therefore, we only need to prove its global attractivity. Since $rT < \theta p/(\mu + \theta K)$, we can choose an $\varepsilon_1 > 0$ small enough such that

$$
(r + \theta \varepsilon_1)T - \frac{\theta p}{\mu + \theta(K + \varepsilon_1)} \doteq \eta < 0. \tag{3.10}
$$

From the first equation of system (1.1), we obtain $S'(t) \leq rS(t)(1 - S(t)/K)$. Consider the comparison equation $\omega'(t) = r\omega(t)(1 - \omega(t)/K)$, $\omega(0) = S(0)$, then we have $S(t) \leq \omega(t)$ and $\omega(t) \to K$ as $t \to \infty$. Thus, there exists an $\varepsilon_1 > 0$ such that $S(t) \leq K + \varepsilon_1$ for t being large enough. Without loss of generality, we assume $S(t) \leq K + \varepsilon_1$ for all $t > 0$.

Note that $V'(t) \geq -[\mu + \theta(K + \varepsilon_1)]V(t)$; by Lemmas 2.3 and 2.5, there exists a n_1 such that for all $t \geq n_1 T$

$$V(t) \geq z^*(t) - \varepsilon_1, \tag{3.11}$$

where $z(t)$ is the solution of

$$z'(t) = -[\mu + \theta(K + \varepsilon_1)]z(t), \quad t \neq nT,$$
$$\Delta z(t) = p, \quad t = nT,$$
$$z(0^+) = V(0^+) > 0. \tag{3.12}$$
$$z^*(t) = \frac{pe^{-[\mu+\theta(K+\varepsilon_1)](t-nT)}}{1 - e^{-[\mu+\theta(K+\varepsilon_1)]T}}, \quad t \in (nT, (n+1)T].$$

Thus we have

$$S'(t) = rS(t)\left(1 - \frac{S(t) + I(t)}{K}\right) - \beta S(t)I(t) - \theta S(t)V(t)$$
$$\leq S(t)[r - \theta V(t)] \tag{3.13}$$
$$\leq S(t)[r - \theta(z^*(t) - \varepsilon_1)].$$

Integrating the above inequality on $((n_1 + k)T, (n_1 + k + 1)T], \ k \in N$, yields

$$S(t) \leq S(n_1 T)e^{\int_{n_1 T}^{(n_1+1)T}[r-\theta(z^*(t)-\varepsilon_1)]dt} \leq S(n_1 T)e^{k\eta}. \tag{3.14}$$

Since $\eta < 0$, we can easily get $S(t) \to 0$ as $t \to \infty$. For $\varepsilon_2 > 0$ small enough being ($\varepsilon_2 < \lambda/\beta$), there must exist an n_2 ($n_2 > n_1$) such that $0 < S(t) < \varepsilon_2$, for $t \geq n_2 T$; then from the second equation of system (1.1), we have $I'(t) \leq (\beta\varepsilon_2 - \lambda)I(t) + \theta M_2\varepsilon_2$, so $\liminf_{t\to\infty} I(t) \leq \limsup_{t\to\infty} \leq \theta M_2\varepsilon_2/(\lambda - \beta\varepsilon_2)$.

In the following, we prove $V(t) \to V^*(t)$, as $t \to +\infty$. From system (1.1), we have

$$(-\mu - \theta\varepsilon_2)V(t) \leq V'(t) \leq \frac{b\lambda\theta M_2\varepsilon_2}{\lambda - \beta\varepsilon_2} - \mu V(t), \tag{3.15}$$

and by Lemmas 2.3, 2.5, and 2.6, there exists an n_3 ($n_3 > n_2$) such that

$$V_2^*(t) - \varepsilon \leq V(t) \leq V_1^*(t) + \varepsilon \quad \forall t \geq nT, \ n > n_3, \tag{3.16}$$

where $V_1^*(t) = b\lambda\theta M_2\varepsilon_2/(\lambda - \beta\varepsilon_2)\mu + pe^{-\mu(t-nT)}/(1 - e^{-\mu T})$, $V_2^*(t) = pe^{-(\mu+\theta\varepsilon_2)(t-nT)}/(1 - e^{-(\mu+\theta\varepsilon_2)T})$. Let $\varepsilon_2 \to 0$, we have $I(t) \to 0$, $V_1^*(t) \to V^*(t)$, $V_2^*(t) \to V^*(t)$. Therefore, $(0, 0, V^*(t))$ is globally attractive. This completes the proof. $\qquad\square$

Corollary 3.3. *If $p > p_1^* = rT(\mu + \theta K)/\theta$ or $T < T_1^* = \theta p/r(\mu + \theta K)$, then the pest-eradication periodic solution $(0, 0, V^*(t))$ is globally asymptotically stable.*

We have proved that, if $p > p_1^* = rT(\mu + \theta K)/\theta$ or $T < T_1^* = \theta p/r(\mu + \theta K)$, the pest-eradication periodic solution $(0, 0, V^*(t))$ is globally asymptotically stable; that is, the pest population is eradicated totally. But in practice, from the view point of keeping ecosystem balance and preserving biological resources, it is not necessary to eradicate the pest population. Next we focus our attention on the permanence of system (1.1). Before starting our result, we give the definition of permanence.

Definition 3.4. System (1.1) is said to be permanent if there are constants m, $M > 0$ (independent of initial value) and a finite time T_0 such that all solutions $z(t) = (S(t), I(t), V(t))$ with initial values $z(0^+) > 0$, $m \leq z(t) \leq M$ hold for all $t \geq T_0$. Here T_0 may depend on the initial values $z(0^+) > 0$.

Theorem 3.5. *Let $(S(t), I(t), V(t))$ be any positive solution of (1.1) with positive initial values $z(0^+) > 0$. Then system (1.1) is permanent provided that*

$$rT > \theta \int_0^T V^*(t)dt. \tag{3.17}$$

Proof. Suppose that $z(t) = (S(t), I(t), V(t))$ is a solution of system (1.1) with initial values $z(0^+) > 0$. By Lemma 2.4, there exists positive constants M_1, M_2 such that $S(t) \leq M_1$, $I(t) \leq M_1$, and $V(t) \leq M_2$ for t being large enough. We may assume $S(t) \leq M_1$, $I(t) \leq M_1$, $V(t) \leq M_2$ for all $t \geq 0$. From (3.11), we know that

$$V(t) \geq z^*(t) - \varepsilon_1 \geq \frac{pe^{-[\mu+\theta(K+\varepsilon_1)]T}}{1 - e^{-[\mu+\theta(K+\varepsilon_1)]T}} - \varepsilon_1 \doteq m > 0 \tag{3.18}$$

for t being large enough. Thus we only need to find $m_1 > 0$, $m_2 > 0$ such that $S(t) \geq m_1$, $I(t) \geq m_2$ for t being large enough. We shall do it in two steps.

Step 1. Since $rT > \theta \int_0^T V^*(t)dt$, that is $rT > \theta p/\mu$, we can select $m_3 > 0$ ($m_3 < \lambda/\beta$), $\varepsilon > 0$ small enough such that

$$\delta \doteq rT - \left[\frac{rm_3 T}{K} + \frac{rT}{K}\left(\frac{\theta m_3 \eta}{\lambda - \beta m_3} + \varepsilon \right) + \beta T\left(\frac{\theta m_3 \eta}{\lambda - \beta m_3} + \varepsilon \right) + \theta \varepsilon T + \frac{\theta b \lambda m_3 T}{\mu} + \frac{\theta p}{\mu} \right] > 0. \tag{3.19}$$

We shall prove that $S(t) + I(t) < m_3$ cannot hold for all $t > 0$. Otherwise, we have that $S(t) < m_3$ for all $t > 0$. Then from the third equation of system (1.1), we get

$$V'(t) = -\theta S(t)V(t) + b\lambda I(t) - \mu V(t) \leq b\lambda m_3 - \mu V(t). \tag{3.20}$$

Then $V(t) \leq u(t)$ and $u(t) \to u^*(t)$ as $t \to \infty$, where $u^*(t)$ is the solution of

$$u'(t) = b\lambda m_3 - \mu u(t), \quad t \neq nT,$$

$$\Delta u(t) = p, \quad t = nT, \tag{3.21}$$

$$u(0^+) = V(0^+) > 0,$$

$$u^*(t) = \frac{b\lambda m_3}{\mu} + \frac{pe^{-\mu(t-nT)}}{1 - e^{-\mu T}}, \quad t \in (nT, (n+1)T]. \tag{3.22}$$

Therefore, there exists a $\tilde{T} > 0$ such that

$$V(t) \leq u(t) \leq u^*(t) + \varepsilon \leq \frac{b\lambda m_3}{\mu} + \frac{p}{1 - e^{-\mu T}} + \varepsilon \doteq \eta \tag{3.23}$$

for $t > \tilde{T}$. From the second equation of system (1.1), we have $I'(t) = \theta S(t)V(t) + \beta S(t)I(t) - \lambda I(t) \leq \theta m_3 \eta + (\beta m_3 - \lambda)I(t)$. Thus, $I(t) \leq \theta m_3 \eta / (\lambda - \beta m_3) + \varepsilon$ for t being large enough. Therefore, there exists $T_1 > \tilde{T}$ such that

$$S'(t) = rS(t)\left(1 - \frac{S(t) + I(t)}{K}\right) - \beta S(t)I(t) - \theta S(t)V(t)$$

$$\geq S(t)\left[r - \frac{rm_3}{K} - \frac{r}{K}\left(\frac{\theta m_3 \eta}{\lambda - \beta m_3} + \varepsilon\right) - \beta\left(\frac{\theta m_3 \eta}{\lambda - \beta m_3} + \varepsilon\right) - \theta V(t)\right] \tag{3.24}$$

$$\geq S(t)\left[r - \frac{rm_3}{K} - \frac{r}{K}\left(\frac{\theta m_3 \eta}{\lambda - \beta m_3} + \varepsilon\right) - \beta\left(\frac{\theta m_3 \eta}{\lambda - \beta m_3} + \varepsilon\right) - \theta(u^*(t) + \varepsilon)\right]$$

for all $t > T_1$. Let $N_0 \in N$ such that $(N_0 - 1)T \geq T_1$. Integrating the above inequality on $((n-1)T, nT]$, $n \geq N_0$, we have

$$S(nT) \geq S((n-1)T)e^{\int_{(n-1)T}^{nT}[r - rm_3/K - (r/K)(\theta m_3 \eta/(\lambda - \beta m_3) + \varepsilon) - \beta(\theta m_3 \eta/(\lambda - \beta m_3) + \varepsilon) - \theta(u^*(t) + \varepsilon)]dt}$$

$$= S((n-1)T)e^{rT - [rm_3 T/K + (rT/K)(\theta m_3 \eta/(\lambda - \beta m_3) + \varepsilon) + \beta T(\theta m_3 \eta/(\lambda - \beta m_3) + \varepsilon) + \theta \varepsilon T + \theta b\lambda m_3 T/\mu + \theta p/\mu]}$$

$$= S((n-1)T)e^{\delta}. \tag{3.25}$$

Then $S((n+k)T) \geq S(nT)e^{k\delta} \to \infty$ as $k \to \infty$, which is a contradiction to the boundedness of $S(t)$. Thus, there exists a $t_1 > 0$ such that $S(t_1) \geq m_3$.

Step 2. If $S(t) \geq m_3$ for all $t \geq t_1$, then our aim is obtained. Hence we need only to consider the situation that $S(t) \geq m_3$ is not always true for $t \geq t_1$, and we denote $t^* = \inf_{t \geq t_1}\{S(t) < m_3\}$.

Then $S(t) \geq m_3$ for $t \in [t_1, t^*)$ and $S(t^*) = m_3$, since $S(t)$ is continuous. Suppose $t^* \in (n_1 T, (n_1 + 1)T]$, $n_1 \in N$. Select $n_2, n_3 \in N$ such that

$$n_2 T > T_2 = \frac{\ln((M + u_0^*)/\varepsilon)}{\mu}, \qquad e^{\delta_1(n_2+1)T} e^{\delta n_3} > 1, \tag{3.26}$$

where $u_0^* = p/(1 - e^{-\mu T}) + b\lambda m_3/\mu$, $\delta_1 = r - r(m_3 + M_1)/K - \beta M_1 - \theta M_2 < 0$. Let $\widehat{T} = (n_2 + n_3)T$. We claim that there must be a $t_2 \in [(n_1 + 1)T, (n_1 + 1)T + \widehat{T}]$ such that $S(t_2) \geq m_3$. Otherwise $S(t) < m_3$, $t_2 \in [(n_1 + 1)T, (n_1 + 1)T + \widehat{T}]$. Consider (3.21) with $u((n_1 + 1)T^+) = V((n_1 + 1)T^+)$. We have

$$u(t) = (u(n_1 + 1)T^+ - u_0^*)e^{-\mu(t - (n_1+1)T)} + u^*(t), \quad t \in (nT, (n+1)T], \ n_1 + 1 \leq n \leq n_1 + 1 + n_2 + n_3. \tag{3.27}$$

Thus

$$|u(t) - u^*(t)| \leq (M + u_0^*)e^{-\mu n_2 T} < \varepsilon,$$

$$V(t) \leq u(t) \leq u^*(t) + \varepsilon, \quad (n_1 + 1 + n_2)T \leq t \leq (n_1 + 1)T + \widehat{T}. \tag{3.28}$$

Thus, we have

$$S'(t) = rS(t)\left(1 - \frac{S(t) + I(t)}{K}\right) - \beta S(t)I(t) - \theta S(t)V(t)$$

$$\geq S(t)\left[r - \frac{rm_3}{K} - \frac{r}{K}\left(\frac{\theta m_3 \eta}{\lambda - \beta m_3} + \varepsilon\right) - \beta\left(\frac{\theta m_3 \eta}{\lambda - \beta m_3} + \varepsilon\right) - \theta V(t)\right] \tag{3.29}$$

$$\geq S(t)\left[r - \frac{rm_3}{K} - \frac{r}{K}\left(\frac{\theta m_3 \eta}{\lambda - \beta m_3} + \varepsilon\right) - \beta\left(\frac{\theta m_3 \eta}{\lambda - \beta m_3} + \varepsilon\right) - \theta(u^*(t) + \varepsilon)\right]$$

for $(n_1 + 1 + n_2)T \leq t \leq (n_1 + 1)T + \widehat{T}$. As in Step 1, we have

$$S((n_1 + 1 + n_2 + n_3)T) \geq S((n_1 + 1 + n_2)T)e^{\delta n_3}. \tag{3.30}$$

On the interval $t \in [t^*, (n_1 + 1 + n_2)T]$, we have

$$
\begin{aligned}
S'(t) &= rS(t)\left(1 - \frac{S(t) + I(t)}{K}\right) - \beta S(t)I(t) - \theta S(t)V(t) \\
&\geq S(t)\left[r - \frac{r(m_3 + M_1)}{K} - \beta I(t) - \theta V(t)\right] \\
&\geq S(t)\left(r - \frac{r(m_3 + M_1)}{K} - \beta M_1 - \theta M_2\right),
\end{aligned}
\tag{3.31}
$$

$$
\begin{aligned}
S((n_1 + 1 + n_2)T) &\geq S(t^*)e^{\int_{t^*}^{(n_1+1+n_2)T} (r - r(m_3 + M_1)/K - \beta M_1 - \theta M_2)\,dt} \\
&\geq m_3 e^{(r - r(m_3 + M_1)/K - \beta M_1 - \theta M_2)(n_2+1)T} \\
&= m_3 e^{\delta_1(n_2+1)T}.
\end{aligned}
$$

Thus $S((n_1+1+n_2+n_3)T) \geq m_3 e^{\delta_1(n_2+1)T} e^{\delta n_3} > m_3$, which is a contradiction. Let $\bar{t} = \inf_{t \geq t^*}\{S(t) \geq m_3\}$, then $S(\bar{t}) \geq m_3$, for $t \in [t^*, \bar{t})$, and we have $S(t) \geq S(t^*)e^{(t-t^*)\delta_1} \geq m_3 e^{(1+n_2+n_3)T\delta_1} \doteq m_1$. For $t > \bar{t}$, the same arguments can be continued, since $S(\bar{t}) \geq m_3$, and m_1, m_3 are t_1-independent. Hence $S(t) \geq m_1$ for all $t \geq t_1$. In the following, we shall prove that there exists $m_2 > 0$ such that $I(t) \geq m_2$ for t being large enough. From the third equation of system (1.1), we have

$$
V'(t) = -\theta S(t)V(t) + b\lambda I(t) - \mu V(t) \geq -\theta M_1 V(t) - \mu V(t).
\tag{3.32}
$$

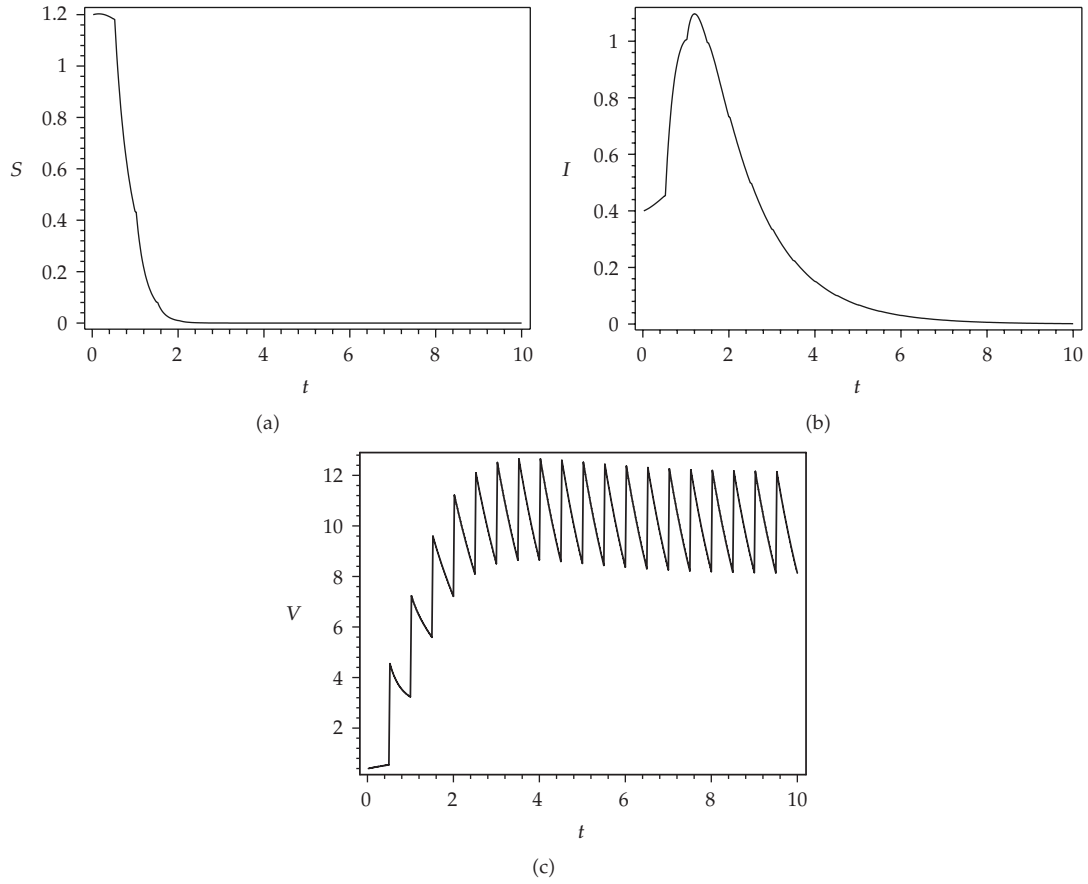Then $V(t) \geq u_1(t)$ and $u_1(t) \to u_1^*(t)$ as $t \to \infty$, where $u_1^*(t)$ is the solution of

$$
\begin{aligned}
u_1'(t) &= -\theta M_1 u_1(t) - \mu u_1(t), \quad t \neq nT, \\
\Delta u_1(t) &= p, \quad t = nT, \\
u(0^+) &= V(0^+) > 0, \\
u_1^*(t) &= \frac{pe^{-(\theta M_1 + \mu)(t - nT)}}{1 - e^{-(\theta M_1 + \mu)T}}, \quad t \in (nT, (n+1)T].
\end{aligned}
\tag{3.33}
$$

Therefore, there exists a $\widetilde{T_1} > t_1 > 0$ such that

$$
V(t) \geq u_1(t) \geq u_1^*(t) - \varepsilon \geq \frac{p}{1 - e^{-(\theta M_1 + \mu)T}} - \varepsilon \doteq \zeta
\tag{3.34}
$$

for $t > \widetilde{T_1}$. Then from the second equation of system (1.1), we have

$$
I'(t) = \beta S(t)I(t) + \theta S(t)V(t) - \lambda I(t) \geq -\lambda I(t) + \theta \zeta m_1,
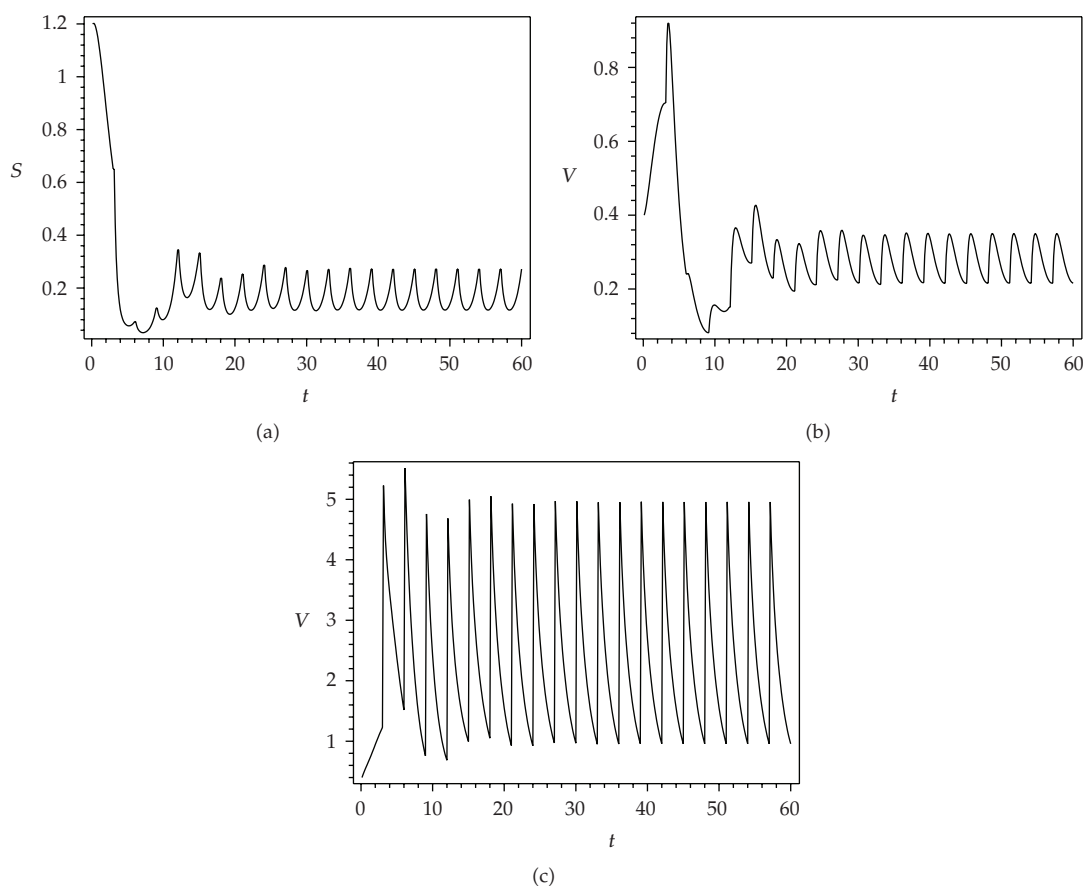\tag{3.35}
$$

(a)

(b)

(c)

Figure 1: Dynamical behavior of the system (1.1) with $r = 1.8, K = 2, \theta = 0.6, \beta = 0.2, p = 4, \lambda = 0.8, \mu = 0.8, b = 3$, and $T = 0.5$.

and then we have $\limsup_{t\to\infty} I(t) \geq \liminf_{t\to\infty} I(t) \geq \theta\zeta m_1/\lambda \doteq m_2$. Therefore $I(t) \geq m_2$ for t being large enough. The proof is complete. □

Corollary 3.6. *If $p < p_2^* = r\mu T/\theta$ or $T > T_2^* = \theta p/r\mu$, then system (1.1) is permanent.*

Example 3.7. Let us consider the following system:

$$S'(t) = 1.8S(t)\left(1 - \frac{S(t) + I(t)}{2}\right) - 0.2S(t)I(t) - 0.6S(t)V(t),$$

$$I'(t) = 0.2S(t)I(t) + 0.6S(t)V(t) - 0.8I(t), \qquad t \neq nT,$$

$$V'(t) = -0.6S(t)V(t) + 2.4I(t) - 0.8V(t),$$

(3.36)

$$\Delta S(t) = 0,$$

$$\Delta I(t) = 0, \quad t = nT, \ n = 1, 2, \ldots,$$

$$\Delta V(t) = p,$$

(a)

(b)

(c)

Figure 2: Dynamical behavior of the system (1.1) with $r = 1.8, K = 2, \theta = 0.6, \beta = 0.2, p = 4, \lambda = 0.8, \mu = 0.8, b = 3$, and $T = 3$.

According to Corollaries 3.3, and 3.6, we know that if $T < 0.67$, then $(0, 0, V^*(t))$ is globally asymptotically stable (see Figure 1), and if $T > 1.67$, then the system is permanent (see Figure 2).

4. Numerical Simulations and Discussion

In this paper, we have investigated the dynamical behavior of a pest management model with periodic releasing virus particles at a fixed time. By using Floquet's theorem, small-amplitude perturbation skills and comparison theorem, we establish the sufficient conditions for the global asymptotical stability of the pest-eradication periodic solution as well as the permanence of the system (1.1). It is clear that the conditions for the global stability and permanence of the system depend on the parameters p, T, which implies that the parameters p, T play a very important role on the model.

From Corollary 3.3, we know that the pest-eradication periodic solution $(0, 0, V^*(t))$ is globally asymptotically stable when $p > p_1^*$ or $T < T_1^*$. In order to drive the pests to extinction, we can determine the impulsive release amount p such that $p > p_1^*$ or the impulsive period T

such that $T < T_1^*$. If we choose parameters as $r = 1.8, K = 2, \theta = 0.6, \beta = 0.2, p = 4, \lambda = 0.8, \mu = 0.8, b = 3$, and $p = 4$, then we have $T_1^* = 0.67$; so we can make the impulsive period T smaller than 0.67 in order to eradicate the pests (see Figure 1). In the same time $T_2^* = 1.67$, so we can make the impulsive period T larger than 1.67 in order to maintain the system permanent (see Figure 2). Similarly, we can fix T and change p in order to achieve the same purpose. However, from a pest control point of view, our aim is to keep pests at acceptably low levels, not to eradicate them, only to control their population size. With regard to this, the optimal control strategy in the management of a pest population is to drive the pest population below a given level and to do so in a manner which minimizes the cost of using the control and the time it takes to drive the system to the target. We hope that our results provide an insight to practical pest management. However, in the real world, for the seasonal damages of pests, should we consider impulsive releasing virus particles on a finite interval? Such work will be beneficial to pest management, and it is reasonable. We leave it as a future work.

Acknowledgment

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