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Research Article

Feedback Control Variables Have No Influence on the Permanence of a Discrete n-Species Schoener Competition System with Time Delays

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We consider a discrete n-species Schoener competition system with time delays and feedback controls. By using difference inequality theory, a set of conditions which guarantee the permanence of system is obtained. The results indicate that feedback control variables have no influence on the persistent property of the system. Numerical simulations show the feasibility of our results.

1. Introduction

The Schoener's competition system has been studied by many scholars. Topics such as existence, uniqueness, and global attractivity of positive periodic solutions of the system were extensively investigated and many excellent results have been derived [1–5].

Liu et al. [1] propose and study the global stability of the following continuous Schoener's competition model with delays:

$$\begin{aligned}\dot{x}_1(t) &= x_1(t)\left[\frac{a_{10}(t)}{x_1(t-\tau_{10})+k_1(t)} - a_{11}(t)x_1(t-\tau_{11}) - a_{12}(t)x_2(t-\tau_{12}) - c_1(t)\right],\\ \dot{x}_2(t) &= x_2(t)\left[\frac{a_{20}(t)}{x_2(t-\tau_{20})+k_2(t)} - a_{21}(t)x_1(t-\tau_{21}) - a_{22}(t)x_2(t-\tau_{22}) - c_2(t)\right],\end{aligned} \tag{1.1}$$

where $\{k_i(t)\}$, $\{a_{ij}(t)\}$, and $\{c_i(t)\}$ are all positive bounded and continuous functions, τ_{ij} $(i = 1,2;\ j = 0,1,2)$ are positive integers, $\tau = \max_{0\le i,j\le 2}\{\tau_{ij}\}, x_i(s) = \phi_i(s) \ge 0$, $s \in [-\tau, 0] \cap Z$, $\phi_i(0) > 0$, $i = 1,2$.

As we all know, though most dynamic behaviors of population models are based on the continuous models governed by differential equations, the discrete time models are more appropriate than the continuous ones when the size of the population is rarely small or the population has nonoverlapping generations. It has been found that the dynamics behaviors of the discrete system are rather complex and contain more rich dynamics than the continuous ones [6]. Recently, more and more scholars paid attention to study the discrete population models (see [4–13] and the references cited therein). For example, [5] considered the permanence and global attractivity of the following discrete Schoener's competition model with delays:

$$
\begin{aligned}
x_1(n+1) &= x_1(n)\exp\left\{\frac{a_{10}(n)}{x_1(n-\tau_{10})+k_1(n)} - a_{11}(n)x_1(n-\tau_{11}) - a_{12}(n)x_2(n-\tau_{12}) - c_1(n)\right\},\\
x_2(n+1) &= x_2(n)\exp\left\{\frac{a_{20}(n)}{x_2(n-\tau_{20})+k_2(n)} - a_{21}(n)x_1(n-\tau_{21}) - a_{22}(n)x_2(n-\tau_{22}) - c_2(n)\right\},
\end{aligned}
\tag{1.2}
$$

where $\{k_i(n)\}$, $\{a_{ij}(n)\}$, and $\{c_i(n)\}$ are real positive bounded sequences, τ_{ij} $(i = 1,2;\ j = 0,1,2)$ are positive integers, $\tau = \max_{0\le i,j\le 2}\{\tau_{ij}\}, x_i(s) = \phi_i(s) \ge 0$, $s \in [-\tau, 0] \cap \mathbb{Z}$, $\phi_i(0) > 0$, $i = 1,2$.

On the other hand, as was pointed out by Huo and Li [14], ecosystem in the real world is continuously disturbed by unpredictable forces which can result in changes in the biological parameters such as survival rates. Practical interest in ecology is the question of whether or not an ecosystem can withstand those unpredictable disturbances which persist for a finite period of time. In the language of control variables, we call the disturbance functions as control variables. During the last decade, many scholars did excellent works on the feedback control ecosystems (see [14–21] and the references cited therein); however, most of those works are concerned with the continuous model and seldom did scholars considered the discrete ecosystem with feedback controls ([13, 15, 21]).

Recently, Li and Yang [15] proposed the following discrete n-species Schoener competition system with time delays and feedback controls:

$$
\begin{aligned}
x_i(k+1) &= x_i(k)\exp\left\{\frac{r_i(k)}{x_i(k-\tau_i)+a_i(k)} - \sum_{j=1}^{n} b_{ij}(k)x_j(k-\tau_j)\right.\\
&\qquad\left. -c_i(k) - d_i(k)u_i(k) - e_i(k)u_i(k-\eta_i)\right\},\\
\Delta u_i(k) &= -\alpha_i(k)u_i(k) + \beta_i(k)x_i(k) + \gamma_i(k)x_i(k-\sigma_i),
\end{aligned}
\tag{1.3}
$$

where $x_i(k)$ $(i = 1,2,\ldots,n)$ is the density of competitive species at kth generations; $u_i(k)$ is the control variable; Δ is the first-order forward difference operator $\Delta u_i(k) = u_i(k+1) - u_i(k)$, $i = 1,2,\ldots,n$.

Throughout this paper, we assume the following.

(H_1) $\alpha_i(k), \beta_i(k), \gamma_i(k), a_i(k), b_{ij}(k), r_i(k), c_i(k), d_i(k), e_i(k), i = 1, 2, \ldots, n$, are all bounded nonnegative sequences such that

$$\begin{gathered}0 < \alpha_i^l \leq \alpha_i^u < 1, \qquad 0 < \beta_i^l \leq \beta_i^u, \qquad 0 < \gamma_i^l \leq \gamma_i^u, \qquad 0 < a_i^l \leq a_i^u, \\ 0 < b_{ij}^l \leq b_{ij}^u, \qquad 0 < r_i^l \leq r_i^u, \qquad 0 < c_i^l \leq c_i^u, \qquad 0 < d_i^l \leq d_i^u, \qquad 0 < e_i^l \leq e_i^u.\end{gathered} \tag{1.4}$$

Here, for any bounded sequence $\{a(k)\}$, $a^u = \sup_{k\in N} a(k)$, $a^l = \inf_{k\in N} a(k)$.

(H_2) $\tau_i, \eta_i, \sigma_i, i = 1, 2, \ldots, n$, are all nonnegative integers.

Let $\tau = \max\{\tau_i, \eta_i, \sigma_i, i = 1, 2, \ldots, n\}$; we consider (1.3) together with the following initial conditions:

$$\begin{gathered}x_i(\theta) = \varphi_i(\theta), \quad \theta \in N[-\tau, 0] = \{-\tau, -\tau+1, \ldots, 0\}, \ \varphi_i(0) > 0, \\ u_i(\theta) = \phi_i(\theta), \quad \theta \in N[-\tau, 0] = \{-\tau, -\tau+1, \ldots, 0\}, \ \phi_i(0) > 0.\end{gathered} \tag{1.5}$$

It is not difficult to see that solutions of (1.3) and (1.5) are well defined for all $k \geq 0$ and satisfy

$$x_i(k) > 0, \quad u_i(k) > 0, \quad k \in Z, \ i = 1, 2, \ldots, n. \tag{1.6}$$

By applying the comparison theorem of difference equation, they obtained a set of sufficient conditions which guarantee the permanence of the system (1.3). Their result shows that feedback control variables play important roles on the persistent property of the system (1.3). But the question is whether or not the feedback control variables have influence on the permanence of the system. The aim of this paper is to apply the analysis technique of Chen et al. [18] to establish sufficient conditions, which is independent of feedback control variables, to ensure the permanence of the system.

The organization of this paper is as follow. In Section 2, we will introduce several lemmas. The permanence of system (1.3) is then studied in Section 3. In Section 4, a suitable example together with its numerical simulations shows the feasibility of our results.

2. Preliminaries

In this section, we will introduce several useful lemmas.

Lemma 2.1 (see [11]). *Assume that $\{x(k)\}$ satisfies $x(k) > 0$ and*

$$x(k+1) \leq x(k) \exp\{a(k) - b(k)x(k)\} \tag{2.1}$$

for $k \in N$, where $a(k)$ and $b(k)$ are nonnegative sequences bounded above and below by positive constants. Then

$$\limsup_{k\to+\infty} x(k) \leq \frac{1}{b^l} \exp(a^u - 1). \tag{2.2}$$

Lemma 2.2 (see [12]). *Assume that* $\{x(k)\}$ *satisfies*

$$x(k+1) \geq x(k)\exp\{a(k) - b(k)x(k)\}, \quad k \geq N_0, \tag{2.3}$$

$\limsup_{k\to+\infty} x(k) \leq x^*$ *and* $x(N_0) > 0$, *where* $a(k)$ *and* $b(k)$ *are nonnegative sequences bounded above and below by positive constants and* $N_0 \in N$. *Then*

$$\liminf_{k\to+\infty} x(k) \geq \min\left\{\frac{a^l}{b^u}\exp\left(a^l - b^u x^*\right), \frac{a^l}{b^u}\right\}. \tag{2.4}$$

Lemma 2.3 (see [13]). *Assume that* $A > 0$, $y(0) > 0$. *Suppose that*

$$y(k+1) \leq Ay(k) + B(k), \quad k = 1, 2, \dots. \tag{2.5}$$

Then for any integer $m \leq k$,

$$y(k) \leq A^m y(k-m) + \sum_{j=0}^{m-1} A^j B(k-j-1). \tag{2.6}$$

If $A < 1$ *and* B *is bounded above with respect to* M, *then*

$$\limsup_{k\to+\infty} y(k) \leq \frac{M}{1-A}. \tag{2.7}$$

Lemma 2.4 (see [13]). *Assume that* $A > 0$, $y(0) > 0$. *Suppose that*

$$y(k+1) \geq Ay(k) + B(k), \quad k = 1, 2, \dots. \tag{2.8}$$

Then for any integer $m \leq k$,

$$y(k) \geq A^m y(k-m) + \sum_{j=0}^{m-1} A^j B(k-j-1). \tag{2.9}$$

If $A < 1$ *and* B *is bounded below with respect to* P, *then*

$$\liminf_{k\to+\infty} y(k) \geq \frac{P}{1-A}. \tag{2.10}$$

3. Permanence

In this section, we establish the following permanence result for system (1.3).

Theorem 3.1. *Assume that* (H_1) *and* (H_2) *hold. Then there exist positive constants* M_i, N_i *which are independent of the solutions of system* (1.3) *such that*

$$\limsup_{k\to\infty} x_i(k) \le M_i, \quad i = 1,2,\dots,n,$$
$$\limsup_{k\to\infty} u_i(k) \le N_i, \quad i = 1,2,\dots,n. \tag{3.1}$$

Proof. Let $x(k) = (x_1(k), x_2(k)\cdots x_n(k), u_1(k), u_2(k)\cdots u_n(k))^T$ be any positive solution of system (1.3) with initial condition (1.5). From the first equation of system (1.3), it follows that

$$x_i(k+1) \le x_i(k)\exp\left\{\frac{r_i(k)}{a_i(k)} - b_{ii}(k)x_i(k-\tau_i)\right\} \le x_i(k)\exp\left\{\frac{r_i^u}{a_i^l}\right\}. \tag{3.2}$$

By using (3.2), one can easily obtain that

$$\prod_{j=k-\tau_i}^{k-1} x_i(j+1) \le \prod_{j=k-\tau_i}^{k-1} x_i(j)\exp\left\{\frac{r_i^u}{a_i^l}\right\}, \tag{3.3}$$

that is,

$$x_i(k) \le x_i(k-\tau_i)\exp\left\{\frac{\tau_i r_i^u}{a_i^l}\right\}. \tag{3.4}$$

Substituting (3.4) into (3.2), it follows that

$$x_i(k+1) \le x_i(k)\exp\left\{\frac{r_i(k)}{a_i(k)} - b_{ii}(k)\exp\left\{-\frac{\tau_i r_i^u}{a_i^l}\right\}x_i(k)\right\}. \tag{3.5}$$

By applying Lemma 2.1, it follows that

$$\limsup_{k\to\infty} x_i(k) \le \frac{\exp\left\{\left(r_i^u/a_i^l\right)(1+\tau_i) - 1\right\}}{b_{ii}^l} =: M_i, \quad i = 1,2,\dots,n. \tag{3.6}$$

Thus, for all $\varepsilon > 0$, there exists a $K_1 > 0$, $K_1 \in N$, for all $k > K_1, x_i(k) \leq M_i + \varepsilon$, $i = 1, 2, \ldots, n$, and so

$$\begin{aligned} u_i(k+1) &= (1-\alpha_i(k))u_i(k) + \beta_i(k)x_i(k) + \gamma_i(k)x_i(k-\sigma_i) \\ &\leq \left(1-\alpha_i^l\right)u_i(k) + \left(\beta_i^u + \gamma_i^u\right)(M_i + \varepsilon) \quad (k > K_1 + \sigma_i). \end{aligned} \tag{3.7}$$

For $0 < 1 - \alpha_i^l < 1$, Lemma 2.3 implies that

$$\limsup_{k\to\infty} u_i(k) \leq \frac{(\beta_i^u + \gamma_i^u)(M_i + \varepsilon)}{\alpha_i^l}. \tag{3.8}$$

Letting $\varepsilon \to 0$ in the above inequality leads to

$$\limsup_{k\to\infty} u_i(k) \leq \frac{(\beta_i^u + \gamma_i^u)M_i}{\alpha_i^l} := N_i. \tag{3.9}$$

The proof of Theorem 3.1 is completed. □

Theorem 3.2. *Assume that* (H_1) *and* (H_2) *hold; assume further that*

$$R_i := \frac{r_i^l}{M_i + a_i^u} - \sum_{j=1, j\neq i}^{n} b_{ij}^u M_j - c_i^u > 0, \quad i = 1, 2, \ldots, n. \tag{H}$$

Then there exist positive constants m_i, n_i *which are independent of the solution of system* (1.3), *such that*

$$\begin{aligned} &\liminf_{k\to\infty} x_i(k) \geq m_i, \quad i = 1, 2, \ldots, n, \\ &\liminf_{k\to\infty} u_i(k) \geq n_i, \quad i = 1, 2, \ldots, n. \end{aligned} \tag{3.10}$$

Proof. Let $x(k) = (x_1(k), x_2(k) \cdots x_n(k), u_1(k), u_2(k) \cdots u_n(k))^T$ be any positive solution of system (1.3) with initial condition (1.5). From Theorem 3.1, for all $0 < \varepsilon < 1$, there exists a $K_2 > K_1$, for all $k > K_2$,

$$x_i(k) \leq M_i + \varepsilon, \quad u_i(k) \leq N_i + \varepsilon, \quad i = 1, 2, \ldots, n, \tag{3.11}$$

$$R_{i\varepsilon} := \frac{r_i^l}{(M_j + \varepsilon) + a_i^u} - \sum_{j=1, j\neq i}^{n} b_{ij}^u (M_j + \varepsilon) - c_i^u > 0, \quad i = 1, 2, \ldots, n. \tag{3.12}$$

From (3.12), we have

$$\begin{aligned} x_i(k+1) &\geq x_i(k)\exp\left\{\frac{r_i^l}{(M_i+\varepsilon)+a_i^u} - \sum_{j=1, j\neq i}^{n} b_{ij}^u(M_j+\varepsilon)\right. \\ &\qquad\left. -c_i^u - d_i^u u_i(k) - e_i^u u_i(k-\eta_i) - b_{ii}^u x_i(k-\tau_i)\right\} \\ &\geq x_i(k)\exp\left\{-\sum_{j=1}^{n} b_{ij}^u(M_j+\varepsilon) - c_i^u - (d_i^u+e_i^u)(N_i+\varepsilon)\right\} \\ &=: x_i(k)\exp\{D_{i\varepsilon}\}. \end{aligned} \tag{3.13}$$

for all $k > K_2 + \tau$, where $D_{i\varepsilon} = -\sum_{j=1}^{n} b_{ij}^u(M_j+\varepsilon) - c_i^u - (d_i^u+e_i^u)(N_i+\varepsilon) < 0$.

Thus, for $\eta < K$, by using (3.13) we obtain

$$x_i(\eta) \leq x_i(k)\exp\{-(k-\eta)D_{i\varepsilon}\}. \tag{3.14}$$

From the second equation of (1.3), we obtain

$$\begin{aligned} u_i(k+1) &= (1-\alpha_i(k))u_i(k) + \beta_i(k)x_i(k) + \gamma_i(k)x_i(k-\sigma_i) \\ &\leq \left(1-\alpha_i^l\right)u_i(k) + \beta_i^u x_i(k) + \gamma_i^u x_i(k-\sigma_i) \\ &\triangleq A_i u_i(k) + B_i(k), \end{aligned} \tag{3.15}$$

where $0 < A_i = 1-\alpha_i^l < 1$ and $B_i(k) = \beta_i^u x_i(k) + \gamma_i^u x_i(k-\sigma_i)$.

Then, Lemma 2.3, (3.14), and (3.15) imply that for any $m \leq K$,

$$\begin{aligned} u_i(k) &\leq A_i^m u_i(k-m) + \sum_{j=0}^{m-1} A_i^j B_i(k-j-1) \\ &= A_i^m u_i(k-m) + \sum_{j=0}^{m-1} A_i^j\{\beta_i^u x_i(k-j-1) + \gamma_i^u x_i(k-j-1-\sigma_i)\} \\ &\leq A_i^m u_i(k-m) \\ &\quad + \sum_{j=0}^{m-1} A_i^j\{\beta_i^u \exp\{-(j+1)D_{i\varepsilon}\} + \gamma_i^u \exp\{-(j+1+\sigma_i)D_{i\varepsilon}\}\}x_i(k) \\ &\leq A_i^m u_i(k-m) + x_i(k)\sum_{j=0}^{m-1} A_i^j(\beta_i^u+\gamma_i^u)\exp\{-(j+1)D_{i\varepsilon}\}. \end{aligned} \tag{3.16}$$

For $0 < A_i < 1, \lim_{m\to\infty}(d_i^u + e_i^u)(N_i + 1)A_i^m = 0$. That is, there exists an $M > 0$, for all $m > M$

$$(N_i + \varepsilon)\left(d_i^u + e_i^u\right)A_i^m < (N_i + 1)\left(d_i^u + e_i^u\right)A_i^m \le \frac{R_{i\varepsilon_0}}{2} \le \frac{R_{i\varepsilon}}{2}. \tag{3.17}$$

For enough small $\varepsilon_0 > 0$, we have

$$R_{i\varepsilon_0} := \frac{r_i^l}{(M_i + \varepsilon_0) + a_i^u} - \sum_{j=1, j\neq i}^{n} b_{ij}^u\left(M_j + \varepsilon_0\right) - c_i^u < R_{i\varepsilon} < R_i, \quad i = 1, 2, \ldots, n. \tag{3.18}$$

We choose $M \ge \max\{1, \log_{A_i}(R_{i\varepsilon_0}/2(N_i + 1)(d_i^u + e_i^u))\}$, $i = 1, 2, \ldots, n$. For fixed M, we get

$$\begin{aligned} u_i(k) &\le A_i^M u_i(k - M) + x_i(k)\sum_{j=0}^{M-1} A_i^j\left(\beta_i^u + \gamma_i^u\right)\exp\{-(j + 1)D_{i\varepsilon}\} \\ &\le A_i^M(N_i + \varepsilon) + E_{i\varepsilon}x_i(k), \end{aligned} \tag{3.19}$$

for all $k \ge K_2 + M$ where $E_{i\varepsilon} = \sum_{j=0}^{M-1} A_i^j(\beta_i^u + \gamma_i^u)\exp\{-(j + 1)D_{i\varepsilon}\}$.

Substituting (3.14) (3.17) and (3.19) into (3.13), for $k \ge K_2 + M + \tau$, one has

$$\begin{aligned} x_i(k+1) &\ge x_i(k)\exp\left\{R_{i\varepsilon} - b_{ii}^u x_i(k - \tau_i) - d_i^u u_i(k) - e_i^u u_i\left(k - \eta_i\right)\right\} \\ &\ge x_i(k)\exp\left\{R_{i\varepsilon} - \left(e_i^u + d_i^u\right)A_i^M(N_i + \varepsilon) - b_{ii}^u x_i(k - \tau_i) - d_i^u E_{i\varepsilon}x_i(k) - e_i^u E_{i\varepsilon}x_i\left(k - \eta_i\right)\right\} \\ &\ge x_i(k)\exp\left[\frac{R_{i\varepsilon}}{2} - b_{ii}^u\exp\{-\tau_i D_{i\varepsilon}\} - d_i^u E_{i\varepsilon} - e_i^u E_{i\varepsilon}\exp\{-\eta_i D_{i\varepsilon}\}\right]x_i(k) \\ &:= x_i(k)\exp\left\{\frac{R_{i\varepsilon}}{2} - Q_{i\varepsilon}x_i(k)\right\}, \end{aligned} \tag{3.20}$$

where $Q_{i\varepsilon} := b_{ii}^u\exp\{-\tau_i D_{i\varepsilon}\} + d_i^u E_{i\varepsilon} + e_i^u E_{i\varepsilon}\exp\{-\eta_i D_{i\varepsilon}\}$.

By applying Lemma 2.2, it follows that

$$\liminf_{k\to\infty} x(k) \ge \min\left\{\frac{R_{i\varepsilon}}{2Q_{i\varepsilon}}\exp\left\{\frac{R_{i\varepsilon}}{2} - Q_{i\varepsilon}M_i\right\}, \frac{R_{i\varepsilon}}{2Q_{i\varepsilon}}\right\}. \tag{3.21}$$

Letting $\varepsilon \to 0$ in the above inequality leads to

$$\liminf_{k\to\infty} x(k) \ge \min\left\{\frac{R_i}{2Q_i}\exp\left\{\frac{R_i}{2} - Q_i M_i\right\}, \frac{R_i}{2Q_i}\right\} := m_i. \tag{3.22}$$

For the above $\varepsilon > 0$, there exists a $K_3 > K_2$, for all $k > K_3$, we have $x_i(k) \geq m_i - \varepsilon$, $i = 1, 2, \ldots, n$. Then for $k > K_3 + \sigma_i$,

$$\begin{aligned} u_i(k+1) &= (1-\alpha_i(k))u_i(k) + \beta_i(k)x_i(k) + \gamma_i(k)x_i(k-\sigma_i) \\ &\geq (1-\alpha_i^u)u_i(k) + \left(\beta_i^l + \gamma_i^l\right)(m_i - \varepsilon). \end{aligned} \tag{3.23}$$

By applying Lemma 2.4, it follows that

$$\liminf_{k\to\infty} u_i(k) \geq \frac{\left(\beta_i^l + \gamma_i^l\right)(m_i - \varepsilon)}{\alpha_i^u}. \tag{3.24}$$

Letting $\varepsilon \to 0$ in the above inequality leads to

$$\liminf_{k\to\infty} u_i(k) \geq \frac{\left(\beta_i^l + \gamma_i^l\right)m_i}{\alpha_i^u} := n_i, \quad i = 1, 2, \ldots, n. \tag{3.25}$$

The proof of Theorem 3.2 is completed. □

Remark 3.3. Theorems 3.1 and 3.2 show that under the assumption (H_1), (H_2) and (H) hold, and system (1.3) is permanent.

4. Example and Numeric Simulations

The following example lends credence to the plausibility of Theorem 3.2.

Example 4.1. Consider the following system:

$$\begin{aligned} x_1(k+1) &= x_1(k)\exp\left\{\frac{0.5}{x_1(k-1)+1} - x_1(k-1) - 0.02(\sin(k)+2)x_2(k-2)\right. \\ &\qquad \left. -0.01 - (\cos(k)+2.5)u_1(k) - (\cos(k)+1.5)u_1(k-1)\right\}, \\ x_2(k+1) &= x_2(k)\exp\left\{\frac{1}{x_2(k-2)+3} - 0.05(\cos(k)+2)x_1(k-1) - 2x_2(k-2)\right. \\ &\qquad \left. -0.05 - (\sin(k)+2)u_2(k) - (\sin(k)+1.1)u_2(k-1)\right\}, \\ \Delta u_1(k) &= -0.5u_1(k) + 0.1(\sin(10+k)+1.5)x_1(k) + 0.1(\sin(k)+1.5)x_1(k-1), \\ \Delta u_2(k) &= -\frac{1}{3}u_2(k) + 0.1(\cos(10+k)+1.5)x_2(k) + 0.1(\cos(k+5)+1.5)x_2(k-1). \end{aligned} \tag{4.1}$$

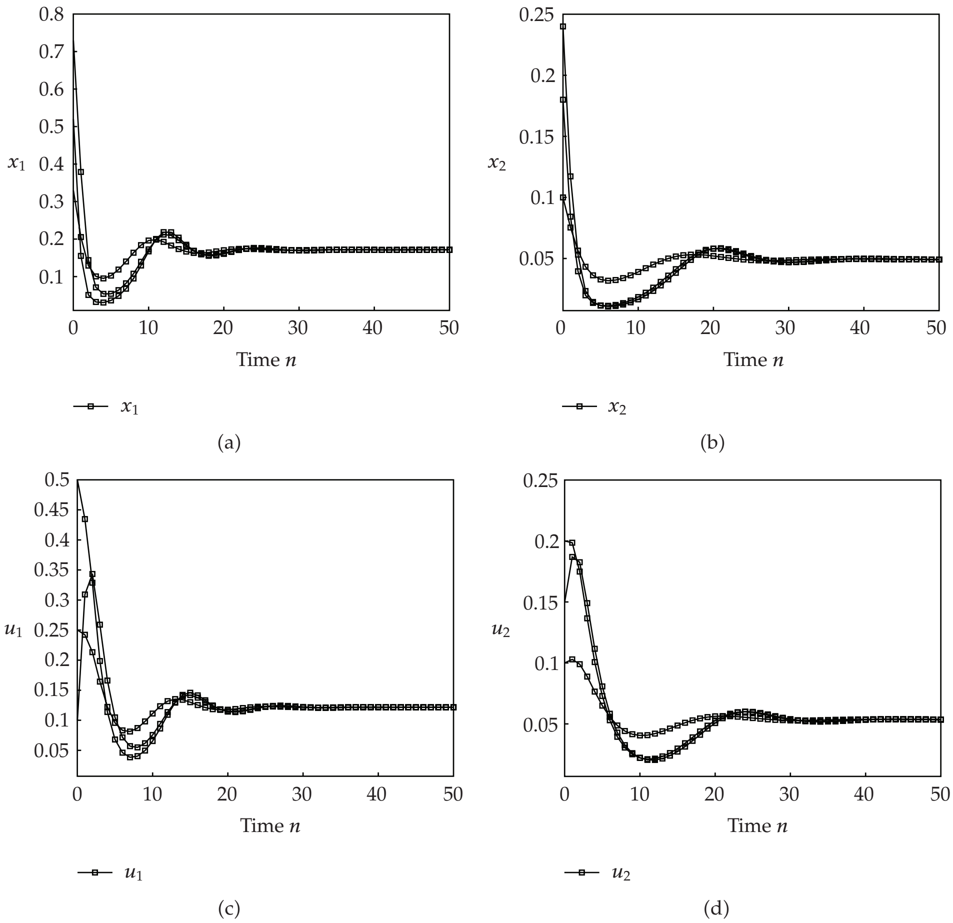

Figure 1: Dynamic behaviors of system (4.1) with initial conditions $(x_1(s), x_2(s), u_1(s), u_2(s))^T = (0.73, 0.1, 0.1, 0.1)^T$, $(0.33, 0.24, 0.25, 0.15)^T$, $(0.52, 0.18, 0.5, 0.2)^T$, $s = -1, 0$.

Here corresponding to system (1.3), we assume that

$$\begin{gathered}
r_1(k) = 0.5, \qquad a_1(k) = 1, \qquad b_{11}(k) = 1, \qquad b_{12}(k) = 0.02(\sin(k) + 2), \\
c_1(k) = 0.01, \qquad \eta_1 = 1, \qquad d_1(k) = \cos(k) + 2.5, \qquad e_1(k) = \cos(k) + 1.5, \\
\alpha_1(k) = 0.5, \qquad \beta_1(k) = 0.1(\sin(k + 10) + 1.5), \\
\gamma_1(k) = 0.1(\sin(k) + 1.5), \qquad \sigma_1 = 1, \\
r_2(k) = 1, \qquad \tau_2 = 2, \qquad a_2(k) = 3, \qquad b_{22}(k) = 2, \qquad b_{21}(k) = 0.05(\cos(k) + 2), \\
c_2(k) = 0.05, \qquad \eta_2 = 1, \qquad d_2(k) = \sin(k) + 2, \qquad e_2(k) = \sin(k) + 1.1, \\
\alpha_2(k) = \frac{1}{3}, \qquad \beta_2(k) = 0.1(\cos(k + 10) + 1.5), \\
\gamma_2(k) = 0.1(\cos(k + 5) + 1.5), \qquad \sigma_2 = 1.
\end{gathered} \tag{4.2}$$

It is easy to see that $M_1 = 1$, $M_2 = 0.5$, and

$$\begin{aligned} &\frac{r_1^l}{M_1 + a_1^u} - b_{12}^u M_2 - c_1^u = 0.21 > 0, \\ &\frac{r_2^l}{M_2 + a_2^u} - b_{21}^u M_1 - c_2^u = \frac{3}{35} > 0. \end{aligned} \tag{4.3}$$

Inequalities (4.3) show that all the conditions of Theorem 3.2 are satisfied; thus, system (1.3) is permanent. Numerical simulation from Figure 1 supports this conclusion.

5. Discussion

In this paper we have attempted to understand the effect of feedback control variables on the permanence of system (1.3). The present work is an extension of an earlier work by Li and Yang [15]. By developing the analysis technique of Chen et al. [18], a set of sufficient conditions are established for the permanence of system (1.3). Theorems 3.1-3.2 and the numerical simulations indicate that feedback control variables have no influence on the permanence of system (1.3).

We would like to mention here that an interesting but challenging problem associated with the study of system (1.3) should be the global attractivity. We leave this for future work.

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