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Research Article

Delay-Dependent H_∞ Filtering for Singular Time-Delay Systems

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This paper deals with the problem of delay-dependent H_∞ filtering for singular time-delay systems. First, a new delay-dependent condition which guarantees that the filter error system has a prescribed H_∞ performance γ is given in terms of linear matrix inequalities (LMIs). Then, the sufficient condition is obtained for the existence of the H_∞ filter, and the explicit expression for the desired H_∞ filter is presented by using LMIs and the cone complementarity linearization iterative algorithm. A numerical example is provided to illustrate the effectiveness of the proposed method.

1. Introduction

Over the past decades, the filtering problem has been widely studied and has found many applications [1, 2]. Current efforts on this topic can be mainly divided into two classes: the Kalman filtering approach and the H_∞ filtering approach. The objective of the latter one is to find a filter such that the resulting error system is asymptotically stable and the L_2-induced norm (for continuous systems) or l_2-induced norm (for discrete systems) from the disturbance input to the filtering error output satisfies a prescribed H_∞ performance level. In contrast to the Kalman filtering, the H_∞ filtering approach does not require the exact knowledge of the statistics of the external noise signals, and it is insensitive to the uncertainties. These features render the H_∞ filtering attracting much attention, and many efforts have been made on this issue [3–6]. The filtering problem for singular systems has also been investigated by many researchers. For example, a necessary and sufficient condition is obtained in [7] for the solvability of the H_∞ filtering problem and the designed filter is proper with a McMillan degree no more than the exponential modes of the plant, while, in [8], a linear normal H_∞ filter is obtained for singular systems. Reduced-order H_∞ filters are designed in [9] for both continuous and discrete singular systems. In [10], a reduced-order H_∞ filter design approach is developed for a class of discrete singular systems with lossy measurements.

On the other hand, for many practical control systems, time delays are frequently encountered and they are often the sources of instability and degradation in control performance. So, recently, there has been increasing interest in H_∞ filtering for time-delay systems. Existing results can be classified into two types: delay-independent ones [11–14] and delay-dependent ones [15–23]; the former do not include any information on the size of delay while the latter employ such information. Generally speaking, delay-dependent results are less conservative than the delay-independent ones, especially when the size of delay is small.

Singular time-delay systems, which are also referred to as implicit time-delay systems, descriptor time-delay systems, or generalized differential-difference equations, often appear in various engineering systems, including aircraft attitude control, flexible arm control of robots, large-scale electric network control, chemical engineering systems, and lossless transmission lines (see, e.g., [24]). Since singular time-delay systems are more general, it is of significance to consider the H_∞ filtering problem for them. Recently, some delay-dependent [25–27] and delay-dependent [28–31] results about H_∞ filters for such systems have been obtained. In [28], the delay-independent filter is of the Luenberger observer type and the decomposition and transformation of the system matrices are involved, which would result in some numerical problems. A full-order filter is designed in [29] for singular systems with communication delays, and H_∞ filtering problems are concerned in [30, 31] for singular systems with time-varying delay in a range.

In this paper, the problem of delay-dependent H_∞ filtering is investigated for singular time-delay systems. We consider the case of discrete delay which is assumed to be constant and known. First, based on the result in [32], we derive a new delay-dependent condition which guarantees that the filter error system has a prescribed H_∞ performance γ; and it can be seen that this new condition is more "efficient" than that in [32] since no redundant variables are involved. Then, the sufficient condition for the existence of the full-order H_∞ filter, which is an admissible singular time-delay system, is obtained and the explicit expression for the desired H_∞ filter is given by using LMIs and the cone complementarity linearization iterative algorithm.

Notations

R^n denotes the n-dimensional Euclidean space and $R^{n\times m}$ denotes the set of all $n \times m$ real matrices, I_n is the n-dimensional identity matrix, and $\operatorname{diag}\{\cdots\}$ is a block-diagonal matrix. For real symmetric matrix X, the notation $X \geq 0$ ($X > 0$) means that the matrix X is positive-semidefinite (positive-definite). The superscript T represents the transpose; the symbol $*$ will be used in some matrix expressions to induce a symmetric structure. $L_2[0,\infty)$ refers to the space of square-integrable vector functions over $[0,\infty)$ with norm $\|f\|_2 := (\int_0^\infty \|f(t)\|^2 dt)^{1/2}$.

2. Problem Statement

Consider the following singular time-delay system:

$$E\dot{x}(t) = Ax(t) + A_\tau x(t-\tau) + Bw(t),$$
$$y(t) = Cx(t) + C_\tau x(t-\tau) + B_1 w(t),$$

$$\begin{aligned} z(t) &= Gx(t) + G_\tau x(t-\tau) + B_2 w(t), \\ x(t) &= \phi(t), \quad t \in [-\tau, 0], \end{aligned} \tag{2.1}$$

where $x(t) \in R^n$ is the state, $w(t) \in R^r$ is the external disturbance signal that belongs to $L_2[0,\infty), y(t) \in R^m$ is the measurement output, and $z(t) \in R^s$ is the signal to be estimated. E, A, A_τ, B, C, C_τ, B_1, G, G_τ, and B_2 are known real constant matrices with appropriate dimensions and $0 < \operatorname{rank} E = p < n$. $\tau > 0$ is the known delay constant and $\phi(t) \in C_{n,\tau}$ is a compatible vector-valued initial function.

Without loss of generality, we assume that $C_\tau = 0$, $B_1 = 0$, $G_\tau = 0$, and $B_2 = 0$. Otherwise, system (2.1) can be equivalently changed into

$$\begin{aligned} \begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ \dot{\zeta}(t) \end{bmatrix} &= \begin{bmatrix} A & 0 \\ 0 & -I_{m+s} \end{bmatrix} \begin{bmatrix} x(t) \\ \zeta(t) \end{bmatrix} \\ &\quad + \begin{bmatrix} A_\tau & | & 0 \\ -- & - & -- \\ C_\tau & | & \\ & & 0_{m+s} \\ G_\tau & | & \end{bmatrix} \begin{bmatrix} x(t-\tau) \\ \zeta(t-\tau) \end{bmatrix} + \begin{bmatrix} B \\ -- \\ B_1 \\ B_2 \end{bmatrix} w(t), \\ \begin{bmatrix} y(t) \\ z(t) \end{bmatrix} &= \begin{bmatrix} C & | & \\ & & I_{m+s} \\ G & | & \end{bmatrix} \begin{bmatrix} x(t) \\ \zeta(t) \end{bmatrix}. \end{aligned} \tag{2.2}$$

Then in the sequel, we discuss the system model as follows:

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + A_\tau x(t-\tau) + Bw(t), \\ y(t) &= Cx(t), \\ z(t) &= Gx(t), \\ x(t) &= \phi(t), \quad t \in [-\tau, 0]. \end{aligned} \tag{2.3}$$

Throughout this paper, we need the following assumption for system (2.3).

Assumption 2.1. *System* (2.3) *is admissible, that is, when* $w(t) \equiv 0$, *system* (2.3) *is regular, impulse free, and asymptotically stable.*

Remark 2.2. About the definitions of regularity, absence of impulses and asymptotical stability for singular time-delay systems, we refer the readers to [33].

For the estimates of $z(t)$, we consider the following linear filter with delay:

$$\begin{aligned} E\dot{\hat{x}}(t) &= A_f \hat{x}(t) + A_{\tau f} \hat{x}(t-\tau) + B_f y(t), \\ \hat{z}(t) &= C_f \hat{x}(t), \\ \hat{x}(t) &= \psi(t), \quad t \in [-\tau, 0], \end{aligned} \tag{2.4}$$

where $\hat{x}(t) \in R^n$ and $\hat{z}(t) \in R^s$ are the state and the output of the filter, respectively. The constant matrices A_f, $A_{\tau f}$, B_f, and C_f are filter parameters to be determined.

Letting

$$e(t) := \left[x^T(t) \;\; \hat{x}^T(t)\right]^T, \qquad \tilde{z}(t) := z(t) - \hat{z}(t), \tag{2.5}$$

one obtains the filter error system

$$\begin{aligned} \widetilde{E}\dot{e}(t) &= \widetilde{A}e(t) + \widetilde{A}_\tau e(t-\tau) + \widetilde{B}w(t), \\ \tilde{z}(t) &= \widetilde{G}e(t), \\ e(t) &= \left[\phi^T(t) \;\; \varphi^T(t)\right]^T, \quad t \in [-\tau, 0], \end{aligned} \tag{2.6}$$

where

$$\begin{aligned} \widetilde{E} &= \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix}, \qquad \widetilde{A} = \begin{bmatrix} A & 0 \\ B_f C & A_f \end{bmatrix}, \\ \widetilde{A}_\tau &= \begin{bmatrix} A_\tau & 0 \\ 0 & A_{\tau f} \end{bmatrix}, \qquad \widetilde{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \qquad \widetilde{G} = \left[G \;\; -C_f\right]. \end{aligned} \tag{2.7}$$

Thus, the filtering problem to be addressed is stated as follows.

H_∞ Filtering Problem

For a given $\gamma > 0$, design a full-order filter with delay of the form of (2.4) such that the filter error system (2.6) has prescribed H_∞ performance γ, that is,

(1) system (2.6) is admissible;

(2) under zero initial condition, for any nonzero $w(t) \in L_2[0, \infty)$, the H_∞ performance $\|z(t)\|_2 \leq \gamma \|w(t)\|_2$ is guaranteed.

Remark 2.3. Similar to [17], it is easy to see that system (2.3) is admissible if the error system (2.6) is admissible. That is why we made Assumption 2.1 on system (2.3).

3. Main Results

At first, we will concentrate our attention on H_∞ performance analysis for the error system (2.6). The following lemma is useful in the proof of Theorem 3.2.

Lemma 3.1 (see [32]). *Given a scalar $\gamma > 0$, the filter error system* (2.6) *has a prescribed H_∞ performance γ if there exist matrices $\widetilde{Q} > 0$, $\widetilde{Z} > 0$, $\widetilde{P}$, $\widetilde{Y}$, and $\widetilde{W}$ satisfying*

$$\widetilde{E}^T\widetilde{P}^T = \widetilde{P}\widetilde{E} \geq 0, \tag{3.1}$$

$$\Phi = \begin{bmatrix} \Phi_1 & \Phi_2 & \tau\widetilde{Y}^T & \widetilde{P}\widetilde{B} & \tau\widetilde{A}^T\widetilde{Z} & \widetilde{G}^T \\ * & \Phi_3 & \tau\widetilde{W}^T & 0 & \tau\widetilde{A}_\tau^T\widetilde{Z} & 0 \\ * & * & -\tau\widetilde{Z} & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & \tau\widetilde{B}^T\widetilde{Z} & 0 \\ * & * & * & * & -\tau\widetilde{Z} & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{3.2}$$

where

$$\Phi_1 = \widetilde{P}\widetilde{A} + \widetilde{A}^T\widetilde{P}^T + \widetilde{Q} - \widetilde{Y}^T\widetilde{E} - \widetilde{E}^T\widetilde{Y}, \qquad \Phi_2 = \widetilde{P}\widetilde{A}_\tau + \widetilde{Y}^T\widetilde{E} - \widetilde{E}^T\widetilde{W},$$
$$\Phi_3 = -\widetilde{Q} + \widetilde{W}^T\widetilde{E} + \widetilde{E}^T\widetilde{W}. \tag{3.3}$$

Based on Lemma 3.1, we will present a new delay-dependent bounded real lemma (BRL) for the performance analysis of system (2.6), which can be shown to be more "efficient" than Lemma 3.1.

Theorem 3.2. *Given a scalar $\gamma > 0$, the filter error system* (2.6) *has a prescribed H_∞ performance γ if there exist matrices $\widetilde{Q} > 0$, $\widetilde{Z} > 0$ and $\widetilde{P}$ satisfying* (3.1) *and*

$$\Omega = \begin{bmatrix} \Omega_1 & \Omega_2 & \widetilde{P}\widetilde{B} & \tau\widetilde{A}^T\widetilde{Z} & \widetilde{G}^T \\ * & \Omega_3 & 0 & \tau\widetilde{A}_\tau^T\widetilde{Z} & 0 \\ * & * & -\gamma^2 I & \tau\widetilde{B}^T\widetilde{Z} & 0 \\ * & * & * & -\tau\widetilde{Z} & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \tag{3.4}$$

where

$$\Omega_1 = \widetilde{P}\widetilde{A} + \widetilde{A}^T\widetilde{P}^T + \widetilde{Q} - \frac{1}{\tau}\widetilde{E}^T\widetilde{Z}\widetilde{E}, \qquad \Omega_2 = \widetilde{P}\widetilde{A}_\tau + \frac{1}{\tau}\widetilde{E}^T\widetilde{Z}\widetilde{E}, \qquad \Omega_3 = -\widetilde{Q} - \frac{1}{\tau}\widetilde{E}^T\widetilde{Z}\widetilde{E}. \tag{3.5}$$

Proof. From Lemma 3.1, if we can prove that the feasibility of $\Omega < 0$ for solution ($\widetilde{Q} > 0, \widetilde{Z} > 0, \widetilde{P}$) is equivalent to that of $\Phi < 0$ for solution ($\widetilde{Q} > 0, \widetilde{Z} > 0, \widetilde{P}, \widetilde{Y}, \widetilde{W}$), then Theorem 3.2 is proved.

Similar to Lemma 4 of [34], take

$$\Psi = \Pi\Phi\Pi^T = \begin{bmatrix} \Omega_1 & \Omega_2 & \tau\widetilde{Y}^T - \widetilde{E}^T\widetilde{Z} & \widetilde{P}\widetilde{B} & \tau\widetilde{A}^T\widetilde{Z} & \widetilde{G}^T \\ * & \Omega_3 & \tau\widetilde{W}^T + \widetilde{E}^T\widetilde{Z} & 0 & \tau\widetilde{A}_\tau^T\widetilde{Z} & 0 \\ * & * & -\tau\widetilde{Z} & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & \tau\widetilde{B}^T\widetilde{Z} & 0 \\ * & * & * & * & -\tau\widetilde{Z} & 0 \\ * & * & * & * & * & -I \end{bmatrix}, \tag{3.6}$$

with

$$\Pi = \begin{bmatrix} I & 0 & \frac{1}{\tau}\widetilde{E}^T & 0 & 0 & 0 \\ 0 & I & -\frac{1}{\tau}\widetilde{E}^T & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}. \tag{3.7}$$

It follows from Schur complement that

$$\Phi < 0 \Longleftrightarrow \Psi < 0 \Longleftrightarrow \widetilde{Z} > 0, \qquad \Omega + \begin{bmatrix} \tau\widetilde{Y}^T - \widetilde{E}^T\widetilde{Z} \\ \tau\widetilde{W}^T + \widetilde{E}^T\widetilde{Z} \\ 0 \\ 0 \\ 0 \end{bmatrix} \left(\tau\widetilde{Z}\right)^{-1} \begin{bmatrix} \tau\widetilde{Y}^T - \widetilde{E}^T\widetilde{Z} \\ \tau\widetilde{W}^T + \widetilde{E}^T\widetilde{Z} \\ 0 \\ 0 \\ 0 \end{bmatrix}^T < 0. \tag{3.8}$$

If there exist $\widetilde{Q} > 0$, $\widetilde{Z} > 0$, $\widetilde{P}$, $\widetilde{Y}$, and $\widetilde{W}$ satisfying $\Phi < 0$, from (3.8) it is easy to see that the above $(\widetilde{Q}, \widetilde{Z}, \widetilde{P})$ is a feasible solution of $\Omega < 0$. Conversely, if there exist $\widetilde{Q} > 0$, $\widetilde{Z} > 0$ and $\widetilde{P}$ such that $\Omega < 0$ holds, via taking $\widetilde{Y} = (1/\tau)\widetilde{Z}\widetilde{E}$ and $\widetilde{W} = -(1/\tau)\widetilde{Z}\widetilde{E}$, $\Phi < 0$ is also feasible for the above $(\widetilde{Q}, \widetilde{Z}, \widetilde{P}, \widetilde{Y}, \widetilde{W})$. This completes the proof. □

The following corollary is easy to be obtained from Theorem 3.2.

Corollary 3.3. *The filter error system* (2.6) *is admissible if there exist matrices* $\widetilde{Q} > 0$, $\widetilde{Z} > 0$ *and* $\widetilde{P}$ *satisfying* (3.1) *and*

$$\begin{bmatrix} \widetilde{P}\widetilde{A} + \widetilde{A}^T\widetilde{P}^T + \widetilde{Q} - \dfrac{1}{\tau}\widetilde{E}^T\widetilde{Z}\widetilde{E} & \widetilde{P}\widetilde{A}_\tau + \dfrac{1}{\tau}\widetilde{E}^T\widetilde{Z}\widetilde{E} & \tau\widetilde{A}^T\widetilde{Z} \\ * & -\widetilde{Q} - \dfrac{1}{\tau}\widetilde{E}^T\widetilde{Z}\widetilde{E} & \tau\widetilde{A}_\tau^T\widetilde{Z} \\ * & * & -\tau\widetilde{Z} \end{bmatrix} < 0. \tag{3.9}$$

Remark 3.4. Theorem 3.2 can also be proved by employing the relationship of two integral inequalities concluded in [35]. In fact, we can see that Lemma 3.1 is obtained by using the integral inequality (7) in [35], while using the integral inequality (9) in [35] yields Theorem 3.2. As shown by [35], the upper bound provided by (9) in [35] is the least upper bound provided by (7) in [35]; therefore introducing more free matrices cannot reduce the conservativeness. Then, Theorem 3.2 can be obtained from Lemma 3.1, and the introduced slack variables $\widetilde{Y}$ and $\widetilde{W}$ in Lemma 3.1 are redundant variables. Hence, from the computational point of view, Theorem 3.2 is more "efficient" than Lemma 3.1.

In the sequel, based on Theorem 3.2, we are devoted to the design of the filter parameters A_f, $A_{\tau f}$, B_f, and C_f. Noticing that (3.4) is nonlinear about the unknown variables $\widetilde{A}$, $\widetilde{A}_\tau$, $\widetilde{P}$, and $\widetilde{Z}$, to reduce the number of the unknown variables, we can do as follows.

From (3.4) we know that

$$\begin{bmatrix} \widetilde{P}\widetilde{A} + \widetilde{A}^T\widetilde{P}^T + \widetilde{Q} - \dfrac{1}{\tau}\widetilde{E}^T\widetilde{Z}\widetilde{E} & \widetilde{P}\widetilde{A}_\tau + \dfrac{1}{\tau}\widetilde{E}^T\widetilde{Z}\widetilde{E} \\ * & -Q - \dfrac{1}{\tau}\widetilde{E}^T\widetilde{Z}\widetilde{E} \end{bmatrix} < 0. \tag{3.10}$$

Multiplying (3.10) by $[I \quad I]$ from the left and by $[I \quad I]^T$ from the right results in

$$\widetilde{P}\left(\widetilde{A} + \widetilde{A}_\tau\right) + \left(\widetilde{A} + \widetilde{A}_\tau\right)^T \widetilde{P}^T < 0, \tag{3.11}$$

which implies that $\widetilde{P}$ is nonsingular. Let

$$\widetilde{P} = \begin{bmatrix} P & P_2 \\ P_3 & P_4 \end{bmatrix}, \quad P \in R^{n\times n},\ P_i \in R^{n\times n},\ i = 2,3,4. \tag{3.12}$$

Without loss of generality, we can assume that $P, P_i, i = 2,3,4$, are all nonsingular [36]. Then, from (3.1), we have that

$$E^T P^T = PE, \qquad E^T P_3^T = P_2 E, \qquad E^T P_4^T = P_4 E. \tag{3.13}$$

Taking

$$T_1 = \begin{bmatrix} I & 0 \\ 0 & PP_3^{-1} \end{bmatrix}, \qquad T_2 = \begin{bmatrix} I & 0 \\ 0 & P_2^{-1}P \end{bmatrix}, \qquad T_3 = \operatorname{diag}\left\{T_1, T_1, I, T_2^T, I\right\} \tag{3.14}$$

and combining with (2.7) and (3.12), we obtain

$$
\begin{gathered}
\overline{E} = T_2^{-1}\widetilde{E}T_1^T = \begin{bmatrix} E & 0 \\ 0 & P^{-1}P_2EP_3^{-T}P^T \end{bmatrix} = \begin{bmatrix} E & 0 \\ 0 & P^{-1}E^TP_3^TP_3^{-T}P^T \end{bmatrix} = \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix}, \\
\overline{P} = T_1\widetilde{P}T_2 = \begin{bmatrix} P & P \\ P & PP_3^{-1}P_4P_2^{-1}P \end{bmatrix}, \\
\overline{A} = T_2^{-1}\widetilde{A}T_1^T = \begin{bmatrix} A & 0 \\ P^{-1}P_2B_fC & P^{-1}P_2A_fP_3^{-T}P^T \end{bmatrix} = \begin{bmatrix} A & 0 \\ \overline{B}_fC & \overline{A}_f \end{bmatrix}, \\
\overline{A}_\tau = T_2^{-1}\widetilde{A}_\tau T_1^T = \begin{bmatrix} A_\tau & 0 \\ 0 & P^{-1}P_2A_{\tau f}P_3^{-T}P^T \end{bmatrix} = \begin{bmatrix} A_\tau & 0 \\ 0 & \overline{A}_{\tau f} \end{bmatrix}, \\
\overline{B} = T_2^{-1}\widetilde{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \\
\overline{G} = \widetilde{G}T_1^{\mathrm{T}} = \begin{bmatrix} G & -C_fP_3^{-T}P^T \end{bmatrix} = \begin{bmatrix} G & -\overline{C}_f \end{bmatrix},
\end{gathered}
\tag{3.15}
$$

where

$$
\overline{A}_f = P^{-1}P_2A_fP_3^{-T}P^T, \qquad \overline{A}_{\tau f} = P^{-1}P_2A_{\tau f}P_3^{-T}P^T, \qquad \overline{B}_f = P^{-1}P_2B_f, \qquad \overline{C}_f = C_fP_3^{-T}P^T, \tag{3.16}
$$

and denote

$$
\overline{Q} = T_1\widetilde{Q}T_1^T, \qquad \overline{Z} = T_2^T\widetilde{Z}T_2. \tag{3.17}
$$

Premultiplying by T_1 and postmultiplying by T_1^T on both sides of (3.1), we have that

$$
T_1\widetilde{E}^TT_2^{-T}T_2^T\widetilde{P}^TT_1^T = T_1\widetilde{P}T_2T_2^{-1}\widetilde{E}T_1^T \geq 0, \tag{3.18}
$$

that is,

$$
\overline{E}^T\overline{P}^T = \overline{P}\,\overline{E} \geq 0. \tag{3.19}
$$

Multiplying (3.4) by T_3 from the left and by T_3^T from the right yields

$$\overline{\Omega} = \begin{bmatrix} \overline{P}\,\overline{A} + \overline{A}^T\overline{P}^T + \overline{Q} - \dfrac{1}{\tau}\overline{E}^T\overline{Z}\,\overline{E} & \overline{P}\,\overline{A}_\tau + \dfrac{1}{\tau}\overline{E}^T\overline{Z}\,\overline{E} & \overline{P}\,\overline{B} & \tau\overline{A}^T\overline{Z} & \overline{G}^T \\ * & -\overline{Q} - \dfrac{1}{\tau}\overline{E}^T\overline{Z}\,\overline{E} & 0 & \tau\overline{A}_\tau^T\overline{Z} & 0 \\ * & * & -\gamma^2 I & \tau\overline{B}^T\overline{Z} & 0 \\ * & * & * & -\tau\overline{Z} & 0 \\ * & * & * & * & -I \end{bmatrix} < 0. \tag{3.20}$$

It can be seen that the systems $(\widetilde{E}, \widetilde{A}, \widetilde{A}_\tau, \widetilde{B}, \widetilde{G})$ and $(\overline{E}, \overline{A}, \overline{A}_\tau, \overline{B}, \overline{G})$ are algebraically equivalent under the r.s.e. (restricted system equivalence) transformation, where T_2^{-1} and T_1^T are taken as the row full rank transformation matrix and the coordinate full rank transformation matrix, respectively, and comparing the coefficient matrices of the two systems, we can see that the difference between them is just the filter parameters $A_f,\ A_{\tau f},\ B_f,\ C_f$, and $\overline{A}_f,\ \overline{A}_{\tau f},\ \overline{B}_f,\ \overline{C}_f$. Moreover, in the r.s.e. transformation, the state and the equation of the filter change while the state and the equation of system (2.3) do not change. So, in the design of the filter, we can directly substitute $\overline{A}_f,\ \overline{A}_{\tau f},\ \overline{B}_f,\ \overline{C}_f$ for A_f, $A_{\tau f},\ B_f,\ C_f$. Noticing that

$$\begin{bmatrix} I & 0 \\ -P_3P^{-1} & I \end{bmatrix}\begin{bmatrix} P & P_2 \\ P_3 & P_4 \end{bmatrix}\begin{bmatrix} I & -P^{-1}P_2 \\ 0 & I \end{bmatrix} = \begin{bmatrix} P & 0 \\ 0 & P_4 - P_3P^{-1}P_2 \end{bmatrix}, \tag{3.21}$$

then $P_4 - P_3P^{-1}P_2$ is nonsingular. Let

$$PP_3^{-1}P_4P_2^{-1}P - P = PP_3^{-1}\left(P_4 - P_3P^{-1}P_2\right)P_2^{-1}P = S^{-1}; \tag{3.22}$$

then $\overline{P}$ can be written as

$$\overline{P} = \begin{bmatrix} P & P \\ P & P + S^{-1} \end{bmatrix}. \tag{3.23}$$

Denote

$$J^T = S + P^{-1}, \qquad T_4 = \begin{bmatrix} J^T & -S \\ I & 0 \end{bmatrix}, \qquad T_5 = T_4\overline{P}\,\overline{Z}^{-1}. \tag{3.24}$$

Since $P(S + P^{-1}) = (P + S^{-1})S = PP_3^{-1}P_4P_2^{-1}PS$ is nonsingular, J is also a nonsingular matrix.

From (3.19), we have that,

$$T_4\overline{E}^T\overline{P}^T T_4^T = T_4\overline{P}\,\overline{E}T_4^T \geq 0. \tag{3.25}$$

Noticing (3.23) and (3.24), we derive

$$T_4\overline{P} = \begin{bmatrix} I & 0 \\ P & P \end{bmatrix},$$

$$T_4\overline{P}\,\overline{E}T_4^T = \begin{bmatrix} EJ & E \\ PEJ - PES^T & PE \end{bmatrix} = \begin{bmatrix} EJ & E \\ PEP^{-T} & PE \end{bmatrix} = \begin{bmatrix} EJ & E \\ E^T & PE \end{bmatrix}, \tag{3.26}$$

$$T_4\overline{E}^T\overline{P}^T T_4^T = \begin{bmatrix} J^T E^T & E \\ E^T & E^T P^T \end{bmatrix};$$

then (3.25) is just

$$EJ = J^T E^T, \qquad PE = E^T P^T, \qquad \begin{bmatrix} EJ & E \\ E^T & PE \end{bmatrix} \geq 0. \tag{3.27}$$

Premultiplying by $\operatorname{diag}\{T_4, T_4, I, T_5, I\}$ and postmultiplying by $\operatorname{diag}\{T_4^T, T_4^T, I, T_5^T, I\}$ on both sides of (3.20), we have

$$\begin{bmatrix} \begin{matrix} T_4\overline{A}^T\overline{P}^T T_4^T + T_4\overline{P}\,\overline{A}T_4^T \\ +T_4\overline{Q}T_4^T - \dfrac{1}{\tau}T_4\overline{E}^T\overline{Z}\,\overline{E}T_4^T \end{matrix} & T_4\overline{P}\,\overline{A}_\tau T_4^T + \dfrac{1}{\tau}T_4\overline{E}^T\overline{Z}\,\overline{E}T_4^T & T_4\overline{P}\,\overline{B} & \tau T_4\overline{A}^T\overline{Z}T_5^T & T_4\overline{G}^T \\ * & -T_4\overline{Q}T_4^T - \dfrac{1}{\tau}T_4\overline{E}^T\overline{Z}\,\overline{E}T_4^T & 0 & \tau T_4\overline{A}_\tau^T\overline{Z}T_5^T & 0 \\ * & * & -\gamma^2 I & \tau\overline{B}^T\overline{Z}T_5^T & 0 \\ * & * & * & -\tau T_5\overline{Z}T_5^T & 0 \\ * & * & * & * & -I \end{bmatrix} < 0. \tag{3.28}$$

Noticing that

$$T_4\overline{P}\,\overline{A}T_4^T = \begin{bmatrix} AJ & A \\ PAJ + P\overline{B}_f CJ - P\overline{A}_f S^T & PA + P\overline{B}_f C \end{bmatrix},$$

$$T_4\overline{P}\,\overline{A}_\tau T_4^T = \begin{bmatrix} A_\tau J & A_\tau \\ PA_\tau J - P\overline{A}_{\tau f} S^T & PA_\tau \end{bmatrix},$$

$$T_4\overline{P}\,\overline{B} = \begin{bmatrix} B \\ PB \end{bmatrix}, \qquad T_4\overline{G}^T = \begin{bmatrix} J^T G^T + S\overline{C}_f^T \\ G^T \end{bmatrix},$$

$$
\begin{aligned}
T_4\overline{A}^T\overline{Z}T_5^T &= T_4\overline{A}^T\overline{Z}\,\overline{Z}^{-1}\overline{P}^TT_4^T = T_4\overline{A}^T\overline{P}^TT_4^T,\\
T_4\overline{A}_\tau^T\overline{Z}T_5^T &= T_4\overline{A}_\tau^T\overline{Z}\,\overline{Z}^{-1}\overline{P}^TT_4^T = T_4\overline{A}_\tau^T\overline{P}^TT_4^T,\\
\overline{B}^T\overline{Z}T_5^T &= \overline{B}^T\overline{Z}\,\overline{Z}^{-1}\overline{P}^TT_4^T = \overline{B}^T\overline{P}^TT_4^T,\\
T_5\overline{Z}T_5^T &= T_4\overline{P}\,\overline{Z}^{-1}\overline{Z}\,\overline{Z}^{-1}\overline{P}^TT_4^T = T_4\overline{P}\,\overline{Z}^{-1}\overline{P}^TT_4^T,
\end{aligned}
\tag{3.29}
$$

denote

$$Q = T_4\overline{Q}T_4^T = \begin{bmatrix} Q_1 & Q_2 \\ Q_2^T & Q_3 \end{bmatrix}, \tag{3.30}$$

$$Z = T_4\overline{P}\,\overline{Z}^{-1}\overline{P}^TT_4^T = \begin{bmatrix} Z_1 & Z_2 \\ Z_2^T & Z_3 \end{bmatrix}, \tag{3.31}$$

$$L = PAJ + P\overline{B}_fCJ - P\overline{A}_fS^T, \qquad L_\tau = PA_\tau J - P\overline{A}_{\tau f}S^T, \tag{3.32}$$

$$W_B = P\overline{B}_f, \qquad W_C = \overline{C}_fS^T. \tag{3.33}$$

Since (3.31) implies that $\overline{Z} = \overline{P}^TT_4^TZ^{-1}T_4\overline{P}$, then

$$T_4\overline{E}^T\overline{Z}\,\overline{E}T_4^T = T_4\overline{E}^T\overline{P}^TT_4^TZ^{-1}T_4\overline{P}\,\overline{E}T_4^T = \begin{bmatrix} EJ & E \\ E^T & PE \end{bmatrix} Z^{-1} \begin{bmatrix} EJ & E \\ E^T & PE \end{bmatrix}. \tag{3.34}$$

Introduce matrix $W = \begin{bmatrix} W_1 & W_2 \\ W_2^T & W_3 \end{bmatrix} \geq 0$ satisfying

$$\tau W \leq \begin{bmatrix} EJ & E \\ E^T & PE \end{bmatrix} Z^{-1} \begin{bmatrix} EJ & E \\ E^T & PE \end{bmatrix}, \tag{3.35}$$

then

$$\begin{bmatrix} -\dfrac{1}{\tau}T_4\overline{E}^T\overline{Z}\,\overline{E}T_4^T & \dfrac{1}{\tau}T_4\overline{E}^T\overline{Z}\,\overline{E}T_4^T \\ * & -\dfrac{1}{\tau}T_4\overline{E}^T\overline{Z}\,\overline{E}T_4^T \end{bmatrix} \leq \begin{bmatrix} -W & W \\ * & -W \end{bmatrix}; \tag{3.36}$$

Obviously, if there exist matrices $Q_1 > 0$, $Q_3 > 0, W_1 \geq 0$, $W_3 \geq 0, Z_1 > 0$, $Z_3 > 0, P,\ J,\ W_B,\ W_C,\ L,\ L_\tau,\ Q_2,\ W_2,$ and Z_2 with $P,\ J$ being nonsingular, satisfying (3.35) and

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & A_\tau J + W_1 & A_\tau + W_2 & B & \tau J^T A^T & \tau L^T & J^T G^T + W_c^T \\ * & \Xi_{22} & L_\tau + W_2^T & PA_\tau + W_3 & PB & \tau A^T & \tau A^T P^T + \tau C^T W_B^T & G^T \\ * & * & -Q_1 - W_1 & -Q_2 - W_2 & 0 & \tau J^T A_\tau^T & \tau L_\tau^T & 0 \\ * & * & * & -Q_3 - W_3 & 0 & \tau A_\tau^T & \tau A_\tau^T P^T & 0 \\ * & * & * & * & -\gamma^2 I & \tau B^T & \tau B^T P^T & 0 \\ * & * & * & * & * & -\tau Z_1 & -\tau Z_2 & 0 \\ * & * & * & * & * & * & -\tau Z_3 & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix} < 0 \quad (3.37)$$

with

$$\begin{gathered} \Xi_{11} = AJ + J^T A^T + Q_1 - W_1, \qquad \Xi_{12} = A + L^T + Q_2 - W_2, \\ \Xi_{22} = PA + A^T P^T + W_B C + C^T W_B^T + Q_3 - W_3, \end{gathered} \quad (3.38)$$

then taking

$$\begin{gathered} S = J^T - P^{-1}, \qquad \overline{B}_f = P^{-1} W_B, \qquad \overline{C}_f = W_C S^{-T}, \\ \overline{A}_f = P^{-1}(PAJ + W_B C J - L) S^{-T}, \qquad \overline{A}_{\tau f} = P^{-1}(PA_\tau J - L_\tau) S^{-T}, \end{gathered} \quad (3.39)$$

one obtains that there are solutions $\overline{Q} > 0$, $\overline{Z} > 0$, and $\overline{P}$ to (3.20).

Hence we get the following theorem for the design of the filter (2.4).

Theorem 3.5. *Given a scalar $\gamma > 0$, if there are matrices $Q_1 > 0$, $Q_3 > 0$, $W_1 \geq 0$, $W_3 \geq 0$, $Z_1 > 0$, $Z_3 > 0$, P, J, W_B, W_C, L, L_τ, Q_2, W_2, Z_2 with P, J being nonsingular, satisfying* (3.27), (3.35), *and* (3.37), *then the H_∞ filter of the form of* (2.4) *exists and the parameters are given by* (3.39).

Remark 3.6. It is worth noting that (3.35) is not an LMI. In order to use the LMI Toolbox in MATLAB to get the solutions, we can do as follows.

Assume that $E = \begin{bmatrix} I_p & 0 \\ 0 & 0 \end{bmatrix}$; otherwise, we can find nonsingular matrices M and N such that $MEN = \begin{bmatrix} I_p & 0 \\ 0 & 0 \end{bmatrix}$. It is worth noting that the feasibility of (3.27), (3.35), and (3.37) is not affected by the selection of M and N. Then, the matrices P, J satisfying (3.27) are of the forms

$$P = \begin{bmatrix} P_{11} & P_{12} \\ 0 & P_{22} \end{bmatrix}, \qquad J = \begin{bmatrix} J_{11} & 0 \\ J_{21} & J_{22} \end{bmatrix}, \quad P_{11} \in R^{p \times p},\ J_{11} \in R^{p \times p} \quad (3.40)$$

with

$$\begin{bmatrix} J_{11} & I \\ I & P_{11} \end{bmatrix} \geq 0. \quad (3.41)$$

Introduce another variable $U > 0$; then (3.35) can be replaced by

$$\tau W \leq \begin{bmatrix} EJ & E \\ E^T & PE \end{bmatrix} U \begin{bmatrix} EJ & E \\ E^T & PE \end{bmatrix}, \tag{3.42}$$

$$UZ = I. \tag{3.43}$$

Write U as

$$U = \begin{bmatrix} U_{11} & U_{12} & U_{13} & U_{14} \\ U_{12}^T & U_{22} & U_{23} & U_{24} \\ U_{13}^T & U_{23}^T & U_{33} & U_{34} \\ U_{14}^T & U_{24}^T & U_{34}^T & U_{44} \end{bmatrix} > 0, \tag{3.44}$$

where

$$U_{11} \in R^{p\times p}, \qquad U_{22} \in R^{(n-p)\times(n-p)}, \qquad U_{33} \in R^{p\times p}, \qquad U_{44} \in R^{(n-p)\times(n-p)}. \tag{3.45}$$

Noticing that

$$\begin{bmatrix} EJ & E \\ E^T & PE \end{bmatrix} U \begin{bmatrix} EJ & E \\ E^T & PE \end{bmatrix} = \begin{bmatrix} \Pi_{11} & 0 & \Pi_{13} & 0 \\ * & 0 & 0 & 0 \\ * & * & \Pi_{33} & 0 \\ * & * & * & 0 \end{bmatrix}, \tag{3.46}$$

where

$$\begin{aligned} \Pi_{11} &= J_{11}U_{11}J_{11} + U_{13}^T J_{11} + J_{11}U_{13} + U_{33}, \\ \Pi_{13} &= J_{11}U_{11} + U_{13}^T + J_{11}U_{13}P_{11} + U_{33}P_{11}, \\ \Pi_{33} &= U_{11} + P_{11}U_{13}^T + U_{13}P_{11} + P_{11}U_{33}P_{11}, \end{aligned} \tag{3.47}$$

we can assume that

$$W = \begin{bmatrix} W_{11} & 0 & W_{21} & 0 \\ 0 & 0 & 0 & 0 \\ W_{21}^T & 0 & W_{31} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \geq 0, \qquad \begin{bmatrix} W_{11} & W_{21} \\ W_{21}^T & W_{31} \end{bmatrix} > 0. \tag{3.48}$$

Then (3.42) is just

$$\tau\begin{bmatrix} W_{11} & W_{21} \\ W_{21}^T & W_{31} \end{bmatrix} \leq \begin{bmatrix} J_{11} & I \\ I & P_{11} \end{bmatrix}\begin{bmatrix} U_{11} & U_{13} \\ U_{13}^T & U_{33} \end{bmatrix}\begin{bmatrix} J_{11} & I \\ I & P_{11} \end{bmatrix}. \tag{3.49}$$

Invoking Schur complement again, we have that (3.49) is equivalent to

$$\begin{bmatrix} \begin{bmatrix} U_{11} & U_{13} \\ U_{13}^T & U_{33} \end{bmatrix} & \begin{bmatrix} J_{11} & I \\ I & P_{11} \end{bmatrix}^{-1} \\ \begin{bmatrix} J_{11} & I \\ I & P_{11} \end{bmatrix}^{-1} & \left(\tau\begin{bmatrix} W_{11} & W_{21} \\ W_{21}^T & W_{31} \end{bmatrix}\right)^{-1} \end{bmatrix} \geq 0. \tag{3.50}$$

Introducing

$$\alpha = \begin{bmatrix} \alpha_1 & \alpha_2 \\ \alpha_2^T & \alpha_3 \end{bmatrix} > 0, \qquad \theta = \begin{bmatrix} \theta_1 & \theta_2 \\ \theta_2^T & \theta_3 \end{bmatrix} > 0, \tag{3.51}$$

then (3.50) can be replaced by

$$\begin{bmatrix} U_{11} & U_{13} & \tau\alpha_1 & \tau\alpha_2 \\ * & U_{33} & \tau\alpha_2^T & \tau\alpha_3 \\ * & * & \tau\theta_1 & \tau\theta_2 \\ * & * & * & \tau\theta_3 \end{bmatrix} \geq 0, \tag{3.52}$$

$$\begin{bmatrix} J_{11} & I \\ I & P_{11} \end{bmatrix}\begin{bmatrix} \alpha_1 & \alpha_2 \\ \alpha_2^T & \alpha_3 \end{bmatrix} = I, \qquad \begin{bmatrix} W_{11} & W_{21} \\ W_{21}^T & W_{31} \end{bmatrix}\begin{bmatrix} \theta_1 & \theta_2 \\ \theta_2^T & \theta_3 \end{bmatrix} = I. \tag{3.53}$$

Therefore, one can consider the H_∞ filter design problem as the following cone complementary problems:

$$\text{Minimize}\left\{ \operatorname{tr}(UZ) + \operatorname{tr}\left(\begin{bmatrix} J_{11} & I \\ I & P_{11} \end{bmatrix}\begin{bmatrix} \alpha_1 & \alpha_2 \\ \alpha_2^T & \alpha_3 \end{bmatrix} + \begin{bmatrix} W_{11} & W_{21} \\ W_{21}^T & W_{31} \end{bmatrix}\begin{bmatrix} \theta_1 & \theta_2 \\ \theta_2^T & \theta_3 \end{bmatrix}\right)\right\} \tag{3.54}$$

subject to LMIs (3.30), (3.31), (3.37), (3.40), (3.41), (3.44), (3.48), (3.51), (3.52), and

$$Q > 0, \quad Z > 0, \qquad \begin{bmatrix} U & I \\ I & Z \end{bmatrix} \geq 0, \qquad \begin{bmatrix} \alpha_1 & \alpha_2 & I & 0 \\ \alpha_2^T & \alpha_3 & 0 & I \\ I & 0 & J_{11} & I \\ 0 & I & I & P_{11} \end{bmatrix} \geq 0, \qquad \begin{bmatrix} \theta_1 & \theta_2 & I & 0 \\ \theta_2^T & \theta_3 & 0 & I \\ I & 0 & W_{11} & W_{21} \\ 0 & I & W_{21}^T & W_{31} \end{bmatrix} \geq 0. \tag{3.55}$$

Then the filer (2.4) can be solved by using the iterative algorithm as [37], in the interests of economy, which is omitted here.

Remark 3.7. Since the filter (2.4) is designed with parameters (3.39) such that inequality (3.20) holds, we have

$$\begin{bmatrix} \overline{P}\,\overline{A}+\overline{A}^T\overline{P}^T+\overline{Q}-\frac{1}{\tau}\overline{E}^T\overline{Z}\,\overline{E} & \overline{P}\,\overline{A}_\tau+\frac{1}{\tau}\overline{E}^T\overline{Z}\,\overline{E} & \tau\overline{A}^T\overline{Z} \\ * & -\overline{Q}-\frac{1}{\tau}\overline{E}^T\overline{Z}\,\overline{E} & \tau\overline{A}_\tau^T\overline{Z} \\ * & * & -\tau\overline{Z} \end{bmatrix}<0. \tag{3.56}$$

By (3.15), (3.16), (3.17), (3.23) and letting $P_f := P+S^{-1}$, $\overline{Q}=\begin{bmatrix}\overline{Q}_1 & \overline{Q}_2\\ * & \overline{Q}_3\end{bmatrix}$, and $\overline{Z}=\begin{bmatrix}\overline{Z}_1 & \overline{Z}_2\\ * & \overline{Z}_3\end{bmatrix}$ with $\overline{Q}_1\in R^{n\times n}$ and $\overline{Z}_1\in R^{n\times n}$, we can conclude from (3.56) that

$$\begin{bmatrix} P_f\overline{A}_f+\overline{A}_f^TP_f^T+\overline{Q}_3-\frac{1}{\tau}E^T\overline{Z}_3E & P_f\overline{A}_{\tau f}+\frac{1}{\tau}E^T\overline{Z}_3E & \tau\overline{A}_f^T\overline{Z}_3 \\ * & -\overline{Q}_3-\frac{1}{\tau}E^T\overline{Z}_3E & \tau\overline{A}_{\tau f}^T\overline{Z}_3 \\ * & * & -\tau\overline{Z}_3 \end{bmatrix}<0. \tag{3.57}$$

In addition, (3.19) implies that

$$E^TP_f^T=P_fE\geq 0. \tag{3.58}$$

Invoking Corollary 3.3, it is obtained that the designed filter (2.4) is admissible, and then it is proper and can be realized in practice.

4. Numerical Examples

Example 4.1. Consider the singular time-delay system given in [25] without uncertainties and distributed delay and with

$$\begin{gathered} E=\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix},\qquad A=\begin{bmatrix}-2&0&0.5\\0.1&-0.9&0.2\\0&0.5&0.3\end{bmatrix},\qquad A_\tau=\begin{bmatrix}0.2&0.1&0\\0.2&0&0.15\\0.1&-0.23&0.1\end{bmatrix},\\ B=\begin{bmatrix}0.2&0&0\\0&0.2&0\\0&0&0.2\end{bmatrix},\qquad C=\begin{bmatrix}1&0&0\end{bmatrix},\qquad G=\begin{bmatrix}1&0.7&0.8\end{bmatrix}. \end{gathered} \tag{4.1}$$

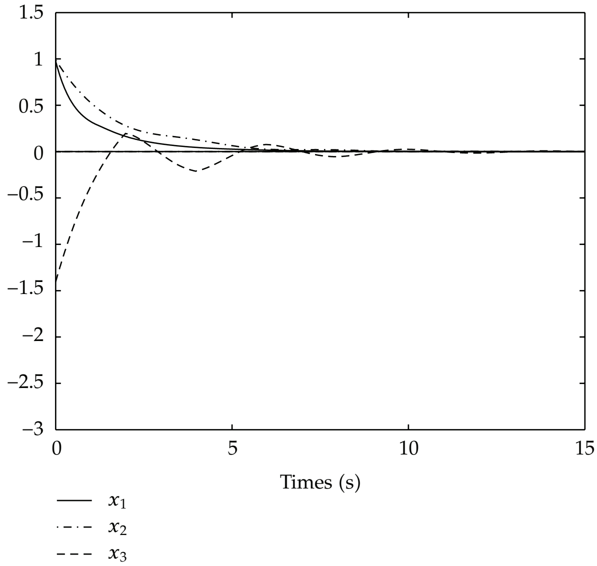

Figure 1: State responses $x(t)$ of the original system.

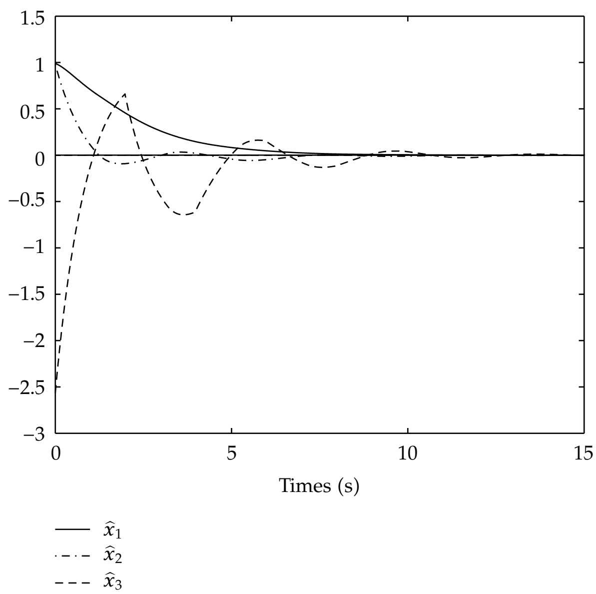

Figure 2: State responses $\hat{x}(t)$ of the filter system.

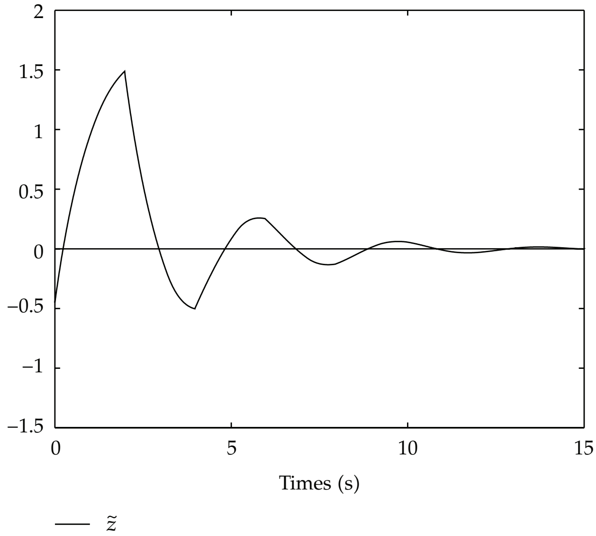

Figure 3: Error estimation signal $\tilde{z}(t)$ with the designed filter.

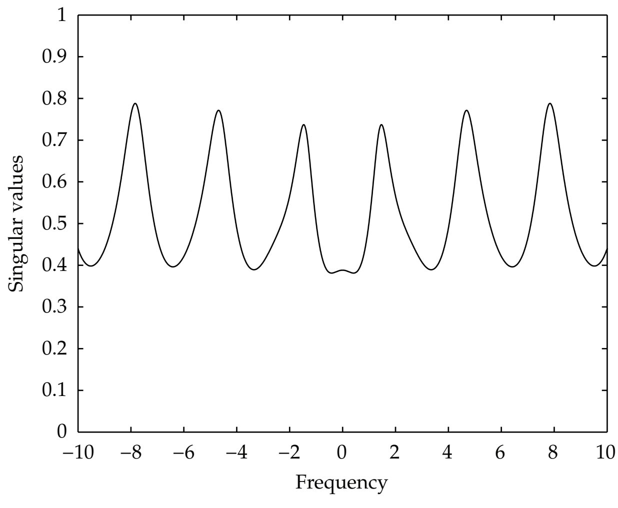

Figure 4: Singular value curve of the filtering error system.

By Theorem 3.2, for $\tau = 2$ and $\gamma = 1$, after 10 iterations, the corresponding filter is obtained with the following parameters:

$$A_f = \begin{bmatrix} -0.9429 & -0.0102 & 0.3206 \\ -0.1042 & -0.8493 & 0.1904 \\ 0.3750 & 0.4870 & 0.3369 \end{bmatrix}, \qquad A_{\tau f} = \begin{bmatrix} 0.0983 & 0.0960 & 0.0046 \\ 0.1248 & -0.0071 & 0.1461 \\ 0.0749 & -0.2438 & 0.0966 \end{bmatrix},$$
$$B_f = \begin{bmatrix} 0.6224 \\ -0.2077 \\ 0.4488 \end{bmatrix}, \qquad C_f = \begin{bmatrix} -0.8379 & -0.7382 & -0.9940 \end{bmatrix}. \tag{4.2}$$

With this filter, Figures 1, 2, and 3 show the state responses $x(t)$ of the original system, the state responses $\hat{x}(t)$ of the filter system, and the error estimation signal $\tilde{z}(t) = z(t) - \hat{z}(t)$ with the initial condition $\phi(t) = [1 \quad 1 \quad -1.425]^T$, $\psi(t) = [1 \quad 1 \quad -2.6341]^T$ for $t \in [-2, 0]$ and the exogenous disturbance input $w(t)$=diag$\{e^{-0.5t}, e^{-0.5t}, e^{-0.5t}\}$. By connecting the filter to the original system, the singular value curve of the resulting filtering error system is also plotted in Figure 4. We can see that all the maximum singular values are less than 1, which illustrate the effectiveness of the proposed method in this paper.

5. Conclusions and Future Works

In this paper, we have studied the H_∞ filtering problem for singular system with a constant discrete delay. Based on an improved BRL, a delay-dependent sufficient condition for the existence of the H_∞ filter with delay is obtained. Then, by using LMIs and the cone complementarity linearization iterative algorithm, the H_∞ filter is designed, which guarantees that the resulting error system is regular, impulse-free, internally stable, and the L_2-induced norm from the disturbance input to the filtering error output satisfies a prescribed H_∞ performance level. It can be seen that the designed filter in this paper is a full-order filter, that is, the finite mode of the filter is equal to rank E. To study the delay-dependent reduced-order H_∞ filtering problem for singular time-delay systems is the key research in the future.

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