Hindawi Publishing Corporation
Discrete Dynamics in Nature and Society
Volume 2011, Article ID 849342, 64 pages
doi:10.1155/2011/849342

*Research Article*

# Profit and Risk under Subprime Mortgage Securitization

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Received 7 March 2011; Accepted 29 April 2011

Academic Editor: Binggen Zhang



We investigate the securitization of subprime residential mortgage loans into structured products such as subprime residential mortgage-backed securities (RMBSs) and collateralized debt obligations (CDOs). Our deliberations focus on profit and risk in a discrete-time framework as they are related to RMBSs and RMBS CDOs. In this regard, profit is known to be an important indicator of financial health. With regard to risk, we discuss credit (including counterparty and default), market (including interest rate, price, and liquidity), operational (including house appraisal, valuation, and compensation), tranching (including maturity mismatch and synthetic) and systemic (including maturity transformation) risks. Also, we consider certain aspects of Basel regulation when securitization is taken into account. The main hypothesis of this paper is that the SMC was mainly caused by the intricacy and design of subprime mortgage securitization that led to information (asymmetry, contagion, inefficiency, and loss) problems, valuation opaqueness and ineffective risk mitigation. The aforementioned hypothesis is verified in a theoretical- and numerical-quantitative context and is illustrated via several examples.

## 1. Introduction

The mid-2007 to 2009 subprime mortgage crisis (SMC) was preceded by a decade of low interest rates that spurred significant increases in both the financing of residential mortgage loans—hereafter, simply called *mortgages*—and house prices. This environment encouraged investors (including investment banks) to pursue instruments that offer yield enhancement. In this regard, subprime mortgages generally offer higher yields than standard mortgages and consequently have been in demand for securitization. In essence, securitization offers the opportunity to transform below investment grade assets (the investment or reference portfolio) into AAA and investment grade liabilities. The demand for increasingly intricate structured mortgage products (SMPs) such as residential mortgage-backed securities (RMBSs) and collateralized debt obligations (CDOs) which embed leverage within their structureexposed investing banks—hereafter, called *investors*—to an elevated risk of default. In the light of relatively low interest rates, rising house prices and investment grade credit ratings (usually AAA) given by the credit rating agencies (CRAs), this risk was not considered to be excessive. A surety wrap—insurance purchased from a monoline insurer—may also be used to ensure such credit ratings.

The process of subprime mortgage securitization is briefly explained below. The first step is where mortgagors—many first-time buyers—or individuals wanting to refinance seeked to exploit the seeming advantages offered by subprime mortgages. Next, mortgage brokers entered the lucrative subprime market with mortgagors being charged high fees. Thirdly, originators offering mortgages solicited funding that was often provided by Wall Street money. After extending mortgages, these originators quickly sold them to dealer (investment) banks and associated special-purpose vehicles (SPVs) for more profits. In this way, originators outsourced credit risk while relying on income from securitization to fund new mortgages. The fourth step involved Wall Street dealer banks pooling risky mortgages that did not meet the standards of the government-sponsored enterprises (GSEs) such as Fannie Mae and Freddie Mac and sold them as "private label," nonagency securities. This is important because the structure of securitization will have special features reflecting the design of the reference mortgage portfolios. Fifthly, CRAs assisted dealer banks trading structured mortgage products (SMPs), so that these banks received the best possible bond ratings, earned exorbitant fees, and made SMPs attractive to investors including money market, mutual and pension funds. However, during the SMC, defaults on reference mortgage portfolios increased, and the appetite for SMPs decreased. The market for these securities came to a standstill. Originators no longer had access to funds raised from pooled mortgages. The wholesale lending market shrunk. Intra- and interday markets became volatile. In the sixth step, the SMPs were sold to investors worldwide thus distributing the risk.

The main hypothesis of this paper is that the SMC was mainly caused by the intricacy and design of subprime structures as well as mortgage origination and securitization that led to information (asymmetry, contagion, inefficiency, and loss) problems, valuation opaqueness and ineffective risk mitigation. More specifically, information was lost due to intricacy resulting from an inability to look through the chain of mortgages and SMPs—reference mortgage portfolios and RMBSs, ABS CDOs, structured investment vehicles (SIVs), etc. This situation was exacerbated by a lack of understanding of the uniqueness of subprime securities and their structural design. It is our opinion that the interlinked security designs that were necessary to make the subprime market operate resulted in information loss among investors as the chain of SMPs stretched longer and longer. Also, asymmetric information arose because investors could not penetrate the SMP portfolio far enough to make a determination of the risk exposure to the financial sector. An additional problem involves information contagion that played a crucial role in shaping defensive retrenchment in interbank as well as mortgage markets during the SMC. As far as valuation problems are concerned, in this contribution, problems with SMPs result from the dependence of valuation on house prices and its independence from the performance of the reference mortgage portfolios. Also, issues related to mortgage and investor valuation are considered. With regard to the latter, we identify a chain of valuations that starts with the valuation of mortgages and SMPs then proceeds to cash flow, profit, and capital valuation and ends up with the valuation of the investors themselves. Finally, we claim that the SMC primarily resulted from mortgage agents' appetite for rapid growth and search for high yields—both of which were very often pursued at the expense of risk mitigation practices. The subprime structure described above is unique to the SMC and will be elaborated upon in the sequel.

### *1.1. Literature Review*

The discussions above and subsequently in Sections 2, 3, 4, and 5 are supported by various strands of existing literature.

The paper [1] examines the different factors that have contributed to the SMC (see, also, [2, 3]). The topics that these papers have in common with our contribution are related to yield enhancement, investment management, agency problems, lax underwriting standards, CRA incentive problems, ineffective risk mitigation, market opaqueness, extant valuation model limitations, and structured product intricacy (see Sections 2 and 3 as well as [4] for more details). Furthermore, our paper discusses the aforementioned issues and offers recommendations to help avoid future crises as in [5, 6].

In [7], light is shed on subprime mortgagors, mortgage design, and their historical performance. Their discussions involve predatory borrowing and lending that are illustrated via real-life examples. The working paper [8] firstly quantifies how different determinants contributed to high delinquency and foreclosure rates for vintage 2006 mortgages—compare with examples in Sections 5.2 and 5.3. More specifically, they analyze mortgage quality as the performance of mortgages adjusted for differences in mortgagor characteristics (such as credit score, level of indebtedness, and ability to provide documentation), mortgage characteristics (such as product type, amortization term, mortgage amount, and interest rate), and subsequent house appreciation (see, also, [3, 4]). Their analysis suggests that different mortgage-level characteristics as well as low house price appreciation were quantitatively too small to explain the bad performance of 2006 mortgages (compare with Table 1 in Section 3). Secondly, they observed a deterioration in lending standards with a commensurate downward trend in mortgage quality and a decrease in the subprime-prime mortgage rate spread during the 2001–2006 period (refer, e.g., Section 5.3). Thirdly, Demyanyk and Van Hemert show that mortgage quality deterioration could have been detected before the SMC (we consider "before the SMC" to be the period prior to July 2007 and "during the SMC" to be the period between July 2007 and December 2009). "After the SMC" is the period subsequent to December 2009. (see, also, [5, 6]).

The literature about mortgage securitization and the SMC is growing with our contribution, for instance, having close connections with [7] where the key structural features of a typical mortgage securitization is presented (compare with Figure 1 in Section 1.2.4). Also, that paper demonstrates how CRAs assign credit ratings to asset-backed securities (ABSs) and how these agencies monitor the performance of reference mortgage portfolios (see Sections 2.1, 2.2, and 2.3). Furthermore, that paper discusses RMBS and CDO architecture and is related to [9] that illustrates how misapplied bond ratings caused RMBSs and ABS CDO market disruptions (see Sections 3.2, 3.3, and 3.4). In [8], it is shown that the subprime mortgage market deteriorated considerably subsequent to 2007 (see, also, [4]). We believe that mortgage standards became slack because securitization gave rise to moral hazard, since each link in the securitization chain made a profit while transferring associated credit risk to the next link (see, e.g., [10]). At the same time, some financial institutions retained significant amounts of the mortgages they originated, thereby retaining credit risk and so were less guilty of moral hazard (see, e.g., [11]). The increased distance between originators

**Table 1:** Global CDO issuance ($millions); source: [22].

| | Global CDO issuance ($millions) | | | | | |
|---|---|---|---|---|---|---|
| | Total issuance | Structured finance | Cash flow and hybrid | Synthetic funded | Arbitrage | Balance sheet |
| Q1:04 | 24 982.5 | NA | 18 807.8 | 6 174.7 | 23 157.5 | 1 825.0 |
| Q2:04 | 42 864.6 | NA | 25 786.7 | 17 074.9 | 39 715.5 | 3 146.1 |
| Q3:04 | 42 864.6 | NA | 36 106.9 | 5 329.7 | 38 207.7 | 3 878.8 |
| Q4:04 | 47 487.8 | NA | 38 829.9 | 8 657.9 | 45 917.8 | 1 569.9 |
| 2004 Tot. | 157 418.5 | NA | 119 531.3 | 37 237.2 | 146 998.5 | 10 419.8 |
| % of Tot. | | | 75.9% | 23.7% | 93.4% | 6.6% |
| Q1:05 | 49 610.2 | 28 171.1 | 40 843.9 | 8 766.3 | 43 758.8 | 5 851.4 |
| Q2:05 | 71 450.5 | 46 720.3 | 49 524.6 | 21 695.9 | 62 050.5 | 9 400.0 |
| Q3:05 | 52 007.2 | 34 517.5 | 44 253.1 | 7 754.1 | 49 636.7 | 2 370.5 |
| Q4:05 | 98 735.4 | 67 224.2 | 71 604.3 | 26 741.1 | 71 957.6 | 26 777.8 |
| 2005 Tot. | 271 803.3 | 176 639.1 | 206 225.9 | 64 957.4 | 227 403.6 | 44 399.7 |
| % of Tot. | | 65.0% | 75.9% | 23.9% | 83.7% | 16.3% |
| Q1:06 | 108 012.7 | 66 220.2 | 83 790.1 | 24 222.6 | 101 153.6 | 6 859.1 |
| Q2:06 | 124 977.9 | 65 019.6 | 97 260.3 | 24 808.4 | 102 564.6 | 22 413.3 |
| Q3:06 | 138 628.7 | 89 190.2 | 102 167.4 | 14 703.8 | 125 945.2 | 12 683.5 |
| Q4:06 | 180 090.3 | 93 663.2 | 131 525.1 | 25 307.9 | 142 534.3 | 37 556.0 |
| 2006 Tot. | 551 709.6 | 314 093.2 | 414 742.9 | 89 042.7 | 472 197.7 | 79 511.9 |
| % of Tot. | | 56.9% | 75.2% | 16.1% | 85.6% | 14.4% |
| Q1:07 | 186 467.6 | 101 074.9 | 140 319.1 | 27 426.2 | 156 792.0 | 29 675.6 |
| Q2:07 | 175 939.4 | 98 744.1 | 135 021.4 | 8 403.0 | 153 385.4 | 22 554.0 |
| Q3:07 | 93 063.6 | 40 136.8 | 56 053.3 | 5 198.9 | 86 331.4 | 6 732.2 |
| Q4:07 | 47 508.2 | 23 500.1 | 31 257.9 | 5 202.3 | 39 593.7 | 7 914.5 |
| 2007 Tot. | 502 978.8 | 263 455.9 | 362 651.7 | 46 230.4 | 436 102.5 | 66 8769.3 |
| % of Tot. | | 52.4% | 72.1% | 9.1% | 86.8% | 13.3% |
| Q1:08 | 12 846.4 | | 12 771.0 | 75.4 | 18 607.1 | 1 294.6 |
| Q2:08 | 16 924.9 | | 15 809.7 | 1 115.2 | 15 431.1 | 6 561.4 |
| Q3:08 | 11 875.0 | | 11 875.0 | — | 10 078.4 | 4 255.0 |
| Q4:08 | 3 290.1 | | 3 140.1 | 150.0 | 3 821.4 | 1 837.8 |
| 2008 Tot. | 44 936.4 | | 43 595.8 | 1 340.6 | 47 938.0 | 13 948.8 |
| % of Tot. | | 32.4% | 91.2.1% | 1.6% | 89.4% | 10.6% |
| Q1:09 | 296.3 | | 196.8 | 99.5 | 658.7 | 99.5 |
| Q2:09 | 1 345.5 | | 1 345.5 | — | 1 886.4 | — |
| Q3:09 | 442.9 | | 337.6 | 105.3 | 208.7 | 363.5 |
| Q4:09 | 730.5 | | 681.0 | 49.5 | 689.5 | 429.7 |
| 2009 Tot. | 2 815.2 | | 2 560.9 | 254.3 | 3 443.3 | 892.7 |
| % of Tot. | | 40.4% | 91.2.1% | 1.6% | 89.4% | 10.6% |
| Q1:10 | 2 420.8 | | 2 378.5 | 42.3 | — | 2 420.7 |
| Q2:10 | 1 655.8 | | 1 655.8 | — | 598.1 | 1 378.9 |
| Q3:10 | 2 002.7 | | 2 002.7 | — | 2002.7 | — |
| Q4:10 | | | — | — | — | — |
| 2010 Tot. | 6 079.3 | | 6 037.0 | 42.3 | 2 600.8 | 3 799.6 |
| % of Tot. | | 44.1% | 91.2.1% | 1.6% | 89.4% | 10.6% |

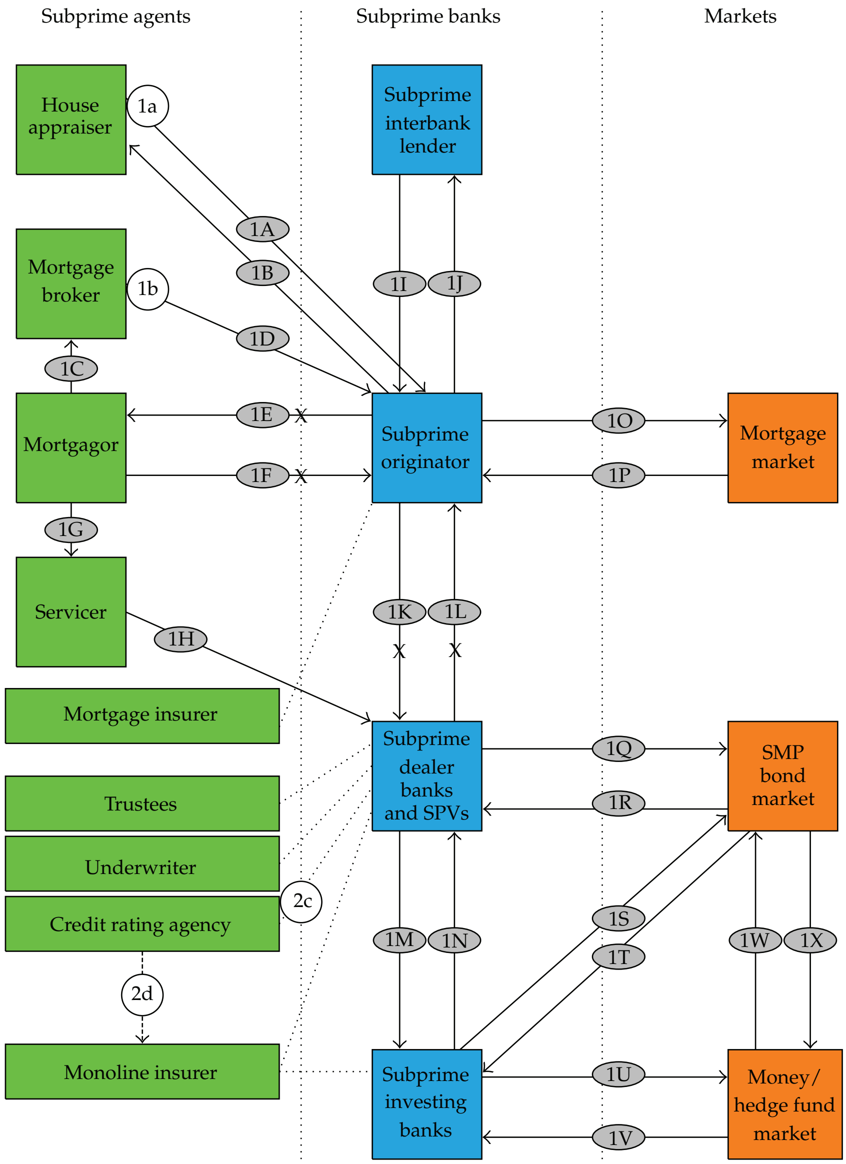


**Figure 1:** A subprime mortgage model with default.

and the ultimate bearers of risk potentially reduced originators' incentives to screen and monitor mortgages (see [12] for a preSMC description). As claimed in the present paper, the SMC and its impact on mortgage prices were magnified by the sale of SMPs. The enhanced intricacy of markets related to these products also reduces investor's ability to value them correctly where the value depends on the correlation structure of default events (see, e.g., [3, 11, 13]). Reference [14] considers parameter uncertainty and the credit risk associated with ABS CDOs (see, also, [4–6]). In [15] it is claimed that ABS CDOs opened up a whole new category of work for monoline insurers who insured the senior tranches of SMPs as part of the credit enhancement (CE) process (see, also, [4]). The working paper [1] asserts that, since the end of 2007, monoline insurers have been struggling to keep their AAA rating (compare with Figure 1 in Section 1.2.4). By the end of 2009, only MBIA and Ambac as well as a few others less exposed to mortgages such as financial security assurance (FSA) and assured guaranty, have been able to inject enough new capital to keep their AAA credit rating. In our paper, the effect of monoline insurance is tracked via the term $c^{i\Sigma}$ (see (2.1) for an example).

Before the SMC, risk management and control put excessive confidence in credit ratings provided by CRAs and failed to provide their own analysis of credit risks in the underlying securities (see, e.g., [16]). The paper [17] investigates the anatomy of the SMC that involves mortgages and their securitization with operational risk as the main issue. At almost every stage in the subprime process—from mortgage origination to securitization—operational risk was insiduously present but not always acknowledged or understood. For instance, when originators originated mortgages, they were outsourcing their credit risk to investors, but what they were left with evolved into something much larger—significant operational and reputational risk (see, e.g., [17]). Before the SMC, the quantity of mortgages originated was more important than their quality while an increased number of mortgages were originated that contained resets. The underwriting of new mortgages embeds credit and operational risk. House prices started to decline and default rates increased dramatically. Also, credit risk was outsourced via mortgage securitization which, in turn, funded new mortgage originations. Securitization of mortgages involves operational, tranching, and liquidity risk. During the SMC, the value of these securities decreased as default rates increased dramatically. The RMBS market froze and returns from these securities were cut off with mortgages no longer being funded. Financial markets became unstable with a commensurate increase in market risk which led to a collapse of the whole financial system (compare with Sections 2.1 and 3.2). The paper [16] discusses several aspects of systemic risk. Firstly, there was excessive maturity transformation through conduits and SIVs—this ended in August 2007. The overhang of SIV ABSs subsequently put additional downward pressure on securities prices. Secondly, as the financial system adjusted to mortgage delinquencies and defaults and to maturity transformation dysfunction, the interplay of market malfunctioning or even breakdown, fair value accounting and the insufficiency of equity capital at financial institutions, and, finally, systemic effects of prudential regulation created a detrimental downward spiral in the overall banking system. Also, [16] argues that these developments have not only been caused by identifiably faulty decisions, but also by flaws in financial system architecture. We agree with this paper that regulatory reform must go beyond considerations of individual incentives and supervision and pay attention to issues of systemic interdependence and transparency. The aforementioned paper also discusses credit, market, and tranching (including maturity mismatch) risks. Furthermore, [4, 18] provides further information about subprime risks such as credit (including counterparty and default), market (including interest rate, price, and liquidity), operational (including house appraisal, valuation, and compensation), tranching (including maturity mismatch and synthetic) and systemic (including maturity transformation) risks (see, the discussion in Section 2.1.1).

Our hypothesis involves the intricacy and design of mortgage origination, securitization and systemic agents as well as information (loss, asymmetry and contagion) problems, valuation opaqueness and ineffective risk mitigation. In this regard, [19] investigates the effects of agency and information asymmetry issues embedded in structural form credit models on bank credit risk evaluation, using American bank data from 2001 to 2005. Findings show that both the agency problem and information asymmetry significantly cause deviations in the credit risk evaluation of structural form models from agency credit ratings (see, also, [3, 16]). Additionally, the aforementioned papers involve both the effects of information asymmetry and debt-equity agency positively relate to the deviation while that of management-equity agency relates to it negatively. The paper [20] is specifically focussed on the issue of counterparty risk and claim that the effects on counterparties in the SMC are remarkably small (see, e.g., Section 2.1.1).

## 1.2. Preliminaries about Subprime Mortgages and Their Securitization

In this subsection, we provide preliminaries about mortgages and risks as well as a subprime mortgage model that describes the main subprime agents. All events take place in either period $t$ or $t+1$.

### 1.2.1. Preliminaries about Subprime Mortgages

Subprime mortgages are financial innovations that aim to provide house ownership to riskier mortgagors. A design feature of these mortgages is that over short periods, mortgagors are able to finance and refinance their houses based on gains from house price appreciation (see [4] for more details). House appraisals were often inflated with originators having too much influence on appraisal companies. No-income-verification, mortgages led to increased cases of fraud and contain resets. Before, during and after the SMC, mortgage brokers were compensated on volume rather than mortgage quality. This increased volume led to a poor credit culture. Before the SMC, house values started to decline. Mortgagors were unable to meet mortgage terms when they reset resulting in increased defaults.

A traditional mortgage model for profit with mortgages at face value is built by considering the difference between cash inflow and outflow in [4] (compare with [21]). For this profit, in period $t$, *cash inflow* is constituted by returns on risky marketable securities, $r_t^B B_t$, mortgages, $r_t^M M_t$ and Treasuries, $r_t^{\mathtt{T}} \mathtt{T}_t$. Furthermore, we denote the recovery amount, mortgage insurance payments per loss and present value of future profits from additional mortgages based on current mortgages by $R_t$, $C(S(\mathcal{C}_t))$, and $\Pi_t^p$, respectively. Also, we consider the cost of funds for $M$, $\bar{c}^{M\omega} M_t$, face value of mortgages in default, $r^S M_t$, recovery value of mortgages in default, $r^R M_t$, mortgage insurance premium, $p^i(\mathcal{C}_t) M_t$, the all-in cost of holding risky marketable securities, $c_t^B B_t$, interest paid to depositors, $r_t^D D_t$, cost of taking deposits, $c^D D_t$, interest paid to investors, $r_t^{\mathtt{B}} \mathtt{B}_t$, the cost of borrowing, $c^{\mathtt{B}} \mathtt{B}_t$, provisions against deposit withdrawals, $P^{\mathtt{T}}(\mathtt{T}_t)$, and the value of mortgage losses, $S(\mathcal{C}_t)$, to collectively comprise *cash outflow*. Here $r^D$ and $c^D$ are the deposit rate and marginal cost of deposits, respectively, while $r^{\mathtt{B}}$ and $c^{\mathtt{B}}$ are the borrower rate and marginal cost of borrowing, respectively. In this case, we have that a traditional model for profit with defaulting, refinancing, and fully amortizing mortgages at face value may be expressed as

$$\begin{aligned}\Pi_t = {} & \left(r_t^M - \bar{c}_t^{M\omega} - p_t^i + c_t^p r_t^f - \left(1 - r_t^R\right) r_t^S\right) M_t + C(\mathbf{E}[S(\mathcal{C}_t)]) \\ & + \left(r_t^B - c_t^B\right) B_t + r_t^{\mathtt{T}} \mathtt{T}_t - P^{\mathtt{T}}(\mathtt{T}_t) - \left(r_t^D + c_t^D\right) D_t - \left(r_t^{\mathtt{B}} + c^{\mathtt{B}}\right) \mathtt{B}_t + \Pi_t^p.\end{aligned} \tag{1.1}$$

From [4], the originator's balance sheet with mortgages at face value may be represented as

$$M_t + B_t + \mathtt{T}_t = (1 - \gamma) D_t + \mathtt{B}_t + K_t. \tag{1.2}$$

Also, the originator's *total capital constraint* for mortgages at face value is given by

$$K_t = n_t E_{t-1} + O_t \geq \rho\Big[\omega(\mathcal{C}_t)M_t + \omega^B B_t + 12.5 f^M(m\text{VaR} + \mathtt{O})\Big], \tag{1.3}$$

where $\omega(\mathcal{C}_t)$ and $\omega^B$ are the risk weights related to $M$ and $B$, respectively, while $\rho$—Basel II pegs $\rho$ at approximately 0.08—is the Basel capital regulation (by *Basel capital regulation*, we mean the regulatory capital framework set out by Basel II and beyond) ratio of regulatory capital to risk weighted assets. Furthermore, for the function

$$\begin{aligned} J_t = {} & \Pi_t + l_t\Big[K_t - \rho\Big(\omega(\mathcal{C}_t)M_t + \omega^B B_t + 12.5 f^M(m\text{VaR} + \mathtt{O})\Big)\Big] - c_t^{dw}[K_{t+1}] \\ & + \mathbf{E}_t[\delta_{t,1} V(K_{t+1}, x_{t+1})], \end{aligned} \tag{1.4}$$

the optimal originator valuation problem is to maximize its value by choosing $r^M$, $D$, $\mathtt{T}$, and $K$, for

$$V(K_t, x_t) = \max_{r_t^M, D_t, \mathtt{T}_t} J_t, \tag{1.5}$$

subject to mortgage, cash flow, balance sheet, and financing constraints given by

$$M_t = m_0 - m_1 r_t^M + m_2 \mathcal{C}_t + \sigma_t^M, \tag{1.6}$$

equations (1.1), (1.2), and

$$K_{t+1} = n_t(d_t + E_t) + \left(1 + r_t^O\right)O_t - \Pi_t + \Delta F_t, \tag{1.7}$$

respectively. In the value function, $l_t$ is the Lagrange multiplier for the capital constraint, $c_t^{dw}$ is the deadweight cost of capital, and $\delta_{t,1}$ is a stochastic discount factor. In the profit function, $\overline{c}^{\Lambda\omega}$ is the constant marginal cost of mortgages (including the cost of monitoring and screening). In each period, banks invest in fixed assets (including buildings and equipment) which we denote by $F_t$. The originator is assumed to maintain these assets throughout its existence, so that it must only cover the costs related to the *depreciation of fixed assets*, $\Delta F_t$. These activities are financed through retaining earnings and eliciting additional debt and equity, $E_t$, so that

$$\Delta F_t = E_t^r + (n_{t+1} - n_t)E_t + O_{t+1}. \tag{1.8}$$

Suppose that $J$ and $V$ are given by (1.4) and (1.5), respectively. When the capital constraint given by (1.3) holds (i.e., $l_t > 0$), a solution to the originator's optimal valuation problem yields an optimal $M$ and $r^M$ of the form

$$M_t^* = \frac{K_t}{\rho\omega(\mathcal{C}_t)} - \frac{\omega^B B_t + 12.5 f^M (m\mathrm{VaR} + 0)}{\omega(\mathcal{C}_t)}, \tag{1.9}$$

$$r_t^{M*} = \frac{1}{m_1}\left(m_0 + m_2\mathcal{C}_t + \sigma_t^M - M_t^*\right), \tag{1.10}$$

respectively. In this case, the originator's corresponding optimal deposits, provisions for deposit withdrawals, and profits are given by

$$\begin{aligned} D_t^* = \frac{1}{1-\gamma}\Bigg(&\overline{D} + \frac{\overline{D}}{r_t^p}\left[r_t^{\mathsf{T}} + \left(r_t^B - c_t^B\right) + \left(r_t^{\mathsf{B}} + c_t^{\mathsf{B}}\right) - \frac{1}{1-\gamma}\left(r_t^D + c_t^D\right)\right] \\ &+ \frac{K_t}{\rho\omega(\mathcal{C}_t)} - \frac{\omega^B B_t + 12.5 f^M (m\mathrm{VaR} + 0)}{\omega(\mathcal{C}_t)} + B_t - K_t - \mathsf{B}_t\Bigg), \end{aligned} \tag{1.11}$$

$$\mathsf{T}_t^* = \overline{D} + \frac{\overline{D}}{r_t^p}\left[r_t^{\mathsf{T}} + \left(r_t^B - c_t^B\right) + \left(r_t^{\mathsf{B}} + c_t^{\mathsf{B}}\right) - \frac{1}{1-\gamma}\left(r_t^D + c_t^D\right)\right], \tag{1.12}$$

$$\begin{aligned} \Pi_t^* = &\left(\frac{K_t}{\rho\omega(\mathcal{C}_t)} - \frac{\omega^B B_t + 12.5 f^M (m\mathrm{VaR} + 0)}{\omega(\mathcal{C}_t)}\right) \\ &\times \Bigg\{\frac{1}{m_1}\left(m_0 - \frac{K_t}{\rho\omega(\mathcal{C}_t)} + \frac{\omega^B B_t + 12.5 f^M (m\mathrm{VaR} + 0)}{\omega(\mathcal{C}_t)} + m_2\mathcal{C}_t + \sigma_t^M\right) \\ &\qquad - \left(\overline{c}_t^{M\omega} - c_t^p r_t^f + p_t^i + \left(1 - r_t^R\right) r_t^S + \left(r_t^D + c_t^D\right)\frac{1}{(1-\gamma)}\right)\Bigg\} \\ &- \left(\left(r_t^D + c_t^D\right)\frac{1}{(1-\gamma)}\right)(B_t - K_t - \mathsf{B}_t) \\ &+ \left(\overline{D} + \frac{\overline{D}}{r_t^p}\left[r_t^{\mathsf{T}} + \left(r_t^B - c_t^B\right) + \left(r_t^{\mathsf{B}} + c_t^{\mathsf{B}}\right) - \frac{1}{1-\gamma}\left(r_t^D + c_t^D\right)\right]\right)\left(r_t^{\mathsf{T}} - \left(r_t^D + c_t^D\right)\frac{1}{(1-\gamma)}\right) \\ &+ \left(r_t^B - c_t^B\right)B_t - \left(r_t^{\mathsf{B}} + c_t^{\mathsf{B}}\right)\mathsf{B}_t + C(\mathbb{E}[S(\mathcal{C}_t)]) - P^{\mathsf{T}}(\mathsf{T}_t) + \Pi_t^p, \end{aligned} \tag{1.13}$$

respectively.

### 1.2.2. *Preliminaries about Residential Mortgage-Backed Securities (RMBSs)*

In this subsection, we discuss the main design features of subprime RMBSs. In particular, we provide a description of SPVs, cost of mortgages, default, collateral, adverse selection, and residual value (see [4] for more details). Further discussions of these features are included in Section 2.

In terms of the organization of the SPV, there are various states that can be associated with corporate forms such as a trust (denoted by $\mathtt{E}^1$), a limited liability corporation (LLC; denoted by $\mathtt{E}^2$), LLP ($\mathtt{E}^3$) or a C-corporation ($\mathtt{E}^4$). Our interest is mainly in $\mathtt{E}^1$ trusts and $\mathtt{E}^2$ LLCs that have their own unique tax benefits and challenges as well as degree of mortgage protection and legal limited liability. For our purposes, the optimal state during the lifetime of $\mathtt{E}^i$, $i \in \{1, 2\}$ is denoted by $\mathtt{E}^*$, such that the deviation from $\mathtt{E}^*$ is given by

$$\left|\mathtt{E}_t^i - \mathtt{E}^*\right|. \tag{1.14}$$

These deviations measure the loss and opportunity costs arising from the use of suboptimal corporate structures such as SPVs. Usually such loss is in the form of increased legal fees, losses due to low limited liability, and the value of additional time spent in dispute resolution. In this case, the formula for $\mathtt{E}$ is given by

$$\mathtt{E}_t = \max\left\{\left|\mathtt{E}_t^i - \mathtt{E}^*\right|,\ 0\right\}. \tag{1.15}$$

Banks may monitor mortgagor activities to see whether they are complying with the restrictive agreements and enforce the agreements if they are not by making sure that mortgagors are not taking on risks at their expense. Securitization dissociates the quality of the original mortgage portfolio from the quality of the cash flows from the reference mortgage portfolio, $f^{\Sigma}M$, to investors, where $f^{\Sigma}$ is the fraction of the face value of mortgages, $M$, that is securitized. Whether on-balance sheet or in the market, the weighted average of the cost of mortgages summarizes the cost of various funding solutions. It is the weighted average of cost of equity and debt on the originator's balance sheet and the weighted average of the costs of securitizing various mortgages. In both cases, we use the familiar *weighted average cost of capital*. As a consequence, cost of funds via securitization, $\overline{c}^{M\Sigma\omega}$ (includes monitoring and transaction costs for $f^{\Sigma}M$ denoted by $c^{m\Sigma}$ and $c^{t\Sigma}$, respectively, as well as the cost of funds in the market through securitizing mortgages), does not have to coincide with the originator's cost of mortgages for $M$, $\overline{c}^{M\omega}$ (includes monitoring and transaction costs for $M$ denoted by $c^m$ and $c^t$, resp.). We note that $c^t$ may include overhead, fixed costs, and variable costs per transaction expressed as a percentage of $M$. This suggests that if

$$\overline{c}^{M\Sigma\omega} < \overline{c}^{M\omega}, \tag{1.16}$$

then the securitization economics is favorable and conversely.

In the sequel, the notation $r_t^{S\Sigma}$ represents the default rate on securitized mortgages, $f_t^{\Sigma}$ denotes the fraction of $M$ that is securitized, while $\widehat{f}_t^{\Sigma}$ denotes the fraction of the originator's reference mortgage portfolio realized as new mortgages in securitization as a result of, for instance, equity extraction via refinancing. In the sequel, collateral is constituted by the reference mortgage portfolio that is securitized and relates to the underlying cash flows. Such flow and credit characteristics of the collateral will determine the performance of the securities and drive the structuring process. Although a wide variety of assets may serve as collateral for securitization, mortgages are the most widely used form of collateral. In the case of a mortgage secured by collateral, if the mortgagor fails to make required payments, the originator has the right to seize and sell the collateral to recover the defaulted amount.

RMBSs mainly use one or both of the sen/sub-shifting of interest structure, sometimes called the *6-pack* structure (with 3 mezz and 3 sub-RMBS bonds junior to the AAA bonds), or an XS/OC structure (see, e.g., [3]). Here, XS and OC denote *excess spread* and *overcollaterization*, respectively. Like sen/sub deals, XS is used to increase OC, by accelerating payments on sen RMBS bonds via sequential amortization—a process known as *turboing*. An *OC target*, $\widetilde{O}^c$, is a fraction of the original mortgage par, $M$, and is designed to be in the second loss position against collateral losses with the interest-only (IO) strip being first. Typically, the initial OC amount, $O^{ci}$, is less than 100% of $\widetilde{O}^c$ and it is then increased over time via the XS until $\widetilde{O}^c$ is reached. When this happens, the OC is said to be *fully funded* and nett interest margin securities can begin to receive cash flows from the RMBS bond deal. Once $\widetilde{O}^c$ has been reached, and subject to certain performance tests, XS can be released for other purposes, including payment to residual (*residual value* is the payout received by the RMBS bond holder—in our case the investor—when bonds have been paid off and cash flows from the reference mortgage portfolio (collateral) are still being generated. Residual value also arises when the proceeds amount from the sale of this reference portfolio as whole mortgages is greater than the amount needed to pay outstanding bonds.) bond holders. In this contribution, we assume that the investor is also a residual bond holder. For our purposes, the symbol $\overline{r}_t^r$, represents the average residual rate in a period $t$ securitization. It is defined as the difference between the average interest rate paid by mortgagors, $\overline{r}^M$, and the present value of interest paid on securitized mortgages, $r^{p\Sigma}$, so that

$$r^r = \overline{r}^M - r^{p\Sigma}. \tag{1.17}$$

*Adverse selection* is the problem created by asymmetric information in originator's mortgage originations. It occurs because high-risk (e.g., subprime) mortgagors that are most likely to default on their mortgages, usually apply for them. In other words, subprime mortgages are extended to mortgagors who are most likely to produce an adverse outcome. We denote the value of the adverse selection problem by $V^a$. For the sake of argument, we set

$$V_t^a = a f_t^\Sigma M_t, \tag{1.18}$$

where $V^a$ is a fraction $a$, of the face value of mortgages in period $t$.

### *1.2.3. Preliminaries about Collateralized Debt Obligations (CDOs)*

RMBS CDOs are sliced into tranches of differing risk-return profiles. SIVs assist hedge funds and banks to pool a number of single RMBS tranches to create one CDO. As with RMBSs, risk associated with CDOs is shifted from sen to subtranches. The funds generated by the sale of CDOs enable CDO issuers to continue to underwrite the securitization of subprime mortgages or continue to purchase RMBSs. Before the SMC, major depository banks around the world used financial innovations such as SIVs to circumvent capital ratio regulations. This type of activity resulted in the failure of Northern Rock, which was nationalized at an estimated cost of $150 billion.

Certain features of RMBS CDOs make their design more intricate (compare with Question 3). For instance, many such CDOs are managed by managers that are to a limited extent allowed to buy and sell RMBS bonds over a given period of time. The reason for this is that CDOs amortize with a longer maturity able to be achieved by reinvestment. In particular, managers are able to use cash that is paid to the CDO from amortization for reinvestment. Under the conditions outlined in Section 1.2, they can sell bonds in the portfolio and buy other bonds with restrictions on the portfolio that must be maintained. CDO managers typically owned all or part of the CDO equity, so they would benefit from higher yielding assets for a given liability structure. In short, CDOs are managed funds with term financing and some constraints on the manager in terms of trading and the portfolio composition. Further discussion of RMBS CDOs is provided in Section 3.

Table 1 below elucidates CDO issuance with Column 1 showing total issuance of CDOs while the next column presents total issuance of RMBS CDOs. This table suggests that CDO issuance has been significant both before and after the SMC with the majority being CDOs with structured notes as collateral. In addition, Table 1 suggests that the motivation for CDO issuance has primarily been arbitrage.

From Table 1, we note that issuance of RMBS CDOs roughly tripled over the period 2005–07 and RMBS CDO portfolios became increasingly concentrated in subprime RMBSs. In this regard, by 2005, spreads on subprime BBB tranches seemed to be wider than other structure mortgage products with the same rating, creating an incentive to arbitrage the ratings between subprime RMBS and CDO tranches ratings. Subprime RMBSs increasingly dominated CDO portfolios, suggesting that the pricing of risk was inconsistent with the ratings. Also, concerning the higher-rated tranches, CDOs may have been motivated to buy large amounts of structured mortgage products, because their AAA tranches would input profitable negative basis trades (According to [3], in a negative basis trade, a bank buys the AAA-rated CDO tranche while simultaneously purchasing protection on the tranche under a physically settled CDS. From the bank's viewpoint, this is the simultaneous purchase and sale of a CDO, which meant that the bank lender could book the nett present value (NPV) of the excess yield on the CDO tranche over the protection payment on the CDS. If the CDS spread is less than the bond spread, the basis is negative. An example of this is given below. Suppose the bank borrows at LIBOR + 5 and buys an AAA-rated CDO tranche which pays LIBOR + 30. Simultaneously, the investor buys protection for 15 bps (basis points). So the investor makes 25 bps over LIBOR nett on the asset, and they have 15 bps in costs for protection, for a 10 bps profit. Note that a negative basis trade swaps the risk of the AAA tranche to a CDS protection writer. Now, the subprime-related risk has been separated from the cash host. Consequently, even if we were able to locate the AAA CDO tranches, this would not be the same as finding out the location of the risk. Refernce [3] suggests that nobody knows the extent of negative basis trades.) As a consequence, the willingness of CDOs to purchase subprime RMBS bonds increased. In the period 2008-2009, during the height of the SMC, there was a dramatic decrease in CDO issuance. During Q1:10 there was a marked increase in RMBS CDO issuance by comparison with Q3:09 and Q4:09 indicating an improvement in the CDO market.

Table 2 shows estimates of the typical collateral composition of sen and mezz RMBS CDOs before the SMC. It is clear that subprime and other RMBS tranches make up a sizeable percentage of both these tranche types.

Table 3 below demonstrates that increased volumes of origination in the mortgage market led to an increase in subprime RMBSs as well as CDO issuance.

**Table 2:** Typical collateral composition of RMBS CDOs (%); source: Citigroup.

| Typical collateral composition of RMBS CDOs (%) | | |
|---|---|---|
| | High-grade RMBS CDOs | Mezzanine RMBS CDOs |
| Subprime RMBS tranches | 50% | 77% |
| Other RMBS tranches | 25 | 12 |
| CDO tranches | 19 | 6 |
| Other | 6 | 5 |

**Table 3:** Subprime-related CDO volumes; source: [23].

| Subprime-related CDO volumes | | | |
|---|---|---|---|
| Vintage | Mezz RMBS CDOs | High srade RMBS CDOs | All CDOs |
| 2005 | 27 | 50 | 290 |
| 2006 | 50 | 100 | 468 |
| 2007 | 30 | 70 | 330 |
| 2008 | 30 | 70 | 330 |

### *1.2.4. Preliminaries about Subprime Mortgage Models*

We introduce a subprime mortgage model with default to encapsulate the key aspects of mortgage securitization.

Figure 1 presents a subprime mortgage model involving nine subprime agents, four subprime banks, and three types of markets. As far as subprime agents are concerned, we note that circles 1a, 1b, 1c, and 1d represent flawed independent assessments by house appraisers, mortgage brokers, CRAs rating SPVs, and monoline insurers being rated by CRAs, respectively. Regarding the former agent, the process of mortgage origination is flawed with house appraisers not performing their duties with integrity and independence. According to [17], this type of fraud is the “linchpin of the house buying transaction” and is an example of operational risk. Also, the symbol X indicates that the cash flow stops as a consequence of defaults. Before the SMC, appraisers estimated house values based on data that showed that the house market would continue to grow (compare with 1A and 1B). In steps 1C and 1D, independent mortgage brokers arrange mortgage deals and perform checks of their own while originators originate mortgages in 1E. Subprime mortgagors generally pay high mortgage interest rates to compensate for their increased risk from poor credit histories (compare with 1F). Next, the servicer collects monthly payments from mortgagors and remits payments to dealers and SPVs. In this regard, 1G is the mortgage interest rate paid by mortgagors to the servicer of the reference mortgage portfolios, while the interest rate 1H (mortgage interest rate minus the servicing fee) is passed by the servicer to the SPV for the payout to investors. Originator mortgage insurers compensate originators for losses due to mortgage defaults. Several subprime agents interact with the SPV. For instance, the trustee holds or manages and invests in mortgages and SMPs for the benefit of another. Also, the underwriter is a subprime agent who assists the SPV in underwriting new SMPs. Monoline insurers guarantee investors’ timely repayment of bond principal and interest when an SPV defaults. In essence, such insurers provide guarantees to SPVs, often in the form of credit wraps, that enhance the credit rating of the SPV. They are so named because they provide services to only one industry. These insurance companies first began providing wraps for municipal bond issues, but now they provide credit enhancement for other types of SMP bonds, such as RMBSs and CDOs. In so doing, monoline insurers act as credit enhancement providers that reduce the risk of subprime mortgage securitization.

The originator has access to mortgage investments that may be financed by borrowing from the lender, represented by 1I. The lender, acting in the interest of risk-neutral shareholders, either invests its deposits in Treasuries or in the originator's mortgage projects. In return, the originator pays interest on these investments to the lender, represented by 1J. Next, the originator deals with the mortgage market represented by 1O and 1P, respectively. Also, the originator pools its mortgages and sells them to dealers and/or SPVs (see 1K). The dealer or SPV pays the originator an amount which is slightly greater than the value of the reference mortgage portfolios as in 1L. A SPV is an organization formed for a limited purpose that holds the legal rights over mortgages transferred by originators during securitization. In addition, the SPV divides this pool into sen, mezz, and jun tranches which are exposed to different levels of credit risk. Moreover, the SPV sells these tranches as securities backed by mortgages to investors (see 1N) that is paid out at an interest rate determined by the mortgage default rate, prepayment and foreclosure (see 1M). Also, SPVs deal with the SMP bond market for investment purposes (compare with 1Q and 1R). Furthermore, originators have SMPs on their balance sheets, that have connections with this bond market. Investors invest in this bond market, represented by 1S and receive returns on SMPs in 1T. The money market and hedge fund market are secondary markets where previously issued marketable securities such as SMPs are bought and sold (compare with 1W and 1X). Investors invest in these short-term securities (see, 1U) to receive profit, represented by 1V. During the SMC, the model represented in Figure 1 was placed under major duress as house prices began to plummet. As a consequence, there was a cessation in subprime agent activities and the cash flows to the markets began to dry up, thus, causing the whole subprime mortgage model to collapse.

We note that the traditional mortgage model is embedded in Figure 1 and consists of mortgagors, lenders and originators as well as the mortgage market. In this model, the lender lends funds to the originator to fund mortgage originations (see, 1I and 1J). Home valuation as well as income and credit checks were done by the originator before issuing the mortgage. The originator then extends mortgages and receives repayments that are represented by 1E and 1F, respectively. The originator also deals with the mortgage market in 1O and 1P. When a mortgagor defaults on repayments, the originator repossesses the house.

### *1.2.5. Preliminaries about Subprime Risks*

The main risks that arise when dealing with SMPs are credit (including counterparty and default), market (including interest rate, price, and liquidity), operational (including house appraisal, valuation, and compensation), tranching (including maturity mismatch and synthetic), and systemic (including maturity transformation) risks. For sake of argument, risks falling in the categories described above are cumulatively known as *subprime risks*. In Figure 2 below, we provide a diagrammatic overview of the aforementioned subprime risks.

The most fundamental of the above risks is *credit* and *market risk*. Credit risk involves the originator's risk of mortgage losses and the possible inability of SPVs to make good on investor payments. This risk category generally includes *counterparty risk* that, in our case, is the risk that a banking agent does not pay out on a bond, credit derivative or credit insurance contract. It refers to the ability of banking agents—such as originators, mortgagors, servicers, investors, SPVs, trustees, underwriters, and depositors—to fulfill their obligations towards

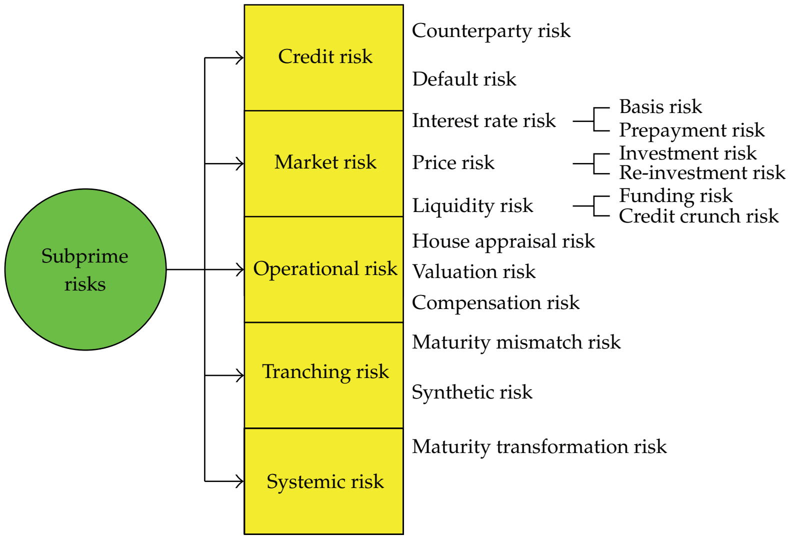


**Figure 2:** Diagrammatic overview of subprime risks.

each other. During the SMC, even banking agents who thought they had hedged their bets by buying insurance—via credit default swap (CDS) or monoline insurance contracts—still faced the risk that the insurer will be unable to pay.

In our case, *market risk* is the risk that the value of the mortgage portfolio will decrease mainly due to changes in the value of securities prices and interest rates (see, e.g., Sections 2.1 and 4.2). *Interest rate risk* arises from the possibility that SMP interest rate returns will change. Subcategories of interest rate risk are basis and prepayment risk. The former is the risk associated with yields on SMPs and costs on deposits which are based on different bases with different rates and assumptions. *Prepayment risk* results from the ability of subprime mortgagors to voluntarily (refinancing) and involuntarily (default) prepay their mortgages under a given interest rate regime. *Liquidity risk* arises from situations in which a banking agent interested in selling (buying) SMPs cannot do it because nobody in the market wants to buy (sell) those SMPs. Such risk includes funding and credit crunch risk. *Funding risk* refers to the lack of funds or deposits to finance mortgages and *credit crunch risk* refers to the risk of tightened mortgage supply and increased credit standards. We consider *price risk* to be the risk that SMPs will depreciate in value, resulting in financial losses, markdowns and possibly margin calls. Subcategories of price risk are valuation risk (resulting from the valuation of long-term SMP investments) and reinvestment risk (resulting from the valuation of short-term SMP investments). Valuation issues are a key concern that must be dealt with if the capital markets are to be kept stable, and they involve a great deal of operational risk.

*Operational risk* is the risk of incurring losses resulting from insufficient or inadequate procedures, processes, systems or improper actions taken. As we have commented before, for mortgage origination, operational risk involves documentation, background checks and progress integrity. Also, subprime mortgage securitization embeds operational risk via misselling, valuation and IB issues. Operational risk related to mortgage origination and securitization results directly from the design and intricacy of mortgages and related structured products. Moreover, investors carry operational risk associated with mark-to-market issues, the worth of SMPs when sold in volatile markets and uncertainty involved in investment payoffs. Also, market reactions include increased volatility leading to behavior that can increase operational risk such as unauthorized trades, dodgy valuations and processing issues. Often additional operational risk issues such as model validation, data accuracy, and stress testing lie beneath large market risk events (see, e.g., [17]).

*Tranching risk* is the risk that arises from the intricacy associated with the slicing of SMPs into tranches in securitization deals. Prepayment, interest rate, price and tranching risk involves the intricacy of subprime SMPs. Another tranching risk that is of issue for SMPs is *maturity mismatch risk* that results from the discrepancy between the economic lifetimes of SMPs and the investment horizons of IBs. *Synthetic risk* can be traded via credit derivatives—like CDSs—referencing individual subprime RMBS bonds, synthetic CDOs or via an index linked to a basket of such bonds.

In banking, *systemic risk* is the risk that problems at one bank will endanger the rest of the banking system. In other words, it refers to the risk imposed by interlinkages and—dependencies in the system where the failure of a single entity or cluster of entities can cause a cascading effect which could potentially bankrupt the banking system or market.

### *1.3. Main Questions and Outline of the Paper*

In this subsection, we identify the main questions solved in and give an outline of the paper.

#### *1.3.1. Main Questions*

The main questions that are solved in this paper may be formulated as follows.

*Question 1* (modeling of profit under subprime mortgage securitization). Can we construct discrete-time subprime mortgage models that incorporate default, monoline insurance, costs of funds and profits under mortgage securitization? (see Sections 2.1 and 3.2).

*Question 2* (modeling of risk under subprime mortgage securitization). Can we identify the risks associated with the different components of the subprime mortgage models mentioned in Question 1? (see Sections 2.1 and 3.2).

*Question 3* (subprime mortgage securitization intricacy and design leading to information problems, valuation opaqueness and ineffective risk mitigation). Was the SMC partly caused by the intricacy and design of mortgage securitization that led to information (asymmetry, contagion, inefficiency and loss) problems, valuation opaqueness and ineffective risk mitigation? (see Sections 2, 3, 4, and 5).

*Question 4* (optimal valuation problem under subprime mortgage securitization). In order to obtain an optimal valuation under subprime mortgage securitization, which decisions regarding mortgage rates, deposits and Treasuries must be made? (see Theorems 2.1 and 3.1 of Sections 2.3 and 3.4, resp.).

#### *1.3.2. Outline of the Paper*

Section 2 contains a discussion of an optimal profit problem under RMBSs. To make this possible, capital, information, risk and valuation for a subprime mortgage model under

RMBSs is analyzed. In this regard, a mechanism for mortgage securitization, RMBS bond structure, cost of funds for RMBSs, financing, adverse selection, monoline insurance contracts for subprime RMBSs as well as residuals underly our discussions. Section 3 is analogous to Section 2 by investigating an optimal profit problem under RMBS CDOs. Section 4 discusses aspects of the relationship between subprime mortgage securitization and Basel regulation. Also, Section 5 provides examples of aspects of the aforementioned issues, while Section 6 discusses important conclusions and topics for future research. Finally, an appendix containing additional information and the proofs of the main results is provided in the appendix.

## 2. Profit, Risk, and Valuation under RMBSs

In this section, we provide more details about RMBSs and related issues such as profit, risk, and valuation. In the sequel, we assume that the notation $\Pi$, $r^M$, $M$, $\overline{c}^{M\omega}$, $p^i$, $c^p$, $r^f$, $r^R$, $r^S$, $S$, $\mathcal{C}$, $C(\mathbf{E}[S(\mathcal{C})])$, $r^B$, $c^B$, $B$, $r^{\mathtt{T}}$, $\mathtt{T}$, $P^{\mathtt{T}}(\mathtt{T})$, $r^D$, $c^D$, $D$, $r^{\mathtt{B}}$, $c^{\mathtt{B}}$, $\mathtt{B}$, $\Pi^p$, $K$, $n$, $E$, $O$, $\omega(\mathcal{C})$, $\omega^B$, $f^M$, $m$VaR, and $\mathtt{O}$ corresponds to that of Section 1.2. Furthermore, the notation $r^{S\Sigma}$ represents the loss rate on RMBSs, $f^{\Sigma}$ is the fraction of $M$ that is securitized and $\widehat{f}^{\Sigma}$ denotes the fraction of $M$, realized as new RMBSs, where $\widehat{f}^{\Sigma} \in f^{\Sigma}$.

The following assumption about the relationship between the investor's and originator's profit is important for subsequent analysis.

*Assumption 1* (relationship between the originator and investor). We suppose that the originator and investor share the same balance sheet in terms of $B$, $\mathtt{T}$, $D$, $\mathtt{B}$ and $K$ (compare with (1.2)). Furthermore, we assume that the investor's mortgages can be decomposed as $M = f^{\Sigma}M + (1 - f^{\Sigma})M$. Finally, we suppose that the investor's profit can be expressed as a function of the variables in the previous paragraph and the securitization components $\mathtt{E}$, $\mathtt{F}$, $r^r$, and $V^a$ (see Section 1.2 for more details).

This assumption enables us to subsequently derive an expression for the investor's profit under RMBSs as in (2.1) from the originator's profit formula given by (1.1). We note that important features of Section 2 are illustrated in Sections 5.1, 5.2, and 5.3.

The key design feature of subprime mortgages involves the ability of mortgagors to finance and refinance their houses based on capital gains due to house price appreciation over short horizons and then turning this into collateral for a new mortgage or extracting equity for consumption. As is alluded to in Section 2, the unique design of mortgages resulted in unique structures for their securitizations (response to Question 3). During the SMC, CRAs were reprimanded for giving investment-grade ratings to RMBSs backed by risky mortgages. Before the SMC, these high ratings enabled such RMBSs to be sold to investors, thereby financing and exacerbating the housing boom. The issuing of these ratings were believed justified because of risk-reducing practices, such as monoline insurance and equity investors willing to bear the first losses. However, during the SMC, it became clear that some role players in rating subprime-related securities knew at the time that the rating process was faulty. Uncertainty in financial markets spread to other subprime agents, increasing the counterparty risk which caused interest rates to increase. Refinancing became almost impossible and default rates exploded. All these operations embed systemic risk which finally caused the banking system to collapse (compare with Section 2.1).

Clearly, during the SMC, the securitization of credit risks was a source of moral hazard that compromised global banking sector stability. Before the SMC, the practice of splitting the claims to a reference mortgage portfolio into tranches was a response to this concern. In this case, sen and mezz tranches can be considered to be senior and junior debt, respectively. *If* originators held equity tranches and *if*, because of packaging and diversification, the probability of default, that is, the probability that reference portfolio returns do not attain the sum of sen and mezz claims, were (close to) zero, we would (almost) be neglecting moral hazard effects. How the banking system failed despite the preceding scenario is explained next (compare with Section 2.1). Unfortunately, in reality, both *ifs* in the statement above were not satisfied. Originators did not, in general, hold the equity tranches of the portfolios that they generated. In truth, as time went on, ever greater portions of equity tranches were sold to external investors. Moreover, default probabilities for sen and mezz tranches were significant because packaging did not provide for sufficient diversification of returns on the reference mortgage portfolios in RMBS portfolios (see, e.g., [16]).

## 2.1. Profit and Risk under RMBSs

In this subsection, we discuss a subprime mortgage model for capital, information, risk, and valuation and its relation to retained earnings.

### 2.1.1. A Subprime Mortgage Model for Profit and Risk under RMBSs

In this paper, a subprime mortgage model for capital, information, risk, and valuation under RMBSs can be constructed by considering the difference between cash inflow and outflow. In period $t$, *cash inflow* is constituted by returns on the residual from mortgage securitization, $r_t^r \widehat{f}_t^\Sigma f_t^\Sigma M_t$, SMPs, $r_t^M(1-\widehat{f}_t^\Sigma)f_t^\Sigma M_t$, unsecuritized mortgages, $r_t^M(1-f_t^\Sigma)M_t$, unsecuritized mortgages that are prepaid, $c_t^p r_t^f(1-f_t^\Sigma)M_t$, $r_t^{\mathrm{T}}\mathtt{T}_t$, $(r_t^B - c_t^B)B_t$, as well as $C(\mathbf{E}[S(\mathcal{C})])$, and the present value of future gains from subsequent mortgage origination and securitizations, $\Pi_t^{\Sigma p}$. On the other hand, in period $t$, we consider the average weighted cost of funds to securitize mortgages, $\overline{c}^{M\Sigma\omega}\widehat{f}_t^\Sigma f_t^\Sigma M_t$, losses from securitized mortgages, $r_t^{S\Sigma}\widehat{f}_t^\Sigma f_t^\Sigma M_t$, forfeit costs related to monoline insurance wrapping RMBSs, $c_t^{i\Sigma}\widehat{f}_t^\Sigma f_t^\Sigma M_t$, transaction cost to originate mortgages, $c_t^t(1-\widehat{f}_t^\Sigma)f_t^\Sigma M_t$, and transaction costs from securitized mortgages $c_t^{t\Sigma}(1-\widehat{f}_t^\Sigma)f_t^\Sigma M_t$ as part of *cash outflow*. Additional components of outflow are weighted average cost of funds for originating mortgages, $\overline{c}_t^{M\omega}(1-f_t^\Sigma)M_t$, mortgage insurance premium for unsecuritized mortgages, $p^i(\mathcal{C}_t)(1-f_t^\Sigma)M_t$, nett losses for unsecuritized mortgages, $(1-r_t^R)r_t^S(1-f_t^\Sigma)M_t$, decreasing value of adverse selection, $af_t^\Sigma M_t$, losses from suboptimal SPVs, $\mathtt{E}_t$ and cost of funding SPVs, $\mathtt{F}_t$. From the above and (1.1), we have that a subprime mortgage model for profit under subprime RMBSs may have the form

$$
\begin{aligned}
\Pi_t^\Sigma = {} & \left(r_t^r - \overline{c}_t^{M\Sigma\omega} - r_t^{S\Sigma} - c_t^{i\Sigma}\right)\widehat{f}_t^\Sigma f_t^\Sigma M_t + \left(r_t^M - c_t^t - c_t^{t\Sigma}\right)\left(1-\widehat{f}_t^\Sigma\right)f_t^\Sigma M_t \\
& + \left(r_t^M - \overline{c}_t^{M\omega} - p_t^i(\mathcal{C}_t) + c_t^p r_t^f - \left(1-r_t^R\right)r_t^S\right)\left(1-f_t^\Sigma\right)M_t - af_t^\Sigma M_t + r_t^{\mathrm{T}}\mathtt{T}_t \\
& - \left(r_t^{\mathrm{B}} + c_t^{\mathrm{B}}\right)\mathtt{B}_t + \left(r_t^B - c_t^B\right)B_t - \left(r_t^D + c_t^D\right)D_t + C(\mathbf{E}[S(\mathcal{C}_t)]) - P^{\mathrm{T}}(\mathtt{T}_t) \\
& + \Pi_t^{\Sigma p} - \mathtt{E}_t - \mathtt{F}_t,
\end{aligned}
\tag{2.1}
$$

where $\Pi_t^{\Sigma p} = \Pi_t^p + \widetilde{\Pi}_t^{\Sigma}$. Furthermore, by considering $\partial S(\mathcal{C}_t)/\partial \mathcal{C}_t^B < 0$ and (2.1), $\Pi^{\Sigma}$ is an increasing function of RMBS credit rating $\mathcal{C}^B$, so that $\partial \Pi_t^{\Sigma}/\partial \mathcal{C}_t^B > 0$. Furthermore, the monoline insurance forfeit cost term, $c^{i\Sigma}$, is a function of SPV's monoline insurance premium and payment terms.

From (2.1), it is clear that bank valuation under RMBSs involves the valuing of the RMBSs themselves. In general, valuing such a vanilla corporate bond is based on default, interest rate and prepayment risks. The number of mortgagors with mortgages underlying RMBSs who prepay, increases when interest rates decrease because they can refinance at a lower fixed interest rate. Since interest rate and prepayment risks are related, it is difficult to solve mathematical models of RMBS value. This level of difficulty increases with the intricacy of the interest rate model and the sophistication of the prepayment-interest rate dependence. As a consequence, to our knowledge, no viable closed-form solutions have been found. In models of this type numerical methods provide approximate theoretical prices. These are also required in most models which specify the credit risk as a stochastic function with an interest rate correlation. Practitioners typically use Monte Carlo method or Binomial Tree numerical solutions. Of course, in (2.1) and hereafter, we assume that the RMBSs can be valued in a reasonably accurate way.

Below we roughly attempt to associate different risk types to different cash inflow and outflow terms in (2.1). We note that the cash inflow terms $r_t^r \widehat{f}_t^{\Sigma} f_t^{\Sigma} M_t$ and $r_t^M(1 - \widehat{f}_t^{\Sigma}) f_t^{\Sigma} M_t$ embed credit, market (in particular, interest rate), tranching and operational risks while $r_t^M(1 - f_t^{\Sigma})M_t$ carries market (specifically, interest rate) and credit risks. Also, $c_t^p r_t^f (1 - f_t^{\Sigma})M_t$ can be associated with market (in particular, prepayment) risk while $(r_t^B - c_t^B)B_t$ mainly embeds market risk. $C(\mathbf{E}[S(\mathcal{C})])$ and $\Pi_t^{\Sigma p}$ involve at least credit (particularly, counterparty) and market (more specifically, interest rate, basis, prepayment, liquidity and price), respectively. In (2.1), the cash outflow terms $\overline{c}^{M\Sigma\omega} \widehat{f}_t^{\Sigma} f_t^{\Sigma} M_t$, $c_t^t(1 - \widehat{f}_t^{\Sigma}) f_t^{\Sigma} M_t$ and $c_t^{t\Sigma}(1 - \widehat{f}_t^{\Sigma}) f_t^{\Sigma} M_t$ involve credit, tranching and operational risks while $\overline{c}_t^{M\omega}(1 - f_t^{\Sigma})M_t$ and $p^i(\mathcal{C}_t)(1 - f_t^{\Sigma})M_t$ carry credit and operational risks. Also, $r_t^{S\Sigma} \widehat{f}_t^{\Sigma} f_t^{\Sigma} M_t$ embeds credit, market (including valuation), tranching and operational risks and $c_t^{i\Sigma} \widehat{f}_t^{\Sigma} f_t^{\Sigma} M_t$ involves credit (in particular, counterparty), tranching and operational risks. Furthermore, $(1 - r_t^R) r_t^S (1 - f_t^{\Sigma}) M_t$ and $a f_t^{\Sigma} M_t$ both carry credit and market (including valuation) risks. Finally, $\mathtt{E}_t$ and $\mathtt{F}_t$ embed credit (in particular, counterparty and valuation) and market and operational risks, respectively. In reality, the risks that we associate with each of the cash inflow and outflow terms in (2.1) are more complicated than presented above. For instance, these risks are interrelated and may be strongly correlated with each other. All of the above risk-carrying terms contribute to systemic risk that affects the entire banking system.

In the early 80s, house financing in the US and many European countries changed from fixed-rate (FRMs) to adjustable-rate mortgages (ARMs) resulting in an interest rate risk shift to mortgagors. However, when market interest rates rose again in the late 80s, originators found that many mortgagors were unable or unwilling to fulfil their obligations at the newly adjusted rates. Essentially, this meant that the interest rate (market) risk that originators thought they had eradicated had merely been transformed into counterparty credit risk. Presently, it seems that the lesson of the 80s that ARMs cause credit risk to be higher, seems to have been forgotten or neglected since the credit risk would affect the RMBS bondholders rather than originators (see, e.g., [16]). Section 2.1 implies that the system of house financing based on RMBSs has some eminently reasonable features. Firstly, this system permits originators to divest themselves from the interest rate risk that is associated with such financing. The experience of the US Savings & Loans debacle has shown that banks cannot cope with this risk. The experience with ARMs has also shown that debtors are not able to bear this risk and that the attempt to burden them with it may merely transform the interest rate risk into counterparty credit risk. Securitization shifts this risk to a third party.

A subprime mortgage model for profit under RMBSs from (2.1) and (2.4) reflects the fact that originators sell mortgages and distribute risk to investors through mortgage securitization. This way of mitigating risks involves at least operational (including valuation and compensation), liquidity (market) and tranching (including maturity mismatch) risk that returned to originators when the SMC unfolded. Originators are more likely to securitize more mortgages if they hold less capital, are less profitable and/or liquid and have mortgages of low quality. This situation was prevalent before the SMC when originators' pursuit of yield did not take decreased capital, liquidity and mortgage quality into account. The investors in RMBSs also embed credit risk which involves bankruptcy if the aforementioned agents cannot raise funds.

### 2.1.2. *Profit under RMBSs and Retained Earnings*

As for originator's profit under mortgages, $\Pi$, we conclude that the investor's profit under RMBSs, $\Pi^{\Sigma}$, are used to meet its obligations, that include dividend payments on equity, $n_t d_t$. The retained earnings, $E_t^r$, subsequent to these payments may be computed by using

$$\Pi_t = E_t^r + n_t d_t + \left(1 + r_t^O\right) O_t. \tag{2.2}$$

After adding and subtracting $(r_t^M - \overline{c}_t^{M\omega} - p_t^i + c_t^p r_t^f - (1 - r_t^R) r_t^S) M_t$ from (2.1), we obtain

$$\begin{aligned}\Pi_t^{\Sigma} = \Pi_t &+ \left(r_t^r - \overline{c}_t^{M\Sigma\omega} - r_t^{S\Sigma} - c_t^{i\Sigma} - r_t^M + c_t^t + c_t^{t\Sigma}\right) f_t^{\Sigma} \widehat{f}_t^{\Sigma} M_t \\ &+ \left(\overline{c}_t^{M\omega} + p_t^i + \left(1 - r_t^R\right) r_t^S - c_t^t - c_t^{t\Sigma} - c_t^p r_t^f - a\right) f_t^{\Sigma} M_t - \mathrm{E}_t - \mathrm{F}_t + \widetilde{\Pi}_t^{\Sigma}.\end{aligned} \tag{2.3}$$

If we replace $\Pi_t$ by using (2.2), $\Pi_t^{\Sigma}$ is given by

$$\begin{aligned}\Pi_t^{\Sigma} = E_t^r + n_t d_t + \left(1 + r_t^O\right) O_t &+ \left(r_t^r - \overline{c}_t^{M\Sigma\omega} - r_t^{S\Sigma} - c_t^{i\Sigma} - r_t^M + c_t^t + c_t^{t\Sigma}\right) f_t^{\Sigma} \widehat{f}_t^{\Sigma} M_t \\ &+ \left(\overline{c}_t^{M\omega} + p_t^i + \left(1 - r_t^R\right) r_t^S - c_t^t - c_t^{t\Sigma} - c_t^p r_t^f - a\right) f_t^{\Sigma} M_t - \mathrm{E}_t - \mathrm{F}_t + \widetilde{\Pi}_t^{\Sigma}.\end{aligned} \tag{2.4}$$

From (1.7) and (2.4), we may derive an expression for the investor's capital of the form

$$\begin{aligned}K_{t+1}^{\Sigma} = n_t(d_t + E_t) - \Pi_t^{\Sigma} + \Delta F_t + \left(1 + r_t^O\right) O_t &+ \left(r_t^r - \overline{c}_t^{M\Sigma\omega} - r_t^{S\Sigma} - c_t^{i\Sigma} - r_t^M + c_t^t + c_t^{t\Sigma}\right) f_t^{\Sigma} \widehat{f}_t^{\Sigma} M_t \\ &+ \left(\overline{c}_t^{M\omega} + p_t^i + \left(1 - r_t^R\right) r_t^S - c_t^t - c_t^{t\Sigma} - c_t^p r_t^f - a\right) f_t^{\Sigma} M_t - \mathrm{E}_t - \mathrm{F}_t + \widetilde{\Pi}_t^{\Sigma},\end{aligned} \tag{2.5}$$

where $K_t$ is defined by (1.2).

In Section 2.1.2, $\Pi^{\Sigma}$ is given by (2.4), while $K^{\Sigma}$ has the form (2.5). It is interesting to note that the formulas for $\Pi^{\Sigma}$ and $K^{\Sigma}$ depend on $\Pi$ and $K$, respectively, and are far more intricate than the latter. Defaults on RMBSs increased significantly as the crisis expanded from the housing market to other parts of the economy, causing $\Pi^{\Sigma}$ (as well as retained earnings in (2.2)) to decrease. During the SMC, capital adequacy ratios declined as $K^{\Sigma}$ levels became depleted while banks were highly leveraged. As a consequence, methods and processes which embed operational risk failed. In this period, such risk rose as banks succeeded in decreasing their capital requirements. Operational risk was not fully understood and acknowledged which resulted in loss of liquidity and failed risk mitigation management (compare with Question 3).

### 2.2. Valuation under RMBSs

If the expression for retained earnings given by (2.4) is substituted into (1.8), the nett cash flow under RMBSs generated by the investor is given by

$$\begin{aligned} N_t^{\Sigma} &= \Pi_t^{\Sigma} - \Delta F_t \\ &= n_t(d_t + E_t) - K_{t+1}^{\Sigma} + \left(1 + r_t^{O}\right)O_t \\ &\quad + \left(r_t^r - \overline{c}_t^{M\Sigma\omega} - r_t^{S\Sigma} - c_t^{i\Sigma} - r_t^{M} + c_t^{t} + c_t^{t\Sigma}\right) f_t^{\Sigma} \widehat{f}_t^{\Sigma} M_t \\ &\quad + \left(\overline{c}_t^{M\omega} + p_t^{i} + \left(1 - r_t^{R}\right) r_t^{S} - c_t^{t} - c_t^{t\Sigma} - c_t^{p} r_t^{f} - a\right) f_t^{\Sigma} M_t - \mathtt{E}_t - \mathtt{F}_t + \widetilde{\Pi}_t^{\Sigma}. \end{aligned} \tag{2.6}$$

We know that valuation is equal to the investor's nett cash flow plus exdividend value. This translates to the expression

$$V_t^{\Sigma} = N_t^{\Sigma} + K_{t+1}^{\Sigma}, \tag{2.7}$$

where $K_t$ is defined by (1.2). Furthermore, the stock analyst evaluates the expected future cash flows in $j$ periods based on a *stochastic discount factor*, $\delta_{t,j}$ such that the investor's value is

$$V_t^{\Sigma} = N_t^{\Sigma} + \mathbf{E}\left[\sum_{j=1}^{\infty} \delta_{t,j} N_{t+j}^{\Sigma}\right]. \tag{2.8}$$

When US house prices declined in 2006 and 2007, refinancing became more difficult and ARMs began to reset at higher rates. This resulted in a dramatic increase in mortgage delinquencies, so that RMBSs began to lose value. Since these mortgage products are on the balance sheet of most banks, their valuation given by (2.8) in Section 2.2 began to decline (see, also, formulas (2.6) and (2.7)). Before the SMC, moderate reference mortgage portfolio delinquency did not affect valuation in a significant way. However, the value of mortgages and related structured products such as RMBSs decreased significantly due incidences of operational, tranching, and liquidity risks during the SMC. The yield from these structured mortgage products decreased as a consequence of high default rates (credit risk) which caused liquidity problems with a commensurate rise in the instances of credit crunch and funding risk (see Section 1.2.5 for more details about these risks).

The imposition of fair value accounting for mortgages and related SMPs such as RMBSs enhances the scope for systemic risk that involves the malfunctioning of the entire banking system. Under this type of accounting, the values at which securities are held in banks' books depend on the prices that prevail in the market (see formulas (2.6), (2.7), and (2.8) for valuations of banks holding such securities). In the event of a change in securities prices, the bank must adjust its books even if the price change is due to market malfunctioning and it has no intention of selling the security, but intends to hold it to maturity. Under currently prevailing capital adequacy requirements, this adjustment has immediate implications for the bank's financial activities. In particular, if market prices of securities held by the bank have decreased, the bank must either recapitalize by issuing new equity or retrench its overall operations. The functioning of the banking system thus depends on how well credit markets are functioning. In short, impairments of the ability of markets to value mortgages and related structured products such as RMBSs can have a large impact on bank valuation (compare with Section 2.2).

### *2.3. Optimal Valuation under RMBSs*

In this subsection, we make use of the modeling of assets, liabilities and capital of the preceding section to solve an optimal valuation problem. The investor's total capital constraint for subprime RMBSs at face value is given by

$$K_t^{\Sigma} = n_t E_{t-1} + O_t \geq \rho\left[\omega^M M_t + \omega\left(C_t^B\right) B_t + 12.5 f^M (m\text{VaR} + 0)\right], \tag{2.9}$$

where $\omega(C_t^B)$ and $\omega^M$ are the risk weights related to subprime RMBSs and mortgages, respectively, while $\rho$—Basel II pegs $\rho$ at approximately 0.08—is the Basel capital regulation ratio of regulatory capital to risk weighted assets. In order to state the investor's optimal valuation problem, it is necessary to assume the following.

*Assumption 2* (subprime investing bank's performance criterion). Suppose that the investor's valuation performance criterion, $J^{\Sigma}$, at $t$ is given by

$$\begin{aligned} J_t^{\Sigma} = \Pi_t^{\Sigma} + l_t^b\left[K_t^{\Sigma} - \rho\left(\omega^M M_t + \omega\left(C_t^B\right) B_t + 12.5 f^M (m\text{VaR} + 0)\right)\right] \\ - c_t^{dw}\left[K_{t+1}^{\Sigma}\right] + \mathbf{E}\left[\delta_{t,1} V\left(K_{t+1}^{\Sigma}, x_{t+1}\right)\right], \end{aligned} \tag{2.10}$$

where $l_t^b$ is the Lagrangian multiplier for the total capital constraint, $K_t^{\Sigma}$ is defined by (2.9), $\mathbf{E}[\cdot]$ is the expectation conditional on the investor's information in period $t$ and $x_t$ is the deposit withdrawals in period $t$ with probability distribution $f(x_t)$. Also, $c_t^{dw}$ is the deadweight cost of total capital that consists of common and preferred equity as well as subordinate debt, $V$ is the value function with a discount factor denoted by $\delta_{t,1}$.

### *2.3.1. Statement of the Optimal Valuation Problem under RMBSs*

The optimal valuation problem is to maximize *investor value* given by (2.8). We can now state the optimal valuation problem as follows.

*Question 5* (statement of the optimal valuation problem under RMBSs). Suppose that the total capital constraint and the performance criterion, $J^{\Sigma}$, are given by (2.9) and (2.10), respectively. The optimal valuation problem under RMBSs is to maximize the investor's value given by (2.8) by choosing the RMBS rate, deposits, and regulatory capital for

$$V^{\Sigma}\left(K_t^{\Sigma}, x_t\right) = \max_{r_t^B, D_t, \Pi_t^{\Sigma}} J_t^{\Sigma}, \tag{2.11}$$

subject to RMBS, balance sheet, cash flow, and financing constraints given by

$$B_t = b_0 + b_1 r_t^B + b_2 \mathcal{C}_t^B + \sigma_t^B, \tag{2.12}$$

$$D_t = \frac{B_t + M_t + \mathtt{T}_t - \mathtt{B}_t - K_t}{1 - \gamma}, \tag{2.13}$$

equations (2.1) and (2.5), respectively.

### *2.3.2. Solution to an Optimal Valuation Problem under RMBSs*

In this subsection, we find a solution to Question 5 when the capital constraint (2.9) holds as well as when it does not. In this regard, the main result can be stated and proved as follows.

**Theorem 2.1** (solution to the optimal valuation problem under RMBSs). *Suppose that $J^{\Sigma}$ and $V^{\Sigma}$ are given by* (2.10) *and* (2.11)*, respectively. When the capital constraint given by* (2.9) *holds (i.e., $l_t^b > 0$), a solution to the optimal valuation problem under RMBSs yields an optimal $B$ and $r^B$ of the form*

$$B_t^* = \frac{K_t^{\Sigma}}{\rho\omega\left(\mathcal{C}_t^B\right)} - \frac{\omega^M M_t + 12.5 f^M (m\text{VaR} + \mathtt{O})}{\omega\left(\mathcal{C}_t^B\right)}, \tag{2.14}$$

$$r_t^{B^*} = -\frac{1}{b_1}\left(b_0 + b_2 \mathcal{C}_t^B + \sigma_t^B - B_t^*\right), \tag{2.15}$$

*respectively. In this case, the investor's optimal deposits and provisions for deposit withdrawals via Treasuries and optimal profits under RMBSs are given by*

$$D_t^{\Sigma^*} = \frac{1}{1-\gamma}\Bigg(\overline{D} + \frac{\overline{D}}{r_t^p}\left[r_t^{\mathrm{T}} + (r_t^{\mathrm{B}} + c_t^{\mathrm{B}}) + \left(r_t^B - c_t^B\right) - \frac{1}{1-\gamma}\left(r_t^D + c_t^D\right)\right]$$
$$+\frac{K_t^{\Sigma}}{\rho\omega(\mathcal{C}_t^B)} - \frac{\omega^M M_t + 12.5 f^M (m\mathrm{VaR}+0)}{\omega(\mathcal{C}_t^B)} + M_t - K_t - \mathrm{B}_t\Bigg), \tag{2.16}$$

$$\mathrm{T}_t^{\Sigma^*} = \overline{D} + \frac{\overline{D}}{r_t^p}\left[r_t^{\mathrm{T}} + (r_t^{\mathrm{B}} + c_t^{\mathrm{B}}) + \left(r_t^B - c_t^B\right) - \frac{1}{1-\gamma}\left(r_t^D + c_t^D\right)\right], \tag{2.17}$$

$$\Pi_t^{\Sigma^*} = \left[\frac{K_t^{\Sigma}}{\rho\omega^M} - \frac{\omega(\mathcal{C}_t^B)B_t + 12.5 f^M(m\mathrm{VaR}+0)}{\omega^M}\right]$$
$$\times\Big[\widehat{f}_t^{\Sigma} f_t^{\Sigma}\left(r_t^r - \overline{c}_t^{M\Sigma\omega} - r_t^{S\Sigma} - c_t^{i\Sigma} - r_t^M + c_t^t + c_t^{t\Sigma}\right)$$
$$+ f_t^{\Sigma}\left(\overline{c}_t^{M\omega} + p_t^i + \left(1 - r_t^R\right)r_t^S - c_t^t - c_t^{t\Sigma} - c_t^p r_t^f - a\right)$$
$$+\left(r_t^M - \overline{c}_t^{M\omega} - p_t^i + c_t^p r_t^f - \left(1 - r_t^R\right)r_t^S\right)\Big]$$
$$+\left[\frac{K_t^{\Sigma}}{\rho\omega(\mathcal{C}_t^B)} - \frac{\omega^M M_t + 12.5 f^M(m\mathrm{VaR}+0)}{\omega(\mathcal{C}_t^B)}\right]$$
$$\times\Bigg[\left(\frac{1}{b_1}\left[\frac{K_t^{\Sigma}}{\rho\omega(\mathcal{C}_t^B)} - \frac{\omega^M M_t + 12.5 f^M(m\mathrm{VaR}+0)}{\omega(\mathcal{C}_t^B)} - b_0 - b_2\mathcal{C}_t^B - \sigma_t^B\right] - c_t^B\right)$$
$$-\left(r_t^D + c_t^D\right)\frac{1}{1-\gamma}\Bigg]$$
$$+\Bigg(\overline{D} + \frac{\overline{D}}{r_t^p}\Bigg[r_t^{\mathrm{T}} + \left(\frac{1}{b_1}\left[\frac{K_t^{\Sigma}}{\rho\omega(\mathcal{C}_t^B)} - \frac{\omega^M M_t + 12.5 f^M(m\mathrm{VaR}+0)}{\omega(\mathcal{C}_t^B)} - b_0 - b_2\mathcal{C}_t^B - \sigma_t^B\right] - c_t^B\right)$$
$$+(r_t^{\mathrm{B}} + c_t^{\mathrm{B}}) - \frac{1}{1-\gamma}\left(r_t^D + c_t^D\right)\Bigg]\Bigg)\left(r_t^{\mathrm{T}} - \left(r_t^D + c_t^D\right)\frac{1}{1-\gamma}\right)$$
$$-\left(\left(r_t^D + c_t^D\right)\frac{1}{1-\gamma}\right)(M_t - K_t - \mathrm{B}_t) - (r_t^{\mathrm{B}} + c_t^{\mathrm{B}})\mathrm{B}_t + C(\mathbf{E}[S(\mathcal{C}_t)])$$
$$-P^{\mathrm{T}}(\mathrm{T}_t) + \Pi_t^{\Sigma p} - \mathrm{E}_t - \mathrm{F}_t, \tag{2.18}$$

*respectively.*

*Proof.* A full proof of Theorem 2.1 is given in Appendix A. □

The next corollary follows immediately from Theorem 2.1.

**Corollary 2.2** (solution to the optimal valuation problem under RMBSs (slack)). *Suppose that $J^{\Sigma}$ and $V^{\Sigma}$ are given by* (2.10) *and* (2.11)*, respectively and $P(\mathcal{C}_t) > 0$. When the capital constraint* (2.9) *does not hold (i.e., $l_t^b = 0$), a solution to the optimal valuation problem under RMBSs posed in Question 5 yields optimal RMBS supply and its rate*

$$\begin{aligned} B_t^{\Sigma n^*} &= \frac{2}{3}\left(b_0 + b_2\mathcal{C}_t^B + \sigma_t^B\right) + \frac{b_1}{3} \\ &\quad\times\Bigg[r_t^M - \overline{c}_t^{M\omega} - p_t^i(\mathcal{C}_t) + c_t^p r_t^f + 2c_t^B - \left(1 - r_t^R\right) r_t^S \\ &\quad + \frac{(r_t^D + c_t^D)}{(1-\gamma)} + \left(r_t^r - \overline{c}_t^{M\Sigma\omega} - r_t^{S\Sigma} - c_t^{i\Sigma} - r_t^M + c_t^t + c_t^{t\Sigma}\right)\widehat{f}_t^{\Sigma} f_t^{\Sigma} \\ &\quad + \left(\overline{c}_t^{M\omega} + p_t^i(\mathcal{C}_t) - c_t^p r_t^f + \left(1 - r_t^R\right) r_t^S - c_t^t - c_t^{t\Sigma} - a\right) f_t^{\Sigma}\Bigg], \end{aligned} \tag{2.19}$$

$$\begin{aligned} r_t^{B^{\Sigma n^*}} &= -\frac{1}{3b_1}\left(b_0 + b_2\mathcal{C}_t^B + \sigma_t^B\right) \\ &\quad + \frac{1}{3}\Bigg[r_t^M - \overline{c}_t^{M\omega} - p_t^i(\mathcal{C}_t) + c_t^p r_t^f + 2c_t^B - \left(1 - r_t^R\right) r_t^S \\ &\quad + \frac{(r_t^D + c_t^D)}{(1-\gamma)} + \left(r_t^r - \overline{c}_t^{M\Sigma\omega} - r_t^{S\Sigma} - c_t^{i\Sigma} - r_t^M + c_t^t + c_t^{t\Sigma}\right)\widehat{f}_t^{\Sigma} f_t^{\Sigma} \\ &\quad + \left(\overline{c}_t^{M\omega} + p_t^i(\mathcal{C}_t) - c_t^p r_t^f + \left(1 - r_t^R\right) r_t^S - c_t^t - c_t^{t\Sigma} - a\right) f_t^{\Sigma}\Bigg], \end{aligned} \tag{2.20}$$

*respectively. In this case, the corresponding $T_t$, deposits and profits under RMBSs are given by*

$$\mathrm{T}_t^{\Sigma n^*} = \overline{D} + \frac{\overline{D}}{r_t^p}\left[r_t^{\mathrm{T}} + (r_t^{\mathrm{B}} + c_t^{\mathrm{B}}) + \left(r_t^B - c_t^B\right) - \frac{1}{1-\gamma}\left(r_t^D + c_t^D\right)\right], \tag{2.21}$$

$$D_t^{\Sigma n^*} = \frac{1}{1-\gamma}\left(\overline{D} + \frac{\overline{D}}{r_t^p}\left[r_t^{\mathrm{T}} + (r_t^{\mathrm{B}} + c_t^{\mathrm{B}}) + \left(r_t^B - c_t^B\right) - \frac{1}{1-\gamma}\left(r_t^D + c_t^D\right)\right] + B_t^{\Sigma n^*} + M_t - K_t - \mathrm{B}_t\right), \tag{2.22}$$

$$\begin{aligned} \Pi_t^{\Sigma n^*} &= M_t\Bigg[\left(r_t^M - \overline{c}_t^{M\omega} - p_t^i(\mathcal{C}_t) + c_t^p r_t^f - \left(1 - r_t^R\right) r_t^S\right) \\ &\quad + \left(r_t^r - \overline{c}_t^{M\Sigma\omega} - r_t^{S\Sigma} - c_t^{i\Sigma} - r_t^M + c^t + c_t^{t\Sigma}\right)\widehat{f}_t^{\Sigma} f_t^{\Sigma} \\ &\quad + \left(\overline{c}_t^{M\omega} + p_t^i(\mathcal{C}_t) + \left(1 - r_t^R\right) r_t^S - c^t - c_t^{t\Sigma} - c_t^p r_t^f - a\right) f_t^{\Sigma}\Bigg] \\ &\quad + \Bigg[\frac{2}{3}\left(b_0 + b_2\mathcal{C}_t^B + \sigma_t^B\right) \end{aligned}$$

$$
\begin{aligned}
&+\frac{b_1}{3}\Bigg(r_t^M-\overline{c}_t^{M\omega}-p_t^i(\mathcal{C}_t)+c_t^p r_t^f+2c_t^B-\left(1-r_t^R\right)r_t^S+\frac{(r_t^D+c_t^D)}{(1-\gamma)}\\
&\qquad+\left(r_t^r-\overline{c}_t^{M\Sigma\omega}-r_t^{S\Sigma}-c_t^{i\Sigma}-r_t^M+c_t^t+c_t^{t\Sigma}\right)\widehat{f}_t^{\Sigma}f_t^{\Sigma}\\
&\qquad+\left(\overline{c}_t^{M\omega}+p_t^i(\mathcal{C}_t)-c_t^p r_t^f+\left(1-r_t^R\right)r_t^S-c_t^t-c_t^{t\Sigma}-a\right)f_t^{\Sigma}\Bigg)\Bigg]\\
&\times\Bigg\{\Bigg[\frac{1}{3}\Big(r_t^M-\overline{c}_t^{M\omega}-p_t^i(\mathcal{C}_t)+c_t^p r_t^f+2c_t^B-\left(1-r_t^R\right)r_t^S\\
&\quad+\frac{(r_t^D+c_t^D)}{(1-\gamma)}+\left(r_t^r-\overline{c}_t^{M\Sigma\omega}-r_t^{S\Sigma}-c_t^{i\Sigma}-r_t^M+c_t^t+c_t^{t\Sigma}\right)\widehat{f}_t^{\Sigma}f_t^{\Sigma}\\
&\quad+\left(\overline{c}_t^{M\omega}+p_t^i(\mathcal{C}_t)-c_t^p r_t^f+\left(1-r_t^R\right)r_t^S-c_t^t-c_t^{t\Sigma}-a\right)f_t^{\Sigma}\Big)\\
&\quad-\frac{1}{3b_1}\left(b_0+b_2\mathcal{C}_t^B+\sigma_t^B\right)-c_t^B\Bigg]-\left(r_t^D+c_t^D\right)\frac{1}{1-\gamma}\Bigg\}\\
&+\Bigg(\overline{D}+\frac{\overline{D}}{r_t^p}\Bigg[r_t^{\mathrm{T}}-\Bigg[\frac{1}{3b_1}\left(b_0+b_2\mathcal{C}_t^B+\sigma_t^B\right)\\
&\qquad\qquad-\frac{1}{3}\Big(r_t^M-\overline{c}_t^{M\omega}-p_t^i(\mathcal{C}_t)+c_t^p r_t^f+2c_t^B-\left(1-r_t^R\right)r_t^S\\
&\qquad\qquad+\frac{(r_t^D+c_t^D)}{(1-\gamma)}+\left(r_t^r-\overline{c}_t^{M\Sigma\omega}-r_t^{S\Sigma}-c_t^{i\Sigma}-r_t^M+c_t^t+c_t^{t\Sigma}\right)\widehat{f}_t^{\Sigma}f_t^{\Sigma}\\
&\qquad\qquad+\left(\overline{c}_t^{M\omega}+p_t^i(\mathcal{C}_t)-c_t^p r_t^f+\left(1-r_t^R\right)r_t^S-c_t^t-c_t^{t\Sigma}-a\right)f_t^{\Sigma}\Big)+c_t^B\Bigg]\\
&\qquad\qquad+(r_t^{\mathrm{B}}+c_t^{\mathrm{B}})-\frac{1}{1-\gamma}\left(r_t^D+c_t^D\right)\Bigg]\Bigg)\left(r_t^{\mathrm{T}}-\left(r_t^D+c_t^D\right)\frac{1}{1-\gamma}\right)\\
&-\left(\left(r_t^D+c_t^D\right)\frac{1}{1-\gamma}\right)(M_t-K_t-\mathrm{B}_t)\\
&-(r_t^{\mathrm{B}}+c_t^{\mathrm{B}})\mathrm{B}_t+C(\mathbf{E}[S(\mathcal{C}_t)])-P^{\mathrm{T}}(\mathrm{T}_t)+\Pi_t^{\Sigma p}-\mathrm{E}_t-\mathrm{F}_t,
\end{aligned}
\tag{2.23}
$$

*respectively.*

*Proof.* A full proof of Corollary 2.2 is given in Appendix B. □

In Section 2.3, the investor's valuation performance criterion, $J^{\Sigma}$, at $t$ is given by (2.10). During the SMC, when valuation was a major issue, $J^{\Sigma}$ was difficult to compute since the valuation of components such as $B$ was not easy to determine. In addition, before the SMC, CRAs used idiosyncratic valuation techniques to give investment-grade ratings to RMBSs despite the fact that the mortgage face value, mortgage rate, the investor's optimal deposits and provisions for deposit withdrawals via Treasuries of the forms (2.14), (2.15), (2.16), and

(2.17), respectively, computed in Theorem 2.1—subject to RMBS, balance sheet, cash flow and financing constraints given by (2.12), (2.13), (2.1), and (2.5), respectively—were clearly suboptimal.

When the capital constraint given by (2.9) holds (i.e., $l_t^b > 0$), a solution to the optimal valuation problem under RMBSs yields optimal profit under RMBSs of the form (2.18). With hindsight, it is clear that the aforementioned subprime parameters did not compare favorably with their optimal counterparts. Also, during the SMC, the financing constraint was violated with not enough capital being held. When the capital constraint (2.9) does not hold (i.e., $l_t^b = 0$) and $P(\mathcal{C}_t) > 0$, then optimal RMBS supply and its rate, (2.19) and (2.20), respectively, are solutions to the optimal valuation problem stated in Corollary 2.2.

## 3. Profit, Risk, and Valuation under RMBS CDOs

In this section, we discuss the relationships between the SMC and profit, risk as well as valuation under RMBS CDOs. In the sequel, we assume that the notation $\Pi$, $r^M$, $M$, $\overline{c}^{M\omega}$, $p^i$, $c^p$, $r^f$, $r^R$, $r^S$, $S$, $\mathcal{C}$, $C(\mathbf{E}[S(\mathcal{C})])$, $r^B$, $c^B$, $B$, $r^{\mathtt{T}}$, $\mathtt{T}$, $P^{\mathtt{T}}(\mathtt{T})$, $r^D$, $c^D$, $D$, $r^{\mathtt{B}}$, $c^{\mathtt{B}}$, $\mathtt{B}$, $\Pi^p$, $K$, $n$, $E$, $O$, $\omega(\mathcal{C})$, $\omega^B$, $f^M$, $m$VaR, $\mathtt{O}$, $r^{S\Sigma}$, $f^{\Sigma}$, and $\widehat{f}^{\Sigma}$, corresponds to that of Sections 1 and 2. Further suppositions about notation are that $r^r$, $\overline{c}^{M\Sigma\omega}$, $c^{i\Sigma}$, $c^t$, $c^{t\Sigma}$, $a$, $\Pi^{\Sigma p}$, $\mathtt{E}$, and $\mathtt{F}$ denote the same parameters as in Section 2.

### *3.1. More Background to RMBS CDOs*

In the sequel, we concentrate on RMBS CDOs where the reference assets of the CDO portfolios are mainly RMBSs. The chain formed by subprime mortgages, RMBSs, and RMBS CDOs is given in Figure 3 below.

Note that as we proceed from left to right in Figure 3, subprime mortgages are securitized into RMBSs that, in turn, get securitized into RMBS CDOs. As far as the latter is concerned, it is clearly shown that RMBS bonds rated AAA, AA, and A constitute *high grade* CDO portfolios. On the other hand, the BBB-rated RMBS bonds are securitized into a *mezz CDO*, since its portfolio mainly consists of BBB-rated RMBSs and their tranches. At the end, if bonds issued by mezz CDOs are put into CDO portfolios, then a type of CDO is known as *CDO squared* or *CDO*$^2$.

*Assumption 3* (senior tranches of RMBSs). We assume that risky marketable securities, $B$, appearing in the balance sheet (1.2), consist entirely of the senior tranches of RMBSs that are wrapped by a monoline insurer. Also, the investor has an incentive to retain an interest in these tranches.

This assumption implies that CDO structure depends on the securitization of senior tranches of RMBSs in particular. We note that important features of Section 3 are illustrated in Sections 5.1, 5.2, and 5.3.

### *3.2. Profit and Risk under RMBS CDOs*

In this subsection, we investigate a subprime mortgage model for profit under RMBS CDOs and its relationship with retained earnings.

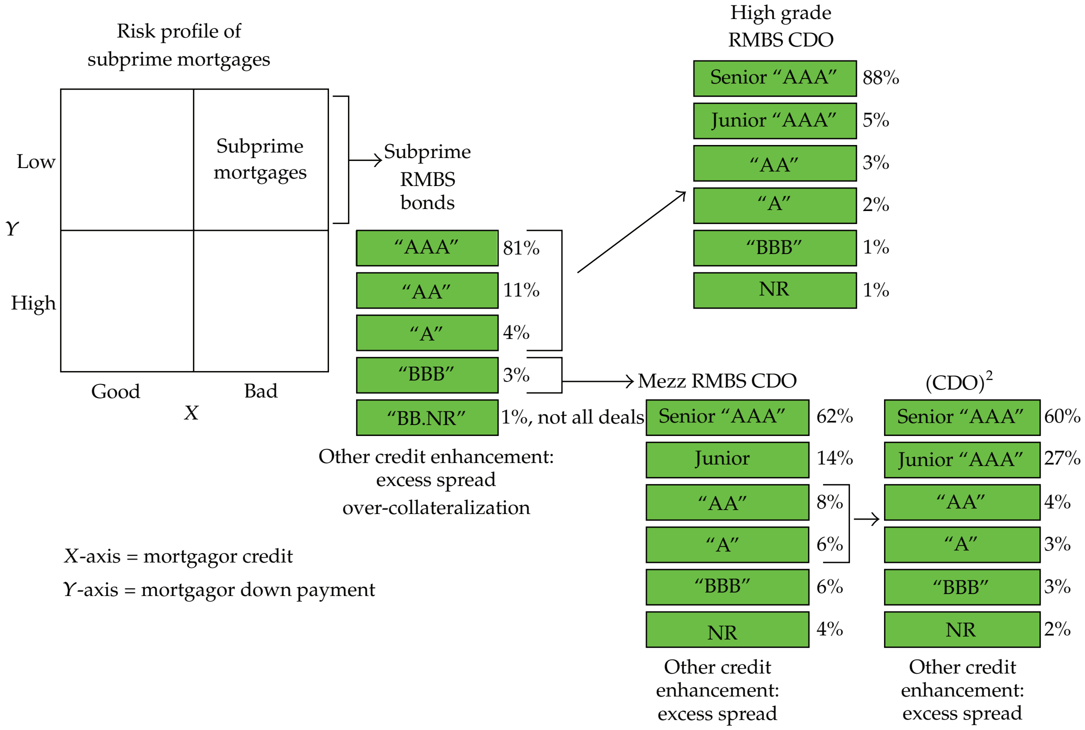


**Figure 3:** Chain of subprime structured mortgage products; compare with [24].

### *3.2.1. A Subprime Mortgage Model for Profit and Risk under RMBS CDOs*

In this paper, a subprime mortgage model for profit under RMBS CDOs can be constructed by considering the difference between cash inflow and outflow. For this profit, in period $t$, *cash inflow* is constituted by returns on the residual from RMBSs securitization, $r_t^{rb}\widehat{f}_t^{\Sigma b} f_t^{\Sigma b} B_t$, securitized subprime RMBSs, $r_t^B(1-\widehat{f}_t^{\Sigma b}) f_t^{\Sigma b} B_t$, unsecuritized securities, $r_t^B(1-f_t^{\Sigma b})B_t$, Treasuries, $r_t^{\mathrm{T}}\mathrm{T}_t$, and mortgages, $r_t^M M_t$, as well as the recovery amount, $R_t$, monoline insurance protection leg payments, $C(S(\mathcal{C}_t))$, and the present value of future gains from subsequent RMBS purchases and their securitizations, $\Pi_t^{\Sigma p}$. On the other hand, we consider the average weighted cost of funds to securitize RMBSs, $\overline{c}^{M\Sigma\omega b}\widehat{f}_t^{\Sigma b} f_t^{\Sigma b}B_t$, losses from securitized RMBSs, $r_t^{S\Sigma b}\widehat{f}_t^{\Sigma b} f_t^{\Sigma b}B_t$, forfeit costs related to monoline insurance wrapping RMBS CDOs, $c_t^{i\Sigma b}\widehat{f}_t^{\Sigma b} f_t^{\Sigma b}B_t$, transaction cost to sell RMBSs, $c_t^{tb}(1-\widehat{f}_t^{\Sigma b})f_t^{\Sigma b}B_t$, and transaction costs from securitized RMBSs $c_t^{t\Sigma b}(1-\widehat{f}_t^{\Sigma b})f_t^{\Sigma b}B_t$ as part of *cash outflow*. Additional components of outflow are weighted average cost of funds for selling RMBSs, $\overline{c}_t^{M\omega b}(1-f_t^{\Sigma b})B_t$, fraction of the face value of unsecuritized RMBSs corresponding to $f_t^{\Sigma b}(1-f_t^{\Sigma b})B_t$, monoline insurance premium for unsecuritized RMBSs losses, $p_t^{ib}(1-f_t^{\Sigma b})B_t$, decreasing value of adverse selection, $a^b f_t^{\Sigma b}B_t$, the all-in cost of holding RMBSs, $c_t^M M_t$, interest paid to depositors, $r_t^D D_t$, the cost of taking deposits, $c^D D_t$, provisions against deposit withdrawals, $P^{\mathrm{T}}(\mathtt{T}_t)$, while $r^{\mathrm{B}}$ and $c^{\mathrm{B}}$ are the borrower rate and marginal cost of borrowing, respectively, losses from suboptimal SPVs, $\mathtt{E}_t$

and costs for funding RMBS securitization, $\mathtt{F}_t$. From the above, we have that model for profit under subprime RMBS CDOs may have the form

$$
\begin{aligned}
\Pi_t^{\Sigma b} = {} & \left(r_t^r - \overline{c}_t^{M\Sigma\omega} - r_t^{S\Sigma} - c_t^{i\Sigma}\right)\widehat{f}_t^{\Sigma} f_t^{\Sigma} M_t + \left(r_t^M - c_t^t - c_t^{t\Sigma}\right)\left(1 - \widehat{f}_t^{\Sigma}\right) f_t^{\Sigma} M_t \\
& + \left(r_t^M - \overline{c}_t^{M\omega} - p_t^i + c_t^p r_t^f - \left(1 - r_t^R\right) r_t^S\right)\left(1 - f_t^{\Sigma}\right) M_t - a f_t^{\Sigma} M_t \\
& + \left(r_t^{rb} - \overline{c}_t^{M\Sigma\omega b} - r_t^{S\Sigma b} - c_t^{i\Sigma b}\right)\widehat{f}_t^{\Sigma b} f_t^{\Sigma b} B_t + \left(r_t^B - c_t^{tb} - c_t^{t\Sigma b}\right)\left(1 - \widehat{f}_t^{\Sigma b}\right) f_t^{\Sigma b} B_t \\
& + \left(r_t^B - \overline{c}_t^{M\omega b} - p_t^{ib} + c_t^{bp} r_t^{fb} - \left(1 - r_t^{Rb}\right) r_t^{Sb}\right)\left(1 - f_t^{\Sigma b}\right) B_t - a^b f_t^{\Sigma b} B_t + r_t^{\mathrm{T}} \mathrm{T}_t \\
& - \left(r_t^{\mathrm{B}} + c_t^{\mathrm{B}}\right)\mathrm{B}_t - \left(r_t^D + c_t^D\right) D_t + C(\mathbf{E}[S(\mathcal{C}_t)]) - P^{\mathrm{T}}(\mathrm{T}_t) + \Pi_t^{\Sigma p} - \mathtt{E}_t - \mathtt{F}_t,
\end{aligned}
\tag{3.1}
$$

where $\Pi_t^{\Sigma p} = \Pi_t^p + \widetilde{\Pi}_t^{\Sigma}$. Furthermore, by considering $\partial S(\mathcal{C}_t)/\partial \mathcal{C}_t^B < 0$ and (3.1), we know that $\Pi^{\Sigma b}$ is an increasing function of RMBS credit rating, $\mathcal{C}^B$.

From the above, we note that in (3.1) the cash inflow terms $r_t^{rb}\widehat{f}_t^{\Sigma b} f_t^{\Sigma b} B_t$ and $r_t^B(1 - \widehat{f}_t^{\Sigma b}) f_t^{\Sigma b} B_t$ carry credit, market (in particular, interest rate), tranching and operational risks, while $r_t^B(1 - f_t^{\Sigma b})B_t$ embed credit (in particular, counterparty) and market (in particular, interest rate) risks. In (3.1), the cash outflow terms $\overline{c}^{M\Sigma\omega b}\widehat{f}_t^{\Sigma b} f_t^{\Sigma b} B_t$,$r_t^{S\Sigma b}\widehat{f}_t^{\Sigma b} f_t^{\Sigma b} B_t$,$c_t^{i\Sigma b}\widehat{f}_t^{\Sigma b} f_t^{\Sigma b} B_t$, $c_t^{tb}(1 - \widehat{f}_t^{\Sigma b}) f_t^{\Sigma b} B_t$, and $c_t^{t\Sigma b}(1 - \widehat{f}_t^{\Sigma b}) f_t^{\Sigma b} B_t$ involve credit (for instance, counterparty), market (specifically, liquidity and valuation), tranching and operational risks. Also, $\overline{c}_t^{M\omega b}(1 - f_t^{\Sigma b})B_t$ and $p_t^{ib}(1 - f_t^{\Sigma b})B_t$ carry credit, market (particularly, liquidity) and operational risks while $a^b f_t^{\Sigma b} B_t$ embeds credit and market (in the form of liquidity and valuation) risks. As before, the risks that we associate with each of the cash inflow and outflow terms in (3.1) are less straightforward. For instance, strong correlations may exist between each of the aforementioned risks. Also, the risk-carrying terms found in (3.1) affect the entire banking system via systemic risk.

### *3.2.2. Profit under RMBS CDOs and Retained Earnings*

We know that profits, $\Pi_t$, are used to meet its obligations, that include *dividend payments on equity*, $n_t d_t$. The *retained earnings*, $E_t^r$, subsequent to these payments may be computed by using (2.2). After adding and subtracting $(r_t^B - \overline{c}_t^{M\omega b} - p_t^{ib} + c_t^{pb} r_t^{fb} - (1 - r_t^{Rb}) r_t^{Sb})B_t$ from (3.1), we get

$$
\begin{aligned}
\Pi_t^{\Sigma b} = \Pi_t & + \left(r_t^r - \overline{c}_t^{M\Sigma\omega} - r_t^{S\Sigma} - c_t^{i\Sigma} - r_t^M + c_t^t + c_t^{t\Sigma}\right) f_t^{\Sigma} \widehat{f}_t^{\Sigma} M_t \\
& + \left(\overline{c}_t^{M\omega} + p_t^i + \left(1 - r_t^R\right) r_t^S - c_t^t - c_t^{t\Sigma} - c_t^p r_t^f - a\right) f_t^{\Sigma} M_t \\
& + \left(r_t^{rb} - \overline{c}_t^{M\Sigma\omega b} - r_t^{S\Sigma b} - c_t^{i\Sigma b} - r_t^B + c_t^{tb} + c_t^{t\Sigma b}\right) f_t^{\Sigma b} \widehat{f}_t^{\Sigma b} B_t \\
& + \left(\overline{c}_t^{M\omega b} + p_t^{ib} + \left(1 - r_t^{Rb}\right) r_t^{Sb} - c_t^{tb} - c_t^{t\Sigma b} - c_t^{pb} r_t^{fb} - a^b\right) f_t^{\Sigma b} B_t
\end{aligned}
$$

$$
\begin{aligned}
&+ \left(r_t^B - \overline{c}_t^{M\omega b} - p_t^{ib} + c_t^{pb} r_t^{fb} - \left(1 - r_t^{Rb}\right) r_t^{Sb}\right) B_t \\
&- \mathtt{E}_t - \mathtt{F}_t + \widetilde{\Pi}_t^{\Sigma}.
\end{aligned}
\tag{3.2}
$$

Replace $\Pi_t$, by using (2.2). In this case, $\Pi_t^{\Sigma b}$ is given by

$$
\begin{aligned}
\Pi_t^{\Sigma b} = {} & E_t^r + n_t d_t + \left(1 + r_t^O\right) O_t + \left(r_t^r - \overline{c}_t^{M\Sigma\omega} - r_t^{S\Sigma} - c_t^{i\Sigma} - r_t^M + c_t^t + c_t^{t\Sigma}\right) f_t^{\Sigma} \widehat{f}_t^{\Sigma} M_t \\
&+ \left(\overline{c}_t^{M\omega} + p_t^i + \left(1 - r_t^R\right) r_t^S - c_t^t - c_t^{t\Sigma} - c_t^p r_t^f - a\right) f_t^{\Sigma} M_t \\
&+ \left(r_t^{rb} - \overline{c}_t^{M\Sigma\omega b} - r_t^{S\Sigma b} - c_t^{i\Sigma b} - r_t^B + c_t^{tb} + c_t^{t\Sigma b}\right) f_t^{\Sigma b} \widehat{f}_t^{\Sigma b} B_t \\
&+ \left(\overline{c}_t^{M\omega b} + p_t^{ib} + \left(1 - r_t^{Rb}\right) r_t^{Sb} - c_t^{tb} - c_t^{t\Sigma b} - c_t^{pb} r_t^{fb} - a^b\right) f_t^{\Sigma b} B_t \\
&+ \left(r_t^B - \overline{c}_t^{M\omega b} - p_t^{ib} + c_t^{pb} r_t^{fb} - \left(1 - r_t^{Rb}\right) r_t^{Sb}\right) B_t \\
&- \mathtt{E}_t - \mathtt{F}_t + \widetilde{\Pi}_t^{\Sigma}.
\end{aligned}
\tag{3.3}
$$

For (3.3) and (1.7), we obtain an expression for capital of the form

$$
\begin{aligned}
K_{t+1}^{\Sigma b} = {} & n_t (d_t + E_t) - \Pi_t^{\Sigma b} + \Delta F_t + \left(1 + r_t^O\right) O_t + \left(r_t^r - \overline{c}_t^{M\Sigma\omega} - r_t^{S\Sigma} - c_t^{i\Sigma} - r_t^M + c_t^t + c_t^{t\Sigma}\right) f_t^{\Sigma} \widehat{f}_t^{\Sigma} M_t \\
&+ \left(\overline{c}_t^{M\omega} + p_t^i + \left(1 - r_t^R\right) r_t^S - c_t^t - c_t^{t\Sigma} - c_t^p r_t^f - a\right) f_t^{\Sigma} M_t \\
&+ \left(r_t^{rb} - \overline{c}_t^{M\Sigma\omega b} - r_t^{S\Sigma b} - c_t^{i\Sigma b} - r_t^B + c_t^{tb} + c_t^{t\Sigma b}\right) f_t^{\Sigma b} \widehat{f}_t^{\Sigma b} B_t \\
&+ \left(\overline{c}_t^{M\omega b} + p_t^{ib} + \left(1 - r_t^{Rb}\right) r_t^{Sb} - c_t^{tb} - c_t^{t\Sigma b} - c_t^{pb} r_t^{fb} - a^b\right) f_t^{\Sigma b} B_t \\
&+ \left(r_t^B - \overline{c}_t^{M\omega b} - p_t^{ib} + c_t^{pb} r_t^{fb} - \left(1 - r_t^{Rb}\right) r_t^{Sb}\right) B_t - \mathtt{E}_t - \mathtt{F}_t + \widetilde{\Pi}_t^{\Sigma},
\end{aligned}
\tag{3.4}
$$

where $K_t$ is defined by (1.2).

A subprime mortgage model for profit under subprime RMBS CDOs has the form (3.1) given in Section 3.2. Under RMBS CDOs, $\Pi_t^{\Sigma b}$ is given by (3.3), while capital is of the form (3.4). In this regard, before the SMC, investors sought higher profits than those offered by US Treasury bonds. Continued strong demand for RMBSs and RMBS CDOs began to drive down lending standards related to originating mortgages destined for reference portfolios in securitization. RMBS CDOs lost most of their value which resulted in a large decline in the capital of many banks and government-sponsored enterprises (GSEs), with a resultant tightening of credit globally.

As we have seen before, subprime risks and profit play a key role in Section 3.2. In this regard, before the SMC, CDOs purchased subprime RMBS bonds because it was profitable. At first, lower-rated BBB tranches of subprime RMBS were difficult to sell since they were thin and, hence, unattractive. Later the thickness of these tranches increased and investment in them became more alluring for investors (compare with the examples contained in Sections 5.2 and 5.3). Despite this, a purchasing CDO may not be aware of the subprime risks inherent in the RMBS deal, including credit and synthetic risk. In this way, risks were underestimated and mortgages and structured mortgage products were overrated. In particular, tranching added intricacy to securitization. This assertion has resonance with the main hypothesis of this contribution involving the intricacy and design of subprime structured mortgage products and their role in information and opaqueness problems as well as risk mismanagement (compare with Question 3).

### *3.3. Valuation under RMBS CDOs*

If the expression for retained earnings given by (3.3) is substituted into (1.8), the nett cash flow generated for a shareholder is given by

$$\begin{aligned}
N_t^{\Sigma b} = \Pi_t^{\Sigma b} - \Delta F_t &= n_t(d_t + E_t) - K_{t+1}^{\Sigma b} + \left(1 + r_t^O\right) O_t \\
&+ \left(r_t^r - \overline{c}_t^{M\Sigma\omega} - r_t^{S\Sigma} - c_t^{i\Sigma} - r_t^M + c_t^t + c_t^{t\Sigma}\right) f_t^{\Sigma} \widehat{f}_t^{\Sigma} M_t \\
&+ \left(\overline{c}_t^{M\omega} + p_t^i + \left(1 - r_t^R\right) r_t^S - c_t^t - c_t^{t\Sigma} - c_t^p r_t^f - a\right) f_t^{\Sigma} M_t \\
&+ \left(r_t^{rb} - \overline{c}_t^{M\Sigma\omega b} - r_t^{S\Sigma b} - c_t^{i\Sigma b} - r_t^B + c_t^{tb} + c_t^{t\Sigma b}\right) f_t^{\Sigma b} \widehat{f}_t^{\Sigma b} B_t \\
&+ \left(\overline{c}_t^{M\omega b} + p_t^{ib} + \left(1 - r_t^{Rb}\right) r_t^{Sb} - c_t^{tb} - c_t^{t\Sigma b} - c_t^{pb} r_t^{fb} - a^b\right) f_t^{\Sigma b} B_t \\
&+ \left(r_t^B - \overline{c}_t^{M\omega b} - p_t^{ib} + c_t^{pb} r_t^{fb} - \left(1 - r_t^{Rb}\right) r_t^{Sb}\right) B_t - \mathrm{E}_t - \mathrm{F}_t + \widetilde{\Pi}_t^{\Sigma}.
\end{aligned} \tag{3.5}$$

We know that valuation is equal to the nett cash flow plus exdividend value. This translates to the expression

$$V_t^{\Sigma b} = N_t^{\Sigma b} + K_{t+1}^{\Sigma b}, \tag{3.6}$$

where $K_t$ is defined by (1.2). Furthermore, under RMBS CDOs, the analyst evaluates the expected future cash flows in $j$ periods based on a *stochastic discount factor*, $\delta_{t,j}$, such that the investor's value is

$$V_t^{\Sigma b} = N_t^{\Sigma b} + \mathbf{E}\left[\sum_{j=1}^{\infty} \delta_{t,j} N_{t+j}^{\Sigma b}\right]. \tag{3.7}$$

In the above, we note that investor value under RMBS CDOs is given by (3.7). In this regard, to our knowledge, there is no standardization of triggers across CDOs with some having sequential cash flow triggers while others have OC trigger calculations based on ratings changes. As far as performing valuations is concerned, in reality, each RMBS CDO must be separately valued which may not be possible (compare with formula (3.5) for nett cash flow under RMBS CDOs). During the SMC, this played a role in the problems investors faced when they attempted a valuation of CDO tranches. Furthermore, RMBS CDOs, widely held by dealer banks and investors, lost most of their value during this period. Naturally, this led to a dramatic decrease in the investor's valuation from holding such structured mortgage products which, in turn, increased the subprime risks in mortgage markets (refer to formulas (3.6) and (3.7) for the investor's valuation under RMBS CDOs).

### 3.4. Optimal Valuation under RMBS CDOs

In this subsection, we make use of the modeling of assets, liabilities, and capital of the preceding section to solve an optimal valuation problem.

#### 3.4.1. Statement of Optimal Valuation Problem under RMBS CDOs

Suppose that the investor's valuation performance criterion, $J^{\Sigma b}$, at $t$ is given by

$$\begin{aligned} J_t^{\Sigma b} &= \Pi_t^{\Sigma b} + l_t^b\left[K_t^{\Sigma b} - \rho\left(\omega^M M_t + \omega\left(C_t^B\right)B_t + 12.5 f^M(m\text{VaR} + 0)\right)\right] \\ &\quad - c_t^{dw}\left[K_{t+1}^{\Sigma b}\right] + \mathbf{E}\left[\delta_{t,1} V\left(K_{t+1}^{\Sigma b}, x_{t+1}\right)\right], \end{aligned} \tag{3.8}$$

where $l_t^b$ is the Lagrangian multiplier for the total capital constraint, $K_t^{\Sigma}$ is defined by (2.9), $\mathbf{E}[\cdot]$ is the expectation conditional on the investor's information in period $t$ and $x_t$ is the deposit withdrawals in period $t$ with probability distribution $f(x_t)$. Also, $c_t^{dw}$ is the deadweight cost of total capital that consists of equity.

The optimal valuation problem is to maximize the *value* given by (3.7). We can now state the optimal valuation problem as follows.

*Question 6* (statement of optimal valuation problem under RMBS CDOs). Suppose that the total capital constraint, $K^{\Sigma b}$, and the performance criterion, $J^{\Sigma b}$, are given by (2.9) and (3.8), respectively. Investor's optimal valuation problem is to maximize its value given by (3.7) by choosing the RMBS rate, deposits, and regulatory capital for

$$V^{\Sigma b}\left(K_t^{\Sigma b}, x_t\right) = \max_{r_t^B, D_t, \Pi_t^{\Sigma b}} J_t^{\Sigma b}, \tag{3.9}$$

subject to RMBS, balance sheet, cash flow, and financing constraints given by (2.12), (2.13), (3.1), and (3.4), respectively.

#### 3.4.2. Solution to an Optimal Valuation Problem under RMBS CDOs

In this subsection, we find a solution to Question 6 when the capital constraint (2.9) holds as well as when it does not. In this regard, the main result can be stated and proved as follows.

**Theorem 3.1** (solution to an optimal valuation problem under RMBS CDOs). *Suppose that $J^{\Sigma b}$ and $V^{\Sigma b}$ are given by* (3.8) *and* (3.9)*, respectively. When the capital constraint given by* (2.9) *holds (i.e., $l_t^b > 0$), a solution to the optimal valuation problem yields an optimal RMBS supply and* *rate given by* (2.14) *and* (2.15)*, respectively. In this case, optimal deposits, provisions for deposit withdrawals via Treasuries, and profits under RMBS CDO securitization are given by*

$$
\begin{aligned}
D_t^{\Sigma b^*} = \frac{1}{1-\gamma}\Bigg(\overline{D} &+ \frac{\overline{D}}{r_t^p}\left[r_t^{\mathrm{T}} + \left(r_t^{\mathrm{B}} + c_t^{\mathrm{B}}\right) - \frac{1}{1-\gamma}\left(r_t^D + c_t^D\right)\right] \\
&+ \frac{K_t^{\Sigma b}}{\rho\omega\left(\mathcal{C}_t^B\right)} - \frac{\omega^M M_t + 12.5 f^B(m\mathrm{VaR} + 0)}{\omega\left(\mathcal{C}_t^B\right)} + M_t - K_t - \mathrm{B}_t\Bigg),
\end{aligned}
\tag{3.10}
$$

$$
\mathrm{T}_t^{\Sigma b^*} = \overline{D} + \frac{\overline{D}}{r_t^p}\left[r_t^{\mathrm{T}} + \left(r_t^{\mathrm{B}} + c_t^{\mathrm{B}}\right) - \frac{1}{1-\gamma}\left(r_t^D + c_t^D\right)\right],
\tag{3.11}
$$

$$
\begin{aligned}
\Pi_t^{\Sigma b^*} = &\left[\frac{K_t^{\Sigma b}}{\rho\omega^M} - \frac{\omega\left(\mathcal{C}_t^B\right)B_t + 12.5 f^M(m\mathrm{VaR} + 0)}{\omega^M}\right] \\
&\times \Big[\widehat{f}_t^{\Sigma} f_t^{\Sigma}\left(r_t^r - \overline{c}_t^{M\Sigma\omega} - r_t^{S\Sigma} - c_t^{i\Sigma} - r_t^M + c_t^t + c_t^{t\Sigma}\right) \\
&\quad + f_t^{\Sigma}\left(\overline{c}_t^{M\omega} + p_t^i + \left(1 - r_t^R\right)r_t^S - c_t^t - c_t^{t\Sigma} - c_t^p r_t^f - a\right) \\
&\quad + \left(r_t^M - \overline{c}_t^{M\omega} - p_t^i + c_t^p r_t^f - \left(1 - r_t^R\right)r_t^S\right)\Big] \\
&+ \left[\frac{K_t^{\Sigma b}}{\rho\omega\left(\mathcal{C}_t^B\right)} - \frac{\omega^M M_t + 12.5 f^M(m\mathrm{VaR} + 0)}{\omega\left(\mathcal{C}_t^B\right)}\right] \\
&\times \Bigg[\widehat{f}_t^{\Sigma b} f_t^{\Sigma b}\Bigg(r_t^{rb} - \overline{c}_t^{M\Sigma\omega b} - r_t^{S\Sigma b} - c_t^{i\Sigma b} \\
&\qquad\qquad + \frac{1}{b_1}\left(b_0 - \frac{K_t^{\Sigma b}}{\rho\omega\left(\mathcal{C}_t^B\right)} + \frac{\omega^M M_t + 12.5 f^M(m\mathrm{VaR} + 0)}{\omega\left(\mathcal{C}_t^B\right)} + b_2\mathcal{C}_t^B + \sigma_t^B\right) \\
&\qquad\qquad + c_t^{tb} + c_t^{t\Sigma b}\Bigg) \\
&\quad + f_t^{\Sigma b}\left(\overline{c}_t^{M\omega b} + p_t^{ib} + \left(1 - r_t^{Rb}\right)r_t^{Sb} - c_t^{tb} - c_t^{t\Sigma b} - c_t^{pb} r_t^{fb} - a^b\right) \\
&\quad - \Bigg(\frac{1}{b_1}\left(b_0 - \frac{K_t^{\Sigma b}}{\rho\omega\left(\mathcal{C}_t^B\right)} + \frac{\omega^M M_t + 12.5 f^M(m\mathrm{VaR} + 0)}{\omega\left(\mathcal{C}_t^B\right)} + b_2\mathcal{C}_t^B + \sigma_t^B\right) \\
&\quad + \overline{c}_t^{M\omega b} + p_t^{ib} - c_t^{pb} r_t^{fb} + \left(1 - r_t^{Rb}\right)r_t^{Sb}\Bigg) - \left(r_t^D + c_t^D\right)\frac{1}{1-\gamma}\Bigg] \\
&\quad + \left(\overline{D} + \frac{\overline{D}}{r_t^p}\left[r_t^{\mathrm{T}} + \left(r_t^{\mathrm{B}} + c_t^{\mathrm{B}}\right) - \frac{1}{1-\gamma}\left(r_t^D + c_t^D\right)\right]\right)
\end{aligned}
$$

$$\times\left(r_t^{\mathrm{T}}-\left(r_t^D+c_t^D\right)\frac{1}{1-\gamma}\right)-\left(\left(r_t^D+c_t^D\right)\frac{1}{1-\gamma}\right)(M_t-K_t-\mathrm{B}_t)$$
$$-\left(r_t^{\mathrm{B}}+c_t^{\mathrm{B}}\right)\mathrm{B}_t+C(\mathbf{E}[S(\mathcal{C}_t)])-P^{\mathrm{T}}(\mathrm{T}_t)+\Pi_t^{\Sigma p}-\mathrm{E}_t-\mathrm{F}_t, \tag{3.12}$$

*respectively.*

*Proof.* A full proof of Theorem 3.1 can be found in Appendix G. □

The next corollary follows immediately from Theorem 3.1.

**Corollary 3.2** (solution to the optimal valuation problem under RMBS CDOs (Slack)). *Suppose that $J^{\Sigma b}$ and $V^{\Sigma b}$ are given by* (3.8) *and* (3.9)*, respectively and* $P(\mathcal{C}_t)>0$. *When the capital constraint* (2.9) *does not hold (i.e., $l_t^b=0$), a solution to the optimal valuation problem under RMBS CDOs stated in Question 6 yields optimal RMBS CDO supply and its rate*

$$\begin{aligned}B_t^{\Sigma bn^*}={}&\frac{1}{2}\left(b_0+b_2\mathcal{C}_t^B+\sigma_t^B\right)+\frac{b_1}{2\left(1-f_t^{\Sigma b}\widehat{f}_t^{\Sigma b}\right)}\\&\times\Bigg[\overline{c}_t^{M\Sigma\omega b}+p_t^{ib}+\left(1-r_t^{Rb}\right)r_t^{Sb}-c_t^{pb}r_t^{fb}+\frac{(r_t^D+c_t^D)}{2(1-\gamma)}\\&\quad-\left(r_t^{rb}-\overline{c}_t^{M\Sigma\omega b}-r_t^{S\Sigma b}-c_t^{i\Sigma b}+c_t^{tb}+c_t^{t\Sigma b}\right)f_t^{\Sigma b}\widehat{f}_t^{\Sigma b}\\&\quad-\left(\overline{c}_t^{M\omega b}+p_t^{ib}+\left(1-r_t^{Rb}\right)r_t^{Sb}-c_t^{tb}-c_t^{t\Sigma b}-c_t^{pb}r_t^{fb}-a^b\right)f_t^{\Sigma b}\Bigg],\end{aligned}\tag{3.13}$$

$$\begin{aligned}r_t^{B^{\Sigma bn^*}}={}&-\frac{1}{2b_1}\left(b_0+b_2\mathcal{C}_t^B+\sigma_t^B\right)+\frac{1}{2\left(1-f_t^{\Sigma b}\widehat{f}_t^{\Sigma b}\right)}\\&\times\Bigg[\overline{c}_t^{M\Sigma\omega b}+p_t^{ib}+\left(1-r_t^{Rb}\right)r_t^{Sb}-c_t^{pb}r_t^{fb}+\frac{(r_t^D+c_t^D)}{2(1-\gamma)}\\&\quad-\left(r_t^{rb}-\overline{c}_t^{M\Sigma\omega b}-r_t^{S\Sigma b}-c_t^{i\Sigma b}+c_t^{tb}+c_t^{t\Sigma b}\right)f_t^{\Sigma b}\widehat{f}_t^{\Sigma b}\\&\quad-\left(\overline{c}_t^{M\omega b}+p_t^{ib}+\left(1-r_t^{Rb}\right)r_t^{Sb}-c_t^{tb}-c_t^{t\Sigma b}-c_t^{pb}r_t^{fb}-a^b\right)f_t^{\Sigma b}\Bigg],\end{aligned}\tag{3.14}$$

*respectively. In this case, the corresponding $\mathrm{T}_t$, deposits and profits under RMBS CDOs are given by*

$$\mathrm{T}_t^{\Sigma bn^*}=\overline{D}+\frac{\overline{D}}{r_t^p}\left[r_t^{\mathrm{T}}+\left(r_t^{\mathrm{B}}+c_t^{\mathrm{B}}\right)-\frac{1}{1-\gamma}\left(r_t^D+c_t^D\right)\right], \tag{3.15}$$

$$D_t^{\Sigma bn^*}=\frac{1}{1-\gamma}\left(\overline{D}+\frac{\overline{D}}{r_t^p}\left[r_t^{\mathrm{T}}+\left(r_t^{\mathrm{B}}+c_t^{\mathrm{B}}\right)-\frac{1}{1-\gamma}\left(r_t^D+c_t^D\right)\right]+B_t^{\Sigma bn^*}+M_t-K_t-\mathrm{B}_t\right), \tag{3.16}$$

$$\begin{aligned}
\Pi_t^{\Sigma bn^*} = M_t\Big[&\Big(r_t^r - \overline{c}_t^{M\Sigma\omega} - r_t^{S\Sigma} - c_t^{i\Sigma} - r_t^M + c_t^t + c_t^{t\Sigma}\Big)\widehat{f}_t^{\Sigma} f_t^{\Sigma} \\
&+ \Big(\overline{c}_t^{M\omega} + p_t^i + \Big(1 - r_t^R\Big) r_t^S - c_t^t - c_t^{t\Sigma} - c_t^p r_t^f - a\Big) f_t^{\Sigma} \\
&+\Big(r_t^M - \overline{c}_t^{M\omega} - p_t^i + c_t^p r_t^f - \Big(1 - r_t^R\Big) r_t^S\Big)\Big] \\
&+ \Bigg[\frac{1}{2}\Big(b_0 + b_2 \mathcal{C}_t^B + \sigma_t^B\Big) + \frac{b_1}{2(1 - f_t^{\Sigma b}\widehat{f}_t^{\Sigma b})} \\
&\quad \times \Bigg(\overline{c}_t^{M\Sigma\omega b} + p_t^{ib} + \Big(1 - r_t^{Rb}\Big) r_t^{Sb} - c_t^{pb} r_t^{fb} + \frac{(r_t^D + c_t^D)}{2(1-\gamma)} \\
&\quad - \Big(r_t^{rb} - \overline{c}_t^{M\Sigma\omega b} - r_t^{S\Sigma b} - c_t^{i\Sigma b} + c_t^{tb} + c_t^{t\Sigma b}\Big) f_t^{\Sigma b}\widehat{f}_t^{\Sigma b} \\
&\quad - \Big(\overline{c}_t^{M\omega b} + p_t^{ib} + \Big(1 - r_t^{Rb}\Big) r_t^{Sb} - c_t^{tb} - c_t^{t\Sigma b} - c_t^{pb} r_t^{fb} - a^b\Big) f_t^{\Sigma b}\Bigg)\Bigg] \\
&\times \Bigg\{ \widehat{f}_t^{\Sigma b} f_t^{\Sigma b}\Bigg(r_t^{rb} - \overline{c}_t^{M\Sigma\omega b} - r_t^{S\Sigma b} - c_t^{i\Sigma b} \\
&\qquad + \Bigg[\frac{1}{2b_1}\Big(b_0 + b_2 \mathcal{C}_t^B + \sigma_t^B\Big) - \frac{1}{2(1 - f_t^{\Sigma b}\widehat{f}_t^{\Sigma b})} \\
&\qquad \times \Bigg(\overline{c}_t^{M\Sigma\omega b} + p_t^{ib} + \Big(1 - r_t^{Rb}\Big) r_t^{Sb} - c_t^{pb} r_t^{fb} + \frac{(r_t^D + c_t^D)}{2(1-\gamma)} \\
&\qquad - \Big(r_t^{rb} - \overline{c}_t^{M\Sigma\omega b} - r_t^{S\Sigma b} - c_t^{i\Sigma b} + c_t^{tb} + c_t^{t\Sigma b}\Big) f_t^{\Sigma b}\widehat{f}_t^{\Sigma b} \\
&\qquad - \Big(\overline{c}_t^{M\omega b} + p_t^{ib} + \Big(1 - r_t^{Rb}\Big) r_t^{Sb} - c_t^{tb} - c_t^{t\Sigma b} - c_t^{pb} r_t^{fb} - a^b\Big) f_t^{\Sigma b}\Bigg)\Bigg] \\
&\qquad + c_t^{tb} + c_t^{t\Sigma b}\Bigg) \\
&\quad + f_t^{\Sigma b}\Big(\overline{c}_t^{M\omega b} + p_t^{ib} + \Big(1 - r_t^{Rb}\Big) r_t^{Sb} - c_t^{tb} - c_t^{t\Sigma b} - c_t^{pb} r_t^{fb} - a^b\Big) \\
&\quad - \Bigg(\Bigg[\frac{1}{2b_1}\Big(b_0 + b_2 \mathcal{C}_t^B + \sigma_t^B\Big) - \frac{1}{2(1 - f_t^{\Sigma b}\widehat{f}_t^{\Sigma b})} \\
&\qquad \times \Bigg(\overline{c}_t^{M\Sigma\omega b} + p_t^{ib} + \Big(1 - r_t^{Rb}\Big) r_t^{Sb} - c_t^{pb} r_t^{fb} + \frac{(r_t^D + c_t^D)}{2(1-\gamma)} \\
&\qquad - \Big(r_t^{rb} - \overline{c}_t^{M\Sigma\omega b} - r_t^{S\Sigma b} - c_t^{i\Sigma b} + c_t^{tb} + c_t^{t\Sigma b}\Big) f_t^{\Sigma b}\widehat{f}_t^{\Sigma b} \\
&\qquad - \Big(\overline{c}_t^{M\omega b} + p_t^{ib} + \Big(1 - r_t^{Rb}\Big) r_t^{Sb} - c_t^{tb} - c_t^{t\Sigma b} - c_t^{pb} r_t^{fb} - a^b\Big) f_t^{\Sigma b}\Bigg)\Bigg] \\
&\qquad + \overline{c}_t^{M\omega b} + p_t^{ib} - c_t^{pb} r_t^{fb} + \Big(1 - r_t^{Rb}\Big) r_t^{Sb}\Bigg) - \Big(r_t^D + c_t^D\Big)\frac{1}{1-\gamma}\Bigg\}
\end{aligned}$$

$$+\left(\overline{D}+\frac{\overline{D}}{r_t^p}\left[r_t^{\mathrm{T}}+(r_t^{\mathrm{B}}+c_t^{\mathrm{B}})-\frac{1}{1-\gamma}\left(r_t^D+c_t^D\right)\right]\right)$$
$$\times\left(r_t^{\mathrm{T}}-\left(r_t^D+c_t^D\right)\frac{1}{1-\gamma}\right)-\left(\left(r_t^D+c_t^D\right)\frac{1}{1-\gamma}\right)(M_t-K_t-\mathrm{B}_t)$$
$$-(r_t^{\mathrm{B}}+c_t^{\mathrm{B}})\mathrm{B}_t+C(\mathbf{E}[S(\mathcal{C}_t)])-P^{\mathrm{T}}(\mathrm{T}_t)+\Pi_t^{\Sigma p}-\mathrm{E}_t-\mathrm{F}_t. \tag{3.17}$$

The optimal valuation problem under RMBSs is to maximize value given by (3.7) by choosing the RMBS rate, deposits, and regulatory capital for (3.9) under RMBSs, balance sheet, cash flow, and financing constraints given by (2.12), (2.13), (3.1), and (3.4), respectively. The optimal valuation formulas mentioned above can be used to illustrate how information is lost due to intricacy. For RMBS CDO investors, the fact that information is lost implies that it is impossible for them to penetrate the chain backwards and value the chain based on the reference mortgage portfolios. RMBS CDO design itself does not allow for valuation based on the reference mortgage portfolio. This is due to the fact that there are at least two layers of structured products in CDOs. Information is lost because of the difficulty of penetrating to the core assets. Nor is it possible for those at the start of the chain to use their information to value the chain "forwards" in a manner of speaking (compare with the analysis in Section 3.4).

When the capital constraint given by (2.9) holds (i.e., $l_t^b > 0$), a solution to the optimal valuation problem yields optimal RMBSs and RMBS rates of the form (2.14) and (2.15), respectively. Most importantly, all the comments about optimal valuation under RMBSs can be repeated for RMBS CDOs.

## 4. Subprime Mortgage Securitization and Capital under Basel Regulation

In this section, we deal with a model where both subprime RMBS default and risk weights are a function of the period $t$ level of credit rating, $\mathcal{C}_t^B$. The capital constraint is described by the expression in (1.3), where the risk weights on RMBSs, $\omega^B \neq 0$, are considered. Also, in this situation, the risk weight on RMBSs, $\omega(\mathcal{C}_t^B)$, is a decreasing function of the period $t$ level of credit rating, that is, $\partial\omega(\mathcal{C}_t^B)/\partial\mathcal{C}_t^B < 0$. In particular, the risk weights for mortgages are kept constant, that is, $\omega^M = 1$. In this case, the capital constraint (1.3) becomes

$$K_t \geq \rho\left[\omega^M M_t + \omega\left(\mathcal{C}_t^B\right)B_t + 12.5 f^M(m\mathrm{VaR}+0)\right]. \tag{4.1}$$

An example of the fact that capital was in short supply at the outset of the SMC in the second half of 2007 is given below. In this period, Citigroup Inc. had its worst-ever quarterly loss of \$ 9.83 billion and had to raise more than \$ 20 billion in capital from outside investors, including foreign-government investment funds. This was done in order to augment the depleted capital on its balance sheet after bad investments in structured mortgage products. According to the Federal Deposit Insurance Corporation (FDIC), at the time, Citigroup held \$80 billion in core capital on its balance sheet to protect against its \$ 1.1 trillion in assets. In the second half of 2007, Citigroup wrote down about \$ 20 billion. Amazingly, at the end of 2007, major US banks like J. P. Morgan Chase & Co., Wachovia Corp., Washington Mutual Inc.

and Citigroup lobbied for leaner, European-style capital cushions. These banks argued that tighter rules would make it tougher for them to compete globally, since more of their money would be tied up in the capital cushion. Eventually, in July 2008, the US Federal Reserve and regulators acceded to the banks' requests by allowing them to follow rules similar to those in Europe. That ruling enabled US banks to hold looser, European-style capital. However, by then, cracks in the global banking system were already spreading rapidly.

### *4.1. Quantity and Pricing of RMBSs and Capital under Basel Regulation*

In this subsection, we firstly examine how capital, $K$, and the quantity and price of RMBSs, $B$, are affected by changes in the level of credit rating, $\mathcal{C}^B$, when risk weight on RMBSs, $\omega(\mathcal{C}_t^B)$, are allowed to vary as in Section 2. Secondly, we will provide an analogue of this result for RMBS CDOs.

**Theorem 4.1** (subprime mortgage securitization and capital under Basel regulation). *Suppose that the assumptions in Section 2 hold and that $B(\mathcal{C}_t^B) > 0$ and the RMBS risk weight, $\omega(\mathcal{C}_t^B)$, are allowed to vary. In this case, we have that*

(1) *if* $\partial\sigma_{t+1}^{B*}/\partial\mathcal{C}_t^B < 0$, *then* $\partial K_{t+1}^{\Sigma}/\partial\mathcal{C}_t^B > 0$,

(2) *if* $\partial\sigma_{t+1}^{B*}/\partial\mathcal{C}_t^B > 0$, *then* $\partial K_{t+1}^{\Sigma}/\partial\mathcal{C}_t^B < 0$.

*Proof.* The full proof of Theorem 4.1 is contained in Appendix H. □

The following corollary represents an analogue of Theorem 4.1 in the case of RMBS CDOs as discussed in Section 3 and follows immediately.

**Corollary 4.2** (subprime mortgage securitization and capital under Basel regulation). *Suppose that the assumptions in Section 3 hold and that $B(\mathcal{C}_t^B) > 0$ and the RMBS risk weight, $\omega(\mathcal{C}_t^B)$, are allowed to vary. In this case, we have that*

(1) *if* $\partial\sigma_{t+1}^{B*}/\partial\mathcal{C}_t^B < 0$, *then* $\partial K_{t+1}^{\Sigma b}/\partial\mathcal{C}_t^B > 0$,

(2) *if* $\partial\sigma_{t+1}^{B*}/\partial\mathcal{C}_t^B > 0$, *then* $\partial K_{t+1}^{\Sigma b}/\partial\mathcal{C}_t^B < 0$.

*Proof.* A proof of Corollary 4.2 in Appendix I. □

During the SMC, as banks adjusted to mortgage delinquencies and defaults and the breakdown of maturity transformation, the interplay of market malfunctioning or even breakdown, fair value accounting and the insufficiency of equity capital, and, finally, systemic effects of prudential regulation created a detrimental downward spiral in the banking system. By contrast, critical securities are now being traded in markets, and market prices determine the day-to-day assessments of equity capital positions of institutions holding them.

Systemic risk explains why the SMC has turned into a worldwide financial crisis unlike the S&L crisis of the late eighties. There were warnings at the peak of the S&L crisis that overall losses of US savings institutions might well amount to \$600–800 billion. This is no less than the IMF's estimates of losses in subprime RMBSs. However, these estimates never translated into market prices and the losses of the S&Ls were confined to the savings institutions and to the deposit insurance institutions that took them over. This difference in institutional arrangements explains why the fallout from the SMC has been so much more severe than that of the S&L crisis.

### 4.2. Subprime RMBSs and Their Rates under Basel Capital Regulation (Slack Constraint)

Next, we consider the effect of a shock to the period $t$ level of RMBS credit rating, $\mathcal{C}_t^B$ on RMBSs, $B$, and the subprime RMBS rate, $r^B$. In particular, we analyze the case where the capital constraint (4.1) is slack.

**Proposition 4.3** (subprime RMBSS under Basel capital regulation (slack constraint)). *under the same hypothesis as Theorem 4.1, when $l_t^b = 0$ we have that*

$$\frac{\partial B_{t+j}^{\Sigma n*}}{\partial \mathcal{C}_t^B} = \frac{1}{3}\mu_j{}^{\mathcal{C}^B}\left[2b_2 - b_1\left(1 - f_t^{\Sigma}\right)\left(\frac{\partial p^i\left(\mathcal{C}_{t+j}^B\right)}{\partial \mathcal{C}_{t+j}^B} + \frac{\partial r_t^S\left(\mathcal{C}_{t+j}^B\right)}{\partial \mathcal{C}_{t+j}^B}\right)\right],$$
$$\frac{\partial r_{t+j}^{B^{\Sigma n*}}}{\partial \mathcal{C}_t^B} = -\frac{1}{3}\mu_j{}^{\mathcal{C}^B}\left[\frac{b_2}{b_1} + \left(1 - f_t^{\Sigma}\right)\left(\frac{\partial p^i\left(\mathcal{C}_{t+j}^B\right)}{\partial \mathcal{C}_{t+j}^B} + \frac{\partial r_t^S\left(\mathcal{C}_{t+j}^B\right)}{\partial \mathcal{C}_{t+j}^B}\right)\right]. \tag{4.2}$$

*Proof.* The proof of Proposition 4.3 can be found in Appendix J. □

Before the SMC, there was a relative decline in equity related to the capital that banks held in fulfilment of capital adequacy requirements as well as the buffers that they held in excess of required capital. A decline in required capital was made possible by changes in statutory rules relating to the prudential regulation of bank capital. The changes in rules provided banks with the option to determine regulatory capital requirements by assessing value-at-risk in the context of their own quantitative risk models, which they had developed for their own risk management. In particular, internationally active banks were able to determine capital requirements for market risks on the basis of these internal models. The amount of capital they needed to hold against any given asset was thereby greatly reduced.

### 4.3. Subprime RMBSs and Their Rates under Basel Capital Regulation (Holding Constraint)

Next, we present results about the effect of changes in the level of credit rating, $\mathcal{C}^B$, on RMBSs when the capital constraint (4.1) holds.

**Proposition 4.4** (subprime RMBSS under Basel capital regulation (holding constraint)). *Assume that the same hypothesis as in Theorem 4.1 holds. If $l_t^b > 0$ then by taking the first derivatives of* (2.14) *with respect to $\mathcal{C}_t^B$ and using the fact that the risk weights for mortgages, $\omega^M$, are constant we obtain*

$$\frac{\partial B_t^*}{\partial \mathcal{C}_t^B} = -\frac{K_t^{\Sigma} - \rho\left(\omega^M M_t + 12.5 f^M (m\mathrm{VaR} + \mathrm{O})\right)}{\left[\omega\left(\mathcal{C}_t^B\right)\right]^2 \rho} \frac{\partial \omega\left(\mathcal{C}_t^B\right)}{\partial \mathcal{C}_t^B}. \tag{4.3}$$

*In this situation, the subprime RMBS payout rate response to changes in the level of credit rating is given by*

$$\frac{\partial r_t^{B*}}{\partial \mathcal{C}_t^B} = -\frac{b_2}{b_1} - \frac{K_t^{\Sigma} - \rho\left(\omega^M M_t + 12.5 f^M (m\text{VaR} + 0)\right)}{\left[\omega(\mathcal{C}_t^B)\right]^2 \rho b_1} \frac{\partial \omega(\mathcal{C}_t^B)}{\partial \mathcal{C}_t^B}. \tag{4.4}$$

*Proof.* In order to prove Proposition 4.4, we can consult Appendix K. □

Section 4.3 present results about the effect of changes in the level of credit rating, $\mathcal{C}^B$, on RMBSs when the capital constraint (4.1) holds. If $l_t^b > 0$ then by taking the first derivatives of (2.14) with respect to $\mathcal{C}_t^B$ and using the fact that the risk weights for mortgages, $\omega^M$, are constant we obtain (4.3). In this situation, the subprime RMBS payout rate response to changes in the level of credit rating is given by (4.4).

Unlike in the 19th century, there is no modern equivalent to clearing houses that allowed information asymmetry, contagion, inefficiency and loss to dissipate. During the SMC, there was no information producing mechanism that was implemented. Instead, accountants follow rules by, for instance, enforcing "marking." Even for earlier vintages, accountants initially seized on the ABX indices in order to determine "price," but were later willing to recognize the difficulties of using ABX indices. However, marking-to-market implemented during the SMC, has very real effects because regulatory capital and capital for CRA purposes is based on generally accepted accounting principles (GAAP). The GAAP measure of capital is probably a less accurate measure of owner-contributed capital than the Basel measure of capital since the latter takes into account banks' exposure to credit, market and operational risk and their off-balance sheet activities. There are no sizeable platforms that can operate ignoring GAAP capital. During the SMC, partly as a result of GAAP capital declines, banks are selling large amounts of assets or are attempting to sell assets to clean up their balance sheets, and in so doing raise cash and delevering. This pushes down prices, and another round of marking down occurs and so on. This downward spiral of prices—marking down then selling then marking down again—is a problem where there is no other side of the market (see, e.g., [16]).

### *4.4. Subprime RMBSs and Their Rates under Basel Capital Regulation (Future Time Periods)*

In the sequel, we examine the effect of a current credit rating shock in future periods on subprime RMBSs, $B$, and their payout rates, $r^B$. If the capital constraint is slack, the response of subprime RMBSs and their rates in period $j \geq 1$ to current fluctuations in the level of credit rating is described by Theorem 4.1. Nevertheless, as time goes by, the impact of the credit rating shock is minimized since $\mu_j^{\mathcal{C}^B} < 1$. In future, if the capital constraint holds, the response of subprime RMBSs and their rates to a change in the level of credit rating, $\mathcal{C}_t^B$, is described by

$$\frac{\partial B^*_{t+j}}{\partial \mathcal{C}_t^B} = \frac{\mu_{j-1}^{\mathcal{C}^B}}{\omega\left(\mathcal{C}_{t+j}^B\right)\rho}\left[\frac{\partial\left(K_{t+j}^{\Sigma} - \rho(\omega^M M_{t+j} + 12.5 f^M(m\mathrm{VaR} + 0))\right)}{\partial \mathcal{C}_{t-1+j}^B}\right]$$
$$- \frac{\mu_{j-1}^{\mathcal{C}^B}}{\omega\left(\mathcal{C}_{t+j}^B\right)\rho}\left[\frac{\mu^{\mathcal{C}^B}}{\omega\left(\mathcal{C}_{t+j}^B\right)}\left(K_{t+j}^{\Sigma} - \rho\left(\omega^M M_{t+j} + 12.5 f^M(m\mathrm{VaR} + 0)\right)\right)\frac{\partial \omega\left(\mathcal{C}_{t+j}^B\right)}{\partial \mathcal{C}_{t+j}^B}\right], \tag{4.5}$$

$$\frac{\partial r^{B^*}_{t+j}}{\partial \mathcal{C}_t^B} = -\frac{b_2}{b_1}\mu_j^{\mathcal{C}^B} + \frac{\mu_{j-1}^{\mathcal{C}^B}}{\omega\left(\mathcal{C}_{t+j}^B\right)\rho b_1}\frac{\partial\left(K_{t+j}^{\Sigma} - \rho(\omega^M M_{t+j} + 12.5 f^M(m\mathrm{VaR} + 0))\right)}{\partial \mathcal{C}_{t-1+j}^B}$$
$$- \frac{\mu_j^{\mathcal{C}^B}}{\left[\omega\left(\mathcal{C}_{t+j}^B\right)\right]^2\rho}\left(K_{t+j}^{\Sigma} - \rho\left(\omega^M M_{t+j} + 12.5 f^M(m\mathrm{VaR} + 0)\right)\right)\frac{\partial \omega\left(\mathcal{C}_{t+j}^B\right)}{\partial \mathcal{C}_{t+j}^B}. \tag{4.6}$$

From (4.5), it can be seen that future subprime RMBSs can either rise or fall in response to positive credit rating shocks. This process depends on the relative magnitudes of the terms in (4.5). If capital rises in response to positive credit rating shocks, subprime RMBSs can fall provided that the effect of the shock on capital is greater than the effect of the shock on subprime RMBS risk weights.

In Section 4.4 we examine the effect of a current credit rating shock in future periods on subprime RMBSs, $B$, and their payout rates, $r^B$. The response of subprime RMBSs and RMBS rates to a change in the level of credit rating, $\mathcal{C}_t^B$, is described by (4.5) if the capital constraint holds. Most importantly, all the comments about mortgages and their rates under Basel capital regulation for RMBSs can be repeated for RMBS CDOs.

The incidence of systemic risk in the SMC has been exacerbated by an insufficiency of equity capital held against future mortgage losses. As the system of risk management on the basis of quantitative risk models was being implemented, banks were becoming more conscious of the desirability of "economizing" on equity capital and of the possibility of using the quantitative risk models for this purpose. Some of the economizing on equity capital involved improvements in the attribution of equity capital to different activities, based on improvements in the awareness and measurement of these activities' risks. Some of the economizing on equity capital led to the relative decline in equity that is one of the elements shaping the dynamics of the downward spiral of the financial system since August 2007. One may assume that the loss of resilience that was caused by the reduction in equity capital was to some extent outweighed by the improvements in the quality of risk management and control. However, there may also have been something akin to the effect that the instalment of seat belts or antiblocking systems in cars induces people to drive more daringly. A greater feeling of protection from harm or a stronger sense of being able to maintain control may induce people to take greater risks.

## 5. Examples Involving Subprime Mortgage Securitization

In this section, we provide examples to illustrate some of the results obtained in the preceding sections. In one way or the other all of the examples in this section support the claim that the SMC was mainly caused by the intricacy and design (refer to Sections 5.1, 5.2, and 5.3) of systemic agents (refer to Section 5.1), mortgage origination (refer to Sections 5.2 and 5.3) and securitization (refer to Sections 5.1, 5.2, and 5.3) that led to information (asymmetry, contagion, inefficiency and loss) problems, valuation opaqueness and ineffective risk mitigation (refer to Sections 5.1, 5.2, and 5.3).

### *5.1. Numerical Example Involving Subprime Mortgage Securitization*

In this subsection, we present a numerical example to highlight some issues in Sections 2 and 3. In particular, we address the role of valuation in house prices. Here we bear in mind that we solve a subprime mortgage securitization maximization problem subject to the financing and regulatory capital constraints, with and without CDO tranching.

The choices of the values of the economic variables in this subsection are justified by considering data from LoanPerformance (LP), Bloomberg, ABSNET, Federal Housing Finance Agency (FHFA; formerly known as OFHEO), Federal Reserve Bank of St Louis (FRBSL) database, Financial Service Research Program's (FSRP) mortgage database, Securities Industry and Financial Markets Association (SIFMA) Research and Statistics as well as Lender Processing Services (LPS; formerly called McDash Analytical) for selected periods before and during the crisis. Additional parameter choices are made by looking at, for instance, [2, 3]. These provide enough information to support the choices for prices, rates and costs while the parameter amounts are arbitrary.

#### *5.1.1. Choices of Subprime Mortgage Securitization Parameters*

In Table 4 below, we make choices for subprime securitization, profit and valuation parameters.

#### *5.1.2. Computation of Subprime Mortgage Securitization Parameters*

We compute important equations by using the values from Table 4. For $\widehat{E}_{t-1} = (500 - 150)/1.75 = 200$ and $\widehat{E}_t = (650 - 150)/2 = 250$, in period $t$, the investor's profit under RMBSs and retained earnings in (2.4) is given by $\Pi_t^{\Sigma} = 2502$. The investor's capital in (2.5) is given by $K_{t+1}^{\Sigma} = 650$ while the nett cash flow under RMBSs given by (2.6) is computed as $N_t^{\Sigma} = 439.7$. Furthermore, valuation in (2.7) is equal to $V_t^{\Sigma} = 1089.7$, while total capital constraint in (2.9) is given by $K_t^{\Sigma} = 500 \geq 136$. Investor's optimal valuation problem under RMBSs is to maximize the value by choosing the RMBS rate, deposits and regulatory capital for (2.11) subject to RMBS, balance sheet, cash flow and financing constraints given by (2.12), (2.13), (2.1) and (2.5), respectively. Here, we have $\sigma_t^B = -6725$ and $D_t = 9300$. The investor's optimal RMBS supply (2.14) and its rate (2.15) are given by $B_t^* = 10400$ and $r_t^{B^*} = 1.925$, respectively. In this case, optimal deposits (2.16), provisions for deposit withdrawals via Treasuries (2.17) and profits under RMBS securitization (2.18) are given by $D_t^{\Sigma^*} = 26855.77$, $\mathrm{T}_t^{\Sigma^*} = 7246.53$ and $\Pi_t^{\Sigma^*} = -28284.7$, respectively.

**Table 4:** Choices of subprime mortgage securitization parameters.

| Parameter | Period $t$ | Period $t+1$ | Parameter | Period $t$ | Period $t+1$ | Parameter | Period $t$ | Period $t+1$ |
|---|---|---|---|---|---|---|---|---|
| $M$ | \$10 000 | \$12 000 | $m_0$ | \$5 000 | \$5 000 | $L$ | 0.909 | 1.043 |
| $\omega^M$ | 0.05 | 0.05 | $m_1$ | \$5 000 | \$5 000 | $L^{1*}$ | 0.9182 | 1.0554 |
| $r^M$ | 0.051 | 0.082 | $m_2$ | \$5 000 | \$5 000 | $L^{n*}$ | 0.3802 | 0.4439 |
| $\overline{c}^{M\omega}$ | 0.0414 | 0.0414 | $\mathcal{C}^{\mathcal{B}}$ | 0.5 | 0.5 | $L^{2*}$ | 0.9201 | 1.0563 |
| $p^i$ | 0.01 | 0.01 | $a$ | 0.05 | 0.05 | $c^{p*}$ | 0.0611 | 0.0633 |
| $c^p$ | 0.05 | 0.05 | $C(\mathbf{E}[S(\mathcal{C}_t)])$ | 400 | 400 | $\Delta F$ | 2 062.3 | — |
| $r^f$ | 0.01 | 0.01 | $r^p$ | 0.1 | 0.1 | $b_0$ | \$5 000 | \$5 000 |
| $r^R$ | 0.5 | 0.5 | $K$ | \$500 | \$650 | $b_1$ | \$5 000 | \$5 000 |
| $r^S$ | 0.15 | 0.25 | $\mathtt{O}$ | \$150 | \$150 | $b_2$ | \$5 000 | \$5 000 |
| $H$ | \$11 000 | \$11 500 | $r^{\mathtt{O}}$ | 0.101 | 0.101 | $c^{i\Sigma}$ | 0.05 | 0.06 |
| $\Lambda$ | \$50 000 | \$63 157.89 | $E$ | \$250 | \$250 | $\overline{c}^{M\Sigma\omega}$ | 0.045 | 0.045 |
| $\rho$ | 0.08 | 0.08 | $E^p$ | \$150 | \$150 | $c^t$ | 0.03 | 0.03 |
| $C$ | \$1 000 | \$1 000 | $E^c$ | \$100 | \$100 | $c^{t\Sigma}$ | 0.04 | 0.04 |
| $r^B$ | 0.105 | 0.105 | $r^{\mathsf{B}}$ | 0.1 | 0.1 | $r^r$ | 0.041 | 0.072 |
| $c^B$ | 0.101 | 0.101 | $c^{\mathsf{B}}$ | 0.09 | 0.09 | $r^{S\Sigma}$ | 0.15 | 0.25 |
| $B$ | \$1 300 | \$1 500 | $\mathtt{B}$ | \$5 200 | \$6 200 | $\widehat{f}^{\Sigma}$ | 0.3 | 0.2 |
| $\omega^B$ | 0.5 | 0.5 | $\Pi^p$ | \$6 000 | \$6 000 | $f^{\Sigma}$ | 0.65 | 0.5 |
| $r^{\mathtt{T}}$ | 0.036 | 0.04 | $D$ | \$9 300 | \$11 100 | $\mathtt{F}$ | 700 | 500 |
| $\mathtt{T}$ | \$2 000 | \$2 000 | $r^D$ | 0.105 | 0.105 | $\mathtt{E}$ | 500 | 300 |
| $O$ | 150 | 150 | $r^O$ | 0.101 | 0.101 | $a^b$ | 0.05 | 0.05 |
| $n$ | 1.75 | 2 | $r^L$ | 0.02 | 0.05 | $\Pi^{\Sigma p}$ | \$8 000 | \$8 000 |
| $d$ | 5.4 | 5.4 | $f^M$ | 0.08 | 0.19 | $\widetilde{\Pi}^{\Sigma}$ | 2 000 | 2 000 |
| $m$ Var | 400 | 400 | $\omega(\mathcal{C}^{\mathcal{B}})$ | 0.5 | 0.5 | $r^{rb}$ | 0.041 | 0.072 |
| $P^{\mathtt{T}}$ | \$800 | \$1 200 | $c^D$ | 0.101 | 0.101 | $\overline{c}^{M\Sigma\omega b}$ | 0.05 | 0.05 |
| $\gamma$ | 0.1828 | 0.2207 | $\varrho$ | 0.031 | 0.032 | $c^{i\Sigma}$ | 0.05 | 0.06 |
| $S$ | 750 | 1 500 | $R$ | 550 | 575 | $r^{S\Sigma b}$ | 0.15 | 0.25 |
| $E^r$ | \$1 849.8 | \$518.65 | $\sigma^M$ | 2 755 | 4 910 | $c^{tb}$ | 0.035 | 0.035 |
| $u$ | 0.03621 | 0.06587 | $v$ | 0.90329 | 0.90329 | $c^{t\Sigma b}$ | 0.045 | 0.045 |
| $w$ | 0.04429 | 0.04398 | $\Pi$ | \$2 024.4 | \$694.6 | $f^{\Sigma b}$ | 0.4 | 0.3 |
| $M^*$ | \$10 100 | \$12 137.5 | $r^{M^*}$ | 0.031 | 0.0545 | $\widehat{f}^{\Sigma b}$ | 0.2 | 0.15 |
| $D^*$ | \$15 387.34 | \$18 631.82 | $T^*$ | \$7 246.53 | \$7 732.28 | $p^{ib}$ | 0.01 | 0.01 |
| $\Pi^*$ | \$656.02 | −\$ 974.36 | $M^{n^*}$ | \$4 182.55 | \$5 104.4 | $r^{Rb}$ | 0.5 | 0.5 |
| $r^{M^{n^*}}$ | 0.8365 | 1.0209 | $T^{n^*}$ | \$7 246.53 | \$7 732.28 | $r^{Sb}$ | 0.15 | 0.25 |
| $D^{n^*}$ | \$8 601.42 | \$9 606.93 | $\Pi^{n^*}$ | \$7 659.27 | \$8 918.64 | $c^{pb}$ | 0.05 | 0.05 |
| $r^{fb}$ | 0.01 | 0.01 | $f^B$ | 0.08 | 0.08 | $c^{i\Sigma b}$ | 0.05 | 0.06 |
| $\overline{c}^{M\omega b}$ | 0.05 | 0.055 | $c^M$ | 0.04 | 0.06 | $r^{p\Sigma}$ | 0.01 | 0.01 |

When the capital constraint (2.9) does not hold, then the solutions for the investor's optimal RMBSs (2.19) and RMBS rate (2.20) are given by $B_t^{\Sigma n^*} = 1094.9$ and $r_t^{B^{\Sigma n^*}} = 0.064$, respectively. In this case, corresponding optimal deposits (2.22) and profits (2.23) are given by $D_t^{\Sigma n^*} = 15469.2$ and $\Pi_t^{\Sigma n^*} = 2198.6$, respectively.

The following values are computed in period $t$ under RMBS CDOs. The investor's profit under RMBS CDOs and retained earnings in (3.3) is given by $\Pi_t^{\Sigma b} = 2463.5$, while the investor's capital in (3.4) is given by $K_{t+1}^{\Sigma b} = 650$. The investor's nett cash flow (3.5) for a shareholder is given by $N_t^{\Sigma b} = 401.2$ Furthermore, the investor's valuation in (3.6) is equal

**Table 5:** Computed subprime mortgage securitization parameters.

| Parameter | Period $t$ | Period $t+1$ | Parameter | Period $t$ | Period $t+1$ |
|---|---|---|---|---|---|
| $\Pi^{\Sigma}$ | \$2 502 | \$1 876 | $K^{\Sigma}$ | 500 | \$650 |
| $N^{\Sigma}$ | \$439.7 | −\$186.3 | $V^{\Sigma}$ | \$1 089.7 | \$ 463.7 |
| $\sigma^{B}$ | −6 725 | −6 525 | $B^{*}$ | \$10 400 | \$12 437.5 |
| $r^{B^{*}}$ | 1.925 | 2.2925 | $D^{\Sigma^{*}}$ | \$26 855.77 | \$32 490.42 |
| $T^{\Sigma^{*}}$ | \$7 246.53 | \$7 732.28 | $\Pi^{\Sigma^{*}}$ | −\$28 284.7 | −\$39 962.7 |
| $B^{\Sigma n^{*}}$ | \$1 094.9 | \$1 268.15 | $r^{B^{\Sigma n^{*}}}$ | 0.064 | 0.0586 |
| $D^{\Sigma n^{*}}$ | \$15 469.2 | \$18 157.9 | $\Pi^{\Sigma n^{*}}$ | \$2 198.6 | \$1 746.33 |
| $\Pi^{\Sigma b}$ | \$2 463.5 | \$1 769.07 | $D^{\Sigma b^{*}}$ | \$26 400.56 | \$31 920.68 |
| $T^{\Sigma b^{*}}$ | \$6 874.53 | \$7 288.28 | $\Pi^{\Sigma b^{*}}$ | \$6 330.82 | \$12 331.2 |
| $B^{\Sigma bn^{*}}$ | \$1 118.64 | \$1 294.25 | $r^{B^{\Sigma bn^{*}}}$ | 0.0687 | 0.0639 |
| $D^{\Sigma bn^{*}}$ | \$15 043 | \$17 621.6 | $\Pi^{\Sigma bn^{*}}$ | \$1 399.96 | \$5 82.72 |

to $V_t^{\Sigma b} = 1051.2$ respectively. Optimal deposits (3.10), provisions for deposit withdrawals via Treasuries (3.11) and profits under RMBS CDO securitization (3.12) are given by $D_t^{\Sigma b^*} = 26400.56, T_t^{\Sigma b^*} = 6874.53$ and $\Pi_t^{\Sigma b^*} = 6330.82$. If capital constraint (2.9) does not hold, then the investor's optimal RMBS CDO supply (3.13) and rate (3.14) are given by $B_t^{\Sigma bn^*} = 1118.64$ and $r_t^{B^{\Sigma bn^*}} = 0.0687$. In this case, corresponding optimal deposits (3.16) and profits (3.17) under RMBS CDO securitization are given by $D_t^{\Sigma bn^*} = 15043$ and $\Pi_t^{\Sigma bn^*} = 1399.96$, respectively.

In period $t+1$, profit under RMBSs and retained earnings in (2.4) is given by $\Pi_{t+1}^{\Sigma} = 1876$, while $\sigma_{t+1}^{B} = -6525$. Optimal RMBS supply (2.14) and its rate (2.15) are given by $B_{t+1}^{*} = 12437.5$ and $r_{t+1}^{B^*} = 2.2925$, respectively. The corresponding Treasuries (2.17), deposits (2.16) and profits under RMBS securitization (2.18) are given by $T_{t+1}^{\Sigma^*} = 7732.28$, $D_{t+1}^{\Sigma^*} = 32490.42$, and $\Pi_{t+1}^{\Sigma^*} = -39962.7$, respectively. When the capital constraint (2.9) does not hold, then $B_{t+1}^{\Sigma n^*} = 1268.15$ and $r_{t+1}^{B^{\Sigma n^*}} = 0.0586$, respectively. In this case, corresponding optimal deposits (2.22) and profits (2.23) are given by $D_{t+1}^{\Sigma n^*} = 18157.9$ and $\Pi_{t+1}^{\Sigma n^*} = 1746.33$, respectively.

The following values are computed in period $t+1$ under RMBS CDOs. The investor's profit under RMBS CDOs and retained earnings in (3.3) is given by $\Pi_{t+1}^{\Sigma b} = 1769.07$, the investor's optimal deposits (3.10), provisions for deposit withdrawals via Treasuries (3.11) and profits under RMBS CDO securitization (3.12) are given by $D_{t+1}^{\Sigma b^*} = 31920.68$, $T_{t+1}^{\Sigma b^*} = 7288.28$ and $\Pi_{t+1}^{\Sigma b^*} = 12331.2$, respectively. If the capital constraint (2.9) does not hold, then $B_{t+1}^{\Sigma bn^*} = 1294.25$ and $r_{t+1}^{B^{\Sigma bn^*}} = 0.0639$, respectively. In this case, corresponding optimal deposits (3.16) and profits (3.17) are given by $D_{t+1}^{\Sigma bn^*} = 17621.6$ and $\Pi_{t+1}^{\Sigma bn^*} = 582.72$, respectively.

We provide a summary of computed profit and valuation parameters under RMBSs and RMBS CDOs in Table 5 below.

The example in Section 5.1 shows that under favorable economic conditions (e.g., where RMBS default rates are low and $C^B$ is high) huge profits can be made from RMBS CDOs as was the case before the SMC. On the other hand, during the SMC, when conditions are less favorable (e.g., where RMBS default rates are high and $C^B$ is low), investors suffer large subprime mortgage securitization losses.

We observe from the numerical example that costs of funds and capital constraints from Basel capital regulation have important roles to play in subprime mortgage securitization, profit and valuation. We see that the profit under securitization in period $t+1$ is less than the profit under securitization in period $t$. This is mainly due to higher reference RMBS

**Table 6:** Structured Asset Investment Loan Trust 2005-6 capital structure; source: [25].

| Structured Asset Investment Loan Trust 2005-6 capital structure | | | | | | | |
|---|---|---|---|---|---|---|---|
| Class | Mortgage reference portfolios | Principal type | Principal amount (dollars) | Tranche thickness (%) | Moody's | S&P | Fitch |
| A1 | 1 | Senior | 455 596 000 | 20.18% | Aaa | AAA | AAA |
| A2 | 1 | Senior | 50 622 000 | 2.24% | Aaa | AAA | AAA |
| A3 | 2 | Senior | 506 116 000 | 22.42% | Aaa | AAA | AAA |
| A4 | 3 | Senior seqntl pay | 96 977 000 | 4.30% | Aaa | AAA | AAA |
| A5 | 3 | Senior seqntl pay | 45 050 000 | 2.00% | Aaa | AAA | AAA |
| A6 | 3<br>3 | Senior seqntl pay | 23 226 000 | 1.03% | Aaa | AAA | AAA |
| A7 | 4 | Senior seqntl pay | 432 141 000 | 19.14% | Aaa | AAA | AAA |
| A8 | 4 | Senior seqntl Pay | 209 009 000 | 9.26% | Aaa | AAA | AAA |
| A9 | 4 | Senior seqntl pay | 95 235 000 | 4.22% | Aaa | AAA | AAA |
| M1 | 1, 2, 3, 4 | Subordinated | 68 073 000 | 3.02% | Aa1 | AA+ | AA+ |
| M2 | 1, 2, 3, 4 | Subordinated | 63 534 000 | 2.81% | Aa2 | AA | AA |
| M3 | 1, 2, 3, 4 | Subordinated | 38 574 000 | 1.71% | Aa3 | AA− | AA− |
| M4 | 1, 2, 3, 4 | Subordinated | 34 036 000 | 1.51% | A1 | A+ | A+ |
| M5 | 1, 2, 3, 4 | Subordinated | 34 036 000 | 1.51% | A2 | A | A |
| M6 | 1, 2, 3, 4 | Subordinated | 26 094 000 | 1.16% | A3 | A− | A− |
| M7 | 1, 2, 3, 4 | Subordinated | 34 036 000 | 1.51% | Baa2 | BBB | BBB |
| M8 | 1, 2, 3, 4 | Subordinated | 22 691 000 | 1.01% | Baa3 | BBB− | BBB− |
| M9 | 1, 2, 3, 4 | Subordinated | 11 346 000 | 0.50% | N/R | BBB− | BBB− |
| M10-A | 1, 2, 3, 4 | Subordinated | 5 673 000 | 0.25% | N/R | BBB− | BB+ |
| M10-F | 1, 2, 3, 4 | Subordinated | 5 673 000 | 0.25% | N/R | BBB− | BB+ |

portfolio defaults as a result of higher RMBS rates in period $t + 1$. This was a major cause of the SMC.

### 5.2. Example of a Subprime RMBS Bond Deal

The example contained in this subsection explains a subprime RMBS bond deal related to the Structured Asset Investment Loan Trust 2005-6 (SAIL 2005-6) issued in July 2005 (compare with [3]). The example considers the evolution of different tranches' riskiness with the refinancing of reference mortgage portfolios affecting the loss triggers for subordinated tranches. This changes the sensitivity of the values of different claims to house prices that drive collateral values. Later the example goes on to discuss the Structured Asset Investment Loan Trust 2006-2 (SAIL 2006-2). The bond capital structure is outlined in Table 6 below.

**Table 7:** Summary of the reference mortgage portfolios' characteristics; source: [3].

| Summary of the Reference Mortgage Portfolios' Characteristics | | | | |
|---|---|---|---|---|
| | Pool 1 | Pool 2 | Pool 3 | Pool 4 |
| % First Lein | 94.12% | 98.88% | 100.00% | 93.96% |
| % 2/28 ARMs | 59.79% | 46.68% | 75.42% | 37.66% |
| % 3/27 ARMs | 20.82% | 19.14% | 19.36% | 9.96% |
| % Fixed rate | 13.00% | 8.17% | 2.16% | 11.46% |
| % Full doc | 59.98% | 56.74% | 44.05% | 35.46% |
| % Stated doc | 39.99% | 37.47% | 34.30% | 33.17% |
| % Primary residence | 90.12% | 90.12% | 80.61% | 82.59% |
| WA FICO | 636 | 615 | 673 | 635 |

From Table 6, we see that the majority of tranches in SAIL 2005-6 have an investment-grade rating of BBB− or higher with Class A1 to A9 certificates being rated AAA. On a pro rata basis, Class A1 and A2 certificates receive principal payments, $\varpi^1 f^\Sigma M$, concurrently, unless cumulative reference mortgage portfolio losses or delinquencies exceed specified levels. In the latter case, these classes will be treated as senior, sequential pay tranches.

The classes of certificates listed in Table 6 were offered publicly by the SAIL 2005-6 prospectus supplement while others like Class P, Class X, and Class R certificates were not. Four types of reference mortgage portfolios constitute the deal with limited cross-collateralization. Principal payments, $\varpi^1 f^\Sigma M$, on the senior certificates will mainly depend on how the reference mortgage portfolios are constituted. However, the senior certificates will have the benefit of CE in the form of overcollateralization (OC) and subordination from each mortgage portfolio. As a consequence, if the rate of loss per reference mortgage portfolio related to any class of sen certificates is low, losses in unrelated mortgages may reduce the loss protection for those certificates.

At initiation, we note that the mezz tranches (AA+ to BBB−) were very thin with minimal defaults. This thinness may be offset by a significant prepayment amount, $\varpi^2 f^p f^\Sigma M$, entering the deal at the outset. An example of this is the M9 tranche with a thickness of 50 bps, but with a BBB− investment-grade rating. Although the rating may not necessarily be wrong, the underlying assumption is that the cash flow dynamics of SAIL 2005-6 has a high probability of success.

Some of the characteristics of the reference mortgage portfolios are shown in Table 7.

From Table 6 in Section 5.2, for the structure of SAIL 2005-6, we can deduce that there are four reference mortgage portfolios with limited cross-collateralization. This deal took place immediately prior to the onset of the SMC in mid-2007. Furthermore, it is obvious that principal payments on the sen certificates will largely depend on collections on the reference mortgage portfolios. Thus, even if the loss rate per reference portfolio related to any sen certificates class is low, losses in unrelated mortgages may reduce the loss protection for those certificates. This is so because the sen certificates will have the benefit of CE in the form of OC and subordination from each mortgage pool. This is typically what happened during the SMC with toxic mortgages reducing protection for sen certificates.

Initially, the mezz tranches are thin and small with respect to defaults. This makes the investment-grade rating BBB− of these tranches somewhat surprising. This may be offset by a significant amount of prepayment, $\varpi^2 f^p f^\Sigma M$, coming into the SAIL 2005-6 deal at the onset. Despite the fact that the underlying supposition is that the deal's cash flow dynamics has a high probability of success, the accuracy of these ratings are being questioned in the light of the SMC. The procedure by which $\varpi^2 f^p f^\Sigma M$ from $f^\Sigma M$ are allocated will differ depending on the occurrence of several different triggers (Some of these triggers were simply ignored before and during the SMC.) given in Section 5.2. As noted in [3] and described in the SAIL 2005-6 prospectus supplement, the triggers have the following specifications.

(i) whether a distribution date occurs before or on or after the *step-down date*, which is the latter of the (1) distribution date in July 2008 and (2) first distribution date on which the ratio

$$\frac{\text{Total principal balance of the subordinate certificates plus any OC amount}}{\text{Total principal balance of the mortgages in the trust fund}} \tag{5.1}$$

equals or exceeds the percentage specified in this prospectus supplement;

(ii) a *cumulative loss trigger event* occurs when cumulative losses on the mortgages are higher than certain levels specified in this prospectus supplement;

(iii) a *delinquency event* occurs when the rate of delinquencies of the mortgages over any 3-month period is higher than certain levels set forth in this prospectus supplement;

(iv) in the case of reference mortgage portfolio 1, a *sequential trigger event* occurs if (a) before the distribution date in July 2008, a cumulative loss trigger event occurs or (b) on or after the distribution date in July 2008, a cumulative loss trigger event or a delinquency event occurs.

### *5.3. Comparisons between Two Subprime RMBS Deals*

In this subsection, we follow [3] by considering the subprime securitization deals Ameriquest Mortgage Securities Inc. 2005-R2 (AMSI 2005-R2) and Structured Assets Investment Loan Trust 2006-2 (SAIL 2006-2). Both AMSI 2005-R2 and SAIL 2006-2 possess the basic structures of securitization deals outlined in Section 5.2, with OC and various triggers determining the features of CE. In this regard, we provide an argument about how the speed of securitization effects the optionality of RMBS CDO tranches with respect to the underlying house prices in both deals. We note that AMSI 2005-R2 consists of three reference portfolios while both deals have OC. Our aim is to compare the performance of AMSI 2005-R2 and SAIL 2006-2 with 2005 vintage mortgages and 2006 vintage mortgages, respectively. For instance, we demonstrate that the latter vintage mortgages underperformed as house prices began to decline in that year. The ensuing examples also demonstrate how the extent of refinancing of the reference mortgage portfolios affects securitization.

#### *5.3.1. Details of AMSI 2005-R2 and SAIL 2006-2*

Tables 8 and 9 present AMSI 2005-R2 deal structure, tranche thickness and ratings at the outset as well as in Q1:07. The initial thickness of the BBB tranches—measured as a percentage of collateral—is extremely thin. Rating agencies do not usually allow such thin tranches, but it was anticipated that these tranches will grow as more sen tranches amortize as a result of refinancing and sequential amortization. Further, we note the subordination percentages for BBB tranches at inception. For instance, the M9 tranche of AMSI 2005-R2 was only 2.95% of subordination. However, as amortization occurs the deals shrink, while CE accumulates and

**Table 8:** Ameriquest Mortgage Securities Inc. 2005-R2 (AMSI 2005-R2) at Issue in 2005; source: [26].

| Ameriquest Mortgage Securities Inc. 2005-R2 (AMSI 2005-R2) At Issue in 2005 | | | | | |
|---|---|---|---|---|---|
| | Size | Related mortgage pool(s) | Ratings (fitch, moody's S&P) | % of collateral | Subordination |
| Publicly-offered certificates | | | | | |
| A-1A | 258 089 000 | I | AAA/Aaa/AAA | 21.5% | 35.48% |
| A-1B | 64 523 000 | I | AAA/Aaa/NR | 5.4% | 19.35% |
| A-2A | 258 048 000 | II | AAA/Aaa/AAA | 21.5% | 35.48% |
| A-2B | 64 511 000 | II | AAA/Aaa/NR | 5.4% | 19.35% |
| A-3A | 124 645 000 | III | AAA/Aaa/AAA | 10.4% | 19.35% |
| A-3B | 139 369 000 | III | AAA/Aaa/AAA | 11.6% | 19.35% |
| A-3C | 26 352 000 | III | AAA/Aaa/AAA | 2.2% | 19.35% |
| A-3D | 32 263 000 | III | AAA/Aaa/NR | 2.7% | 19.35% |
| M1 | 31 200 000 | I, II, III | AA+/Aa1/AA+ | 2.6% | 16.75% |
| M2 | 49 800 000 | I, II, III | AA/Aa2/AA | 4.1% | 12.6% |
| M3 | 16 800 000 | I, II, III | AA−/Aa3/AA− | 1.4% | 11.2% |
| M4 | 28 800 000 | I, II, III | A+/A1/A+ | 2.4% | 8.8% |
| M5 | 16 800 000 | I, II, III | A/A2/A | 1.4% | 7.4% |
| M6 | 12 000 000 | I, II, III | A−/A3/A− | 1.0% | 6.4% |
| M7 | 19 200 000 | I, II, III | BBB+/Baa1/BBB+ | 1.6% | 4.8% |
| M8 | 9 000 000 | I, II, III | BBB/Baa2/BBB | 0.7% | 4.05% |
| M9 | 13 200 000 | I, II, III | BBB/Baa2/BBB− | 1.1% | 2.95% |
| Nonpublicly-offered certificates | | | | | |
| M10 | 7 800 000 | I, II, III | BB+/Ba1/BB+ | 1.0% | 1.3% |
| M11 | 12 000 000 | I, II, III | BB/Ba2/BB | 1.3% | 0.0% |
| CE | 15 600 000 | | NR/NR/NR | | |
| Total | 1 200 000 000 | | | | |
| Collateral | 1 200 000 147 | | | | |

reference mortgages refinance. Also, after the step-down date, the BBB tranches will seem attractive—depending on $H$.

In Tables 8 and 9, the abbreviations PIF, WR, and NR are for tranches *paid in full*, *withdrawn rating* and *no rating*, respectively.

Tables 10 and 11 present SAIL 2006-2 deal structure, tranche thickness, and ratings at the outset as well as in Q1:07. Once again, the initial thickness of the BBB tranches—measured as a percentage of collateral—are extremely thin. As far as the subordination percentages for BBB tranches at inception are concerned, for instance, the M8 tranche of SAIL 2006-2 has only 0.7% subordination. As before, as amortization occurs, CE accumulates and reference mortgages refinance, this situation could improve.

In Tables 10 and 11, in addition, there are Class X, P, LT-R and R certificates. The former are entitled to any monthly excess cashflow left over after contracted payouts to offered certificates and Class B1 and B2 certificates and payments to the SPV (includes paying swap counterparties) as well as on and after the distribution date on April 2016 to deposit any final maturity reserve amount in the final maturity reserve account. On the other hand, Class P certificates will solely be entitled to receive all prepayment premiums from reference

**Table 9:** Ameriquest Mortgage Securities Inc. 2005-R2 (AMSI 2005-R2) in Q1:07; source: [26].

| Ameriquest Mortgage Securities Inc. 2005-R2 (AMSI 2005-R2) In Q1:07 | | | | | |
|---|---|---|---|---|---|
| | Size | Related mortgage pool(s) | Ratings (fitch, moody's S&P) | % of collateral | Subordination |
| Publicly-offered certificates | | | | | |
| A-1A | 30 091 837 | I | AAA/Aaa/AAA | 8.3% | 91.67% |
| A-1B | 7 523 047 | I | AAA/Aaa/NA | 2.1% | 89.58% |
| A-2A | 43 208 414 | II | AAA/Aaa/AAA | 12.0% | 77.62% |
| A-2B | 10 801 936 | II | AAA/Aaa/NA | 3.0% | 63.56% |
| A-3A | — | III | PIF/WR/NR | 0.0% | 63.56% |
| A-3B | 9 597 506 | III | AAA/Aaa/AAA | 2.7% | 63.56% |
| A-3C | 26 352 000 | III | AAA/Aaa/AAA | 7.3% | 63.56% |
| A-3D | 3 994 403 | III | AAA/Aaa/AAA | 1.1% | 63.56% |
| M1 | 31 200 000 | I, II, III | AA+/Aa1/AA+ | 8.6% | 54.92% |
| M2 | 49 800 000 | I, II, III | AA/Aa2/AA | 13.8% | 41.13% |
| M3 | 16 800 000 | I, II, III | AA−/Aa3/AA− | 4.7% | 36.48% |
| M4 | 28 800 000 | I, II, III | A+/A1/A+ | 8.0% | 28.50% |
| M5 | 16 800 000 | I, II, III | A/A2/A | 4.7% | 23.85% |
| M6 | 12 000 000 | I, II, III | BBB/A3/A− | 3.3% | 20.53% |
| M7 | 19 200 000 | I, II, III | B/Baa1/BBB+ | 5.3% | 15.21% |
| M8 | 9 000 000 | I, II, III | B/Baa2/BBB | 2.5% | 12.72% |
| M9 | 13 200 000 | I, II, III | B/Baa3/BBB− | 3.7% | 9.06% |
| Nonpublicly-offered certificates | | | | | |
| M10 | 7 800 000 | I, II, III | CCC/Ba1/BB+ | 2.2% | 6.90% |
| M11 | 12 000 000 | I, II, III | CCC/Ba2/BB | 3.3% | 3.58% |
| CE | 12 928 188 | | NR/NR/NR | 3.6% | 0.00% |
| Total | 361 097 331 | | | | |
| Collateral | 361 097 430 | | | | |

mortgage portfolios. Such amounts are not available for payouts to holders of other certificate classes or to servicers as additional servicing compensation. The Class LT-R and R certificates will represent the remaining interest in the assets of the SPV after the required payouts to all other classes of certificates are made. These classes will evidence the residual interests in the REMIC (A REMIC (Real Estate Mortgage Investment Conduit) is an investment vehicle—a legal structure that can hold commercial mortgage loans and mortgages, in trust, and issue securities representing undivided interests in these mortgages. Besides a trust, a REMIC can be a corporation, association or partnership.).

### *5.3.2. Comparisons between AMSI 2005-R2 and SAIL 2006-2*

Judging from Q1:07, the two deals differ dramatically. AMSI 2005-R2 is older than SAIL 2006-2 and by Q1:07, AMSI 2005-R2 has passed its triggers. As expected, the tranche thicknesses and subordination levels have increased. For example, initially M9 from the AMSI 2005-R2 deal had a 2.95% subordination (1.1% collateral) level, but by Q1:07 its subordination

**Table 10:** Structured Asset Investment Loan Trust 2006-2 (SAIL 2006-2) At Issue in 2006; Source: [25].

| Structured Asset Investment Loan Trust 2006-2 (SAIL 2006-2) At Issue in 2006 | | | | | |
|---|---|---|---|---|---|
| | Size | Related Mortgage Pool(s) | Ratings (Fitch, Moody's S&P) | % of Collateral | Subordination |
| Publicly-Offered Certificates | | | | | |
| A1 | 607 391 000 | I | Aaa/AAA/AAA | 45.3% | 16.75% |
| A2 | 150 075 000 | I | Aaa/AAA/AAA | 5.4% | 19.35% |
| A3 | 244 580 000 | II | Aaa/AAA/AAA | 21.5% | 35.48% |
| A4 | 114 835 000 | II | Aaa/AAA/AAA | 5.4% | 19.35% |
| M1 | 84 875 000 | III | Aa2/AA/AA | 10.4% | 19.35% |
| M2 | 25 136 000 | III | Aa3/AA−/AA− | 11.6% | 19.35% |
| M3 | 20 124 000 | III | A1/A+/A+ | 2.2% | 19.35% |
| M4 | 20 124 000 | III | A2/A/A | 2.7% | 19.35% |
| M5 | 15 428 000 | I, II, III | A3/A−/A− | 2.6% | 16.75% |
| M6 | 15 428 000 | I, II, III | Baa1/BBB+/BBB+ | 4.1% | 12.6% |
| M7 | 11 404 000 | I, II, III | Baa2/BBB/BBB | 1.4% | 11.2% |
| M8 | 10 733 000 | I, II, III | Baa3/BBB−/BBB− | 0.7% | 4.05% |
| Non-Publicly-Offered Certificates | | | | | |
| B1 | 7 379 000 | I, II, III | Ba1/Ba1/Ba1 | 0.6% | 1.05% |
| B2 | 7 379 000 | I, II, III | Ba2/Ba1/Ba1 | 0.6% | 0.5% |
| CE | 6 708 733 | | | | |
| Total | 1 341 599 733 | | | | |

(collateral) is 9.06% (3.7%). Despite this, at the time Fitch downgraded the BBB tranches to B. By contrast, SAIL 2006-2 is younger than AMSI 2005-R2 and took place during a period where house prices were flat and it was more difficult to refinance. By Q1:07, neither tranche thickness nor subordination had increased significantly since inception in 2006 thus weakening the SAIL 2006-2 deal. This is reflected by the mezz tranche ratings.

From the example involving the deals AMSI 2005-R2 and SAIL 2006-2 in Section 5.3, it is clear that the thin tranches and low subordination levels at inception are acceptable provided that the reference mortgages refinance in the anticipated way. In this situation, the RMBS bond deals shrink as amortization occurs. Also, depending on house prices, CE will build up, and after the step-down date, investors will consider investing in the BBB tranches.

The features of AMSI 2005-R2 and SAIL 2006-2 illustrate the differences between subprime and standard securitizations with their fixed tranche sizes and use of XS to create CE through reserve fund build-up. However, this is not the primary form of CE. In the subprime case, Section 5.3 illustrates how the *option* on house prices implicitly embedded in subprime reference mortgage portfolios resulted in the behavior of subprime RMBSs being sensitive to house prices. By contrast to standard securitization, the tranche thickness and extent of CE—depend on cash flow coming into the deal from $\varpi^2 f^p f^{\Sigma} M$ on $f^{\Sigma} M$ via refinancing that is also house price dependent.

The deals AMSI 2005-R2 and SAIL 2006-2 illustrate this link to house price very effectively. The former passed its triggers and achieved the CE and subordination levels hoped for at inception—largely due to reference mortgage refinancing and prepayments. By contrast, the SAIL 2006-2 deal deteriorated. In 2006, subprime mortgagors did not accumulate

**Table 11:** Structured Asset Investment Loan Trust 2006-2 (SAIL 2006-2) In Q1:07; source: [25].

| Structured Asset Investment Loan Trust 2006-2 (SAIL 2006-2) in Q1:07 | | | | | |
|---|---|---|---|---|---|
| | Size | Related mortgage pool(s) | Ratings (fitch, moody's S&P) | % of collateral | Subordination |
| Publicly-offered certificates | | | | | |
| A1 | 89 285 238 | I | Aaa/AAA/AAA | 11.0% | 26.16% |
| A2 | 150 075 000 | I | Aaa/AAA/AAA | 18.5% | 26.16% |
| A3 | 244 580 000 | II | Aaa/AAA/AAA | 30.2% | 26.16% |
| A4 | 114 835 000 | II | Aaa/A/A | 14.2% | 26.16% |
| M1 | 84 875 000 | III | Ba3/CCC/B | 10.5% | 15.70% |
| M2 | 25 136 000 | III | B3/CCC/CCC | 3.1% | 12.60% |
| M3 | 20 124 000 | III | Caa2/CCC/CCC | 2.5% | 10.12% |
| M4 | 20 124 000 | III | Caa3/CC/CC | 2.5% | 7.36% |
| M5 | 15 428 000 | I, II, III | Ca/CC/CC | 1.9% | 5.73% |
| M6 | 15 428 000 | I, II, III | C/CC/CC | 1.9% | 3.83% |
| M7 | 11 404 000 | I, II, III | C/CC/C | 1.4% | 2.42% |
| M8 | 10 733 000 | I, II, III | C/D/C | 1.3% | 1.10% |
| Nonpublicly-offered certificates | | | | | |
| B1 | 7 379 000 | I, II, III | C/D/C | 0.9% | 0.19% |
| B2 | 1 534 646 | I, II, III | WR/NR/NR | 0.2% | 0.09% |
| CE | 98 | I, II, II | | 11.0% | 88.99% |
| Total | 810 940 982 | | | | |

enough house equity to refinance with the result that they defaulted on their repayments. Consequently, SAIL 2006-2 was not able to pass its triggers.

## 6. Conclusions and Future Directions

This paper investigates modeling aspects of the securitization of subprime mortgages into SMPs such as RMBSs and CDOs (compare with Questions 1 and 2). In this regard, our discussions in Sections 2 and 3 focus on profit, risk and valuation as well as the role of capital under RMBSs and RMBS CDOs, respectively. As posed in Question 3, the main hypothesis of this paper is that the SMC was largely caused by the intricacy and design of mortgage securitization that led to information (loss, asymmetry, and contagion) problems, valuation opaqueness (compare with Question 4) and ineffective risk mitigation. This claim is illustrated via the examples presented in Section 5.

Our paper has connections with credit, maturity mismatch, basis, counterparty, liquidity, synthetic, prepayment, interest rate, price, tranching and systemic risks. On the face of it, the securitization of housing finance through SMPs appears, in principle, to be a efficient way of shifting risks resulting from the mismatch between the economic lifetimes of SMPs and investors' horizons away from originators and their debtors without impairing originators' incentives to originate mortgages. Securitization would thus appear to provide a substantial improvement in risk allocation in the global banking system. The question is then what went wrong. In several important respects, the practice was different from the theory. Firstly, moral hazard in origination was not eliminated, but was actually enhanced by several developments. Secondly, many of the SMPs did not end up in the portfolios of insurance companies or pension funds, but in the portfolios of highly leveraged investors that engaged in substantial maturity transformation and were in constant need of refinancing. Finally, the markets for refinancing these highly leveraged banks broke down during the SMC. In the years since 2000, with low interest rates, low intermediation margins and depressed stock markets, many private investors were eagerly looking for structured products offering better yields and many banks were looking for better margins and fees. The focus on yields and on growth blinded them to the risk implications of what they were doing. In particular, they found it convenient to rely on CRA assessments of credit risks, without appreciating that these assessments involved some obvious flaws. Given investors' appetite for securitization and high-yielding securities, there was little to contain moral hazard in mortgage origination, which, indeed, seems to have risen steadily from 2001 to 2007. For a while, the flaws in the system were hidden because house prices were rising, partly in response to the inflow of funds generated by this very system. However, after house prices began to fall in the summer of 2006, the credit risk in the reference mortgage portfolios became apparent. Often additional operational risk issues such as model validation, data accuracy and stress testing lie beneath large market risk events. Market events demonstrated that risk cannot always be eliminated and can rarely be completely outsourced. It tends to come back in a different and often more virulent form. For instance, Countrywide Financial had outsourced its credit risk through packaging and selling of mortgages. However, in doing so, the company created sizeable operational risks through its business practices and strategy.

As far as subprime risks are concerned, we identify that investors carry credit, market and operational risks involving mark-to-market issues, subprime mortgage securitizations valuation when sold in volatile markets, uncertainty involved in investment payoffs and the intricacy and design of structured products. Market reactions include market risk, operational risk involving increased volatility leading to behavior that can increase operational risk such as unauthorized trades, dodgy valuations and processing issues and credit risk related to the possibility of bankruptcies if originators, dealer banks and investors cannot raise funds. Recent market events, which demonstrate how credit, market and operational risks come together to create volatility and losses, suggest that it is no longer relevant to dissect, delineate and catalogue credit and market risk in distinct categories without considering their interconnection with operational risk. Underlying many of the larger credit events are operational risk practices that have been overlooked such as documentation, due diligence, suitability and compensation.

A shortcoming of this paper is that it does not provide a complete description of what would happen if the economy were to deteriorate or improve from one period to the next. This is especially interesting in the light of the fact that in the real economy one has yield curves that are not flat and so describe changes in the dynamics of the structured note market. More specifically, we would like to know how this added structure will affect the results obtained in this paper. This is a question for future consideration. Also, we would like to investigate the relationships between subprime agents more carefully. Also, scenarios need to be more robust. They need to account for what will happen when more than one type of risk actualizes. For instance, a scenario can be constructed from 2007-2008 events that could include misselling of products to investors, securitizing that same product, the bringing of lawsuits by retail investors who bought the product and investors who acquired the securitized mortgages. Several questions that need urgent answers arise. What would this scenario affect originators as well as lender, dealer and investment banks? What would the liquidity consequences be? Can the real-life examples be readily studied? Future studies should also consider how the information about house prices and delinquencies and foreclosures was linked to valuations of the various links of the chain.

## Appendix

In this section, we prove some of the main results in the paper.

## A. Proof of Theorem 2.1

An immediate consequence of the prerequisite that the capital constraint (2.9) holds is that RMBS supply is closely related to the capital adequacy constraint and is given by (2.14). Also, the dependence of changes in the RMBS rate on credit rating may be fixed as

$$\frac{\partial r_t^{B*}}{\partial C_t^B} = \frac{b_2}{b_1}. \tag{A.1}$$

Equation (2.14) follows from (2.9) and the fact that the capital constraint holds. This also leads to equality in (2.9). In (2.15), we substituted the optimal value for $B_t$ into control law (2.12) to get the optimal default rate. We obtain the optimal $T_t$ using the following steps. Firstly, we rewrite (1.2) to make deposits the dependent variable, so that

$$D_t = \frac{M_t + B_t + \mathtt{T}_t - \mathtt{B}_t - K_t}{1 - \gamma}. \tag{A.2}$$

Next, we note that the first-order conditions (for verification of these conditions, see the appendix below) are given by

$$\frac{\partial \Pi_t^{\Sigma}}{\partial r_t^B}\left[1 + c_t^{dw} - \mathbf{E}\left\{\int_{\underline{B}}^{\overline{B}} \delta_{t,1} \frac{\partial V^{\Sigma}}{\partial (K_{t+1}^{\Sigma})} dF\left(\sigma_{t+1}^B\right)\right\}\right] - l_t^b \rho b_1 \omega\left(C_t^B\right) = 0, \tag{A.3}$$

$$\frac{\partial \Pi_t^{\Sigma}}{\partial D_t}\left[1 + c_t^{dw} - \mathbf{E}\left\{\int_{\underline{B}}^{\overline{B}} \delta_{t,1} \frac{\partial V^{\Sigma}}{\partial (K_{t+1}^{\Sigma})} dF\left(\sigma_{t+1}^B\right)\right\}\right] = 0, \tag{A.4}$$

$$\rho\left[\omega\left(C_t^B\right) B_t + \omega^M M_t + 12.5 f^M (m\mathrm{VaR} + \mathtt{0})\right] \le K_t^{\Sigma}, \tag{A.5}$$

$$-c_t^{dw} + \mathbf{E}\left\{\int_{\underline{B}}^{\overline{B}} \delta_{t,1} \frac{\partial V^{\Sigma}}{\partial (K_{t+1}^{\Sigma})} dF\left(\sigma_{t+1}^B\right)\right\} = 0. \tag{A.6}$$

Here $F(\cdot)$ is the cumulative distribution of the shock to the RMBS. Using (A.6), we can see that (A.4) becomes

$$\frac{\partial \Pi_t^{\Sigma}}{\partial D_t} = 0. \tag{A.7}$$

Looking at the form of $\Pi_t^{\Sigma}$ given in (2.1) and the equation

$$P^{\mathrm{T}}(\mathrm{T}_t) = \frac{r_t^p}{2\overline{D}}\left[\overline{D} - T_t\right]^2, \tag{A.8}$$

it follows that

$$\begin{aligned}
\Pi_t^{\Sigma} = {} & \left(r_t^r - \overline{c}_t^{M\Sigma\omega} - r_t^{S\Sigma} - c_t^{i\Sigma}\right)\widehat{f}_t^{\Sigma} f_t^{\Sigma} M_t + \left(r_t^M - c_t^t - c_t^{t\Sigma}\right)\left(1 - \widehat{f}_t^{\Sigma}\right) f_t^{\Sigma} M_t \\
& + \left(r_t^M - \overline{c}_t^{M\omega} - p_t^i + c_t^p r_t^f - \left(1 - r_t^R\right) r_t^S\right)\left(1 - f_t^{\Sigma}\right) M_t - a f_t^{\Sigma} M_t + r_t^{\mathrm{T}} \mathrm{T}_t \\
& + \left(r_t^B - c_t^B\right) B_t - \left(r_t^{\mathrm{B}} + c_t^{\mathrm{B}}\right)\mathrm{B}_t - \left(r_t^D + c_t^D\right) D_t + C(\mathbf{E}[S(\mathcal{C}_t)]) - \frac{r_t^p}{2\overline{D}}\left[\overline{D} - T_t\right]^2 \\
& + \Pi_t^{\Sigma p} - \mathrm{E}_t - \mathrm{F}_t.
\end{aligned} \tag{A.9}$$

Finding the partial derivatives of the investor's profit, $\Pi_t^{\Sigma}$, with respect to deposit, $D_t$, we have that

$$\frac{\partial \Pi_t^{\Sigma}}{\partial D_t} = (1 - \gamma)\left(r_t^{\mathrm{T}} + \left(r_t^{\mathrm{B}} + c_t^{\mathrm{B}}\right) + \left(r_t^B - c_t^B\right) + \frac{r_t^p}{\overline{D}}\left(\overline{D} - \mathrm{T}_t\right)\right) - \left(r_t^D + c_t^D\right). \tag{A.10}$$

This would then give us the optimal value for $D_t$. Using (1.2) and all the optimal values calculated to date, we can find optimal deposits as well as optimal profits.

## B. Proof of Corollary 2.2

For the situation where capital constraint (2.9) does not hold (i.e., $l_t^b = 0$), using (A.6) and the fact that $l_t^b = 0$, we can see that (A.3) becomes $\partial \Pi_t^{\Sigma} / \partial r_t^B = 0$. As in the proof of Theorem 2.1, looking at the form of $\Pi_t^{\Sigma}$ given in (2.1) and (A.8), we have (A.9). Therefore,

$$\begin{aligned}
\frac{\partial \Pi_t^{\Sigma}}{\partial r_t^B} = B_t - b_1\Big[ & \left(r_t^r - \overline{c}_t^{M\Sigma\omega} - r_t^{S\Sigma} - c_t^{i\Sigma} - r_t^M + c_t^t + c_t^{t\Sigma}\right)\widehat{f}_t^{\Sigma} f_t^{\Sigma} \\
& + \left(\overline{c}_t^{M\omega} + p_t^i(\mathcal{C}_t) - c_t^p r_t^f + \left(1 - r_t^R\right) r_t^S - c_t^t - c_t^{t\Sigma} - a\right) f_t^{\Sigma} \\
& + \left(r_t^M - \overline{c}_t^{M\omega} - p_t^i(\mathcal{C}_t) + c_t^p r_t^f - \left(1 - r_t^R\right) r_t^S\right)\Big] \\
+ b_1\Big[ & \left(r_t^B - c_t^B\right) - r_t^{\mathrm{T}} - \left(r_t^{\mathrm{B}} + c_t^{\mathrm{B}}\right)\Big] - \frac{r_t^p}{\overline{D}}\left(\overline{D} - \mathrm{T}_t\right) b_1 = 0.
\end{aligned} \tag{B.1}$$

Substituting (A.10) into (B.1) and using (2.12) would give us optimal RMBSs and RMBS rate given by (2.19) and (2.20), respectively. Furthermore, we can find the investor's optimal deposit, deposit withdrawals, and profits. To derive equations (A.3) to (A.6), we rewrite (2.11) to become

$$V^{\Sigma}\left(K_t^{\Sigma}, x_t\right) = \max_{r_t^B, D_t, \Pi_t^{\Sigma}} \left\{\Pi_t^{\Sigma} + l_t^b\left[K_t^{\Sigma} - \rho\left(\omega\left(\mathcal{C}_t^B\right)B_t + \omega^M M_t + 12.5 f^M (m\text{VaR} + \mathtt{O})\right)\right] - c_t^{dw}\left[K_{t+1}^{\Sigma}\right] + \mathbf{E}\left[\delta_{t,1} V\left(K_{t+1}^{\Sigma}, x_{t+1}\right)\right]\right\}. \tag{B.2}$$

By substituting (2.12) and (2.5), (B.2) becomes

$$\begin{aligned} V^{\Sigma}\left(K_t^{\Sigma}, x_t\right) &= \max_{r_t^B, D_t, \Pi_t^{\Sigma}} \Big\{ n_t(d_t + E_t) - K_{t+1}^{\Sigma} + \Delta F_t + \left(1 + r_t^O\right) O_t \\ &\quad + \left(r_t^r - \overline{c}_t^{M\Sigma\omega} - r_t^{S\Sigma} - c_t^{i\Sigma} - r_t^M + c_t^t + c_t^{t\Sigma}\right) f_t^{\Sigma} \hat{f}_t^{\Sigma}\left(m_0 - m_1 r_t^M + m_2 \mathcal{C}_t + \sigma_t^M\right) \\ &\quad + \left(\overline{c}_t^{M\omega} + p_t^i + \left(1 - r_t^R\right) r_t^S - c_t^t - c_t^{t\Sigma} - c_t^p r_t^f - a\right) \\ &\quad \times f_t^{\Sigma}\left(m_0 - m_1 r_t^M + m_2 \mathcal{C}_t + \sigma_t^M\right) - \mathtt{E}_t - \mathtt{F}_t + \widetilde{\Pi}_t^{\Sigma} \\ &\quad + l_t^b\Big[K_t^{\Sigma} - \rho\Big[\omega\left(\mathcal{C}_t^B\right)\left(b_0 + b_1 r_t^B + b_2 \mathcal{C}_t^B + \sigma_t^B\right) + \omega^M\left(m_0 - m_1 r_t^M + m_2 \mathcal{C}_t + \sigma_t^M\right) \\ &\quad + 12.5 f^M (m\text{VaR} + \mathtt{O})\Big]\Big] \\ &\quad - c_t^{dw}\left[K_{t+1}^{\Sigma}\right] + \mathbf{E}\left[\delta_{t,1} V\left(K_{t+1}^{\Sigma}, x_{t+1}\right)\right]\Big\}. \end{aligned} \tag{B.3}$$

Finding the partial derivative of the investor's value in (B.3), with respect to the capital constraint, $K_{t+1}^{\Sigma}$, we have

$$\frac{\partial V^{\Sigma}}{\partial K_{t+1}^{\Sigma}} = -1 - c_t^{dw} + E_t\left[\int_{\underline{B}}^{\overline{B}} \delta_{t,1} \frac{\partial V^{\Sigma}}{\partial K_{t+1}^{\Sigma}} dF\left(\sigma_{t+1}^B\right)\right]. \tag{B.4}$$

Next, we discuss the formal derivation of the first-order conditions (A.3) to (A.6).

## C. First-Order Condition (A.3)

Choosing the RMBS rate, $r_t^B$, from (B.3) and using (B.4) above, the first-order condition (A.3) for Question 5 is

$$\frac{\partial \Pi_t^{\Sigma}}{\partial r_t^B}\left[1 + c_t^{dw} - \mathbf{E}\left\{\int_{\underline{B}}^{\overline{B}} \delta_{t,1} \frac{\partial V^{\Sigma}}{\partial (K_{t+1}^{\Sigma})} dF\left(\sigma_{t+1}^B\right)\right\}\right] - l_t^b \rho b_1 \omega\left(\mathcal{C}_t^B\right) = 0. \tag{C.1}$$

## D. First-Order Condition (A.4)

Choosing the deposits, $D_t$, from (B.3) and using (B.4) above, the first-order condition (A.4) for Question 5 is

$$\frac{\partial \Pi_t^{\Sigma}}{\partial D_t}\left[1 + c_t^{dw} - \mathbf{E}\left\{\int_{\underline{B}}^{\overline{B}} \delta_{t,1}\frac{\partial V^{\Sigma}}{\partial (K_{t+1}^{\Sigma})} dF\left(\sigma_{t+1}^{B}\right)\right\}\right] = 0. \tag{D.1}$$

## E. First-Order Condition (A.5)

We now find the partial derivative of the investor's value in (B.3) with respect to the Lagrangian multiplier, $l_t^b$

$$\frac{\partial V^{\Sigma}}{\partial l_t^b} = K_t^{\Sigma} - \rho\left[\omega\left(C_t^B\right)B_t + \omega^M M_t + 12.5 f^M (m\mathrm{VaR} + \mathrm{O})\right]. \tag{E.1}$$

In this case, the first-order condition (A.5) for Question 5 is given by

$$\rho\left[\omega\left(C_t^B\right)B_t + \omega^M M_t + 12.5 f^M (m\mathrm{VaR} + \mathrm{O})\right] \le K_t^{\Sigma}. \tag{E.2}$$

## F. First-Order Condition (A.6)

Choosing the regulatory capital, $\Pi_t^{\Sigma}$, from (B.3) and using (B.4) above, the first-order condition (A.6) for Question 5 is

$$-1 - c_t^{dw} + \mathbf{E}\left\{\int_{\underline{B}}^{\overline{B}} \delta_{t,1}\frac{\partial V^{\Sigma}}{\partial (K_{t+1}^{\Sigma})} dF\left(\sigma_{t+1}^{B}\right)\right\} + 1 = 0, \tag{F.1}$$

which is the same as

$$-c_t^{dw} + \mathbf{E}\left\{\int_{\underline{B}}^{\overline{B}} \delta_{t,1}\frac{\partial V^{\Sigma}}{\partial (K_{t+1}^{\Sigma})} dF\left(\sigma_{t+1}^{B}\right)\right\} = 0. \tag{F.2}$$

## G. Proof of Theorem 3.1

An immediate consequence of the prerequisite that the capital constraint (2.9) holds, is that RMBS CDO supply is closely related to the capital adequacy constraint and is given by (2.14). Also, the dependence of changes in the RMBS CDO bond rate on credit rating may be fixed as

$$\frac{\partial r_t^B}{\partial C_t^B} = \frac{b_2}{b_1}. \tag{G.1}$$

Equation (2.14) follows from (2.9) and the fact that the capital constraint holds. This also leads to equality in (2.9). In (2.15) we substituted the optimal value for $B_t$ into control law (2.12) to get the optimal default rate. We obtain the optimal $T_t$ using the following steps. Firstly, we rewrite (1.2) to make deposits the dependent variable, so that

$$D_t = \frac{M_t + B_t + \mathtt{T}_t - \mathtt{B}_t - n_t E_{t-1} - O_t}{1-\gamma}. \tag{G.2}$$

Next, we note that the first-order conditions are given by

$$\begin{aligned}
&\frac{\partial \Pi_t^{\Sigma b}}{\partial r_t^B}\left[1 + c_t^{dw} - \mathbf{E}\left\{\int_{\underline{B}}^{\overline{B}} \delta_{t,1} \frac{\partial V^{\Sigma b}}{\partial\left(K_{t+1}^{\Sigma b}\right)} dF\left(\sigma_{t+1}^B\right)\right\}\right] \\
&\quad + b_1\Big[\left(r_t^{rb} - \overline{c}_t^{M\Sigma\omega b} - r_t^{S\Sigma b} - c_t^{i\Sigma b} - r_t^B + c_t^{tb} + c_t^{t\Sigma b}\right) f_t^{\Sigma b} \widehat{f}_t^{\Sigma b} \\
&\qquad + \left(\overline{c}_t^{M\omega b} + p_t^{ib} + \left(1 - r_t^{Rb}\right) r_t^{Sb} - c_t^{tb} - c_t^{t\Sigma b} - c_t^{pb} r_t^{fb} - a^b\right) f^{\Sigma b} \\
&\qquad + \left(r_t^B - \overline{c}_t^{M\omega b} - p_t^{ib} + c_t^{pb} r_t^{fb} - \left(1 - r_t^{Rb}\right) r_t^{Sb}\right)\Big] \\
&\qquad + \left(1 - f_t^{\Sigma b} \widehat{f}_t^{\Sigma b}\right) B_t - l_t^b \rho b_1 \omega\left(\mathcal{C}_t^B\right) = 0,
\end{aligned} \tag{G.3}$$

$$\frac{\partial \Pi_t^{\Sigma b}}{\partial D_t}\left[1 + c_t^{dw} - \mathbf{E}\left\{\int_{\underline{B}}^{\overline{B}} \delta_{t,1} \frac{\partial V^{\Sigma b}}{\partial\left(K_{t+1}^{\Sigma b}\right)} dF\left(\sigma_{t+1}^B\right)\right\}\right] = 0, \tag{G.4}$$

$$\rho\left[\omega\left(\mathcal{C}_t^B\right) B_t + \omega^M M_t + 12.5 f^M (m\text{VaR} + \mathtt{0})\right] \leq K_t^{\Sigma b}, \tag{G.5}$$

$$-c_t^{dw} + \mathbf{E}\left\{\int_{\underline{B}}^{\overline{B}} \delta_{t,1} \frac{\partial V^{\Sigma b}}{\partial\left(K_{t+1}^{\Sigma b}\right)} dF\left(\sigma_{t+1}^B\right)\right\} = 0. \tag{G.6}$$

Here $F(\cdot)$ is the cumulative distribution of the shock to the RMBS CDOs. Using (G.6) we can see that (G.4) becomes

$$\frac{\partial \Pi_t^{\Sigma b}}{\partial D_t} = 0. \tag{G.7}$$

Looking at the form of $\Pi_t^{\Sigma b}$ given in (3.1) and the equation

$$P^{\mathtt{T}}(\mathtt{T}_t) = \frac{r_t^p}{2\overline{D}}\left[\overline{D} - T_t\right]^2, \tag{G.8}$$

it follows that

$$\begin{aligned}\Pi_t^{\Sigma b} = & \left(r_t^r - \overline{c}_t^{M\Sigma\omega} - r_t^{S\Sigma} - c_t^{i\Sigma}\right)\widehat{f}_t^{\Sigma} f_t^{\Sigma} M_t + \left(r_t^M - c_t^t - c_t^{t\Sigma}\right)\left(1 - \widehat{f}_t^{\Sigma}\right) f_t^{\Sigma} M_t \\ & + \left(r_t^M - \overline{c}_t^{M\omega} - p_t^i + c_t^p r_t^f - \left(1 - r_t^R\right) r_t^S\right)\left(1 - f_t^{\Sigma}\right) M_t - a f_t^{\Sigma} M_t \\ & + \left(r_t^{rb} - \overline{c}_t^{M\Sigma\omega b} - r_t^{S\Sigma b} - c_t^{i\Sigma b}\right)\widehat{f}_t^{\Sigma b} f_t^{\Sigma b} B_t + \left(r_t^B - c_t^{tb} - c_t^{t\Sigma b}\right)\left(1 - \widehat{f}_t^{\Sigma b}\right) f_t^{\Sigma b} B_t \\ & + \left(r_t^B - \overline{c}_t^{M\omega b} - p_t^{ib} + c_t^{bp} r_t^{fb} - \left(1 - r_t^{Rb}\right) r_t^{Sb}\right)\left(1 - f_t^{\Sigma b}\right) B_t - a^b f_t^{\Sigma b} B_t + r_t^{\mathrm{T}} \mathrm{T}_t \\ & - \left(r_t^{\mathrm{B}} + c_t^{\mathrm{B}}\right)\mathrm{B}_t - \left(r_t^D + c_t^D\right) D_t + C(\mathbf{E}[S(\mathcal{C}_t)]) - \frac{r_t^p}{2\overline{D}}\left[\overline{D} - T_t\right]^2 + \Pi_t^{\Sigma p} - \mathrm{E}_t - \mathrm{F}_t.\end{aligned} \tag{G.9}$$

Finding the partial derivatives of the investor's profit under RMBS CDOs, $\Pi_t^{\Sigma b}$, with respect to deposit, $D_t$, we have that

$$\frac{\partial \Pi_t^{\Sigma b}}{\partial D_t} = (1-\gamma)\left(r_t^{\mathrm{T}} + \left(r_t^{\mathrm{B}} + c_t^{\mathrm{B}}\right) + \frac{r_t^p}{\overline{D}}\left(\overline{D} - \mathrm{T}_t\right)\right) - \left(r_t^D + c_t^D\right). \tag{G.10}$$

This would then give us the optimal value for $D_t$. Using (1.2) and all the optimal values calculated to date, we can find optimal deposits as well as optimal profits.

## H. Proof of Theorem 4.1

We equate the investor's optimal RMBSs for the problems with $l_t^b = 0$ and $l_t^b > 0$ in order to obtain

$$\begin{aligned}& \frac{2}{3}\left(b_0 + b_2 \mathcal{C}_t^B + \sigma_t^B\right) \\ & \quad + \frac{b_1}{3}\Bigg[r_t^M - \overline{c}_t^{M\omega} - p_t^i(\mathcal{C}_t) + c_t^p r_t^f + 2c_t^B - \left(1 - r_t^R\right) r_t^S + \frac{\left(r_t^D + c_t^D\right)}{(1-\gamma)} \\ & \qquad + \left(r_t^r - \overline{c}_t^{M\Sigma\omega} - r_t^{S\Sigma} - c_t^{i\Sigma} - r_t^M + c_t^t + c_t^{t\Sigma}\right)\widehat{f}_t^{\Sigma} f_t^{\Sigma} \\ & \qquad + \left(\overline{c}_t^{M\omega} + p_t^i(\mathcal{C}_t) - c_t^p r_t^f + \left(1 - r_t^R\right) r_t^S - c_t^t - c_t^{t\Sigma} - a\right) f_t^{\Sigma}\Bigg] \\ & = \frac{K_t^{\Sigma}}{\rho\omega\left(\mathcal{C}_t^B\right)} - \frac{M_t + 12.5 f^M(m\mathrm{VaR} + 0)}{\omega\left(\mathcal{C}_t^B\right)}.\end{aligned} \tag{H.1}$$

Solving for $\sigma_t^B$, we obtain

$$\sigma_t^{B^*} = 3\left(\frac{K_t^\Sigma}{\rho\omega(\mathcal{C}_t^B)} - \frac{M_t + 12.5f^M(m\text{VaR} + 0)}{\omega(\mathcal{C}_t^B)}\right) - 2\left(b_0 + b_2\mathcal{C}_t^B\right)$$
$$- b_1\Bigg[r_t^M - \overline{c}_t^{M\omega} - p_t^i(\mathcal{C}_t) + c_t^p r_t^f + 2c_t^B - \left(1 - r_t^R\right)r_t^S + \frac{(r_t^D + c_t^D)}{(1-\gamma)}$$
$$+ \left(r_t^r - \overline{c}_t^{M\Sigma\omega} - r_t^{S\Sigma} - c_t^{i\Sigma} - r_t^M + c_t^t + c_t^{t\Sigma}\right)\widehat{f}_t^\Sigma f_t^\Sigma$$
$$+\left(\overline{c}_t^{M\omega} + p_t^i(\mathcal{C}_t) - c_t^p r_t^f + \left(1 - r_t^R\right)r_t^S - c_t^t - c_t^{t\Sigma} - a\right)f_t^\Sigma\Bigg]. \tag{H.2}$$

Using (A.3) and substituting (2.17), we obtain

$$b_1\rho\omega\left(\mathcal{C}_t^B\right)l_t^b = b_1\Bigg[2\left(r_t^B - c_t^B\right) - \left(r_t^r - \overline{c}_t^{M\Sigma\omega} - r_t^{S\Sigma} - c_t^{i\Sigma} - r_t^M + c_t^t + c_t^{t\Sigma}\right)\widehat{f}_t^\Sigma f_t^\Sigma$$
$$- \left(\overline{c}_t^{M\omega} + p_t^i(\mathcal{C}_t) - c_t^p r_t^f + \left(1 - r_t^R\right)r_t^S - c_t^t - c_t^{t\Sigma} - a\right)f_t^\Sigma$$
$$-\left(r_t^M - \overline{c}_t^{M\omega} - p_t^i(\mathcal{C}_t) + c_t^p r_t^f - \left(1 - r_t^R\right)r_t^S\right) - \frac{(r_t^D + c_t^D)}{1-\gamma}\Bigg] + B_t^*. \tag{H.3}$$

Substitute $r_t^{B^*}$ and $B_t^*$ into the expression above to obtain

$$l_t^{b^*} = \frac{2\sigma_t^B - \sigma_t^{B^*}}{\omega(\mathcal{C}_t^B)\rho b_1}. \tag{H.4}$$

Using (2.11) to find the partial derivative of the value function with respect to the investor capital we obtain

$$\frac{\partial V^\Sigma}{\partial K_t^\Sigma} = \begin{cases} l_t^b + \dfrac{1}{1-\gamma}(r_t^D + c_t^D), & \\ \dfrac{1}{1-\gamma}(r_t^D + c_t^D), & \text{for } \underline{B} \le \sigma_t^B \le \sigma_t^{B^*}, \\ \dfrac{1}{1-\gamma}(r_t^D + c_t^D) + \dfrac{2\sigma_t^B - \sigma_t^{B^*}}{\omega(\mathcal{C}_t^B)\rho b_1}, & \text{for } \sigma_t^{B^*} \le \sigma_t^B \le \overline{B}. \end{cases} \tag{H.5}$$

By substituting the above expression into the optimal condition for total capital (A.6), we obtain

$$c_t^{dw} - \mathbf{E}\left[\delta_{t,1}\frac{1}{1-\gamma}\left(r_t^D + c_t^D\right)\right] - \frac{1}{\omega(\mathcal{C}_{t+1}^B)\rho b_1}\mathbf{E}\left[\int_{\sigma_{t+1}^{B^*}}^{\overline{B}} \delta_{t,1}\left(2\sigma_{t+1}^B - \sigma_{t+1}^{B^*}\right)dF\left(\sigma_{t+1}^B\right)\right] = 0. \tag{H.6}$$

We denote the left-hand side of the above expression by $\Upsilon$, so that

$$\Upsilon = \frac{1}{\omega(C_{t+1}^{B})\rho b_1}\mathbf{E}\left[\int_{\sigma_{t+1}^{B^*}}^{\overline{B}} \delta_{t,1}\left(2\sigma_{t+1}^{B} - \sigma_{t+1}^{B^*}\right)dF\left(\sigma_{t+1}^{B}\right)\right]. \tag{H.7}$$

From the implicit function theorem, we can calculate $\partial\Upsilon/\partial C_t^B$ by using (H.7) in order to obtain

$$\begin{aligned}\frac{\partial \Upsilon}{\partial C_t^B} &= -\frac{1}{\rho b_1}\frac{\left(-\mu^{C_t^B}\right)(\partial\omega/\partial C_{t+1}^B)}{[\omega(C_{t+1}^B)]^2}\mathbf{E}\left[\int_{\sigma_{t+1}^{B^*}}^{\overline{B}} \delta_{t,1}\left(2\sigma_{t+1}^{B} - \sigma_{t+1}^{B^*}\right)dF\left(\sigma_{t+1}^{B}\right)\right] \\ &\quad - \frac{1}{\rho b_1\omega(C_{t+1}^B)}\frac{\partial\sigma_{t+1}^{B^*}}{\partial C_t^B}\mathbf{E}\left[\int_{\sigma_{t+1}^{B^*}}^{\overline{B}} \delta_{t,1}dF\left(\sigma_{t+1}^{B}\right)\right],\end{aligned} \tag{H.8}$$

where

$$\begin{aligned}\frac{\partial\sigma_{t+1}^{B^*}}{\partial C_t^B} &= -\frac{3}{\rho}\left(\frac{K_t^{\Sigma} - \rho(M_t + 12.5f^M(m\text{VaR} + 0))}{[\omega(C_{t+1}^B)]^2}\right)\mu^{C_t^B}\frac{\partial\omega}{\partial C_{t+1}^B} \\ &\quad - 2b_2\mu^{C_t^B} + b_1\mu^{C_t^B}\left(1 - f_t^{\Sigma}\right)\left[\frac{\partial p_t^i}{\partial C_{t+1}^B} + \frac{\partial r_t^S}{\partial C_{t+1}^B}\right],\end{aligned} \tag{H.9}$$

$$\frac{\partial \Upsilon}{\partial K_{t+1}^{\Sigma}} = \frac{3}{b_1[\omega(C_{t+1}^B)\rho]^2}\mathbf{E}\left[\int_{\sigma_{t+1}^{B^*}}^{\overline{B}} \delta_{t,1}dF\left(\sigma_{t+1}^{B}\right)\right]. \tag{H.10}$$

As a consequence, we have that $\partial K_{t+1}^{\Sigma}/\partial C_t^B > 0$ only if $\partial\sigma_{t+1}^{B^*}/\partial C_t^B < 0$.

## I. Proof of Corollary 4.2

We equate the investor's optimal RMBS CDOs for the problems with $l_t^b = 0$ and $l_t^b > 0$ in order to obtain

$$\begin{aligned}&\frac{1}{2}\left(b_0 + b_2C_t^B + \sigma_t^B\right) + \frac{b_1}{2\left(1 - f_t^{\Sigma b}\hat{f}_t^{\Sigma b}\right)} \\ &\quad\times\left[\overline{c}_t^{M\Sigma\omega b} + p_t^{ib} + \left(1 - r_t^{Rb}\right)r_t^{Sb} - c_t^{pb}r_t^{fb} + \frac{(r_t^D + c_t^D)}{2(1-\gamma)}\right. \\ &\quad\quad - \left(r_t^{rb} - \overline{c}_t^{M\Sigma\omega b} - r_t^{S\Sigma b} - c_t^{i\Sigma b} + c_t^{tb} + c_t^{t\Sigma b}\right)f_t^{\Sigma b}\hat{f}_t^{\Sigma b}\end{aligned}$$

$$-\left(\overline{c}_t^{M\omega b} + p_t^{ib} + \left(1 - r_t^{Rb}\right) r_t^{Sb} - c_t^{tb} - c_t^{t\Sigma b} - c_t^{pb} r_t^{fb} - a^b\right) f_t^{\Sigma b}\Bigg]$$

$$= \frac{K_t^{\Sigma b}}{\rho\omega(\mathcal{C}_t^B)} - \frac{M_t + 12.5 f^M (m\mathrm{VaR} + 0)}{\omega(\mathcal{C}_t^B)}. \tag{I.1}$$

Solving for $\sigma_t^B$, we obtain

$$\begin{aligned}
\sigma_t^{B^*} = {} & 2\left(1 - f_t^{\Sigma b} \widehat{f}_t^{\Sigma b}\right)\left(\frac{K_t^{\Sigma b}}{\rho\omega(\mathcal{C}_t^B)} - \frac{M_t + 12.5 f^M (m\mathrm{VaR} + 0)}{\omega(\mathcal{C}_t^B)}\right) - \left(1 - f_t^{\Sigma b} \widehat{f}_t^{\Sigma b}\right)\left(b_0 + b_2 \mathcal{C}_t^B\right) \\
& - b_1 \Bigg[\overline{c}_t^{M\Sigma\omega b} + p_t^{ib} + \left(1 - r_t^{Rb}\right) r_t^{Sb} - c_t^{pb} r_t^{fb} + \frac{(r_t^D + c_t^D)}{2(1-\gamma)} \\
& - \left(r_t^{rb} - \overline{c}_t^{M\Sigma\omega b} - r_t^{S\Sigma b} - c_t^{i\Sigma b} + c_t^{tb} + c_t^{t\Sigma b}\right) f_t^{\Sigma b} \widehat{f}_t^{\Sigma b} \\
& - \left(\overline{c}_t^{M\omega b} + p_t^{ib} + \left(1 - r_t^{Rb}\right) r_t^{Sb} - c_t^{tb} - c_t^{t\Sigma b} - c_t^{pb} r_t^{fb} - a^b\right) f_t^{\Sigma b}\Bigg].
\end{aligned} \tag{I.2}$$

Using (G.3) and substituting (3.11), we obtain

$$\begin{aligned}
b_1 \rho\omega\left(\mathcal{C}_t^B\right) l_t^b = 2b_1 \Bigg[ & \left(r_t^B - \overline{c}_t^{M\omega b} - p_t^{ib} + c_t^{pb} r_t^{fb} - \left(1 - r_t^{Rb}\right) r_t^{Sb}\right) \\
& + \left(r_t^{rb} - \overline{c}_t^{M\Sigma\omega b} - r_t^{S\Sigma b} - c_t^{i\Sigma b} - r_t^B + c_t^{tb} + c_t^{t\Sigma b}\right) f_t^{\Sigma b} \widehat{f}_t^{\Sigma b} \\
& + \left(\overline{c}_t^{M\omega b} + p_t^{ib} + \left(1 - r_t^{Rb}\right) r_t^{Sb} - c_t^{tb} - c_t^{t\Sigma b} - c_t^{pb} r_t^{fb} - a^b\right) f_t^{\Sigma b} \\
& - \frac{(r_t^D + c_t^D)}{2(1-\gamma)} \Bigg] + 2\left(1 - f_t^{\Sigma b} \widehat{f}_t^{\Sigma b}\right) B_t^*.
\end{aligned} \tag{I.3}$$

Substitute $r_t^{B^*}$ and $B_t^*$ into the expression above to obtain

$$l_t^{b^*} = \frac{\sigma_t^B - \sigma_t^{B^*}}{\omega(\mathcal{C}_t^B)\rho b_1}. \tag{I.4}$$

Using (3.9) to find the partial derivative of the value function with respect to investor capital we obtain

$$\frac{\partial V^{\Sigma b}}{\partial K_t^{\Sigma b}} = \begin{cases} l_t^b + \dfrac{1}{1-\gamma}\left(r_t^D + c_t^D\right), \\ \dfrac{1}{1-\gamma}\left(r_t^D + c_t^D\right), & \text{for } \underline{B} \le \sigma_t^B \le \sigma_t^{B^*}, \\ \dfrac{1}{1-\gamma}\left(r_t^D + c_t^D\right) + \dfrac{\sigma_t^B - \sigma_t^{B^*}}{\omega\left(\mathcal{C}_t^B\right)\rho b_1}, & \text{for } \sigma_t^{B^*} \le \sigma_t^B \le \overline{B}. \end{cases} \tag{I.5}$$

By substituting the above expression into the optimal condition for total capital (G.6), we obtain

$$c_t^{dw} - \mathbf{E}\left[\delta_{t,1}\frac{1}{1-\gamma}\left(r_t^D + c_t^D\right)\right] - \frac{1}{\omega\left(\mathcal{C}_{t+1}^B\right)\rho b_1}\mathbf{E}\left[\int_{\sigma_{t+1}^{B^*}}^{\overline{B}} \delta_{t,1}\left(\sigma_{t+1}^B - \sigma_{t+1}^{B^*}\right) dF\left(\sigma_{t+1}^B\right)\right] = 0. \tag{I.6}$$

We denote the left-hand side of the above expression by $\Upsilon$, so that

$$\Upsilon = \frac{1}{\omega\left(\mathcal{C}_{t+1}^B\right)\rho b_1}\mathbf{E}\left[\int_{\sigma_{t+1}^{B^*}}^{\overline{B}} \delta_{t,1}\left(\sigma_{t+1}^B - \sigma_{t+1}^{B^*}\right) dF\left(\sigma_{t+1}^B\right)\right]. \tag{I.7}$$

From the implicit function theorem, we can calculate $\partial\Upsilon/\partial\mathcal{C}_t^B$ by using (I.7) in order to obtain

$$\begin{aligned} \frac{\partial \Upsilon}{\partial \mathcal{C}_t^B} = {} & -\frac{1}{\rho b_1}\frac{\left(-\mu^{\mathcal{C}_t^B}\right)\left(\partial\omega/\partial\mathcal{C}_{t+1}^B\right)}{\left[\omega\left(\mathcal{C}_{t+1}^B\right)\right]^2}\mathbf{E}\left[\int_{\sigma_{t+1}^{B^*}}^{\overline{B}} \delta_{t,1}\left(\sigma_{t+1}^B - \sigma_{t+1}^{B^*}\right) dF\left(\sigma_{t+1}^B\right)\right] \\ & - \frac{1}{\rho b_1 \omega\left(\mathcal{C}_{t+1}^B\right)}\frac{\partial \sigma_{t+1}^{B^*}}{\partial \mathcal{C}_t^B}\mathbf{E}\left[\int_{\sigma_{t+1}^{B^*}}^{\overline{B}} \delta_{t,1} dF\left(\sigma_{t+1}^B\right)\right], \end{aligned} \tag{I.8}$$

where

$$\begin{aligned} \frac{\partial \sigma_{t+1}^{B^*}}{\partial \mathcal{C}_t^B} = {} & -\frac{2}{\rho}\left(\frac{K_t^{\Sigma b} - \rho\left(M_t + 12.5 f^M (m\mathrm{VaR} + 0)\right)}{\left[\omega\left(\mathcal{C}_{t+1}^B\right)\right]^2}\right)\mu^{\mathcal{C}_t^B}\frac{\partial\omega}{\partial\mathcal{C}_{t+1}^B} \\ & - b_2\mu^{\mathcal{C}_t^B} - b_1\mu^{\mathcal{C}_t^B}\left(1 - f_t^{\Sigma b}\right)\left[\frac{\partial p_t^{ib}}{\partial \mathcal{C}_{t+1}^B} + \frac{\partial r_t^{Sb}}{\partial \mathcal{C}_{t+1}^B}\right], \end{aligned} \tag{I.9}$$

$$\frac{\partial \Upsilon}{\partial K_{t+1}^{\Sigma b}} = \frac{2}{b_1\left[\omega\left(\mathcal{C}_{t+1}^B\right)\rho\right]^2}\mathbf{E}\left[\int_{\sigma_{t+1}^{B^*}}^{\overline{B}} \delta_{t,1} dF\left(\sigma_{t+1}^B\right)\right]. \tag{I.10}$$

As a consequence, we have that $\partial K_{t+1}^{\Sigma b}/\partial\mathcal{C}_t^B > 0$ only if $\partial\sigma_{t+1}^{B^*}/\partial\mathcal{C}_t^B < 0$.

## J. Proof of Proposition 4.3

In order to prove Proposition 4.3, we find the partial derivatives of the investor's RMBS supply, $B^{\Sigma n^*}$, and the subprime RMBS rate, $r^{B^{\Sigma n^*}}$, with respect to the period $t$ RMBS credit rating, $\mathcal{C}_t^B$. Here, we consider (2.19), (2.20) and the conditions $\partial r_t^S(\mathcal{C}_{t+j}^B)/\partial \mathcal{C}_{t+j}^B < 0$, and $r_t^R = 0$. We are now able to calculate

$$\begin{aligned}
&\frac{\partial B_{t+j}^{\Sigma n^*}}{\partial \mathcal{C}_t^B}\Bigg[\frac{2}{3}\left(b_0 + b_2\mathcal{C}_t^B + \sigma_t^B\right)\\
&\quad + \frac{b_1}{3}\Bigg[\left(r_t^M - \overline{c}_t^{M\omega} - p_t^i(\mathcal{C}_t) + c_t^p r_t^f + 2c_t^B - \left(1 - r_t^R\right)r_t^S + \frac{(r_t^D + c_t^D)}{1-\gamma}\right)\\
&\qquad + \left(r_t^r - \overline{c}_t^{M\Sigma\omega} - r_t^{S\Sigma} - c_t^{i\Sigma} - r_t^M + c_t^t + c_t^{t\Sigma}\right)\widehat{f}_t^{\Sigma} f_t^{\Sigma}\\
&\qquad + \left(\overline{c}_t^{M\omega} + p_t^i(\mathcal{C}_t) - c_t^p r_t^f + \left(1 - r_t^R\right)r_t^S - c_t^t - c_t^{t\Sigma} - a\right)f_t^{\Sigma}\Bigg]\Bigg]\\
&= \frac{1}{3}\mu_j{}^{\mathcal{C}^B}\left(2b_2 - b_1\left(1 - f_t^{\Sigma}\right)\left[\frac{\partial p^i\left(\mathcal{C}_{t+j}^B\right)}{\partial \mathcal{C}_{t+j}^B} + \frac{\partial r_t^S\left(\mathcal{C}_{t+j}^B\right)}{\partial \mathcal{C}_{t+j}^B}\right]\right),\\
&\frac{\partial r_{t+j}^{B^{\Sigma n^*}}}{\partial \mathcal{C}_t^B}\Bigg[-\frac{1}{3b_1}\left(b_0 + b_2\mathcal{C}_t^B + \sigma_t^B\right)\\
&\quad + \frac{1}{3}\Bigg[\left(r_t^M - \overline{c}_t^{M\omega} - p_t^i(\mathcal{C}_t) + c_t^p r_t^f + 2c_t^B - \left(1 - r_t^R\right)r_t^S + \frac{(r_t^D + c_t^D)}{1-\gamma}\right)\\
&\qquad + \left(r_t^r - \overline{c}_t^{M\Sigma\omega} - r_t^{S\Sigma} - c_t^{i\Sigma} - r_t^M + c_t^t + c_t^{t\Sigma}\right)\widehat{f}_t^{\Sigma} f_t^{\Sigma}\\
&\qquad + \left(\overline{c}_t^{M\omega} + p_t^i(\mathcal{C}_t) - c_t^p r_t^f + \left(1 - r_t^R\right)r_t^S - c_t^t - c_t^{t\Sigma} - a\right)f_t^{\Sigma}\Bigg]\Bigg]\\
&= -\frac{1}{3}\mu_j{}^{\mathcal{C}^B}\left(\frac{b_2}{b_1} + \left(1 - f_t^{\Sigma}\right)\left[\frac{\partial p^i\left(\mathcal{C}_{t+j}^B\right)}{\partial \mathcal{C}_{t+j}^B} + \frac{\partial r_t^S\left(\mathcal{C}_{t+j}^B\right)}{\partial \mathcal{C}_{t+j}^B}\right]\right).
\end{aligned} \tag{J.1}$$

## K. Proof of Proposition 4.4

In order to prove Proposition 4.4, we find the partial derivatives of the optimal RMBS supply, $B^*$, and subprime RMBS rate, $r^B$, with respect to $\mathcal{C}_t^B$. This involves using the equations (2.14) and (2.15) and the condition $\partial\omega(\mathcal{C}_{t+j}^B)/\partial\mathcal{C}_{t+j}^B < 0$ in order to find $\partial B_t^*/\partial\mathcal{C}_t^B$ and $\partial r_t^{B^*}/\partial\mathcal{C}_t^B$, respectively. We are now able to determine that

$$\begin{aligned}
&\frac{\partial B_t^*}{\partial\mathcal{C}_t^B}\left(\frac{K_t^\Sigma}{\rho\omega(\mathcal{C}_t^B)}-\left[\frac{\omega^M M_t+12.5f^M(m\mathrm{VaR}+\mathrm{O})}{\omega(\mathcal{C}_t^B)}\right]\right)\\
&\quad=-\frac{K_t^\Sigma-\rho\left(12.5f^M(m\mathrm{VaR}+\mathrm{O})+\omega^M M_t\right)}{\left[\omega(\mathcal{C}_t^B)\right]^2\rho}\frac{\partial\omega(\mathcal{C}_t^B)}{\partial\mathcal{C}_t^B},\\
&\frac{\partial r_t^{B^*}}{\partial\mathcal{C}_t^B}\left(-\frac{1}{b_1}\left(b_0+b_2\mathcal{C}_t^B+\sigma_t^B-\frac{K_t^\Sigma}{\rho\omega(\mathcal{C}_t^B)}+\frac{\omega^M M_t+12.5f^M(m\mathrm{VaR}+\mathrm{O})}{\omega(\mathcal{C}_t^B)}\right)\right)\\
&\quad=-\frac{b_2}{b_1}-\frac{K_t^\Sigma-\rho\left(12.5f^M(m\mathrm{VaR}+\mathrm{O})+\omega^M M_t\right)}{\left[\omega(\mathcal{C}_t^B)\right]^2\rho b_1}\frac{\partial\omega(\mathcal{C}_t^B)}{\partial\mathcal{C}_t^B},
\end{aligned}\tag{K.1}$$

as required to complete the proof of Proposition 4.4.

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