Hindawi Publishing Corporation
Discrete Dynamics in Nature and Society
Volume 2012, Article ID 908451, 9 pages
doi:10.1155/2012/908451

Research Article

Endogenous Reactivity in a Dynamic Model of Consumer's Choice

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Received 16 June 2011; Revised 10 December 2011; Accepted 12 December 2011

Academic Editor: Vimal Singh

We move from a boundedly rational consumer model (Naimzada and Tramontana, 2008, 2010) characterized by a gradient-like decisional process in which, under particular parameters conditions, the asymptotical convergence to the optimal choice does not happen but it does under a least squared learning mechanism. In the present paper, we prove that even a less sophisticated learning mechanism leads to convergence to the rational choice and also prove that convergence is ensured when both learning mechanisms are available. The stability results that we obtain give more strength to the rational behavior assumption of the original model; in fact, the less demanding is the learning mechanism ensuring convergence to the rational behavior, the higher is the probability that even quite naive consumers will learn the composition of their optimum consumption bundles.

1. Introduction

In the neoclassical economics, a fully rational representative consumer makes his/her choices maximizing a utility function subject to a budget constraint. The representative consumer has a complete knowledge of the utility function and has got the computational skills for solving an optimization problem. Behavioral economists refuse this approach and, among the other assumptions, introduce *bounded rationality*. Recently, D'Orlando and Rodano [1], Naimzada and Tramontana [2, 3] proposed decisional mechanisms based on repeated choices following a dynamic adaptive process. The convergence, in the long run, of these adaptive processes to the equilibrium point of the traditional static consumer model may be seen as an "evolutionary explanation" of the assumption of rational behavior.

In particular, Naimzada and Tramontana [2] present a consumer that adopts a simple adaptive decisional mechanism. They assume that at each time period the consumer updates his/her choice in the direction of increasing/decreasing utility using a gradient-like decisional process. In discrete time evolution, they obtain a one-dimensional map whose unique steady state represents the optimal choice of the rational consumer. In such a model, the rational choice is asymptotically stable when the preferences are such that the weight given to each good and/or the reactivity parameter of the decisional process are relatively weak, otherwise we have a transition, via flip bifurcation, to complex dynamics. The adopted learning mechanism can be considered a particular implementation of the iterative gradient method used to detect maxima of functions in numerical analysis [4] and to find Nash equilibria in economics [5]. This kind of algorithms has been also used for the representation of the decision-making process of fully rational economic agents (see [6–11]).

In Naimzada and Tramontana [3], the authors show that by endowing the consumer with a variable reactivity and a least squared learning mechanism, it is possible to get back convergence to the optimal rational choice.

Least squares learning mechanisms are also applied in the seminal paper of Bullard [12] where it is proved that the learning mechanism itself is potentially able to create new equilibria different from the rational expectations one, the so-called *learning equilibria*. This paper and those directly inspired by it like Schönhofer [13] and Tuinstra and Wagener [14], let the researchers ask not only if a learning mechanism that permits convergence to the rational behavior exists, but also which learning mechanisms are able to do so and which are not. Another question is how far we can go by relaxing the assumptions of rationality of the economic agents and the amount of information used in order to still be able to learn the rational behavior? In fact, in a bounded rationality approach, it appears difficult to justify how the consumer becomes able to perform a least squared analysis that requires to know some econometrics.

Moving from these research questions, in the present work, we suppose that after some long period of chaotic or periodic consumption experiences, the consumer becomes able to identify the levels of utility induced by a relatively high number of consumption bundles and that he/she regulates the reactivity parameter in the following manner: when the utility reached is relatively low, the consumer changes drastically the vector of choices, whereas if he/she experiences a relatively high level of utility (close to the optimal choice), he/she reacts in a weak way. Consumers who choose subsets from a larger choice set are also those of the models of Pancras [15, 16]. This mechanism is clearly less sophisticated with respect to the least squares mechanism introduced in Naimzada and Tramontana [3] and requires a lower amount of past information to be implemented. We show that the dynamical system still owns a unique fixed point equivalent to the optimal choice which is always locally asymptotically stable, and, in this way, the consumer becomes able to approach, and finally, the system converges to the optimal choice of the corresponding static framework. This result strengthens the assumption of rational behavior of the consumer in fact, the less the learning mechanism implemented is demanding for the consumer, the more realistic appears that consumers soon or later will learn how to maximize their utility functions, at least under this framework.

The paper is organized as follows. In Section 2, we introduce the benchmark model with a perfectly rational consumer. In the same section, we also present the results obtained by removing some degree of rationality and endowing the consumer with a learning mechanism characterized by the application of a gradient method with exogenous reactivity [2]. In Section 3, we explore a simple way to make endogenous the reactivity parameter.

2. The Benchmark Model

We have a representative consumer whose preferences for the goods x and y are represented by a Cobb-Douglas utility function

$$U(x, y) = x^{\alpha} y^{1-\alpha}, \tag{2.1}$$

where $\alpha \in [0, 1]$ measures the preference for the good x.

The budget constraint takes this form

$$px + y = m, \tag{2.2}$$

where the price of the good y is normalized to 1.

The consumer knows all the prices and his/her own utility function and is able to solve a maximization problem. His/Her optimal choice is given by the following vector:

$$(\hat{x}, \hat{y}) = \left(\alpha \frac{m}{p}, (1 - \alpha)m\right). \tag{2.3}$$

In neoclassical economics, the consumer is perfectly rational, and his/her preferences are fixed, so he/she chooses the vector (2.3) in one shot.

In Naimzada and Tramontana [2], the present authors supposed that the consumer knows prices and income, but he/she does not know globally his/her own utility function. Only after a consumption experience, the consumer acquires information about his/her utility function. Information acquired is local, that is, they concern the utility function in a neighbourhood of the quantity of good experienced. In particular, he/she exactly evaluates the utility given by the actual consumption level and becomes able to foresee the variation of his/her utility given by a little increasing/decreasing of the quantity of good consumed.

It is useful for our purposes to express the utility function in terms of the good x,

$$V(x) = x^{\alpha} (m - px)^{1-\alpha}, \tag{2.4}$$

which is unimodal, concave, and with a unique maximum given by the optimal choice $\hat{x}$. If we assume that in the period t the consumption of the good x is equal to x_t, our assumption implies that at the beginning of the following period the consumer knows the value of the derivative $V'(x_t)$ and uses it to decide the new consumption bundle.

Starting from this minimal hypothesis, we can draw the following decisional mechanism: at each time period t, if the current consumption experience indicates that, locally, an increasing (resp., decreasing) in the utility derives from an increasing in the consumption of the good x, then the consumer in the period $t+1$ will change the composition of the vector of choices in favour of the good x (resp., y).

Our decisional mechanism involves the value of x in different time periods and gives rise to a dynamic model. The dynamic of the decisional process can be described by the following first order difference equation:

$$x_{t+1} = x_t + \phi\left[V'(x_t)\right], \tag{2.5}$$

with $\phi[0] = 0$ and $\phi'[0] > 0$.

We proposed a linear specification of (2.5),

$$x_{t+1} = f(x_t) = x_t + \gamma V'(x_t), \tag{2.6}$$

where the parameter γ measures the intensity of the reaction to the information given by $V'(x_t)$. In the case of the Cobb-Douglas utility function, we had the following dynamical system:

$$x_{t+1} = f(x_t) = x_t + \gamma\left[\alpha x_t^{\alpha-1}(m - px_t)^{1-\alpha} - (1-\alpha)px_t^{\alpha}(m - px_t)^{-\alpha}\right]. \tag{2.7}$$

The dynamic process represented by (2.7) owns only one fixed point which is the optimal choice.

Proposition 2.1 (See [2]). *The fixed point $\widehat{x}$ is locally stable if*

$$\gamma < 2\frac{\alpha^{1-\alpha}(1-\alpha)^{\alpha}m}{p^{2-\alpha}}. \tag{2.8}$$

Proposition 2.1 states that relatively high values of p and low levels of m have a stabilizing effect. The effect of variations of the preference parameter α is showed in the bifurcation diagram in Figure 1. As we can see if the preferences for the two goods are well balanced (α is not too distant from 0.5), the optimal choice is stable, whereas in the other cases, the fixed point can lose stability via flip bifurcation and if the preference for one good is marked enough, we have a chaotic sequence of consumption choices (see Figure 2).

Moreover, low levels of the reactivity parameter γ have a stabilizing effect.

3. Endogenous Reactivity

Let us focus on the situations in which the optimal consumption bundle is not a stable fixed point. After a long sequence of consumption experiences, it appears quite realistic to suppose that the consumer may also learn to regulate his/her reactivity. In fact, the consumer, over time, can reach an understanding, certainly not precise but qualitatively correct, of the function V, in the sense that he/she is able to identify on the budget line constraint a set of consumption bundles, around the optimal consumption vector, that lead to relatively high levels of utility and other subintervals that lead relatively to low level of utility. In particular, we assume that when the consumer realizes that the actual consumption bundle provides a low level of utility, he/she is very responsive to reach consumption bundles with a higher utility, that is, the reactivity parameter γ has a high value. At the opposite, perceiving that the actual utility reached is relatively high, the consumer becomes reluctant to radical changes in the composition of the consumption bundle. He/she understands that the optimum is not far away and so performs only gradual changes, and this means that the reactivity parameter γ has a low value. In this way, the learning process defines an endogenous reactivity parameter $\gamma(x)$, which depends on the consumption vectors. In particular, the values taken by $\gamma(x)$ around the optimal choice are very low and higher in the other subsets of the budget line. Formally, we can impose the following conditions on $\gamma(x)$:

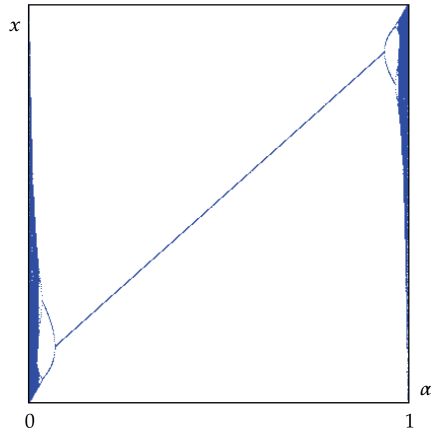


Figure 1: Bifurcation diagram.

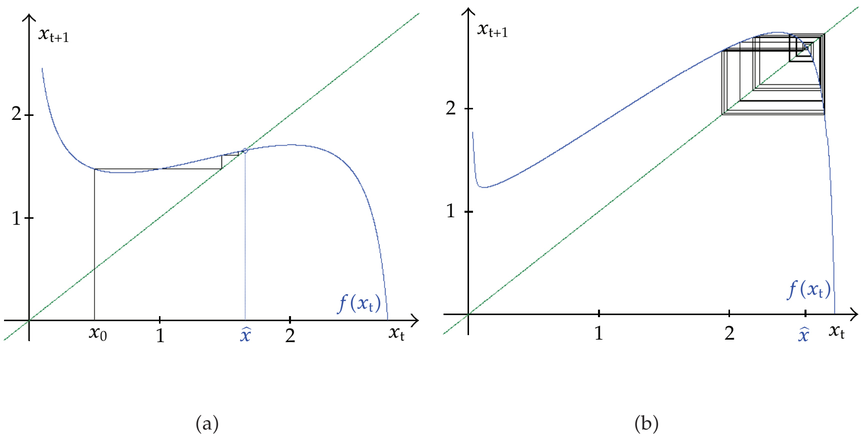


Figure 2: (a) Obtained with this set of parameters: $m = 4, p = 1.4, \gamma = 0.9$, and $\alpha = 0.58$, we have that the optimal choice $\hat{x}$ is locally stable. (b) The preference for the good x is marked ($\alpha = 0.91$), the convergence to the optimal choice fails, and the consumption choices vary chaotically around it.

(1) continuity,

(2) the domain of the function $\gamma(x)$ is $[0, m/p]$ and the codomain is formed by non-negative real numbers,

(3) $\gamma'(x) < 0$ for $x \in [0, \hat{x}[, \gamma'(x) > 0$ for $x \in]\hat{x}, m/p], \gamma'(x) = 0$ for $x = \hat{x}$ and

(4) $\gamma(\hat{x}) = 0$.

Continuity comes from the fact that the changes in the composition of consumption do not imply sudden changes in the reactivity parameter. The second property follows for construction of the model. The third property formalises the assumptions made on particular learning process proposed. The last property also assures us that the optimal consumption vector is still the only fixed point and that the reactivity around it is very low. The endogenous form may be the one represented in Figure 3.

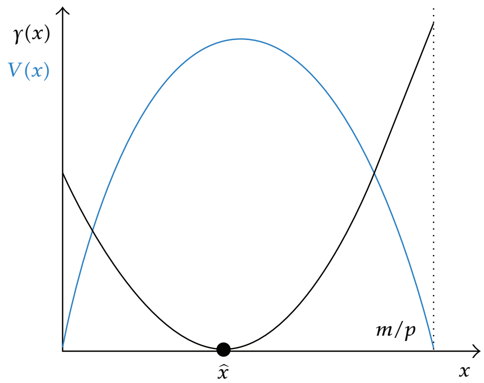


Figure 3: Consumer without learning versus consumer endowed with learning.

The dynamical behaviour of the system is represented by the following first order nonlinear difference equation:

$$x_{t+1} = f(x_t) = x_t + \gamma(x_t)V'(x_t). \tag{3.1}$$

Again the present dynamical system has a unique fixed point, exactly the optimal choice, as stated by the following proposition.

Proposition 3.1. *The unique equilibrium point of map* (3.1) *is the optimal choice* $\hat{x} = \alpha m/p$.

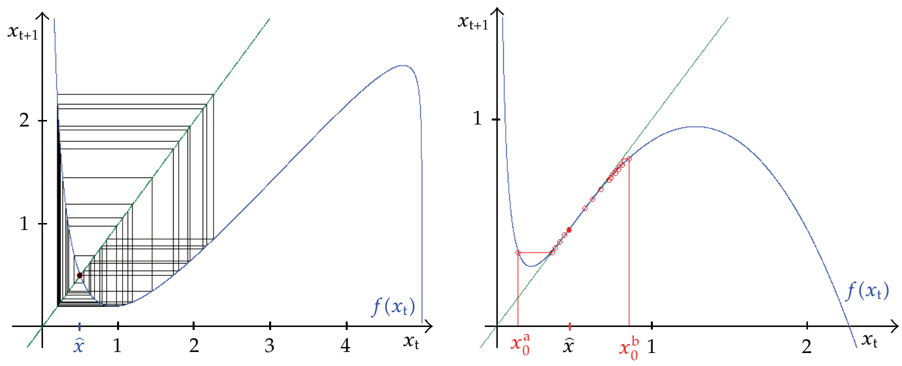
Proof. Imposing the steady state condition, $x_{t+1} = x_t = x^*$, we obtain $\gamma(x^*)V'(x^*) = 0$ which is only verified by the optimal choice. □

Differently from the model with reactivity parameter γ exogenously given, in this case the dynamics is strongly stable with monotonic convergent trajectories in a neighbourhood of the optimal choice.

Proposition 3.2. *The optimal choice is an asymptotically stable equilibrium point of the map* (3.1).

Proof. The derivative of map (3.1) evaluated in the equilibrium point is $f(x^*) = 1$, and thus, the map in the neighbourhood of the equilibrium is increasing. In particular, before the equilibrium, the map is above the diagonal because it is defined by the sum of the diagonal and the term $\gamma(x_t)V'(x_t)$, which is positive; after the equilibrium, the map is below the diagonal because is defined by the sum of the diagonal and the term $\gamma(x_t)V'(x_t)$, which is negative. Starting with an initial condition x_0 belonging to the interval $(0, x^*)$, there will be an increasing and bounded sequence converging to the optimal choice. In the same way starting with an initial condition x_0 belonging to the interval (x^*, ∞), there will be a decreasing and bounded sequence converging to the optimal choice. □

In Figure 4, we can see a comparison between a situation with exogenous reactivity and chaotic dynamics (Figure 4(a)) and a situation characterized by the same



(a) (b)

Figure 4: (a) obtained with this set of parameters: $m = 7, p = 1.4, \alpha = 0.1, \gamma = 1.276$ the optimal choice $\hat{x}$ is unstable and the dynamics are chaotic. (b) the reactivity parameter γ is endogenous and there is convergence both starting from an initial condition lower (x_0^a) and higher than the optimal choice (x_0^b).

economic parameters (income, price, and preference) but endogenous reactivity parameter (Figure 4(b)). In particular, in order to obtain the figure, we used the specification:

$$\gamma(x) = \frac{p}{2}x^2 - \alpha m x + \frac{1}{2}\left(\frac{\alpha^2 m^2}{p}\right). \tag{3.2}$$

It is not difficult to verify that the specification (3.2) satisfies the four conditions previously given.

As we can see, with endogenous reactivity, the vector of choices converges to the optimal one, both starting from a consumption lower and higher than x^*.

As stated by Proposition 3.2, (3.2) is only one of the suitable specifications of the equation regulating the way in which the consumption influences the reactivity, and any other specification of $\gamma(x)$ that follows those conditions permits to obtain the same dynamic results of (3.2).

3.1. *Competition of Learning Mechanisms*

It could be interesting to take into consideration a scenario in which the learning mechanism studied in the previous section coexists with the more sophisticated least squares learning mechanism introduced in Naimzada and Tramontana [3]. In particular, as in Tuinstra and Wagener [14], we want to study the effects of an evolutionary competition between these two learning mechanisms.

We remember that in Naimzada and Tramontana [3], the way in which the reactivity parameter γ endogenously varies follows the following dynamic equation:

$$\gamma_t = \omega(x_t - x_{s,t})^2 \quad \text{with } \omega > 0, \tag{3.3}$$

where $x_{s,t}$ depends upon the n remembered bundles.

In order to perform such exercise, we need to assume that initially two kinds of consumers endowed with identical utility functions coexist: a sophisticated group of consumers that are able to perform a least squares analysis, and a less sophisticated one that use the learning mechanism presented in this paper. Nevertheless, it is possible to imitate the behavior of the other group, and this permits the shares of the two groups to be variable. In an evolutionary competition framework, the share of the group whose perform measure is higher than the other will increase to the detriment of the share of the other group.

Following Brock and Hommes [17], we can define the share of population adopting the mechanism h at period $t+1(n_{h,t+1})$ as

$$n_{h,t+1} = \frac{\exp \beta\phi_{h,t}}{\sum_h \exp \beta\phi_{h,t}}, \tag{3.4}$$

where the parameter β is the intensity of choice measuring how fast consumers choose between different learning mechanisms, while $\phi_{h,t}$ is the performance measure.

In our context, it appears obvious to consider the level of utility reached the best performance measure, so

$$\phi_{h,t} = U_{h,t}. \tag{3.5}$$

Note that (3.4) does not influence the performance of the representative agent of each type, because in our framework the share of population choosing each strategy does not influence the environment in which the consumers operate (i.e., the market). This is a consequence of the assumption of an exogenous market price. So, the introduction of (3.4) does not affect the global stability properties of both the learning mechanisms. In other words, they both still permit the consumers to converge towards their optimal consumption bundle.

This has a crucial consequence for our exercise. In fact, after the transient, the performance measures tend to asymptotically coincide with the maximum level of utility reachable, leading the population to be equally distributed between the two mechanisms.

This means that neither the less sophisticated nor the least squares learning mechanism finally prevails against the other one.

We leave for future research the analysis of what happens when the market prices are endogenous and in particular when they are influenced by the shares of consumers adopting one learning mechanism or the other.

4. Conclusions

Naimzada and Tramontana [2] presented a model of consumer's choice with a boundedly rational consumer with variable preferences. They proved that complicated dynamics in the consumption choices may arise, causing a failure in the convergence toward the optimal consumption bundle. The same authors showed that by adopting a least squared learning mechanism, it is possible to recover the convergence to the choice of a fully rational consumer [3]. In this paper, we prove that it is not necessary to introduce such a sophisticated learning mechanism; it is enough to endow the consumer with the skill of recognizing when his/her utility is relatively low/high and regulating his/her reactivity as a consequence of this signal. When the utility reached is relatively low, the consumer changes drastically the vector of choices, whereas if he/she experiences a high level of utility (close to the optimal choice), he/she reduces his/her reaction and becomes able to approach and finally converge to the optimal choice. We also show that when both learning mechanisms are available and compete in an evolutionary setting, they both survive, and consumers equally distribute themselves between them.

Acknowledgments

The authors thank an anonymous referee for his/her helpful comments. This work has been performed within the activity of the PRIN project "Local interactions and global dynamics in economics and finance: models and tools," MIUR, Italy.

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