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Research Article

Studies on a Double Poisson-Geometric Insurance Risk Model with Interference

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This paper mainly studies a generalized double Poisson-Geometric insurance risk model. By martingale and stopping time approach, we obtain adjustment coefficient equation, the Lundberg inequality, and the formula for the ruin probability. Also the Laplace transformation of the time when the surplus reaches a given level for the first time is discussed, and the expectation and its variance are obtained. Finally, we give the numerical examples.

1. Introduction

In insurance mathematics, the classical risk model has been the center of focus for decades [1–3]. The surplus $U(t)$ in the classical model at time t can be expressed as

$$U(t) = u + ct - \sum_{i=1}^{N_1(t)} Y_i, \quad (1)$$

where $u = U(0) > 0$ is the initial capital, $c > 0$ is the constant rate of premium, and $\{N_1(t), t \geq 0\}$ is a Poisson process, with Poisson rate $\lambda_1 > 0$ denoting the number of claims up to time t. The individual claim sizes $Y_1, Y_2, \ldots$, independent of $\{N_1(t), t \geq 0\}$, are independent and identically distributed nonnegative random variables with common distribution function $F(y)$ with mean μ_Y, variance σ_Y^2, and moment generating function $M_Y(r) = E[e^{rY}]$.

But in the Poisson process, the expectation and variance are equal. This is obviously not consistent with actual situation. So recently there is a huge amount of literature devoted to the generalization of the classical model in different ways. Lu and Li [4] consider a Markov-modulated risk model in which the claim interarrivals, claim sizes, and premiums are influenced by an external Markovian environment process. Tan and Yang [5] discuss the compound binomial risk model with an interest on the surplus under a constant dividend barrier and periodically paying dividends. Vellaisamy and Upadhye [6] study the convolution of compound negative binomial distributions with arbitrary parameters. The exact expression and also a random parameter representation are obtained. Cossette et al. [7] present a compound Markov binomial model, which is an extension of the compound binomial model. The compound Markov binomial model is based on the Markov Bernoulli process which introduces dependency between claim occurrences. Recursive formulas are provided for the computation of the ruin probabilities over finite- and infinite-time horizons. A Lundberg exponential bound is derived for the ruin probability, and numerical examples are also provided. Yang and Zhang [8] investigate a Sparre Andersen risk model in which the inter-claim times are generalized Erlang(n) distributed. Czarna and Palmowski [9] focus on a general spectrally negative Levy insurance risk process. For this class of processes, they analyze the so-called Parisian ruin probability, which arises when the surplus process stays below 0 longer than a fixed amount of time $t > 0$.

In this paper, we will consider a double Poisson-Geometric risk model with diffusion in which the arrival of policies is a Poisson-Geometric process and the claims process follows the compound Poisson-Geometric process. For more details and new developments on the Poisson-Geometric risk model, the interested readers can refer to [10–13].

The rest of the paper is organized as follows. In Section 2, the risk model is introduced. In Section 3, we obtain the adjustment coefficient equation and the formula of ruin probability. Then we present the effect of the related parameters on the adjustment coefficient. In Section 4, using the martingale method, the time when the surplus reaches a level firstly is considered, and the expectation and its variance are obtained. Numerical illustrations are also given.

2. The Risk Model

Definition 1 (see [10]). A distribution is said to be Poisson-Geometric distributed, denoted by $PG(\lambda, \rho)$, if its generating function is

$$\exp\left\{\frac{\lambda(t-1)}{1-\rho t}\right\}, \tag{2}$$

where $\lambda > 0$, $0 \leq \rho < 1$. Note that if $\rho = 0$, then the Poisson-Geometric distribution degenerates into Poisson distribution.

Definition 2 (see [10]). Let $\lambda > 0$ and $0 \leq \rho < 1$, then $\{N(t), t \geq 0\}$ is said to be a Poisson-Geometric process with parameters λ, ρ if it satisfies

(1) $N(0) = 0$;

(2) $\{N(t), t \geq 0\}$ has stationary and independent increments;

(3) for all $t > 0$, $N(t)$ is a Poisson-Geometric distributed with parameters λ, ρ, and $E[N(t)] = \lambda t/(1-\rho)$, $\operatorname{Var}[N(t)] = \lambda t(1+\rho)/(1-\rho)^2$.

The corresponding moment generating function of $N(t)$ is $M_{N(t)}(r) = \exp[\lambda t(e^r - 1)/(1-\rho e^r)]$.

Then the double Poisson-Geometric risk model with interference is defined as

$$U(t) = u + cN_2(t) - \sum_{k=1}^{N_3(t)} Y_k + \sigma W(t), \tag{3}$$

where $N_2(t)$ is the number of premium up to time t and follows a Poisson-Geometric distribution with parameters λ_2 and ρ_2; $N_3(t)$ is the number of claims up to time t and follows a Poisson-Geometric distribution with parameters λ_3 and ρ_3. $W(t)$ is the standard Brownian motion and σ is a constant, representing the diffusion volatility parameters. Throughout this paper, we assume that $N_2(t)$, $N_3(t)$, $W(t)$, and $\{Y_k\}$ are mutually independent.

In order to ensure the insurance company's stable operation, we assume

$$E\left[cN_2(t) - \sum_{k=1}^{N_3(t)} Y_k + \sigma W(t)\right] > 0, \tag{4}$$

which implies

$$\frac{\lambda_2 c}{1-\rho_2} - \frac{\lambda_3 \mu_Y}{1-\rho_3} > 0. \tag{5}$$

Let

$$\frac{\lambda_2 c}{1-\rho_2} = (1+\theta)\frac{\lambda_3 \mu_Y}{1-\rho_3}. \tag{6}$$

Then $\theta > 0$ is the relative security loading factor.

For the risk model (3), the time to ruin, denoted by T, is defined as

$$T = \inf\{t \geq 0 \mid U(t) < 0\}. \tag{7}$$

And define the ruin probability with an initial surplus $u > 0$ by $\psi(u)$, namely,

$$\psi(u) = \Pr(T < \infty \mid U(0) = u). \tag{8}$$

3. The Ruin Probability

Define the profits process by $\{S(t); t \geq 0\}$; that is,

$$S(t) = cN_2(t) - \sum_{k=1}^{N_3(t)} Y_k + \sigma W(t). \tag{9}$$

Obviously we have

$$\begin{aligned} E[S(t)] &= \left[\frac{\lambda_2 c}{1-\rho_2} - \frac{\lambda_3 \mu_Y}{1-\rho_3}\right] t, \\ \operatorname{Var}[S(t)] &= c^2 \operatorname{Var}[N_2(t)] + \operatorname{Var}[N_3(t)] \cdot E^2[Y_k] \\ &\quad + E[N_3(t)] \cdot \operatorname{Var}[Y_k] + \sigma^2 \operatorname{Var}[W(t)] \\ &= \left[\frac{\lambda_2 c^2(1+\rho_2)}{(1-\rho_2)^2} + \frac{\lambda_3(1+\rho_3)\mu_Y^2}{(1-\rho_3)^2}\right. \\ &\quad \left. + \frac{\lambda_3 \sigma_Y^2}{1-\rho_3} + \sigma^2\right] t. \end{aligned} \tag{10}$$

Let

$$\begin{aligned} \alpha &= \frac{\lambda_2 c}{1-\rho_2} - \frac{\lambda_3 \mu_Y}{1-\rho_3}, \\ \beta &= \frac{\lambda_2 c^2(1+\rho_2)}{(1-\rho_2)^2} + \frac{\lambda_3(1+\rho_3)\mu_Y^2}{(1-\rho_3)^2} + \frac{\lambda_3 \sigma_Y^2}{1-\rho_3} + \sigma^2. \end{aligned} \tag{11}$$

Then

$$\begin{aligned} E[S(t)] &= \alpha t, \\ \operatorname{Var}[S(t)] &= \beta t. \end{aligned} \tag{12}$$

Lemma 3. *The profits process $\{S(t); t \geq 0\}$ has the following properties:*

(1) *$S(0) = 0$;*

(2) *$\{S(t); t \geq 0\}$ has stationary and independent increments.*

Theorem 4. *For the profits process $\{S(t); t \geq 0\}$, there is a function $g(r)$ such that*

$$E\left[e^{-rS(t)}\right] = e^{tg(r)}. \tag{13}$$

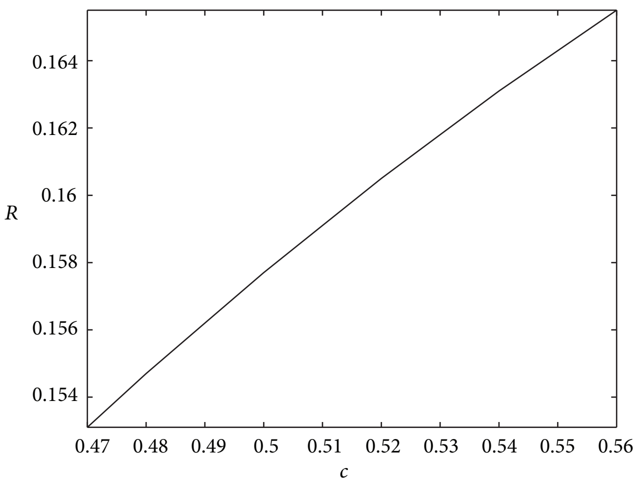

Figure 1: The impact of c on R.

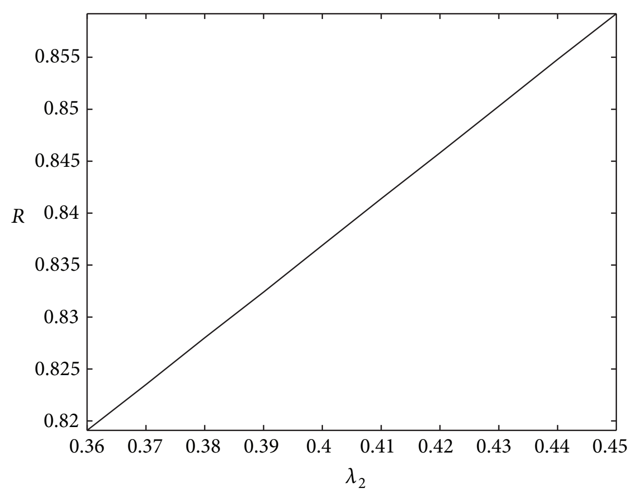

Figure 2: The impact of λ_2 on R.

Proof. Consider

$$\begin{aligned} E\left[e^{-rS(t)}\right] &= E\left\{\exp\left[-rcN_2(t)\right]\right\} \cdot E \\ &\quad\times \left\{\exp\left[r\sum_{k=1}^{N_3(t)} Y_k\right]\right\} \cdot E\left\{\exp\left[-r\sigma W(t)\right]\right\} \\ &= \exp\left\{t\left[\frac{\lambda_2\left(e^{-rc}-1\right)}{1-\rho_2 e^{-rc}} \right.\right. \\ &\quad \left.\left. + \lambda_3 \frac{M_Y(r)-1}{1-\rho_3 M_Y(r)} + \frac{1}{2}\sigma^2 r^2\right]\right\}. \end{aligned} \tag{14}$$

Let

$$g(r) = \frac{\lambda_2\left(e^{-rc}-1\right)}{1-\rho_2 e^{-rc}} + \lambda_3 \frac{M_Y(r)-1}{1-\rho_3 M_Y(r)} + \frac{1}{2}\sigma^2 r^2. \tag{15}$$

Then we obtain (13). □

Theorem 5. *Equation*

$$g(r) = 0 \tag{16}$$

has a unique positive solution $r = R > 0$, and (16) *is said to be an adjustment coefficient equation of the risk model* (3) *and $R > 0$ is said to be an adjustment coefficient.*

Proof. From (15), we have $g(0) = 0$, and since

$$\begin{aligned} g'(r) &= \frac{c\lambda_2 e^{-rc}\left(\rho_2-1\right)}{\left(1-\rho_2 e^{-rc}\right)^2} + \frac{\lambda_3\left(1-\rho_3\right)E\left[Ye^{rY}\right]}{\left(1-\rho_3 M_Y(r)\right)^2} + \sigma^2 r, \\ g''(r) &= \frac{c^2\lambda_2 e^{-rc}\left(1-\rho_2\right)\left(1+\rho_2 e^{-rc}\right)}{\left(1-\rho_2 e^{-rc}\right)^3} \\ &\quad + \frac{\lambda_3\left(1-\rho_3\right)\left[1-\rho_3 M_Y(r)\right]}{\left[1-\rho_3 M_Y(r)\right]^4} \\ &\quad \times \left\{\left(1-\rho_3 M_Y(r)\right)E\left[Y^2 e^{rY}\right] + 2\rho\left[E\left(Ye^{rY}\right)\right]^2\right\} + \sigma^2, \end{aligned} \tag{17}$$

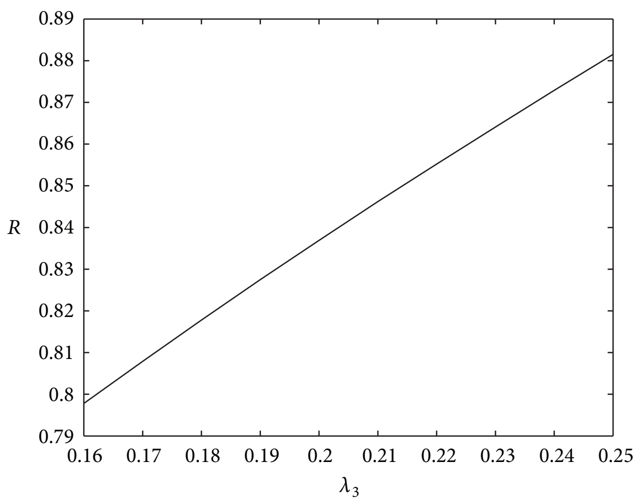

Figure 3: The impact of λ_3 on R.

which imply

$$g'(0) = -\frac{\lambda_2 c}{1-\rho_2} + \frac{\lambda_3\mu_Y}{1-\rho_3} = -\theta\frac{\lambda_3\mu_Y}{1-\rho_3} < 0. \tag{18}$$

It is easy to see that the moment generating function $M_Y(r)$ is an increasing function. Due to $0 < \rho_3 < 1$, there exists an r_1 such that $M_Y(r_1) = 1/\rho_3$; that is, $1-\rho_3 M_Y(r) > 0$ when $0 < r < r_1$. So when $0 < r < r_1$, $g''(r) > 0$ and $g(r)$ is a convex function with $\lim_{r\to+\infty} g(r) = +\infty$. Then it can be shown that $g(r)$ has a unique positive solution on $(0, +\infty)$. □

Example 6. Suppose $c = 0.5$, $\lambda_2 = 0.4$, $\lambda_3 = 0.2$, $\rho_2 = 0.9$, and $\rho_3 = 0.6$, $\alpha = 0.9$, $\sigma = 1.4$. By (16), we obtain the adjustment coefficient $R = 0.158$. Moreover, we give the effect of related parameters on adjustment coefficient R; see Figures 1, 2, 3, 4, 5, 6, and 7.

For the profits process $\{S(t); t \geq 0\}$, let $F_t^s = \sigma\{S(v); v \leq t\}$.

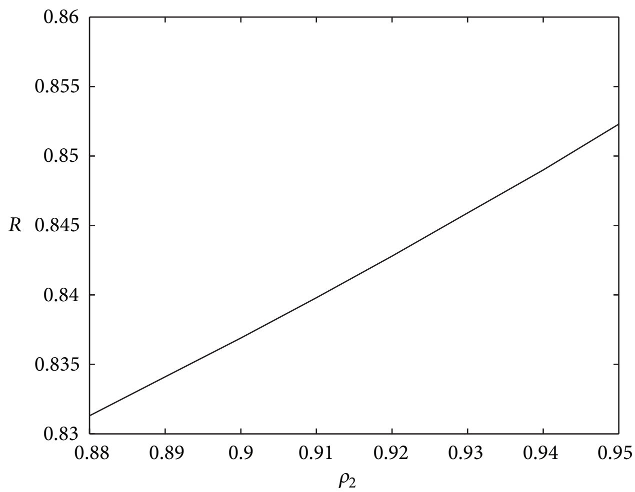

Figure 4: The impact of ρ_2 on R.

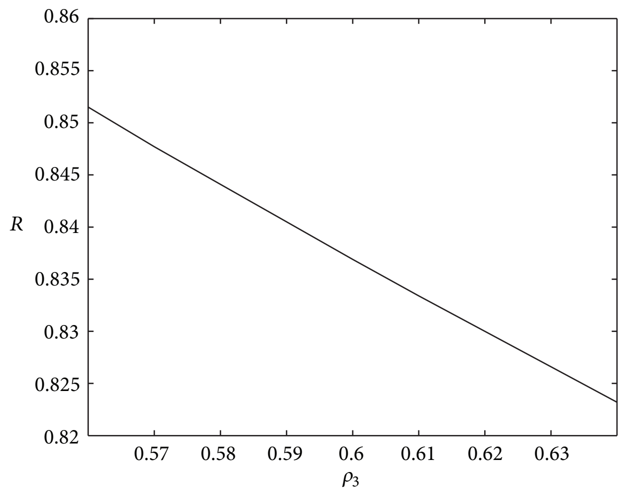

Figure 5: The impact of ρ_3 on R.

Theorem 7. *$\{H_u(t); F_t^s; t \geq 0\}$ is a martingale, where $H_u(t) = e^{-r(u+S(t))}/e^{tg(r)}$.*

Proof. Consider

$$\begin{aligned} E\left[H_u(t) \mid F_v^s\right] &= E\left[\frac{e^{-r(u+S(t))}}{e^{tg(r)}} \mid F_v^s\right] \\ &= E\left[\frac{e^{-r(u+S(v))}}{e^{vg(r)}}\frac{e^{-r(S(t)-S(v))}}{e^{(t-v)g(r)}} \mid F_v^s\right] \\ &= H_u(v)\, E\left[\frac{e^{-r(S(t)-S(v))}}{e^{(t-v)g(r)}} \mid F_v^s\right] \\ &= H_u(v). \end{aligned} \tag{19}$$

□

Theorem 8. *If r and s satisfy the equation $g(r) = s$, then the surplus $\{e^{-rS(t)-ts}; t \geq 0\}$ is a martingale.*

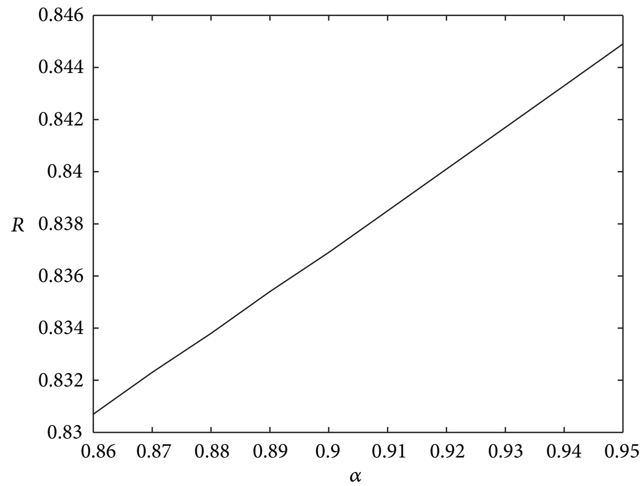

Figure 6: The impact of α on R.

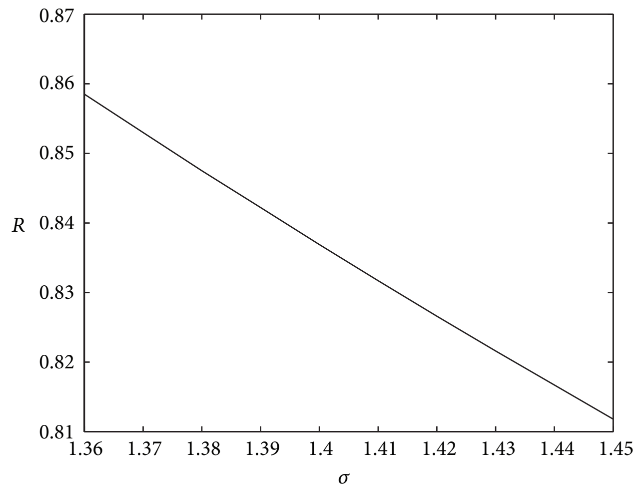

Figure 7: The impact of σ on R.

Proof. Consider

$$\begin{aligned} E\left[e^{-rS(t)-ts} \mid F_v^s\right] &= E\left[e^{-rS(t)-tg(r)} \mid F_v^s\right] \\ &= E\left[e^{-rS(v)-tg(r)-r[S(t)-S(v)]-(t-v)g(r)} \mid F_v^s\right] \\ &= e^{-rS(v)-tg(r)} \cdot E\left[e^{-r[S(t)-S(v)]-(t-v)g(r)} \mid F_v^s\right] \\ &= e^{-rS(v)-ts}. \end{aligned} \tag{20}$$

□

Lemma 9. *The ruin time T is the stopping time of F_t^s.*

Theorem 10. *For for all r, the ultimate ruin probability satisfies*

$$\psi(u) \leq e^{-ru} B(r), \tag{21}$$

where $B(r) = E[\sup_{t\geq 0}\{\exp[tg(r)]\}]$.

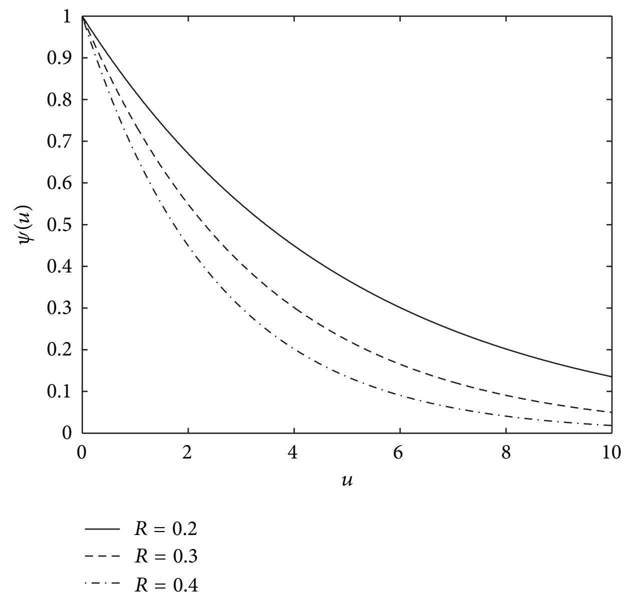

Figure 8: The impact of R on ruin probability $\psi(u)$.

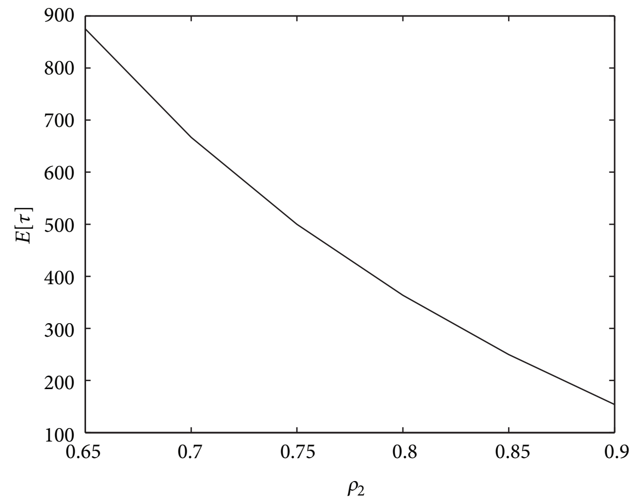

Figure 9: The impact of ρ_2 on $E[\tau]$.

Proof. For a fixed time t_0, $t_0 \wedge T$ is a bounded stopping time; using the theorem of martingale and stopping time, we have

$$\begin{aligned} e^{-ru} &= E\left[H_u(0)\right] = E\left[H_u\left(T \wedge t_0\right)\right] \\ &= E\left[H_u(T) \mid T \le t_0\right] \Pr\left(T \le t_0\right) \\ &\quad + E\left[H_u(t) \mid T > t_0\right] \Pr\left(T > t_0\right) \\ &\ge E\left[H_u(T) \mid T \le t_0\right] \Pr\left(T \le t_0\right), \end{aligned} \tag{22}$$

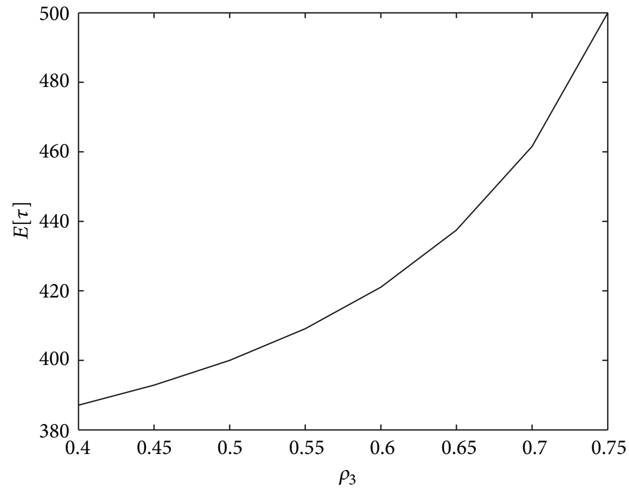

Figure 10: The impact of ρ_3 on $E[\tau]$.

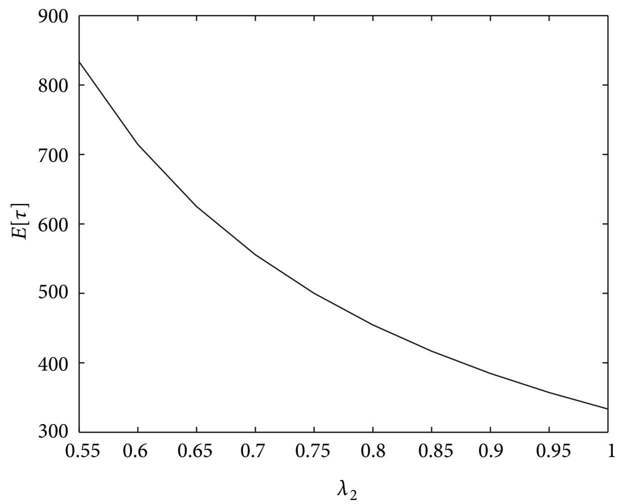

Figure 11: The impact of λ_2 on $E[\tau]$.

which implies

$$\begin{aligned} \Pr\left(T \le t_0\right) &= \frac{e^{-ru}}{E\left[H_u(T) \mid T < t_0\right]} \le \frac{e^{-ru}}{\inf_{0 \le t \le t_0} \exp\left[-tg(r)\right]} \\ &= e^{-ru} \sup_{0 \le t \le t_0} \left\{\exp\left[tg(r)\right]\right\}, \end{aligned} \tag{23}$$

by expectation on both sides of (23), and letting $t_0 \to +\infty$, we can obtain (21). □

Theorem 11. *The probability of the risk model* (3) *is*

$$\psi(u) = \frac{e^{-Ru}}{E\left[e^{-RU(T)} \mid T < \infty\right]}. \tag{24}$$

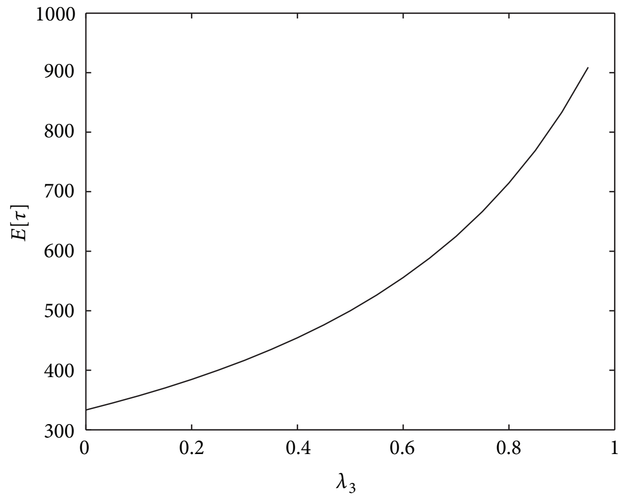

FIGURE 12: The impact of λ_3 on $E[\tau]$.

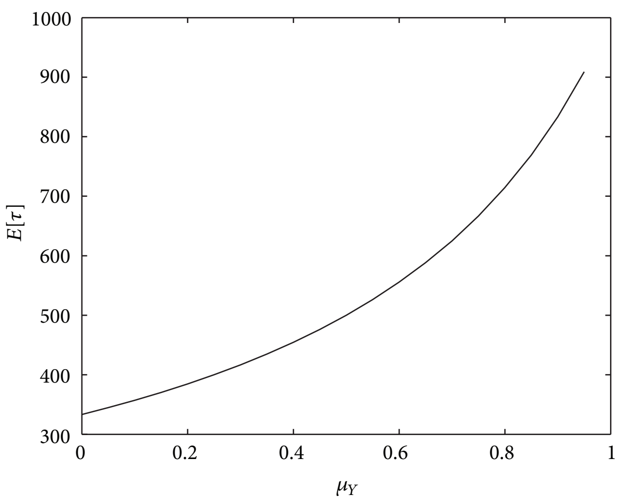

FIGURE 13: The impact of μ_Y on $E[\tau]$.

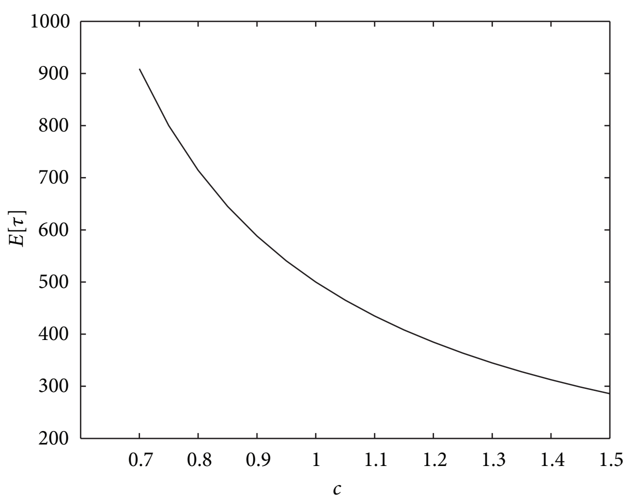

FIGURE 14: The impact of c on $E[\tau]$.

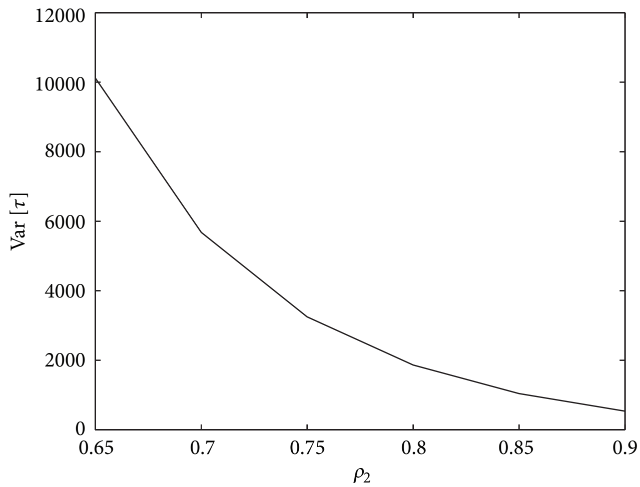

FIGURE 15: The impact of ρ_2 on Var$[\tau]$.

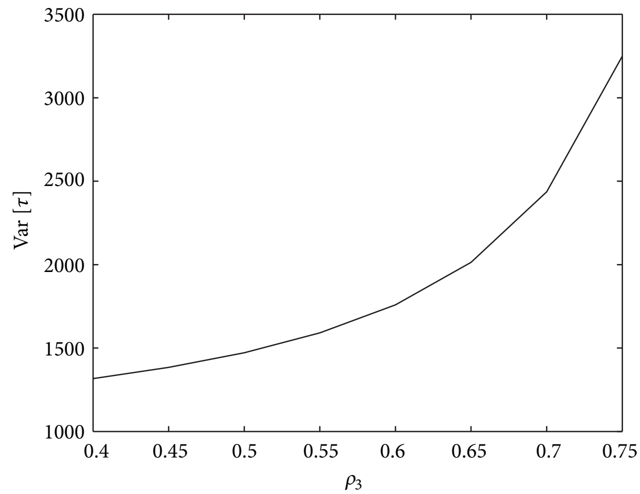

FIGURE 16: The impact of ρ_3 on Var$[\tau]$.

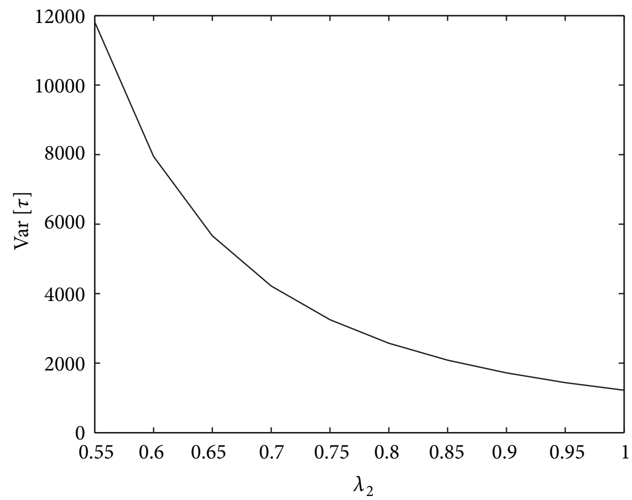

FIGURE 17: The impact of λ_2 on Var$[\tau]$.

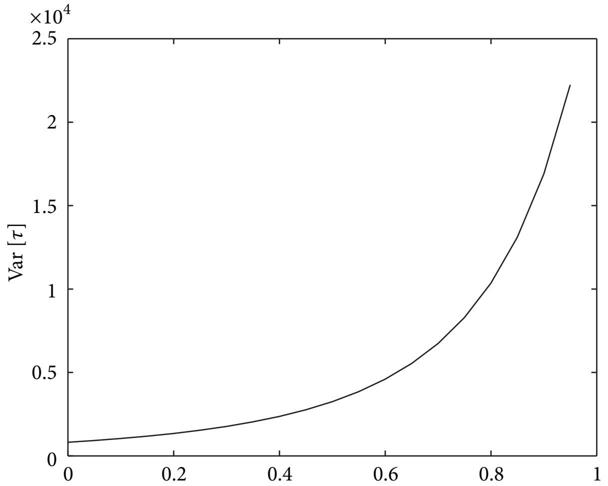

Figure 18: The impact of λ_3 on Var$[\tau]$.

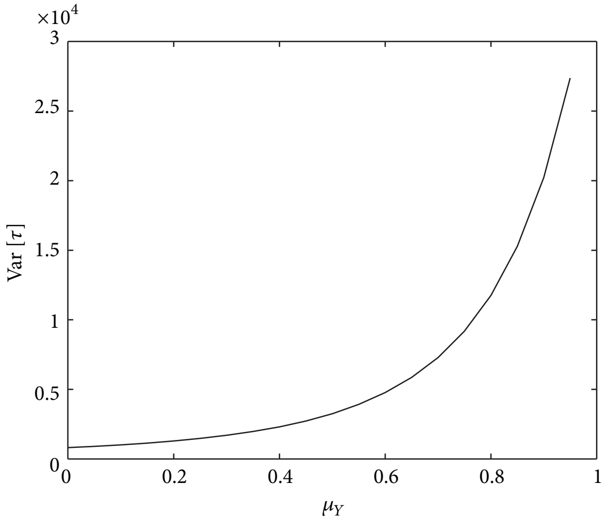

Figure 19: The impact of μ_Y on Var$[\tau]$.

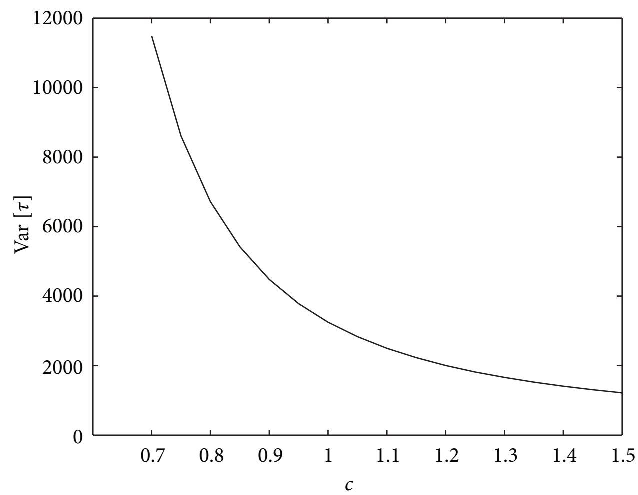

Figure 20: The impact of c on Var$[\tau]$.

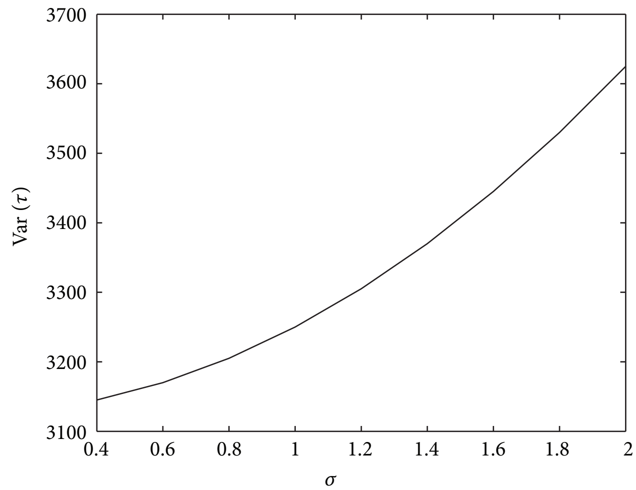

Figure 21: The impact of σ on Var$[\tau]$.

Proof. T is a ruin time and for a fixed time t_0, $T_u \wedge t_0$ is a bounded stopping time. Using the theorem of martingale and stopping time, we have

$$\begin{aligned} e^{-ru} &= H_u(0) = E\left[H_u(T \wedge t_0)\right] \\ &= E\left[H_u(T \wedge t_0) \mid T \le t_0\right] \Pr(T \le t_0) \\ &\quad + E\left[H_u(T \wedge t_0) \mid T > t_0\right] \Pr(T > t_0). \end{aligned} \tag{25}$$

Let $r = R$, we have

$$\begin{aligned} e^{-Ru} = {} & E\left[e^{-RU(T)} \mid T \le t_0\right] \Pr(T \le t_0) \\ & + E\left[e^{-RU(T)} \mid T > t_0\right] \Pr(T > t_0). \end{aligned} \tag{26}$$

If $I(A)$ is an indicator function of the event A, we get

$$\begin{aligned} 0 &\le E\left[e^{-RU(T)} \mid T > t_0\right] \Pr(T > t_0) \\ &= E\left[e^{-RU(T)} I(T > t_0)\right] \le E\left[e^{-RU(t_0)} I(U(t_0) \ge 0)\right]. \end{aligned} \tag{27}$$

Since

$$0 \le e^{-RU(t_0)} I(U(t_0) \ge 0) \le 1, \tag{28}$$

by the law of large numbers, when $t_0 \to \infty$, $U(t_0) \to \infty$ (a.s.). By dominated convergence theorem, we have

$$\lim_{t_0 \to \infty} E\left[e^{-RU(T)} \mid T > t_0\right] \Pr(T > t_0) = 0, \quad \text{(a.s.)}. \tag{29}$$

Then when $t_0 \to \infty$ in (26), we can obtain (24). □

Corollary 12. *Consider*

$$\psi(u) \le e^{-Ru}. \tag{30}$$

Example 13. Suppose $R = 0.2$, $R = 0.3$, and $R = 0.4$. By (30), we give the effect of adjustment coefficient R on the upper bound of the ruin probability; see Figure 8.

4. The Time to Reach a Given Level

Let

$$\tau = \inf \{t \geq 0 \mid U(t) = x\}. \tag{31}$$

Then τ is the time when the surplus reaches a given level firstly.

Theorem 14. *The Laplace transform of τ is*

$$E\left[e^{-s\tau}\right] = e^{rx}, \tag{32}$$

where r and s satisfy

$$g(r) = s. \tag{33}$$

Proof. For the surplus process $\{U(t); t \geq 0\}$, using the theorem of martingale and stopping time, we see that τ is a stopping rime of F_t^s. Let $Q(t) = e^{-rU(t)-ts}$. By Theorem 8, the surplus process $\{Q(t); t \geq 0\}$ is a martingale; hence, we have

$$E[Q(\tau)] = E[Q(0)], \tag{34}$$

implying that

$$E\left[e^{-rU(t)-ts}\right] = 1. \tag{35}$$

Since $U(t) = x$, so we get

$$E\left[e^{-s\tau}\right] = e^{rx}. \tag{36}$$

□

Theorem 15. *The expectation and variance of τ satisfy*

$$E[\tau] = \frac{x}{\alpha}, \qquad \operatorname{Var}[\tau] = \frac{x\beta}{\alpha^3}. \tag{37}$$

Proof. Let $\varphi(s) = \ln E[e^{-s\tau}]$. Using Theorem 11, we have $\varphi(s) = rx$. Then

$$\begin{aligned} \frac{d\varphi(s)}{ds} &= \frac{d\varphi(s)}{dr} \cdot \frac{dr}{ds} = \frac{d\varphi(s)}{dr} \cdot \frac{1}{dg(r)/dr} = \frac{x}{g'(r)}, \\ \frac{d^2\varphi(s)}{ds^2} &= \frac{d\varphi'(s)}{ds} = \frac{d\varphi'(s)}{dr} \cdot \frac{dr}{ds} = \frac{d\varphi'(s)}{dr} \cdot \frac{1}{dg(r)/dr} \\ &= -\frac{xg''(r)}{[g'(r)]^2} \cdot \frac{1}{g'(r)} = -\frac{xg''(r)}{[g'(r)]^3}. \end{aligned} \tag{38}$$

Let $s = r = 0$. We have

$$\begin{aligned} E[\tau] &= -\left.\frac{d\varphi(s)}{ds}\right|_{s=0} = -\frac{x}{g'(0)} = \frac{x}{\alpha}, \\ \operatorname{Var}[\tau] &= -\left.\frac{d^2\varphi(s)}{ds^2}\right|_{s=r=0} = \frac{x\beta}{\alpha^3}. \end{aligned} \tag{39}$$

□

Example 16. Suppose $\rho_2 = 0.75$, $\rho_3 = 0.75$, $\lambda_2 = 0.75$, $\lambda_3 = 0.5$, $\mu_Y = 0.5$, $\sigma_Y = 0.5$, $\sigma = 1$, and $c = 1$. By (37), we give the effect of related parameters on $E[\tau]$ and $\operatorname{Var}[\tau]$; see Figures 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, and 21.

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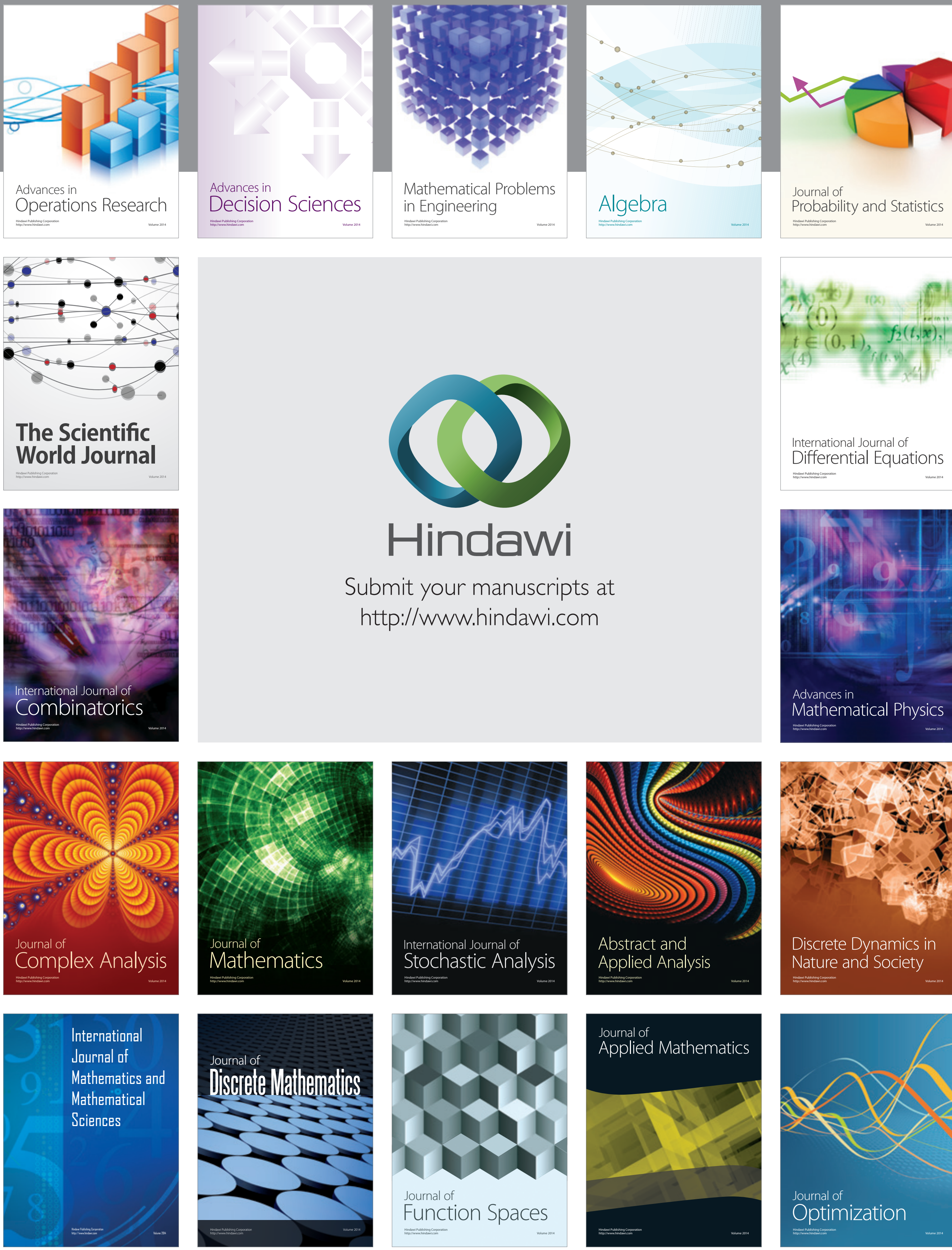
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