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On "Open-Economy Macroeconomics Using Models of Closed Systems"

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International macroeconomic crises occur because of the enlargement of the dimension of the state space within which economies operate. Our focus is the recent financial turbulence worldwide in which (i) banks had a role to play and (ii) whole systems collapsed. We validate these propositions in the context of the qualitative theory of differential equations. The economic framework used is that of Wynne Godley.

Keywords: Qualitative differential equations; Open-economy macroeconomics

1. INTRODUCTION

The Bretton Woods arrangements were constructed to reduce the dimension of the state space within economies can operate. The objective was to minimize the variability of exchange rates to, at most, a crawl and consequently take away the freedom to determine monetary policy from central banks. The dismantling of the system has eventually resulted in what LeBaron and McCulloch, 2000, call "the post-1973 international monetary non-system". Volatility of exchange rates has been excessive and, for some time, monetary surprises were large. Arbitrariness in monetary policy making can be said to be vanishing for reasons independent of international economics. The discussion spawned by the literature on credibility and time consistency has ensured that the behavior of central bankers is orderly and predictable. No particular position on instruments and targets is entailed here but it might be noted that there is a wide consensus that the short-run instrument should be the interest rate and the unique target should be inflation.

Variability of exchange rates per se would not be a matter of concern if there was no negative impact on real variables. The standard results on the nice properties of freely floating exchange rates and the ability of central banks to choose Pareto-superior equilibria in the short run, given discretion, are not in question. The problem is that the wild swings in exchange rates seem to impact on the second moments of trade flows. The earlier debate was engaged with the relationship between exchange

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rate volatility and the level (first moment) of trade flows (Barkoulas, Baum and Caglayan, 2000). The connection could arise from the timing of decisions to import. It takes time for imports to be delivered and the exchange rate that applies to the billing in foreign currency is determined at the point of delivery. Decisions on quantity are based on anticipations of future exchange rates that cannot be hedged. Both exporters and importers frame state-dependent contracts. Under present dispensations the problem of extracting signal from noise is acute. Since the observation of the state vector is imperfect, agents are prone to respond to rumours and fads and thereby stampede into or exit, en masse, out of markets. The reasons for the high volatility of exchange rates are not clear. Some argue that the best predictor of the exchange rate is a random walk. There has also been a rigorous line of enquiry into monetary policy noise as an explanatory variable. The thesis can explain the famous Mussa puzzle, that is, (i) that industrial countries that transited from fixed to flexible exchange rates experienced inexplicable variability in their exchange rates and (ii) nominal and exchange rates are strongly correlated. The abandonment of systematic monetary policy in the form of an interest rate rule, for example, assigning increasing weights to the deviations of the nominal exchange rate from a theoretical parity might drive the real exchange rate. At any rate, exchange rate based explanations are superior to real business cycle accounts that have not been able to model the high volatilities of trade variables. The work of Zimmerman, 1994 and 1996, is an econometric vindication of these hypotheses. He calibrates a model for three trading regions: Europe, North America and Japan. These countries account for the bulk of world trade and trade mostly with each other.

It turns out that countries that are linked by trade are also, naturally, linked by banking relationships. Corresponding with regional trading blocs there are common creditor country financial blocs. In connection with the recent meltdowns, US banks had an important role to play in the debt crisis in the early eighties and Japanese banks were a link in the propagation of shocks in the Asian crisis of 1997. Banks are a fundamental component of modern financial systems and no particular view need be taken concerning their role in financial collapses. In one line of argument, in their lending practices, they are the prime contributors to instability rather than portfolio investments in equities and bonds (Baily, Farrell and Lund, 2000). The reason is that the majority of bank lending today is in the form of short-term interbank loans rather than long-term project finance. An opposite view is that bank responses in calling loans and drying up credit lines are the natural outcome of a system-wide crisis. If a bank is faced with a sharp rise in its nonperforming assets in one country, it will reduce the worldwide risk of its portfolio by calling in high risk loans made in other countries (Kaminsky and Reinhart, 2000). These spillovers will be exacerbated as it recapitalizes and makes provisions and therefore lends less. In addition, there remains the problem of bank runs brought about by a collapse of confidence that agents place in the system. Explanations in terms of stochastic shocks are not necessary. The deterministic interaction of heteregenous agents are sufficient to cause a breakdown of the banking system.

2. NONLINEAR DYNAMICS AND OPEN-ECONOMY MACROECONOMICS

The terminology used to describe the collapse of regimes is evocative and significant: the tequila crisis of 1994, 1995, the Asian flu of 1997, the Russian virus of 1998 and so on. The suggestion is that these implosions are best modeled as the instability of macroeconomic systems as a whole rather than by the device of the representative agent responding to exogenous shocks. For instance, the imagery of all agents acting in concert as herds is often employed to analyze the sudden, instantaneous collapse of financial systems. The framework to be used then is clearly nonlinear. In linear systems, large fluctuations are the result of large shocks or the sum of small shocks. Since the fluctuations of interest are characterised by a zero mean, only the former case applies. The analysis of instability has, in that case, to be confined to epoch-marking events like oil price shocks. On the other hand, if the system under consideration is nonlinear, small shocks may generate sharp changes. If systems are strongly nonlinear, the strong law of large numbers does not apply and the aggregation problem becomes nontrivial (Delli Gatti, Gallegati, Giulioni and Palestrini, 2000).

We develop a methodology on the lines of Wynne Godley, 1983 and 1997, that does not require explicit microfoundations to examine the subject. All the same, the account below is consistent with portfolio choice by agents and optimal inventory adjustment by firms. The framework is short-run and structural. Fundamental macroeconomic relationships are represented in the balance-sheets of banks. The system is in mixed differential-difference equation form. The sectors and relationships are described in the following way:

The *private sector financial surplus* of the economy is represented by the following equation where dots on variables denote time derivatives.

$$YP - PE = B\dot{D} + B\dot{P} - P\dot{L}$$

YP is private disposable income, PE is private sector purchases, BD is the change in bank deposits, BP is the change in private holdings of bonds and PL is the increase in bank loans.

The *budget deficit*, ignoring changes in banks' reserve assets, is given by

$$G - YG = B\dot{B} + B\dot{P}$$

where G is government expenditure, YG is net government income, BB is bank holdings of domestic bonds and BP is private holdings of bonds.

Using the familiar identity between the budget deficit and the private sector surplus we get

$$BB_t + PL_t = BD_t$$

Recall that the other familiar identity, that is, between the budget deficit and the current account deficit is subsumed.

The above equation is also the *balance sheet of the banking system*.

Allowing for capital gains, we derive

$$BB_t + \dot{r}^d BB_{t-1} + PL_t + \dot{r}^l PL_{t-1} = BD_t + \dot{r}^{dd} BD_{t-1}$$

where r^d is the rate of interest on domestic bonds, r^l is the rate of interest on loans, r^{dd} is the rate of interest on deposits. In the present model flows between the home country and the foreign country is entirely due to exports and imports.

In Godley's Model 3 (1999) we allow for free capital flows between two countries. We can proceed in the identical manner above and arrive at the following equation in an expanded balance sheet of the banking system. To that end, let e denote the exchange rate, r^f the rate of interest on a foreign bond, BB^f a bond issued by a foreign monetary authority and held by a domestic bank, PL^f loans made by foreign banks to domestic borrowers and BD^f deposits made by foreigners in domestic banks. We get

$$\begin{aligned}&BB_t + \dot{r}^d BB_{t-1} + BB^f_t + \\&+ \dot{r}^f eBB^f_{t-1} + \dot{e}r^f BB^f_{t-1} + \\&+ PL_t + \dot{r}^l PL_{t-1} + PL^f_t + \\&+ \dot{r}^l ePL^f_{t-1} + \dot{e}r^l PL^f_{t-1} = \\&= BD_t + \dot{r}^{dd} BD_{t-1} + BD^f_t + \\&+ \dot{r}^{dd} eBD^f_{t-1} + \dot{e}r^{dd} BD^f_{t-1}\end{aligned}$$

The first model can be described as a one-dimensional system. In such a system the phase diagram can capture the significant properties of the dynamics. Moving to the higher dimensional system of Model 3, the global dynamics becomes more complex. Dynamical systems theory is being extensively used in mainstream work but the workhorse still remains the overlapping generations model in which agents, albeit heterogenous, respond to exogenous shocks. For our purposes, since the signs of the behavioural relationships are unknown, a qualitative approach is necessary.

We proceed to translate the above system into a nonlinear dynamic system. To that end, let $r^d BB \equiv x_1$, $r^l PL \equiv x_2$, $r^{dd} BD \equiv x_3$. We posit the following primitive relationship

$$Y = f(X),$$

where $X \equiv (x_1, x_2, x_3)$. Therefore

$$\frac{dY}{dt} = \frac{df}{dX}\frac{dX}{dt}.$$

In like manner in the case of Model 3, we denote $r^f eBB \equiv x_4$, $r^l ePL^f \equiv x_5$, $r^{dd} eBD^f \equiv x_6$. Our primitive is now defined over a higher dimensional vector space $X \equiv (x_1, x_2, x_3, x_4, x_5, x_6)$.

The earlier discussion motivates the following treatment of the problem (Nemytskii and Stepanov, 1960, hereafter N-S). For ease of reference, as far as possible, the notations are unchanged. We are given a function

$$f(p, t)$$

which is a mapping from R^2 and any real number $t(-\infty < t < +\infty)$ onto R^2. The parameter t is called *time*. The function is assumed to be continuous with respect to p and t.

The function $f(p, t)$ for fixed p is called a *motion*. In what follows we will be concerned, without loss of generality, with the following set

$$\{f(p, t); 0 < t < \infty\}$$

called the *positive half-trajectory*. Negative half-trajectories and, in general, trajectories can be defined in the obvious manner. We will use the following definitions.

Definition 1 A motion is called *positively stable according to Lagrange* if the closure of the positive half-trajectory is a compact set.

Take any bounded increasing sequence of values of t:

$$0 \leq t_1 < t_2 < \cdots < t_n < \cdots, \lim_{n\to\infty} t_n = +\infty.$$

If the sequence of points

$$f(p, t_1), f(p, t_2), \ldots, f(p, t_n), \ldots$$

has a limit point q, then the limit point is called the *ω-limit* point of the motion. Let Ω_p denote the set of all ω-limit points. Clearly for a motion positively stable according to Lagrange, the set Ω_p is not empty.

A point p is called *wandering* if there exists a neighborhood $U(p)$ of it and a positive number T such that

$$U(p) \cdot f(U(p), t) = 0 \quad \text{for all } t \geq T.$$

This leads to

Definition 2 A dynamical system is called *completely unstable* if all its points are wandering.

The simplest example of a completely unstable system is a dynamical system along a family of parallel lines. The N-S agenda is to establish necessary and sufficient conditions under which there exists a topological transformation of the given metric space into E^∞ under which the trajectories of a given system map into a family of parallel lines.

For our purposes the following projection will suffice. Let $x = \Phi(p)$, where $p \in R^2$ and $x = (\xi_0, \xi_1, \xi_2, \xi_3)$, be a mapping of the space R^2 onto a subset $X \subset R^4$, under which the trajectories $f(p, t)$ pass into the points $x = (\xi_0, q_1, q_2, q_3)$ where q_1 is the first component of a trajectory vector, q_2 is the second component and q_3 corresponds to zero in the domain. The variable coordinate ξ_0 increases with t. It is clear that the mapping Φ is one-to-one and bi-continuous. The following variation of the fundamental results of N-S can now be stated:

THEOREM *The* (i) *Lagrange instability and* (ii) *complete instability of a dynamical system in* R^2 *is a necessary condition for the Lagrange instability and complete instability of a dynamical system in* $X \subset R^4$ *under the projection* Φ.

Proof

(i) Let there be a sequence of points $\{x_n\} \subset X$. Now

$$x_n = f(x, t_n).$$

Suppose that a convergent subsequence is chosen from the sequence $\{t_{n.}\}$. For convenience, let $\{t_n\}$ be that subsequence. Then

$$\lim_{n\to\infty} f(x, t_n) = f(x, \varsigma) = y$$

and

$$y \notin \Omega_x$$

where Ω_x is the set of ω-limit points of the motion in X.

By virtue of the mapping Φ we have

$$\lim_{n\to\infty} f(p, t_n) = f(p, \varsigma) = q$$

and $q \notin \Omega_p$.

(ii) In an abuse of notation the dynamical system in X is defined by

$$f(x, t) = \Phi(f(p, t)).$$

Consider an arbitrary point $p \in R^2$. Take the closure $\overline{U(p, \varepsilon)}$ of its compact neighbourhood. The image of this set under Φ is a closed, compact set in R^4. By assumption we can find a $T > 0$, such that for $t > T$,

$$\Phi(\overline{U(p,\varepsilon)}) \cdot f(\Phi(\overline{U(p,\varepsilon)}, t) = \Phi(\overline{U(p,\varepsilon)}) \cdot \Phi(f(\overline{U(p,\varepsilon)}, t).$$

Passing to the transformation Φ^{-1} in the space R^2, we get

$$\overline{U(p,\varepsilon)} \cdot f(\overline{U(p,\varepsilon)}, t) = 0 \quad \text{for } t \geq T.$$

Q.E.D.

Sufficient conditions are more subtle and require the introduction of an array of concepts and results elaborated in N-S. In other words, instability of Model 3 in the form of the changing exchange rate, the changing price of the foreign bonds, the variability of loans made by foreign banks to domestic borrowers and deposits made by foreigners in domestic banks destabilises the autarkic economy even if in an initial condition of equilibrium.

3. CONCLUSION

A regime of bounds on the fluctuations of exchange rates and the attendant requirement of monetary authorities to tie their hands to a strong and resolute leader resulted in the intertemporal consumption and production decisions of agents being grounded in sound fundamentals. The dismantling of the regime has resulted in an increase in the dimension of the choice set over which agents must make their plans. Since the additional elements, like exchange rates and monetary policy, are free to vary arbitrarily, forecasting their future values becomes hazardous. A collapse in the higher-order system can precipitate a collapse in the lower-order system even in economies when the trade variables are in a virtuous relationship. We show that the language of the qualitative theory of differential equations can be used to model this phenomenon.

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