Hindawi Publishing Corporation
Fixed Point Theory and Applications
Volume 2008, Article ID 543154, 8 pages
doi:10.1155/2008/543154

Research Article

Coincidence Point, Best Approximation, and Best Proximity Theorems for Condensing Set-Valued Maps in Hyperconvex Metric Spaces

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Received 8 October 2008; Accepted 9 December 2008

Recommended by William A. Kirk

In hyperconvex metric spaces, we first present a coincidence point theorem for condensing set-valued self-maps. Then we consider the best approximation problem and the best proximity problem for set-valued mappings that are condensing. As an application, we derive a coincidence point theorem for nonself-condensing set-valued maps.

1. Introduction and preliminaries

The best approximation problem in a hyperconvex metric space consists of finding conditions for given set-valued mappings F and G and a set X such that there is a point $x_0 \in X$ satisfying $d(G(x_0), F(x_0)) \leq d(x, F(x_0))$ for $x \in X$. When $G = I$, the identity mapping, and when the set X is compact, best approximation theorems for mappings in hyperconvex metric spaces are given for the single-valued case in [1–4], for the set-valued case in [1, 3, 5–9]. Some results for condensing set-valued maps were given in [2].

Given subsets A, B, set-valued mappings $F : A \multimap B$, and $G : A \multimap A$ the best proximity problem consists of finding conditions on F, G, A, and B implying that there is a point $x_0 \in A$ such that $d(G(x_0), F(x_0)) = d(A, B)$. Then $(G(x_0), F(x_0))$ is called a *best proximity pair*, see [2, 10]. For A, B nonempty subsets of a metric space M, we define the following sets

$$A_0 = \{x \in A : d(x, y) = d(A, B) \text{ for some } y \in B\},$$
$$B_0 = \{y \in B : d(x, y) = d(A, B) \text{ for some } x \in A\}. \tag{1.1}$$

A metric space (M,d) is said to be a *hyperconvex metric space* [11] if for any collection of points x_α of M and any collection r_α of nonnegative real numbers with $d(x_\alpha,x_\beta)\le r_\alpha+r_\beta$, we have

$$\bigcap_\alpha B(x_\alpha,r_\alpha)\neq\varnothing. \tag{1.2}$$

The *admissible* subsets of a hyperconvex metric space M are sets of the form $\bigcap_\alpha B(x_\alpha,r_\alpha)$, that is, the family of all ball intersections in M. Every admissible subset of a hyperconvex metric space is hyperconvex. For a subset A of M, $N_\epsilon(A)$ denotes the closed ϵ-neighborhood of A, that is, $N_\epsilon(A)=\{x\in M : d(x,A)\le\epsilon\}$, where $d(x,A)=\inf_{y\in A}d(x,y)$. If A is admissible, then $N_\epsilon(A)$ is admissible [12].

A subset A of a metric space M is said to be *externally hyperconvex* if given any family x_α of points in M and the family r_α of nonnegative real numbers satisfying

$$d(x_\alpha,x_\beta)\le r_\alpha+r_\beta,\qquad d(x_\alpha,A)\le r_\alpha, \tag{1.3}$$

it follows that

$$\bigcap_\alpha B(x_\alpha,r_\alpha)\cap A\neq\varnothing. \tag{1.4}$$

Every externally hyperconvex subset of a metric space is hyperconvex [13, Theorem 3.10]. Let (M,d) be a metric space and X be a nonempty subset of M. X is said to be a *proximal nonexpansive retract* of M if there exists a nonexpansive retraction $r : M \to X$ with the property

$$d(x,r(x))=d(x,X),\quad\text{for every } x\in X. \tag{1.5}$$

Every admissible set is externally hyperconvex and the externally hyperconvex sets are proximinal nonexpansive retracts of M [14].

For each $A,B\subseteq M$, let

$$d(A,B)=\inf\{d(a,b) : a\in A,\ b\in B\}. \tag{1.6}$$

It is well know that if A and B are compact subsets of M then there exist $a_0\in A$ and $b_0\in B$ such that $d(A,B)=d(a_0,b_0)$. Therefore, in this case

$$d(A,B)=0 \Longleftrightarrow A\cap B\neq\varnothing. \tag{1.7}$$

Let X and Y be topological spaces with $A\subseteq X$ and $B\subseteq Y$. Let $F : X \multimap Y$ be a set-valued map with nonempty values. The image of A under F is the set $F(A)=\bigcup_{x\in A}F(x)$ and the inverse image of B under F is $F^-(B)=\{x\in X : F(x)\cap B\neq\varnothing\}$. Now F is said to be

(i) lower semicontinuous if for each open set $B\subseteq Y$, $F^-(B)=\{x\in X : F(x)\cap B\neq\varnothing\}$ is open in X;

(ii) upper semicontinuous if for each closed set $B \subseteq Y$, $F^{-}(B) = \{x \in X : F(x) \cap B \neq \emptyset\}$ is closed in X;

(iii) continuous if F is both lower semicontinuous and upper semicontinuous.

Let M be a metric space and let $\mathcal{M}$ denote the family of nonempty, closed bounded subsets of M. Let $A, B \in \mathcal{M}$. The Hausdorff metric D on $\mathcal{M}$ is defined by

$$D(A,B) = \inf\{\epsilon > 0 : A \subseteq N_{\epsilon}(B),\ B \subseteq N_{\epsilon}(A)\}. \tag{1.8}$$

Let X be a nonempty subset of M. A set-valued map $F : X \multimap \mathcal{M}$ is called *Hausdorff continuous* if it is continuous with respect to Hausdorff metric.

A topological space is said to be *acyclic* if all of the reduced Čech homology groups over rationals vanish. Every hyperconvex metric space is acyclic [15]. Let X be an admissible subset of M. A set-valued map $F : X \multimap M$ is said to be *quasiadmissible* if the set $F^{-}(A)$ is closed acyclic for each admissible set A of M.

Let $\mathcal{B}(M)$ denote the set of all bounded subsets of M. The *Kuratowski measure of noncompactness* $\alpha : \mathcal{B}(M) \to [0,\infty)$ is defined by

$$\alpha(A) = \inf\left\{\delta > 0 : A \subseteq \bigcup_{i=1}^{n} A_i,\ \operatorname{diam}(A_i) < \delta\right\}. \tag{1.9}$$

A mapping $F : M \to \mathcal{B}(M)$ is said to be *condensing* provided that $\alpha(F(A)) < \alpha(A)$, for any $A \in \mathcal{B}(M)$ with $\alpha(A) > 0$. If $\alpha(F(A)) \leq \alpha(A)$ for any $A \in \mathcal{B}(M)$, then F is called 1-set contraction.

The following fixed point theorem, which will be used in the next section, is due to Amini-Harandi et al. [6].

Theorem 1.1. *Let M be a hyperconvex metric space. Suppose that $F : M \multimap M$ is an upper semicontinuous condensing set-valued map with nonempty closed acyclic values. Then F has a fixed point.*

2. Coincidence point

Now we present a coincidence point theorem for condensing set-valued self-maps.

Theorem 2.1. *Let M be a hyperconvex metric space and $F : M \multimap M$ be an upper semicontinuous condensing set-valued map with nonempty closed acyclic values. Let $G : M \multimap M$ be an onto, quasiadmissible set-valued map for which $G(A)$ is closed for each closed set $A \subseteq M$. Assume that $G^{-} : M \multimap M$ is a 1-set contraction. Then there exists an $x_0 \in M$ with*

$$F(x_0) \cap G(x_0) \neq \emptyset. \tag{2.1}$$

Proof. Since

$$F(x_0) \cap G(x_0) \neq \emptyset \Longleftrightarrow x_0 \in G^{-}(F(x_0)) = \{x \in M : G(x) \cap F(x_0) \neq \emptyset\}, \tag{2.2}$$

then the conclusion follows if we show that the set-valued map $H(x) = G^{-}(F(x)) : M \multimap M$ has a fixed point. Since G is onto, then $H(x) \neq \emptyset$. Since $F(x)$ is admissible and G is quasiadmissible, then $H(x)$ is closed acyclic. Now we show that H is upper semicontinuous. To show this, let A be a closed subset of M. Then

$$\begin{aligned} H^{-}(A) &= \{x \in M : H(x) \cap A \neq \emptyset\} \\ &= \{x \in M : \{t \in M : G(t) \cap F(x) \neq \emptyset\} \cap A \neq \emptyset\} \\ &= \{x \in M : \exists\, a \in A \text{ such that } G(a) \cap F(x) \neq \emptyset\} \\ &= \{x \in M : F(x) \cap G(A) \neq \emptyset\} \\ &= F^{-}(G(A)). \end{aligned} \tag{2.3}$$

Since F is upper semicontinuous and $G(A)$ is closed, then $H^{-}(A) = F^{-}(G(A))$ is closed. Hence H is upper semicontinuous. Now we show that H is condensing. To show this, let $A \subseteq M$ with $\alpha(A) > 0$. Since G^{-} is 1-set contraction and F is condensing, then

$$\alpha(H(A)) = \alpha(G^{-}(F(A)) \leq \alpha(F(A)) < \alpha(A). \tag{2.4}$$

Therefore, H satisfies all conditions of Theorem 1.1 and so it has a fixed point. □

Corollary 2.2. *Let M be a hyperconvex metric space and $f : M \to M$ be a continuous condensing map. Let $G : M \multimap M$ be an onto, quasiadmissible set-valued map for which $G(A)$ is closed for each closed set $A \subseteq M$. Assume that $G^{-} : M \multimap M$ is a 1-set contraction. Then there exists an $x_0 \in M$ with*

$$f(x_0) \in G(x_0). \tag{2.5}$$

3. Best approximation

In this section, we extend some well-known best approximation theorems by involving a second set-valued map G.

Theorem 3.1. *Let M be a hyperconvex metric space and X be a nonempty admissible subset of M. Let $F : X \multimap M$ be a Hausdorff continuous condensing set-valued map with nonempty bounded externally hyperconvex values and $G : X \multimap X$ be an onto, quasiadmissible set-valued map for which $G(A)$ is closed for each closed set $A \subseteq X$. Assume that $G^{-} : X \multimap X$ is a 1-set contraction. Then there exists an $x_0 \in X$ such that*

$$d(G(x_0), F(x_0)) = \inf_{x \in X} d(x, F(x_0)). \tag{3.1}$$

Proof. Define a mapping $H : X \multimap M$ by

$$H(x) = \bigcap_{\epsilon > \epsilon(x)} (N_{\epsilon}(X) \cap F(x)), \tag{3.2}$$

where $\epsilon(x) = \inf\{\epsilon > 0 : N_\epsilon(X) \cap F(x) \neq \emptyset\}$. The values of H are nonempty and externally hyperconvex [13, page 408, Theorem 5.4]. From [8, Lemma 1],

$$D\big(N_{\epsilon(x)}(X) \cap F(x), N_{\epsilon(y)}(X) \cap F(y)\big) \leq D\big(F(x), F(y)\big). \tag{3.3}$$

Hence $D(H(x), H(y)) \leq D(F(x), F(y))$. Since F is Hausdorff continuous, this implies that H is also continuous in the Hausdorff metric. By a selection result in [16, Theorem 1], there is a mapping $h : X \to M$ such that $h(x) \in H(x)$ for each $x \in X$ and $d(h(x), h(y)) \leq D(H(x), H(y))$ for each $x, y \in X$. Note h is continuous. Since $h(x) \in H(x) \subseteq F(x)$, h is also condensing. The admissible set X is a proximinal nonexpansive retract of M [14] and we denote the retraction by $P_X : M \to X$. It follows that the mapping $P_X(h(\cdot)) : X \to X$ is continuous and condensing, and therefore, by Corollary 2.2, there exists an $x_0 \in X$ such that $P_X(h(x_0)) \in G(x_0)$. Fix $x \in X$. Now we show that $\epsilon(x) = d(X, F(x))$. Let $\epsilon > \epsilon(x)$ and let $y_\epsilon \in N_\epsilon(X) \cap F(x)$. Then $d(X, F(x)) \leq d(X, y_\epsilon) \leq \epsilon$. We can do this argument for each $\epsilon > \epsilon(x)$ so, therefore, $d(X, F(x)) \leq \epsilon(x)$. Suppose now that $d(X, F(x)) < \epsilon(x)$. Then there exists a $y \in F(x)$ such that $d(X, F(x)) \leq d(X, y) \equiv \epsilon < \epsilon(x)$. Thus $y \in N_\epsilon(X) \cap F(x) \neq \emptyset$. This is a contradiction. Fix $n \in \{1, 2, \ldots\}$ and let $\epsilon_n = d(X, F(x_0)) + 1/n$; note $\epsilon_n > \epsilon(x_0)$. Then since $h(x_0) \in H(x_0)$, we have $h(x_0) \subseteq N_{\epsilon_n}(X)$ so $d(X, h(x_0)) \leq \epsilon_n = d(X, F(x_0)) + 1/n$. We can do this for each n so

$$d\big(X, h(x_0)\big) \leq d\big(X, F(x_0)\big). \tag{3.4}$$

Since $h(x_0) \in F(x_0)$ we get

$$d\big(X, h(x_0)\big) = d\big(X, F(x_0)\big). \tag{3.5}$$

Therefore, we have since $P_X(h(x_0)) \in G(x_0))$ and $h(x_0) \in F(x_0)$ that

$$\begin{aligned} d\big(G(x_0), F(x_0)\big) &\leq d\big(P_X\big(h(x_0)\big), F(x_0)\big) \\ &\leq d\big(P_X\big(h(x_0)\big), h(x_0)\big) \\ &= d\big(X, h(x_0)\big), \end{aligned} \tag{3.6}$$

since X is a proximity retract of M. Thus

$$d\big(G(x_0), F(x_0)\big) \leq d\big(X, h(x_0)\big) = d\big(X, F(x_0)\big). \tag{3.7}$$

Since $G(x_0) \subseteq X$ then

$$d\big(G(x_0), F(x_0)\big) = \inf_{x \in X} d\big(x, F(x_0)\big). \tag{3.8}$$

□

Remark 3.2. Let X be a nonempty compact admissible subset of a hyperconvex metric space M and let $G : X \to X$ be an isometry. We show that G satisfies all the conditions of

Theorem 3.1. Since X is compact and $G : X \to X$ is an isometry, then G is onto. Now we show that G is quasiadmissible. Let A be an admissible subset of X. Since G is an isometry, then $G^{-}(A) = G^{-1}(A)$ is admissible and so is closed and acyclic. Let $A \subsetneq X$ be closed, then A is compact. Since G is continuous, then $G(A)$ is compact and so is closed. Since X is compact, then $G^{-1} : X \multimap X$ is a 1-set contraction (note for each $A \subsetneq X$, $\alpha(G^{-1}(A)) = \alpha(A) = 0$).

If we take $G = I$, then Theorem 3.1 reduces to the following result of Markin and Shahzad [2].

Corollary 3.3. *Let M be a hyperconvex metric space and X be a nonempty admissible subset of M and $F : X \multimap M$ be a Hausdorff continuous condensing set-valued map with nonempty bounded externally hyperconvex values. Then there exists an $x_0 \in X$ such that*

$$d(x_0, F(x_0)) = \inf_{x \in X} d(x, F(x_0)). \tag{3.9}$$

Proof. It suffices to show that $G = I$ satisfies the conditions of Theorem 3.1. The identity mapping $I : M \to M$ is onto and $I(A) = A$ is closed for each closed set $A \subsetneq M$. Let A be an admissible subset of M. Then $I^{-}(A) = A$ is admissible and so is acyclic [15, Lemma 5.2]. Thus I is a quasiadmissible map. Finally, since $\alpha(I^{-}(A)) = \alpha(A)$ for each subset A of M, then $I^{-} : M \to M$ is a 1-set contraction map. □

The following is a coincidence point theorem for condensing nonself-set-valued maps.

Corollary 3.4. *Let M be a hyperconvex metric space and X be a nonempty admissible subset of M. Assume the mappings F, G are compact valued and satisfy the conditions of Theorem 3.1. Assume that $F(x) \cap X \neq \varnothing$ for $x \in X$. Then there exists an $x_0 \in X$ such that*

$$F(x_0) \cap G(x_0) \neq \varnothing. \tag{3.10}$$

Proof. By Theorem 3.1, there exists an $x_0 \in X$ with $d(G(x_0), F(x_0)) = \inf_{x \in X} d(x, F(x_0))$. Since $F(x_0) \cap X \neq \varnothing$, then $\inf_{x \in X} d(x, F(x_0)) = 0$. Thus $d(G(x_0), F(x_0)) = 0$. Therefore, $F(x_0) \cap G(x_0) \neq \varnothing$. □

4. Best proximity pairs

In this section, we obtain a best proximity pair theorem for condensing set-valued maps in hyperconvex metric spaces.

Theorem 4.1. *Let M be a hyperconvex metric space, A be an admissible subset, and B be a bounded externally hyperconvex subset of M. Let $G : A_0 \multimap A_0$ an onto, quasiadmissible set-valued map for which $G(C)$ is closed for each closed set $C \subsetneq A_0$. Assume that $G^{-} : A_0 \multimap A_0$ is a 1-set contraction. Assume the mapping $F : A \multimap B$ is condensing, Hausdorff continuous with nonempty admissible values. Assume that $F(x) \cap B_0 \neq \varnothing$ for each $x \in A_0$. Then there exists an $x_0 \in A_0$ such that*

$$d(G(x_0), F(x_0)) = d(A, B). \tag{4.1}$$

Proof. By [2, Lemma 5.1], A_0 and B_0 are externally hyperconvex and nonempty. Define a mapping $H : A_0 \multimap B_0$ by $H(x) = F(x) \cap B_0$. Since $A_0 = \bigcap_{n=1}^{\infty} N_{d(A,B)+1/n}(B) \cap A = A \cap N_{d(A,B)}(B)$

and $B_0 = \bigcap_{n=1}^{\infty} N_{d(A,B)+1/n}(A) \cap B = B \cap N_{d(A,B)}(A)$ [2, Lemma 5.1], then by [9, Lemma 1], we have $D(F(x) \cap B_0, F(y) \cap B_0) \leq D(F(x), F(y))$. Since F is Hausdorff continuous, this implies that H is continuous in the Hausdorff metric. Since $H(x)$ is externally hyperconvex for each $x \in A_0$, by a selection result in [16], there is a continuous mapping $h : A_0 \to B_0$ such that $h(x) \in H(x)$ for each $x \in A_0$. Since $h(x) \in F(x)$, h is also condensing. The admissible set A is a proximinal nonexpansive retract of M and we denote the retraction by $P_A : M \to A$. Note $P_A(B_0) \subseteq A_0$. To see this note, if $y \in B_0$, then there is an $x \in A$ such that $d(x, y) = d(A, B)$. Thus $d(y, P_A(y)) = d(y, A) \leq d(y, x) = d(A, B)$ so we have $d(y, P_A(y)) = d(A, B)$ and so $P_A(y) \in A_0$. Since externally hyperconvex subset of M is hyperconvex [13, page 398, Theorem 3.10], then A_0 is a hyperconvex metric space. Now the mapping $P_A(h(\cdot)) : A_0 \to A_0$ is continuous and condensing, and therefore, by Corollary 2.2, there exists an $x_0 \in A$ such that $P_A(h(x_0)) \in G(x_0)$. Therefore, since $P_A(h(x_0)) \in A_0$, we have $d(P_A(h(x_0)), h(x_0)) \leq d(x, h(x_0))$, for each $x \in A_0$. Since $h(x_0) \in B_0$, there is an $a_0 \in A$ such that $d(a_0, h(x_0)) = d(A, B)$, and therefore, $B(h(x_0), d(A, B)) \neq \varnothing$. Furthermore, since $A_0 = A \cap N_{d(A,B)}(B)$, then it follows from the external hyperconvexity of $N_{d(A,B)}(B)$ that $(B(h(x_0), d(A, B)) \cap A) \cap N_{d(A,B)}(B) \neq \varnothing$ (note $B(h(x_0), d(A, B)) \cap A$ is admissible) [16, Lemma 2]. Let $a_1 \in B(h(x_0), d(A, B)) \cap A \cap N_{d(A,B)}(B)$. Then $a_1 \in A$ and $d(a_1, h(x_0)) = d(A, B)$. Since $h(x_0) \in B_0 \subseteq B$, then we have $a_1 \in A_0$. Therefore, from the above, we have

$$d(P_A(h(x_0)), h(x_0)) \leq d(a_1, h(x_0)) = d(A, B). \tag{4.2}$$

However, note also since $G(x_0) \subseteq A$, $F(x_0) \subseteq B$, $P_A(h(x_0)) \in G(x_0)$ and $h(x_0) \in F(x_0)$ that

$$\begin{aligned} d(A, B) &\leq d(G(x_0), F(x_0)) \\ &\leq d(P_A(h(x_0)), h(x_0)) \\ &\leq d(a_1, h(x_0)) \\ &= d(A, B). \end{aligned} \tag{4.3}$$

Thus

$$d(G(x_0), F(x_0)) = d(A, B). \tag{4.4}$$

□

As a special case of Theorem 4.1, we obtain the following result of Markin and Shahzad [2].

Theorem 4.2. *Let M be a hyperconvex metric space, A be an admissible subset, and B be a bounded externally hyperconvex subset of M. Assume the mapping $F : A \multimap B$ is condensing, Hausdorff continuous with nonempty admissible values. Assume that $F(x) \cap B_0 \neq \varnothing$ for each $x \in A_0$. Then there exists an $x_0 \in A_0$ such that*

$$d(x_0, F(x_0)) = d(A, B). \tag{4.5}$$

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