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Research Article

Fixed Point Theorems for Monotone Mappings on Partial Metric Spaces

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Matthews (1994) introduced a new distance p on a nonempty set X, which is called partial metric. If (X, p) is a partial metric space, then $p(x, x)$ may not be zero for $x \in X$. In the present paper, we give some fixed point results on these interesting spaces.

1. Introduction

There are a lot of fixed and common fixed point results in different types of spaces. For example, metric spaces, fuzzy metric spaces, and uniform spaces. One of the most interesting is partial metric space, which is defined by Matthews [1]. In partial metric spaces, the distance of a point in the self may not be zero. After the definition of partial metric space, Matthews proved the partial metric version of Banach fixed point theorem. Then, Valero [2], Oltra and Valero [3], and Altun et al. [4] gave some generalizations of the result of Matthews. Again, Romaguera [5] proved the Caristi type fixed point theorem on this space.

First, we recall some definitions of partial metric spaces and some properties of theirs. See [1–3, 5–7] for details.

A partial metric on a nonempty set X is a function $p : X \times X \to \mathbb{R}^+$ such that for all $x, y, z \in X$:

(p_1) $x = y \Leftrightarrow p(x, x) = p(x, y) = p(y, y)$,

(p_2) $p(x, x) \leq p(x, y)$,

(p_3) $p(x,y) = p(y,x)$,

(p_4) $p(x,y) \leq p(x,z) + p(z,y) - p(z,z)$.

A partial metric space is a pair (X,p) such that X is a nonempty set and p is a partial metric on X. It is clear that if $p(x,y) = 0$, then from (p_1) and (p_2) $x = y$. But if $x = y$, $p(x,y)$ may not be 0. A basic example of a partial metric space is the pair $(\mathbb{R}^+, p)$, where $p(x,y) = \max\{x,y\}$ for all $x,y \in \mathbb{R}^+$. Other examples of partial metric spaces, which are interesting from a computational point of view, may be found in [1, 8].

Each partial metric p on X generates a T_0 topology τ_p on X, which has as a base the family open p-balls $\{B_p(x,\varepsilon) : x \in X, \varepsilon > 0\}$, where $B_p(x,\varepsilon) = \{y \in X : p(x,y) < p(x,x) + \varepsilon\}$ for all $x \in X$ and $\varepsilon > 0$.

If p is a partial metric on X, then the function $p^s : X \times X \to \mathbb{R}^+$ given by

$$p^s(x,y) = 2p(x,y) - p(x,x) - p(y,y) \tag{1.1}$$

is a metric on X.

Let (X,p) be a partial metric space, then we have the following.

(i) A sequence $\{x_n\}$ in a partial metric space (X,p) converges to a point $x \in X$ if and only if $p(x,x) = \lim_{n\to\infty} p(x,x_n)$.

(ii) A sequence $\{x_n\}$ in a partial metric space (X,p) is called a Cauchy sequence if there exists (and is finite) $\lim_{n,m\to\infty} p(x_n,x_m)$.

(iii) A partial metric space (X,p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges, with respect to τ_p, to a point $x \in X$ such that $p(x,x) = \lim_{n,m\to\infty} p(x_n,x_m)$.

(iv) A mapping $F : X \to X$ is said to be continuous at $x_0 \in X$, if for every $\varepsilon > 0$, there exists $\delta > 0$ such that $F(B_p(x_0,\delta)) \subseteq B_p(Fx_0,\varepsilon)$.

Lemma 1.1 (see [1, 3]). *Let (X,p) be a partial metric space.*

(a) *$\{x_n\}$ is a Cauchy sequence in (X,p) if and only if it is a Cauchy sequence in the metric space (X,p^s).*

(b) *A partial metric space (X,p) is complete if and only if the metric space (X,p^s) is complete. Furthermore, $\lim_{n\to\infty} p^s(x_n,x) = 0$ if and only if*

$$p(x,x) = \lim_{n\to\infty} p(x_n,x) = \lim_{n,m\to\infty} p(x_n,x_m). \tag{1.2}$$

On the other hand, existence of fixed points in partially ordered sets has been considered recently in [9], and some generalizations of the result of [9] are given in [10–15] in a partial ordered metric spaces. Also, in [9], some applications to matrix equations are presented; in [14, 15], some applications to ordinary differential equations are given. Also, we can find some results on partial ordered fuzzy metric spaces and partial ordered uniform spaces in [16–18], respectively.

The aim of this paper is to combine the above ideas, that is, to give some fixed point theorems on ordered partial metric spaces.

2. Main Result

Theorem 2.1. *Let $(X, \preceq)$ be partially ordered set, and suppose that there is a partial metric p on X such that (X, p) is a complete partial metric space. Suppose $F : X \to X$ is a continuous and nondecreasing mapping such that*

$$p(Fx, Fy) \leq \Psi\left(\max\left\{p(x,y), p(x,Fx), p(y,Fy), \frac{1}{2}[p(x,Fy) + p(y,Fx)]\right\}\right) \tag{2.1}$$

for all $x, y \in X$ with $y \preceq x$, where $\Psi : [0,\infty) \to [0,\infty)$ is a continuous, nondecreasing function such that $\sum_{n=1}^{\infty} \Psi^n(t)$ is convergent for each $t > 0$. If there exists an $x_0 \in X$ with $x_0 \preceq Fx_0$, then there exists $x \in X$ such that $x = Fx$. Moreover, $p(x,x) = 0$.

Proof. From the conditions on Ψ, it is clear that $\lim_{n\to\infty} \Psi^n(t) = 0$ for $t > 0$ and $\Psi(t) < t$. If $Fx_0 = x_0$, then the proof is finished, so suppose $x_0 \neq Fx_0$. Now, let $x_n = Fx_{n-1}$ for $n = 1, 2, \ldots$. If $x_{n_0} = x_{n_0+1}$ for some $n_0 \in \mathbb{N}$, then it is clear that x_{n_0} is a fixed point of F. Thus, assume $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}$. Notice that since $x_0 \preceq Fx_0$ and F is nondecreasing, we have

$$x_0 \preceq x_1 \preceq x_2 \preceq \cdots \preceq x_n \preceq x_{n+1} \preceq \cdots. \tag{2.2}$$

Now, since $x_{n-1} \preceq x_n$, we can use the inequality (2.1) for these points, then we have

$$\begin{aligned}
&p(x_{n+1}, x_n) \\
&\quad = p(Fx_n, Fx_{n-1}) \\
&\quad \leq \Psi\left(\max\left\{p(x_n, x_{n-1}), p(x_n, Fx_n), p(x_{n-1}, Fx_{n-1}), \frac{1}{2}[p(x_n, Fx_{n-1}) + p(x_{n-1}, Fx_n)]\right\}\right) \\
&\quad \leq \Psi\left(\max\left\{p(x_n, x_{n-1}), p(x_n, x_{n+1}), \frac{1}{2}[p(x_{n-1}, x_n) + p(x_n, x_{n+1})]\right\}\right) \\
&\quad = \Psi(\max\{p(x_n, x_{n-1}), p(x_n, x_{n+1})\})
\end{aligned} \tag{2.3}$$

since

$$p(x_n, x_n) + p(x_{n-1}, x_{n+1}) \leq p(x_{n-1}, x_n) + p(x_n, x_{n+1}) \tag{2.4}$$

and Ψ is nondecreasing. Now, if

$$\max\{p(x_n, x_{n-1}), p(x_n, x_{n+1})\} = p(x_n, x_{n+1}) \tag{2.5}$$

for some n, then from (2.3) we have

$$p(x_{n+1}, x_n) \leq \Psi(p(x_n, x_{n+1})) < p(x_n, x_{n+1}), \tag{2.6}$$

which is a contradiction since $p(x_n, x_{n+1}) > 0$. Thus

$$\max\{p(x_n, x_{n-1}), p(x_n, x_{n+1})\} = p(x_n, x_{n-1}) \tag{2.7}$$

for all n. Therefore, we have

$$p(x_{n+1}, x_n) \leq \Psi(p(x_n, x_{n-1})), \tag{2.8}$$

and so

$$p(x_{n+1}, x_n) \leq \Psi^n(p(x_1, x_0)). \tag{2.9}$$

On the other hand, since

$$\max\{p(x_n, x_n), p(x_{n+1}, x_{n+1})\} \leq p(x_n, x_{n+1}), \tag{2.10}$$

then from (2.9) we have

$$\max\{p(x_n, x_n), p(x_{n+1}, x_{n+1})\} \leq \Psi^n(p(x_1, x_0)). \tag{2.11}$$

Therefore,

$$\begin{aligned} p^s(x_n, x_{n+1}) &= 2p(x_n, x_{n+1}) - p(x_n, x_n) - p(x_{n+1}, x_{n+1}) \\ &\leq 2p(x_n, x_{n+1}) + p(x_n, x_n) + p(x_{n+1}, x_{n+1}) \\ &\leq 4\Psi^n(p(x_1, x_0)). \end{aligned} \tag{2.12}$$

This shows that $\lim_{n\to\infty} p^s(x_n, x_{n+1}) = 0$. Now, we have

$$\begin{aligned} p^s(x_{n+k}, x_n) &\leq p^s(x_{n+k}, x_{n+k-1}) + \cdots + p^s(x_{n+1}, x_n) \\ &\leq 4\Psi^{n+k-1}(p(x_1, x_0)) + \cdots + 4\Psi^n(p(x_1, x_0)). \end{aligned} \tag{2.13}$$

Since $\sum_{n=1}^{\infty} \Psi^n(t)$ is convergent for each $t > 0$, then $\{x_n\}$ is a Cauchy sequence in the metric space (X, p^s). Since (X, p) is complete, then, from Lemma 1.1, the sequence $\{x_n\}$ converges in the metric space (X, p^s), say $\lim_{n\to\infty} p^s(x_n, x) = 0$. Again, from Lemma 1.1, we have

$$p(x, x) = \lim_{n\to\infty} p(x_n, x) = \lim_{n,m\to\infty} p(x_n, x_m). \tag{2.14}$$

Moreover, since $\{x_n\}$ is a Cauchy sequence in the metric space (X, p^s), we have $\lim_{n,m\to\infty} p^s(x_n, x_m) = 0$, and, from (2.11), we have $\lim_{n\to\infty} p(x_n, x_n) = 0$, thus, from definition p^s, we have $\lim_{n,m\to\infty} p(x_n, x_m) = 0$. Therefore, from (2.14), we have

$$p(x, x) = \lim_{n\to\infty} p(x_n, x) = \lim_{n,m\to\infty} p(x_n, x_m) = 0. \tag{2.15}$$

Now, we claim that $Fx = x$. Suppose $p(x, Fx) > 0$. Since F is continuous, then, given $\varepsilon > 0$, there exists $\delta > 0$ such that $F(B_p(x,\delta)) \subseteq B_p(Fx,\varepsilon)$. Since $p(x,x) = \lim_{n\to\infty} p(x_n,x) = 0$, then there exists $k \in \mathbb{N}$ such that $p(x_n,x) < p(x,x)+\delta$ for all $n \geq k$. Therefore, we have $x_n \in B_p(x,\delta)$ for all $n \geq k$. Thus, $F(x_n) \in F(B_p(x,\delta)) \subseteq B_p(Fx,\varepsilon)$, and so $p(Fx_n,Fx) < p(Fx,Fx)+\varepsilon$ for all $n \geq k$. This shows that $p(Fx,Fx) = \lim_{n\to\infty} p(x_{n+1},Fx)$. Now, we use the inequality (2.1) for $x = y$, then we have

$$p(Fx,Fx) \leq \Psi\bigl(\max\{p(x,x),p(x,Fx)\}\bigr) = \Psi\bigl(p(x,Fx)\bigr). \tag{2.16}$$

Therefore, we obtain

$$\begin{aligned} p(x,Fx) &\leq p(x,x_{n+1}) + p(x_{n+1},Fx) - p(x_{n+1},x_{n+1}) \\ &\leq p(x,x_{n+1}) + p(x_{n+1},Fx), \end{aligned} \tag{2.17}$$

and letting $n \longrightarrow \infty$, we have

$$\begin{aligned} p(x,Fx) &\leq \lim_{n\to\infty} p(x,x_{n+1}) + \lim_{n\to\infty} p(x_{n+1},Fx) \\ &= p(Fx,Fx) \\ &\leq \Psi(p(x,Fx)) \\ &< p(x,Fx), \end{aligned} \tag{2.18}$$

which is a contradiction since $p(x,Fx) > 0$. Thus, $p(x,Fx) = 0$, and so $x = Fx$. $\square$

In the following theorem, we remove the continuity of F. Also, The contractive condition (2.1) does not have to be satisfied for $x = y$, but we add a condition on X.

Theorem 2.2. *Let $(X,\preceq)$ be a partially ordered set, and suppose that there is a partial metric p on X such that (X,p) is a complete partial metric space. Suppose $F : X \longrightarrow X$ is a nondecreasing mapping such that*

$$p(Fx,Fy) \leq \Psi\left(\max\left\{p(x,y),p(x,Fx),p(y,Fy),\frac{1}{2}[p(x,Fy)+p(y,Fx)]\right\}\right) \tag{2.19}$$

for all $x,y \in X$ with $y \prec x$ (i.e., $y \preceq x$ and $y \neq x$), where $\Psi : [0,\infty) \longrightarrow [0,\infty)$ is a continuous, nondecreasing function such that $\sum_{n=1}^{\infty} \Psi^n(t)$ is convergent for each $t > 0$. Also, the condition

$$\text{If } \{x_n\} \subset X \text{ is a increasing sequence with } x_n \longrightarrow x \text{ in } X, \text{ then } x_n \prec x, \quad \forall n \tag{2.20}$$

holds. If there exists an $x_0 \in X$ with $x_0 \preceq Fx_0$, then there exists $x \in X$ such that $x = Fx$. Moreover, $p(x,x) = 0$.

Proof. As in the proof of Theorem 2.1, we can construct a sequence $\{x_n\}$ in X by $x_n = Fx_{n-1}$ for $n = 1, 2, \ldots$. Also, we can assume that the consecutive terms of $\{x_n\}$ are different. Otherwise we are finished. Therefore, we have

$$x_0 \prec x_1 \prec x_2 \prec \cdots \prec x_n \prec x_{n+1} \prec \cdots . \tag{2.21}$$

Again, as in the proof of Theorem 2.1, we can show that $\{x_n\}$ is a Cauchy sequence in the metric space (X, p^s), and, therefore, there exists $x \in X$ such that

$$p(x,x) = \lim_{n\to\infty} p(x_n, x) = \lim_{n,m\to\infty} p(x_n, x_m) = 0. \tag{2.22}$$

Now, we claim that $Fx = x$. Suppose $p(x, Fx) > 0$. Since the condition (2.20) is satisfied, then we can use (2.19) for $y = x_n$. Therefore, we obtain

$$\begin{aligned}
&p(Fx, Fx_n)\\
&\quad\le \Psi\left(\max\left\{p(x,x_n), p(x,Fx), p(x_n,Fx_n), \frac{1}{2}\left[p(x,Fx_n) + p(x_n,Fx)\right]\right\}\right)\\
&\quad\le \Psi\left(\max\left\{p(x,x_n), p(x,Fx), p(x_n,x_{n+1}), \frac{1}{2}\left[p(x,x_{n+1}) + p(x_n,x) + p(x,Fx) - p(x,x)\right]\right\}\right)\\
&\quad= \Psi\left(\max\left\{p(x,x_n), p(x,Fx), p(x_n,x_{n+1}),\ \frac{1}{2}\left[p(x,x_{n+1}) + p(x_n,x) + p(x,Fx)\right]\right\}\right),
\end{aligned} \tag{2.23}$$

using the continuity of Ψ and letting $n \to \infty$, we have $\lim_{n\to\infty} p(Fx, Fx_n) \le \Psi(p(x, Fx))$. Therefore, we obtain

$$\begin{aligned}
p(x,Fx) &\le \lim_{n\to\infty} p(x,x_{n+1}) + \lim_{n\to\infty} p(x_{n+1},Fx)\\
&= \lim_{n\to\infty} p(x,x_{n+1}) + \lim_{n\to\infty} p(Fx_n,Fx)\\
&\le \Psi(p(x,Fx))\\
&< p(x,Fx),
\end{aligned} \tag{2.24}$$

which is a contradiction. Thus, $p(x, Fx) = 0$, and so $x = Fx$. □

Example 2.3. Let $X = [0,\infty)$ and $p(x,y) = \max\{x,y\}$, then it is clear that (X,p) is a complete partial metric space. We can define a partial order on X as follows:

$$x \preceq y \Longleftrightarrow x = y \quad \text{or} \quad \{x, y \in [0,1] \text{ with } x \le y\}. \tag{2.25}$$

Let $F : X \to X$,

$$Fx = \begin{cases} \dfrac{x^2}{1+x}, & x \in [0,1], \\ 2x, & x \in (1,\infty), \end{cases} \tag{2.26}$$

and $\Psi : [0,\infty) \to [0,\infty)$, $\Psi(t) = t^2/(1+t)$. Therefore, Ψ is continuous and nondecreasing. Again we can show by induction that $\Psi^n(t) \le t(t/(1+t))^n$, and so we have $\sum_{n=1}^{\infty} \Psi^n(t)$ that is convergent. Also, F is nondecreasing with respect to $\preceq$, and for $y \prec x$, we have

$$\begin{aligned} p(Fx,Fy) &= \max\left\{\frac{x^2}{1+x}, \frac{y^2}{1+y}\right\} \\ &= \frac{x^2}{1+x} \\ &= \Psi(p(x,y)) \\ &\le \Psi\left(\max\left\{p(x,y), p(x,Fx), p(y,Fy), \frac{1}{2}[p(x,Fy)+p(y,Fx)]\right\}\right), \end{aligned} \tag{2.27}$$

that is, the condition (2.19) of Theorem 2.2 is satisfied. Also, it is clear that the condition (2.20) is satisfied, and for $x_0 = 0$, we have $x_0 \preceq Fx_0$. Therefore, all conditions of Theorem 2.2 are satisfied, and so F has a fixed point in X. Note that if $x = 1$ and $y = 2$, then

$$p(Fx,Fy) = 4 \nleq \frac{16}{5} = \Psi\left(\max\left\{p(x,y), p(x,Fx), p(y,Fy), \frac{1}{2}[p(x,Fy)+p(y,Fx)]\right\}\right). \tag{2.28}$$

This shows that the contractive condition of Theorem 1 of [4] is not satisfied.

Theorem 2.4. *If one uses the following condition instead of* (2.1) *in Theorem 2.1, one has the same result.*

$$p(Fx,Fy) \le \Psi\left(\max\left\{p(x,y), \frac{1}{2}[p(x,Fx)+p(y,Fy)], \frac{1}{2}[p(x,Fy)+p(y,Fx)]\right\}\right) \tag{2.29}$$

for all $x, y \in X$ with $y \preceq x$.

In what follows, we give a sufficient condition for the uniqueness of the fixed point in Theorem 2.4, this condition is

$$\text{for } x, y \in X \text{ there exists a lower bound or an upper bound.} \tag{2.30}$$

In [15], it was proved that condition (2.30) is equivalent to

$$\text{for } x, y \in X \text{ there exists } z \in X \text{ which is comparable to } x \text{ and } y. \tag{2.31}$$

Theorem 2.5. *Adding condition* (2.31) *to the hypotheses of Theorem 2.4, one obtains uniqueness of the fixed point of F.*

Proof. Suppose that there exists z and that $y \in X$ are different fixed points of F, then $p(z, y) > 0$. Now, we consider the following two cases.

(i) If z and y are comparable, then $F^n z = z$ and $F^n y = y$ are comparable for $n = 0, 1, \ldots$. Therefore, we can use the condition (2.1), then we have

$$\begin{aligned} p(z, y) &= p(F^n z, F^n y) \\ &\le \Psi\Big(\max\Big\{p\big(F^{n-1}z, F^{n-1}y\big), \frac{1}{2}\Big[p\big(F^{n-1}z, F^n z\big) + p\big(F^{n-1}y, F^n y\big)\Big], \\ &\qquad \frac{1}{2}\Big[p\big(F^{n-1}z, F^n y\big) + p\big(F^{n-1}y, F^n z\big)\Big]\Big\}\Big) \\ &= \Psi\Big(\max\Big\{p(z, y), \frac{1}{2}[p(z, z) + p(y, y)]\Big\}\Big) \\ &= \Psi(p(z, y)) \\ &< p(z, y), \end{aligned} \tag{2.32}$$

which is a contradiction.

(ii) If z and y are not comparable, then there exists $x \in X$ comparable to z and y. Since F is nondecreasing, then $F^n x$ is comparable to $F^n z = z$ and $F^n y = y$ for $n = 0, 1, \ldots$. Moreover,

$$\begin{aligned} p(z, F^n x) &= p(F^n z, F^n x) \\ &\le \Psi\Big(\max\Big\{p\big(F^{n-1}z, F^{n-1}x\big), \frac{1}{2}\Big[p\big(F^{n-1}z, F^n z\big) + p\big(F^{n-1}x, F^n x\big)\Big], \\ &\qquad \frac{1}{2}\Big[p\big(F^{n-1}z, F^n x\big) + p\big(F^{n-1}x, F^n z\big)\Big]\Big\}\Big) \\ &= \Psi\Big(\max\Big\{p\big(z, F^{n-1}x\big), \frac{1}{2}\Big[p(z, z) + p\big(F^{n-1}x, F^n x\big)\Big], \frac{1}{2}\Big[p(z, F^n x) + p\big(F^{n-1}x, z\big)\Big]\Big\}\Big) \\ &\le \Psi\Big(\max\Big\{p\big(z, F^{n-1}x\big), \frac{1}{2}\Big[p\big(F^{n-1}x, z\big) + p(z, F^n x)\Big], \frac{1}{2}\Big[p(z, F^n x) + p\big(F^{n-1}x, z\big)\Big]\Big\}\Big) \\ &= \Psi\Big(\max\Big\{p\big(z, F^{n-1}x\big), \frac{1}{2}\Big[p\big(F^{n-1}x, z\big) + p(z, F^n x)\Big]\Big\}\Big). \end{aligned} \tag{2.33}$$

Now, if $p(z, F^{n-1}x) < p(z, F^n x)$ for some n, then we have

$$p(z, F^n x) \leq \Psi\big(p(z, F^n x)\big) < p(z, F^n x), \tag{2.34}$$

which is a contradiction. Thus, $p(z, F^{n-1}x) \geq p(z, F^n x)$ for all n, and so

$$p(z, F^n x) \leq \Psi\Big(p\Big(z, F^{n-1}x\Big)\Big) < p\Big(z, F^{n-1}x\Big). \tag{2.35}$$

This shows that $p(z, F^n x)$ is a nonnegative and nondecreasing sequence and so has a limit, say $\alpha \geq 0$. From the last inequality, we can obtain

$$\alpha \leq \Psi(\alpha) < \alpha, \tag{2.36}$$

hence $\alpha = 0$. Similarly, it can be proven that, $\lim_{n\to\infty} p(y, F^n x) = 0$. Finally,

$$\begin{aligned} p(z, y) &\leq p(z, F^n x) + p(F^n x, y) - p(F^n x, F^n x) \\ &\leq p(z, F^n x) + p(F^n x, y), \end{aligned} \tag{2.37}$$

and taking limit $n \to \infty$, we have $p(z, y) = 0$. This contradicts $p(z, y) > 0$.

Consequently, F has no two fixed points. □

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