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LAMB'S PLANE PROBLEM IN A THERMO-VISCO-ELASTIC MICROPOLAR MEDIUM WITH THE EFFECT OF GRAVITY

PRAVANGSU SEKHAR DAS
KTPP Township, Midnapore - 721171, India

P.R. SENGUPTA

Department of Mathematics
University of Kalyani,
Kalyani, West Bengal, India

and

LOKENATH DEBNATH

Department of Mathematics
University of Central Florida
Orlando, Florida 32816, U.S.A.

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Abstract. A study is made of the Lamb plane problem in an infinite thermo-visco-elastic micropolar medium with the effect of gravity. The visco-elasticity is characterized by the rate dependent theory of micro-visco-elasticity generalizing the classical Kelvin-Voigt theory. The action of time harmonic loading is treated in detail. The solutions for the displacement fields, couple stresses and the temperature field are obtained in general and particular cases.

Key Words and Phrases: Lamb's problem, Fourier transforms and effect of gravity.
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1. INTRODUCTION. Eringen [1-4] has developed a general theory of linear micropolar continuous media and of linear micropolar visco-elastic media. The classical Lamb problem in an elastic medium has received considerable attention in various elastic media with different kinds of loading. A selected reference including Sengupta and his associates [5-7], Chadha et al. [8], Rajneesh Kumar et al. [9] is cited for the reader.

The purpose of this paper is to study the effect of gravity on Lamb's plane problem in a micropolar thermo-visco-elastic medium. As far as we know, this problem has not yet received any attention.

2. FORMULATION OF THE PROBLEM. We consider a homogeneous micropolar thermo-visco-elastic semi-infinite medium with the influence of gravity under the action of loading $g(x_1, t)$ free plane boundary $x_3 = 0$. It is assumed that the medium is free to exchange heat with the material in the region $x_3 > 0$. It is everywhere at the constant absolute temperature T_0 prior to the appearance of any disturbance.

Since we consider the plane problem, we assume that the displacement $\underline{u} = (u_1, 0, u_3)$ and rotation $\underline{\omega} = (0, \omega_2, 0)$ which are functions of (x_2, x_3, t). The displacements are related to the displacement potentials $\phi(x_1, x_3, t)$ and $\psi(x_1, x_3, t)$ as follows:

$$u_1 = \phi_{x_1} - \psi_{x_3}, \quad u_3 = \phi_{x_3} + \psi_{x_1} \tag{2.1ab}$$

so that

$$e = \nabla^2\phi, \quad \nabla^2\psi = \frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1} \tag{2.2ab}$$

where

$$e = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3} \quad \text{and} \quad \nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2} \tag{2.3ab}$$

The basic dynamical equations of motion in a micropolar-visco-elastic solid medium under the influence of temperature are

$$\left[(\mu_0 + \alpha_0) + (\mu_1 + \alpha_1)\frac{\partial}{\partial t}\right]\nabla^2\underline{u} + \left[(\lambda_0 + \mu_0 - \alpha_0) + (\lambda_1 + \mu_1 - \alpha_1)\frac{\partial}{\partial t}\right]\text{grad}\, div\underline{u}$$

$$+ 2\left(\alpha_0 + \alpha_1\frac{\partial}{\partial t}\right)rot\underline{\omega} - \left[(3\lambda_0 + 2\mu_0) + (3\lambda_1 + 2\mu_1)\frac{\partial}{\partial t}\right]\alpha_t\, \text{grad}\,\theta = \rho\underline{\ddot{u}} \tag{2.4}$$

$$\left[(\gamma_0 + \varepsilon_0) + (\gamma_1 + \varepsilon_1)\frac{\partial}{\partial t}\right]\nabla^2\underline{\omega} + [(\gamma_0 + \beta_0 - \varepsilon_0) + (\gamma_1 + \beta_1 - \varepsilon_1)\frac{\partial}{\partial t}\,\text{grad}\, div\,\underline{\omega}$$

$$- 4\left(\alpha_0 + \alpha_1\frac{\partial}{\partial t}\right)\underline{\omega} + 2\left(\alpha_0 + \alpha_1\frac{\partial}{\partial t}\right)\text{rot}\,\underline{u} = J\underline{\ddot{\omega}} \tag{2.5}$$

where λ_0, μ_0 are the Lame elastic constants while $\alpha_0, \beta_0, \gamma_0, \varepsilon_0$ are the other material constants, $\lambda_1, \mu_1, \alpha_1, \beta_1, \gamma_1, \varepsilon_1$ are the parameters representing the effects of viscosity. Also these elastic moduli $\lambda_0, \mu_0, \alpha_0, \beta_0, \gamma_0, \varepsilon_0$ and viscosities $\lambda_1, \mu_1, \alpha_1, \beta_1, \gamma_1, \varepsilon_1$ are subjected to the following restrictions

$$0 \le 3\lambda_0 + 2\mu_0 + \alpha_0, \quad 0 \le 3\beta_0 + 2\gamma_0, \quad 0 \le \mu_0, \quad -\gamma_0 \le \varepsilon_0 \le \gamma_0, \quad 0 \le \alpha_0, \quad 0 \le \gamma_0$$

$$0 \le 3\lambda_1 + 2\mu_1 + \alpha_1, \quad 0 \le 3\beta_1 + 2\gamma_1, \quad 0 \le \mu_1, \quad -\gamma_1 \le \varepsilon_1 \le \gamma_1, \quad 0 \le \alpha_1, \quad 0 \le \gamma_1 \tag{2.6}$$

and α_t is the coefficient of linear expansion of solid, ρ is the density, $\theta = T - T_0$ (= absolute temperature-initial absolute temperature), and J is the rotational inertia.

For the present study, it is convenient to rewrite equations (2.4)-(2.5) as

$$\left[(\mu_0 + \alpha_0) + (\mu_1 + \alpha_1)\frac{\partial}{\partial t}\right]\nabla^2 u_1 + \left[(\lambda_0 + \mu_0 - \alpha_0) + (\lambda_1 + \mu_1 - \alpha_1)\frac{\partial}{\partial t}\right]\frac{\partial e}{\partial x_1}$$

$$- 2\left(\alpha_0 + \alpha_1\frac{\partial}{\partial t}\right)\frac{\partial\omega_2}{\partial x_3} - \left[(3\lambda_0 + 2\mu_0) + (3\lambda_1 + 2\mu_1)\frac{\partial}{\partial t}\right]\alpha_t\frac{\partial\theta}{\partial x_1} + \rho g u_{3,1} = \rho\ddot{u}_1 \tag{2.7}$$

$$\left[(\mu_0 + \alpha_0) + (\mu_1 + \alpha_1)\frac{\partial}{\partial t}\right]\nabla^2 u_3 + \left[(\lambda_0 + \mu_0 - \alpha_0) + (\lambda_1 + \mu_1 - \alpha_1)\frac{\partial}{\partial t}\right]\frac{\partial e}{\partial x_3}$$

$$+ 2\left(\alpha_0 + \alpha_1\frac{\partial}{\partial t}\right)\frac{\partial\omega_2}{\partial x_1} - \left[(3\lambda_0 + 2\mu_0) + (3\lambda_1 + 2\mu_1)\frac{\partial}{\partial t}\right]\alpha_t\frac{\partial\theta}{\partial x_3} - \rho g u_{1,1} = \rho\ddot{u}_3$$

$$\left[(\gamma_0 + \varepsilon_0) + (\gamma_1 + \varepsilon_1)\frac{\partial}{\partial t}\right]\nabla^2\omega_2 + 2\left(\alpha_0 + \alpha_1\frac{\partial}{\partial t}\right)\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) - 4\left(\alpha_0 + \alpha_1\frac{\partial}{\partial t}\right)\omega_2 = J\ddot{\omega}_2 \tag{2.8}$$

The temperature θ satisfies the Fourier's law of heat conduction

$$\kappa\nabla^2\theta = \rho C_\varepsilon \frac{\partial\theta}{\partial t} + T_0\left[(3\lambda_0 + 2\mu_0) + (3\lambda_1 + 2\mu_1)\frac{\partial}{\partial t}\right]\alpha_t \frac{\partial}{\partial t}(\nabla^2\phi), \tag{2.9}$$

where κ is the thermal conductivity and C_ε is the specific heat at constant strain.

Substituting (2.2ab) into (2.7)-(2.9) gives

$$\left[\left(C_1^2 + C'^2_1\frac{\partial}{\partial t}\right)\nabla^2 - \frac{\partial^2}{\partial t^2}\right]\phi + g\frac{\partial\psi}{\partial x_1} - \left(p_1^2 + p_1'^2\frac{\partial}{\partial t}\right)\theta = 0 \tag{2.10}$$

$$\left[\left(C_2^2 + C_2'^2\frac{\partial}{\partial t}\right)\nabla^2 - \frac{\partial^2}{\partial t^2}\right]\psi - g\frac{\partial\phi}{\partial x_1} + \left(p_2^2 + p_2'^2\frac{\partial}{\partial t}\right)\omega_2 = 0 \tag{2.11}$$

$$\left[\left(C_4^2 + C_4'^2\frac{\partial}{\partial t}\right)\nabla^2 - \left(C_5^2 + C_5'^2 + \frac{\partial^2}{\partial t^2}\right)\right]\omega_2 - \frac{1}{2}\left(C_5^2 + C_5'^2\frac{\partial}{\partial t}\right)\nabla^2\psi = 0 \tag{2.12}$$

$$\left[\left(C_3^2\nabla^2 - \frac{\partial}{\partial t}\right)\right]\theta - r\left(p_1^2 + p_1'^2\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}(\nabla^2\phi) = 0 \tag{2.13}$$

where

$$C_1^2 = \frac{\lambda_0 + 2\mu_0}{\rho}, \quad C_1'^2 = \frac{\lambda_1 + 2\mu_1}{\rho}, \quad C_2^2 = \frac{\mu_0 + \alpha_0}{\rho}, \quad C_2'^2 = \frac{\mu_1 + \alpha_1}{\rho} \tag{2.14}$$

$$C_3^2 = \frac{\kappa}{\rho C_\varepsilon}, \quad C_4^2 = \frac{\gamma_0 + \varepsilon_0}{J}, \quad C_4'^2 = \frac{\gamma_1 + \varepsilon_1}{J}, \quad C_5^2 = \frac{4\alpha_0}{J}, \quad C_5'^2 = \frac{4\alpha_1}{J} \tag{2.15}$$

$$p_1^2 = \frac{(3\lambda_0 + 2\mu_0)\alpha_t}{\rho}, \quad p_1'^2 = \frac{(3\lambda_1 + 2\mu_1)\alpha_t}{\rho}, \quad p_2^2 = \frac{2\alpha_0}{\rho}, \quad p_2'^2 = \frac{2\alpha_1}{\rho}, \quad r = \frac{T_0}{C_\varepsilon} \tag{2.16}$$

3. BOUNDARY CONDITIONS.

The stress-strain relations in considered medium are

$$\sigma_{ij} = [(\mu_0 + \alpha_0) + (\mu_1 + \alpha_1)\frac{\partial}{\partial t}\gamma_{ij} + \left[(\mu_0 - \alpha_0) + (\mu_1 - \alpha_1)\frac{\partial}{\partial t}\right]\gamma_{ij}$$
$$+\left[\left(\lambda_0 + \lambda_1\frac{\partial}{\partial t}\right)\gamma_{\kappa\kappa} - \left\{(3\lambda_0 + 2\mu_0) + (3\lambda_1 + 2\mu_1)\frac{\partial}{\partial t}\right\}\alpha_t\theta\right]\delta_{ij} \tag{3.1}$$

$$\mu_{ij} = \left[(\gamma_0 + \varepsilon_0) + (\gamma_1 + \varepsilon_1)\frac{\partial}{\partial t}\right]\chi_{ij} + \left[(\gamma_0 + \varepsilon_0) + (\gamma_1 - \varepsilon_1)\frac{\partial}{\partial t}\right]\chi_{ij} + \left(\beta_{0,} + \beta_1\frac{\partial}{\partial t}\right)\chi_{\kappa\kappa}\,\delta_{ij}, \tag{3.2}$$

in which

$$\gamma_{ji} = u_{i,j} - \varepsilon_{\kappa ji}\omega_\kappa, \quad \chi_{ji} = \omega_{i,j}; \; (i, j, \kappa = 1,2,3) \tag{3.3}$$

$\varepsilon_{\kappa ji}$ is the unit antisymmetric tensor, and σ_{ij} is the Kronecker delta.

Hence the boundary conditions are

$$\sigma_{33} = -f(x_1, t), \quad \sigma_{31} = 0, \quad \mu_{32} = 0 \quad \text{on} \quad x_3 = 0 \tag{3.4}$$

where

$$\sigma_{33} = 2\left(\mu_0 + \mu_1\frac{\partial}{\partial t}\right)[\phi_{,33} - \psi_{,13}] + \left(\lambda_0 + \lambda_1\frac{\partial}{\partial t}\right)\nabla^2\phi$$
$$-(3\lambda_0 + 2\mu_0) + (3\lambda_1 + 2\mu_1)\frac{\partial}{\partial t}]\alpha_t\theta. \tag{3.5}$$

$$\sigma_{31} = \left(\mu_0 + \mu_1 \frac{\partial}{\partial t}\right)[2\phi,_{13} + \psi,_{33} - \psi,_{11}] + \left(\alpha_0 + \alpha_1 \frac{\partial}{\partial t}\right)(\nabla^2\psi - 2\omega_2) \tag{3.6}$$

$$\mu_{32} = \left[(\gamma_0 + \varepsilon_0) + (\gamma_1 + \varepsilon_1)\frac{\partial}{\partial t}\right]\frac{\partial \omega_2}{\partial x_3} \tag{3.7}$$

The thermal condition is

$$\frac{\partial \theta}{\partial x_3} + h\theta = 0 \quad \text{on} \quad x_3 = 0 \tag{3.8}$$

4. SOLUTION OF THE PROBLEM. We introduce Fourier's double integral transform pair defined by (see Debnath and Myint-U [10])

$$\hat{\phi}(x_3, \xi, \eta) = \frac{1}{2\pi}\int\int_{-\infty}^{\infty} \phi(x_1, x_3, t) e^{i(x_1\xi + t\eta)} dx_1 dt$$

$$\phi(x_1, x_3, t) = \frac{1}{2\pi}\int\int_{-\infty}^{\infty} \hat{\phi}(x_3, \xi, \eta) e^{-i(x_1\xi + t\eta)} d\xi d\eta \tag{4.1ab}$$

and similarly for $\hat{\psi}, \hat{\omega}_2, \hat{\theta}$.

Application of this transform reduces (2.10)-(2.13) into the form

$$\left[(C_1^2 - i\eta C_1'^2)\left(\frac{d^2}{dx_3^2} - \xi^2\right) + \eta^2\right]\hat{\phi} - i\xi g\hat{\psi} - (p_1^2 - i\eta p_1'^2)\hat{\theta} = 0 \tag{4.2}$$

$$\left[(C_2^2 - i\eta C_2'^2)\left(\frac{d^2}{dx_3^2} - \xi^2\right) + \eta^2\right]\hat{\psi} + i\xi g\hat{\phi} - (p_2^2 - i\eta p_2'^2)\hat{\omega}_2 = 0 \tag{4.3}$$

$$\left[(C_4^2 - i\eta C_4'^2)\left(\frac{d^2}{dx_3^2} - \xi^2\right) - (C_5^2 - i\eta C_5'^2 - \eta^2)\right]\hat{\omega}_2 - \frac{1}{2}(C_5^2 - i\eta C_5'^2)\left(\frac{d^2}{dx_3^2} - \xi^2\right)\hat{\psi} = 0 \tag{4.4}$$

$$\left[C_3^2\left(\frac{d^2}{dx_3^2} - \xi^2\right) + i\eta\right]\hat{\theta} + ri\eta(p_1^2 - i\eta p_1'^2)\left(\frac{d^2}{dx_3^2} - \xi^2\right)\hat{\phi} = 0 \tag{4.5}$$

provided that in the case of exponential solutions of the equation the following conditions are satisfied

$$\phi, \psi, \omega_2, \theta \to 0, \quad \text{as} \quad x_3 \to \infty$$

Therefore the solutions of the equations (4.2)-(4.5) given by

$$\hat{\phi} = Ae^{-x_3\sqrt{\zeta_1}} + Be^{-x_3\sqrt{\zeta_2}} + Ce^{-x_3\sqrt{\zeta_3}} + De^{-x_3\sqrt{\zeta_4}} \tag{4.6}$$

$$\hat{\psi} = A_1e^{-x_3\sqrt{\zeta_1}} + B_1e^{-x_3\sqrt{\zeta_2}} + C_1e^{-x_3\sqrt{\zeta_3}} + D_1e^{-x_3\sqrt{\zeta_4}} \tag{4.7}$$

$$\hat{\omega}_2 = A_2e^{-x_3\sqrt{\zeta_1}} + B_2e^{-x_3\sqrt{\zeta_2}} + C_2e^{-x_3\sqrt{\zeta_3}} + D_2e^{-x_3\sqrt{\zeta_4}} \tag{4.8}$$

$$\hat{\theta} = A_3e^{-x_3\sqrt{\zeta_1}} + B_3e^{-x_3\sqrt{\zeta_2}} + C_3e^{-x_3\sqrt{\zeta_3}} + D_3e^{-x_3\sqrt{\zeta_4}} \tag{4.9}$$

where ζ_j^2 $(j = 1,2,3,4)$, are the roots of the equation

$$
\begin{aligned}
&\{(C_1^2 - i\eta C_1'^2)(\zeta^2-\xi^2)+\eta^2\}[\{(C_2^2 - i\eta C_2'^2)(\zeta^2-\xi^2)+\eta^2\}\{(C_4^2 - i\eta C_4'^2)(\zeta^2-\xi^2)\\
&\quad -(C_5^2 - i\eta C_5'^2-\eta^2)\}\{C_3^2(\zeta^2-\xi^2)+i\eta\} - \frac{1}{2}(p_2^2 - i\eta p_2'^2)(C_5^2 - i\eta C_5'^2)(\zeta^2-\xi^2)\{C_3^2(\zeta^2-\xi^2)+i\eta\}]\\
&\quad -g^2\xi^2\{(C_4^2 - i\eta C_4'^2)(\zeta^2-\xi^2)-(C_5^2 - i\eta C_5'^2-\eta^2)\}\{C_3^2(\zeta^2-\xi^2)+i\eta\}\\
&\quad +(p_1^2 - i\eta p_1'^2)\Big[\{(C_2^2 - i\eta C_2'^2)(\zeta^2-\xi^2)+\eta^2\}\\
&\quad \times\{(C_4^2 - i\eta C_4'^2)(\zeta^2-\xi^2)-(C_5^2 - i\eta C_5'^2-\eta^2)\}\{ri\eta(p_1^2 - i\eta p_1'^2)(\zeta^2-\xi^2)\}\\
&\quad +\frac{1}{2}ri\eta(C_5^2 - i\eta C_5'^2)(p_1^2 - i\eta p_1'^2)(\zeta^2-\xi^2)^2\Big] = 0
\end{aligned}
\tag{4.10}
$$

and the constants $A, B, C, D, A_1, B_1, C_1, D_1, \ldots$ etc. are related by

$$
\begin{aligned}
A_1 &= \alpha_1^* A, & A_2 &= \beta_1^* A, & A_3 &= \gamma_1^* A\\
B_1 &= \alpha_2^* B, & B_2 &= \beta_2^* B, & B_3 &= \gamma_2^* B\\
C_1 &= \alpha_3^* C, & C_2 &= \beta_3^* C, & C_3 &= \gamma_3^* C\\
D_1 &= \alpha_4^* D, & D_2 &= \beta_4^* D, & D_3 &= \gamma_4^* D
\end{aligned}
\tag{4.11}
$$

where

$$\alpha_j = \frac{1}{i\xi g}[(C_1^2 - i\eta C_1'^2)(\zeta_j-\xi^2)+\eta^2-(p_1^2 - i\eta p_1'^2)\gamma_j^*] \tag{4.12}$$

$$\beta_j = \frac{\frac{1}{2}(C_5^2 - i\eta C_5'^2)(\zeta_j-\xi^2)\alpha_j^*}{(C_4^2 - i\eta C_4'^2)(\zeta_j-\xi^2)-(C_5^2 - i\eta C_5'^2-\eta^2)} \tag{4.13}$$

$$\gamma_j = \frac{ri\eta(p_1^2 - i\eta p_1'^2)(\xi^2-\zeta_j)}{C_3^2(\zeta_j-\xi^2)+i\eta}, \qquad (j = 1,2,3,4) \tag{4.14}$$

Substitution of (4.6)-(4.9), (4.1ab) into (3.4) yields

$$
\begin{aligned}
p_1A + p_2B + p_3C + p_4D &= -\hat{f}(\xi,\eta)\\
q_1A + q_2B + q_3C + q_4D &= 0\\
r_1A + r_2B + r_3C + r_4D &= 0\\
s_1A + s_2B + s_3C + s_4D &= 0
\end{aligned}
\tag{4.15abcd}
$$

where

$$
\begin{aligned}
p_j = {} & 2(\mu_0 - i\eta\mu_1)(\zeta_j - i\xi\alpha_j^*\sqrt{\zeta_j}) + (\lambda_0 - i\eta\lambda_1)(\zeta_j-\xi^2)\\
& -\{(3\lambda_0+2\mu_0) - i\eta(3\lambda_1+2\mu_1)\}\alpha_t\gamma_j^*,
\end{aligned}
\tag{4.16}
$$

$$q_j = (\mu_0 - i\eta\mu_1)[(\zeta_j+\xi^2)\alpha_j^* + 2i\xi\sqrt{\zeta_j} + (\alpha_0 - i\eta\alpha_1)[\alpha_j^*(\zeta_j-\xi^2) - 2\beta_j^*], \tag{4.17}$$

$$\gamma_j = \beta_j^*\sqrt{\zeta_j}, \quad s_j = \gamma_j^*(\sqrt{\zeta_j}-h), \quad (j = 1,2,3,4) \tag{4.18ab}$$

Solving the equations (4.15abcd), we obtain

$$A = \frac{\Delta_1}{\Delta}\hat{f}, \quad B = \frac{\Delta_2}{\Delta}\hat{f}, \quad C_3 = \frac{\Delta_3}{\Delta}\hat{f}, \quad D = \frac{\Delta_4}{\Delta}\hat{f} \tag{4.19abcd}$$

where

$$\Delta = \begin{vmatrix} p_1 & p_2 & p_3 & p_4 \\ q_1 & q_2 & q_3 & q_4 \\ r_1 & r_2 & r_3 & r_4 \\ s_1 & s_2 & s_3 & s_4 \end{vmatrix}, \quad \Delta_1 = \begin{vmatrix} -1 & p_2 & p_3 & p_4 \\ 0 & q_2 & q_3 & q_4 \\ 0 & r_2 & r_3 & r_4 \\ 0 & s_2 & s_3 & s_4 \end{vmatrix} \tag{4.20}$$

Similarly, $\Delta_2, \Delta_3, \Delta_4$ are obtained, eliminating second, third and fourth column of Δ by the column $(-1, 0, 0, 0)$.

Inserting the values of A, B, C, D and (4.6)-(4.9) and using (4.1b) we obtain

$$\phi = \frac{1}{2\pi} \int_{-\infty}^{\infty}\!\!\int \frac{\hat{f}}{\Delta}\left[\Delta_1 e^{-x_3\sqrt{\zeta_1}} + \Delta_2 e^{-x_3\sqrt{\zeta_2}} + \Delta_3 e^{-x\sqrt{\zeta_3}} + \Delta_4 e^{-x_3\sqrt{\zeta_4}}\right] e^{-i(x_1\xi + t\eta)} d\xi d\eta \tag{4.21}$$

$$\psi = \frac{1}{2\pi} \int_{-\infty}^{\infty}\!\!\int \frac{\hat{f}}{\Delta}\left[\alpha_1^{*}\Delta_1 e^{-x_3\sqrt{\zeta_1}} + \alpha_2^{*}\Delta_2 e^{-x_3\sqrt{\zeta_2}} + \alpha_3^{*}\Delta_3 e^{-x_3\sqrt{\zeta_3}} + \alpha_4^{*}\Delta_4 e^{-x_3\sqrt{\zeta_4}}\right] e^{-i(x_1\xi + t\eta)} d\xi d\eta \tag{4.22}$$

$$\omega_2 = \frac{1}{2\pi} \int_{-\infty}^{\infty}\!\!\int \frac{\hat{f}}{\Delta}\left[\beta_1^{*}\Delta_1 e^{-x_3\sqrt{\zeta_1}} + \beta_2^{*}\Delta_2 e^{-x_3\sqrt{\zeta_2}} + \beta_3^{*}\Delta_3 e^{-x_3\sqrt{\zeta_3}} + \beta_4^{*}\Delta_4 e^{-x_3\sqrt{\zeta_4}}\right] e^{-i(x_1\xi + t\eta)} d\xi d\eta \tag{4.23}$$

$$\theta = \frac{1}{2\pi} \int_{-\infty}^{\infty}\!\!\int \frac{\hat{f}}{\Delta}\left[\gamma_1^{*}\Delta_1 e^{-x_3\sqrt{\zeta_1}} + \gamma_2^{*}\Delta_2 e^{-x_3\sqrt{\zeta_2}} + \gamma_3^{*}\Delta_3 e^{-x_3\sqrt{\zeta_3}} + \gamma_4^{*}\Delta_4 e^{-x_3\sqrt{\zeta_4}}\right] e^{-i(x_1\xi + t\eta)} d\xi d\eta \tag{4.24}$$

We can now easily determine the components of displacement and the non-zero components of force stress as well as couple stress tensor.

5. LOADING FUNCTION HARMONIC IN TIME

In this case, the boundary conditions (11) take the following forms

$$\sigma_{33} = -f(x_1)e^{-i\omega t}, \quad \sigma_{31} = 0, \quad \mu_{32} = 0 \quad \text{on} \quad x_3 = 0 \tag{5.1}$$

The formula for displacements u_1, u_3 may be found by using the relations (2.1ab) and the equations for ϕ and ψ in (4.21)-(4.22). Therefore we have

$$u_1 = -\frac{1}{2\pi} \int_{-\infty}^{\infty}\!\!\int \frac{\bar{f}}{\Delta}[(i\xi - \alpha_1^{*}\sqrt{\zeta_1})\Delta_1 e^{-x_3\sqrt{\zeta_1}} + (i\xi - \alpha_2\sqrt{\zeta_2})\Delta_2 e^{-x_3\sqrt{\zeta_2}}$$
$$+(i\xi - \alpha_3^{*}\sqrt{\zeta_3})\Delta_3 e^{-x_3\sqrt{\zeta_3}} + (i\xi - \alpha_4^{*}\sqrt{\zeta_4})\Delta_4 e^{-x_3\sqrt{\zeta_4}}]e^{-i(x_1\xi + t\eta)} d\xi d\eta, \tag{5.2}$$

$$u_3 = -\frac{1}{2\pi} \int_{-\infty}^{\infty}\!\!\int \frac{\hat{f}}{\Delta}[(i\xi\alpha_1^{*} + \sqrt{\zeta_1})\Delta_1 e^{-x_3\sqrt{\zeta_1}} + (i\xi\alpha_2^{*} + \sqrt{\zeta_2})\Delta_2 e^{-x_3\sqrt{\zeta_2}}$$
$$+(i\xi\alpha_3^{*} - \sqrt{\zeta_3})\Delta_3 e^{-x_3\sqrt{\zeta_3}} + (i\xi\alpha_4^{*} - \sqrt{\zeta_4})\Delta_4 e^{-x_3\sqrt{\zeta_4}}]e^{-i(x_1\xi + t\eta)} d\xi d\eta, \tag{5.3}$$

We also find

$$\mu_{32} = -\frac{1}{2\pi}\{(\gamma_0 + \varepsilon_0) - i\eta(\gamma_1 + \varepsilon_1)\} \int_{-\infty}^{\infty}\!\!\int \frac{1}{\Delta}[\beta_1^{*}\sqrt{\zeta_1}\,\Delta_1 e^{-x_3\sqrt{\zeta_1}}$$
$$+\beta_2^{*}\sqrt{\zeta_2}\,\Delta_2 e^{-x_3\sqrt{\zeta_2}} + \beta_3^{*}\sqrt{\zeta_3}\,\Delta_3 e^{-x_3\sqrt{\zeta_3}} + \beta_4^{*}\sqrt{\zeta_4}\,\Delta_4 e^{-x_3\sqrt{\zeta_4}}]\, e^{-i(x_1\xi + t\eta)} d\xi d\eta \tag{5.4}$$

$$\hat{f}(\xi,\eta)=\frac{1}{2\pi}\int\int_{-\infty}^{\infty} f(x_1)e^{-i\omega t}e^{i(\xi x_1+\eta t)}dx_1dt=\frac{1}{2\pi}\int_{-\infty}^{\infty} f(x_1)e^{i\xi x_1}dx_1\int_{-\infty}^{\infty} e^{-it(\omega-\eta)}dt$$

$$=\sqrt{2\pi}\,\delta(\eta-\omega)f^*(\xi) \tag{5.5}$$

where

$$f^*(\xi)=\frac{1}{2\pi}\int_{-\infty}^{\infty} f(x_1)e^{i\xi x_1}dx_1 \tag{5.6}$$

and $\delta(x)$ is the Dirac delta function defined by

$$\int_{-\infty}^{\infty} e^{ipt}dt=\sqrt{2\pi}\,\delta(p) \tag{5.7}$$

Substitution of (5.5) in (5.2)-(5.4) and in (4.24) gives, after integration,

$$u_1=-\frac{e^{-i\omega t}}{\sqrt{2\pi}}\int\int_{-\infty}^{\infty}\Bigg[\frac{1}{\Delta}\Big\{(i\xi-\alpha_1^*\sqrt{\zeta_1})\Delta_1e^{-x_3\sqrt{\zeta_1}}+(i\xi-\alpha_2^*\sqrt{\zeta_2})\Delta_2e^{-x_3\sqrt{\zeta_2}}$$
$$+(i\xi-\alpha_3^*\sqrt{\zeta_3})\Delta_3e^{-x_3\sqrt{\zeta_3}}+(i\xi-\alpha_4^*\sqrt{\zeta_4})\Delta_4e^{-x_3\sqrt{\zeta_4}}\Big\}\Bigg]_{\eta=\omega}f^*(\xi)e^{-i\xi x_1}d\xi, \tag{5.8}$$

$$u_3=-\frac{e^{-i\omega t}}{\sqrt{2\pi}}\int\int_{-\infty}^{\infty}\Bigg[\frac{1}{\Delta}\Big\{(i\xi\alpha_1^*+\sqrt{\zeta_1})\Delta_1e^{-x_3\sqrt{\zeta_1}}+(i\xi\alpha_2^*+\sqrt{\zeta_2})\Delta_2e^{-x_3\sqrt{\zeta_2}}$$
$$+(i\xi\alpha_3^*+\sqrt{\zeta_3})\Delta_3e^{-x_3\sqrt{\zeta_3}}+(i\xi\alpha_4^*+\sqrt{\zeta_4})\Delta_4e^{-x_3\sqrt{\zeta_4}}\Big\}\Bigg]_{\eta=\omega}f^*(\xi)e^{-i\xi x_1}d\xi, \tag{5.9}$$

$$\mu_{32}=-\frac{e^{-i\omega t}}{\sqrt{2\pi}}\{(\gamma_0+\varepsilon_0)-i\eta(\gamma_1+\varepsilon_1)\}\int\int_{-\infty}^{\infty}\Bigg[\frac{1}{\Delta}\Big\{\beta_1^*\sqrt{\zeta_1}\,\Delta_1e^{-x_3\sqrt{\zeta_1}}$$
$$+\beta_2^*\sqrt{\zeta_2}\,\Delta_2e^{-x_3\sqrt{\zeta_2}}+\beta_3^*\sqrt{\zeta_3}\,\Delta_3e^{-x_3\sqrt{\zeta_3}}+\beta_4^*\sqrt{\zeta_4}\,\Delta_4e^{-x_3\sqrt{\zeta_4}}\Big\}\Bigg]_{\eta=\omega}f^*(\xi)e^{-i\xi x_1}d\xi, \tag{5.10}$$

$$\theta=\frac{e^{-i\omega t}}{\sqrt{2\pi}}\int\int_{-\infty}^{\infty}\Bigg[\frac{1}{\Delta}\Big\{\gamma_1^*\Delta_i\,e^{-x_3\sqrt{\zeta_1}}+\gamma_2^*\Delta_2e^{-x_3\sqrt{\zeta_2}}+\gamma_3^*\Delta_3e^{-x_3\sqrt{\zeta_3}}+\gamma_4^*\Delta_4e^{-x_3\sqrt{\zeta_4}}\Big\}\Bigg]_{\eta=\omega}f^*(\xi)e^{-i\xi x_1}d\xi \tag{5.11}$$

In particular, when the applied load is a horizontal concentrated force acting at the origin, that is $f(x_1)=P\delta(x_1)$ so that $f^*(\xi)=\frac{P}{\sqrt{2\pi}}$. Hence solutions (5.8)-(5.11) assume simpler forms. The upshot of this analysis is that solutions modified by gravity, viscosity and temperature field.

6. CLOSING REMARKS

With regard to the general character of the harmonic loading function utilized as a disturbance in the theory, it can be added as a concluding remark that the present analysis is sufficiently general, and in addition, it incorporates other forms of harmonic disturbances of physical interest.

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