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Research Article

On Semicompact Sets and Associated Properties

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We continue the study of semicompact sets in a topological space. Several properties, mapping properties of semicompact sets are studied. A special interest to SCS spaces is given, where a space X is SCS if every subset of X which is semicompact in X is semiclosed; we study several properties of such spaces, it is mainly shown that a semi-T_2 semicompact space is SCS if and only if it is extremally disconnected. It is also shown that in an os-regular space X if every point has an SCS neighborhood, then X is SCS.

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1. Introduction and Preliminaries

A subset A of a space X is called semi-open [1] if $A \subset \overline{\operatorname{Int} A}$, or equivalently, if there exists an open subset U of X such that $U \subset A \subset \overline{U}$; A is called semiclosed if $X \setminus A$ is semi-open. The semiclosure $\operatorname{scl}(A)$ of a subset A of a space X is the intersection of all semiclosed subsets of X that contain A or equivalently the smallest semiclosed subset of X that contains A. Clearly, A is semiclosed if and only if $\operatorname{scl}(A) = A$; it is also clear that if A is a subset of a space X and $x \in X$, then $x \in \operatorname{scl}(A)$ if and only if $S \cap A \neq \phi$ for each semi-open subset S of X containing x. A subset A of a space X is called preopen [2] (resp., α-open [3]) if $A \subset \operatorname{Int} \overline{A}$ (resp., $A \subset \operatorname{Int} \overline{\operatorname{Int} A}$). Njastad [3] pointed out that the family of all α-open subsets of a space (X, τ), denoted by τ^α, is a topology on X finer than τ. We will denote the families of semi-open (resp., preopen, α-open) subsets of a space X by $SO(X)$ (resp., $PO(X)$, $\alpha O(X)$). If (X, τ) is a topological space, we will denote the space (X, τ^α) by X^α. Janković [4] pointed out that $PO(X) = PO(X^\alpha)$, $SO(X) = SO(X^\alpha)$ and $\alpha O(X) = \alpha O(X^\alpha)$. Reilly and Vamanamurthy observed in [5] that $\tau^\alpha = SO(X) \cap PO(X)$. It is known that the intersection of a semi-open (resp., preopen) set with an α-open set is semi-open (resp., preopen) and that the arbitrary union of semi-open (resp., preopen) sets is semi-open (resp., preopen).

A space X is called semicompact [6] (resp., semi-Lindelöf [7]) if any semi-open cover of X has a finite (resp., countable) subcover. A subset A of a space X will be called semicompact (resp., semi-Lindelöf) if it is semicompact (resp., semi-Lindelöf) as a subspace.

A function f from a space X into a space Y is called semi-continuous [1] if the inverse image of each open subset of Y is semi-open in X, irresolute [8] if the inverse image of each semi-open subset of Y is semi-open in X and f is called pre-semi-open (resp., pre-semiclosed [8]) if it maps semi-open (resp., semiclosed) subsets of X onto semi-open (resp., semiclosed) subsets of Y.

A space X is called semi-T_2 [9] if for each distinct points x and y of X, there exist two disjoint semi-open subsets U and V of X containing x and y, respectively.

A space X is called extremally disconnected [10] if the closure of each open subset of X is open or equivalently if every regular closed subset of X is preopen.

Throughout this paper, a space X stands for a topological space, and if X is a space and $A \subset X$, then $\overline{A}$ and $\operatorname{Int} A$ stand respectively for the closure of A in X and the interior of A in X. For the concepts not defined here, we refer the reader to [11].

In concluding this section, we recall the following facts for their importance in the material of our paper.

Proposition 1.1. *Let $A \subset B \subset X$, where X is a space. Then*

(i) *If A is semi-open in X, then A is semi-open in B;*
(ii) *[12] If A is semi-open in B and B is semi-open in X, then A is semi-open in X.*

Proposition 1.2. *Let $A \subset B \subset X$, where X is a space and B is preopen in X. Then A is semi-open (resp., semiclosed) in B if and only if $A = S \cap B$, where S is semi-open (resp., semi-closed) in X.*

2. Semicompact Sets

This section is mainly devoted to continue the study of semicompact sets. We also introduce and study semi-Lindelöf sets.

Definition 2.1 (see [13]). A subset A of a space X is called semicompact relative to X if any semi-open cover of A in X has a finite subcover of A.

By semicompact in X, we will mean semicompact relative to X.

Definition 2.2. A subset A of a space X is called semi-Lindelöf in X if any semi-open cover of A in X has a countable subcover of A.

Remark 2.3. It is easy to see from the fact that $SO(X) = SO(X^{\alpha})$, that a subset A of a space X is semicompact (resp., semi-Lindelöf) in X if and only if it is semicompact (resp., semi-Lindelöf) in X^{α}.

The proof of the following proposition is straightforward, and thus omitted.

Proposition 2.4. *The finite (resp., countable) union of semicompact (resp., semi-Lindelöf) sets in a space X is semicompact (resp., semi-Lindelöf) in X.*

Proposition 2.5. *Let B be a preopen subset of a space X and $A \subset B$. If A is semicompact (resp., semi-Lindelöf) in X, then A is semicompact (resp., semi-Lindelöf) in B.*

Proof. We will show the case when A is semicompact in X, the other case is similar. Suppose that $\mathscr{A} = \{A_\alpha : \alpha \in \Lambda\}$ is a cover of A by semi-open sets in B. By Proposition 1.2, $A_\alpha = S_\alpha \cap B$ for each $\alpha \in \Lambda$, where S_α is semi-open in X for each $\alpha \in \Lambda$. Thus $\mathcal{S} = \{S_\alpha : \alpha \in \Lambda\}$ is a cover of A by semi-open sets in X, but A is semicompact in X, so there exist $\alpha_1, \alpha_2, \ldots, \alpha_n \in \Lambda$ such that $A \subset \bigcup_{i=1}^{i=n} S_{\alpha_i}$, and thus $A \subset \bigcup_{i=1}^{i=n}(S_{\alpha_i} \cap B) = \bigcup_{i=1}^{i=n} A_{\alpha_i}$. Hence, A is semicompact in B. □

Corollary 2.6. *Let A be subset of a space X. If A is semicompact (resp., semi-Lindelöf) in X, then A is semicompact (resp., semi-Lindelöf).*

Proposition 2.7. *Let B be a preopen subset of a space X and $A \subset B$. Then A is semicompact (resp., semi-Lindelöf) in X if and only if A is semicompact (resp., semi-Lindelöf) in B.*

Proof. Necessity. It follows from Proposition 2.5.

Sufficiency. We will show the case when A is semicompact in B, the other case is similar. Suppose that $\mathcal{S} = \{S_\alpha : \alpha \in \Lambda\}$ is a cover of A by semi-open sets in X. Then $\mathscr{A} = \{S_\alpha \cap B : \alpha \in \Lambda\}$ is a cover of A. Since S_α is semi-open in X for each $\alpha \in \Lambda$ and B is preopen in X, it follows from Proposition 1.2 that $S_\alpha \cap B$ is semi-open in B for each $\alpha \in \Lambda$, but A is semicompact in B, so there exist $\alpha_1, \alpha_2, \ldots \alpha_n \in \Lambda$ such that $A \subset \bigcup_{i=1}^{i=n}(S_{\alpha_i} \cap B) \subset \bigcup_{i=1}^{i=n} S_{\alpha_i}$. Hence, A is semicompact in X. □

Corollary 2.8. *A preopen subset A of a space X is semicompact (resp., semi-Lindelöf) if and only if A is semicompact (resp., semi-Lindelöf) in X.*

Proposition 2.9. *Let A be a semicompact (resp., semi-Lindelöf) set in a space X and B be a semi-closed subset of X. Then $A \cap B$ is semicompact (resp., semi-Lindelöf) in X. In particular, a semi-closed subset A of a semicompact (resp., semi-Lindelöf) space X is semicompact (resp., semi-Lindelöf) in X.*

Proof. We will show the case when A is semicompact in X, the other case is similar. Suppose that $\mathcal{S} = \{S_\alpha : \alpha \in \Lambda\}$ is a cover of $A \cap B$ by semi-open sets in X. Then $\mathscr{A} = \{S_\alpha : \alpha \in \Lambda\} \cup \{X \setminus B\}$ is a cover of A by semi-open sets in X, but A is semicompact in X, so there exist $\alpha_1, \alpha_2, \ldots, \alpha_n \in \Lambda$ such that $A \subset (\bigcup_{i=1}^{i=n} S_{\alpha_i}) \cup (X \setminus B)$. Thus $A \cap B \subset \bigcup_{i=1}^{i=n}(S_{\alpha_i} \cap B) \subset \bigcup_{i=1}^{i=n} S_{\alpha_i}$. Hence, $A \cap B$ is strongly compact in X. □

Proposition 2.10. *Let $f : X \to Y$ be an irresolute function. Then*

(i) *[13] If A is semicompact in X, then $f(A)$ is semicompact in Y;*

(ii) *If A is semi-Lindelöf in X, then $f(A)$ is semi-Lindelöf in Y.*

Proof. (ii) The proof is similar to that of (i). We will, however, show it for the convenience of the reader. Suppose that $\mathcal{S} = \{S_\alpha : \alpha \in \Lambda\}$ is a cover of $f(A)$ by semi-open sets in Y. Then $\mathscr{A} = \{f^{-1}(S_\alpha) : \alpha \in \Lambda\}$ is a cover of A, but f is irresolute, so $f^{-1}(S_\alpha)$ is semi-open in X for each $\alpha \in \Lambda$. Since A is semi-Lindelöf in X, there exist $\alpha_1, \alpha_2, \alpha_3, \ldots \in \Lambda$ such that $A \subset \bigcup_{i=1}^{i=\infty} f^{-1}(S_{\alpha_i})$. Thus $f(A) \subset \bigcup_{i=1}^{i=\infty} f(f^{-1}(S_{\alpha_i})) \subset \bigcup_{i=1}^{i=\infty} S_{\alpha_i}$. Hence, $f(A)$ is semi-Lindelöf in X. □

Proposition 2.11. *Let $f : X \to Y$ be a pre-semi-closed surjection. If for each $y \in Y$, $f^{-1}(y)$ is semicompact (resp., semi-Lindelöf) in X, then $f^{-1}(A)$ is semicompact (resp., semi-Lindelöf) in X whenever A is semicompact (resp., semi-Lindelöf) in Y.*

Proof. We will show the case when A is semicompact in X, the other case is similar. Suppose that $\mathcal{S} = \{S_\alpha : \alpha \in \Lambda\}$ is a cover of $f^{-1}(A)$ by semi-open sets in X. Then it follows by assumption that for each $y \in A$, there exists a finite subcollection $\mathcal{S}^y$ of $\mathcal{S}$ such that $f^{-1}(y) \subset \bigcup \mathcal{S}^y$. Let $V_y = \cup \mathcal{S}^y$. Then V_y is semi-open in X as any union of semi-open sets is semi-open. Let $H_y = Y \setminus f(X \setminus V_y)$. Then H_y is semi-open in Y as f is pre-semi-closed, also $y \in H_y$ for each $y \in A$ as $f^{-1}(y) \subset V_y$. Thus, $\mathscr{H} = \{H_y : y \in A\}$ is a cover of A by semi-open sets in Y, but A is semicompact in Y, so there exist $y_1, y_2, \ldots, y_n \in A$ such that $A \subset \bigcup_{i=1}^{i=n} H_{y_i}$. Thus, $f^{-1}(A) \subset \bigcup_{i=1}^{i=n} f^{-1}(H_{y_i}) \subset \bigcup_{i=1}^{i=n} V_{y_i}$. Since S^{y_i} is a finite subcollection of $\mathcal{S}$ for each $i \in \{1, 2, \ldots, n\}$, it follows that $\bigcup_{i=1}^{i=n} \mathcal{S}^{y_i}$ is a finite subcollection of $\mathcal{S}$. Hence, $f^{-1}(A)$ is semicompact in X. □

3. *SCS* Spaces

Definition 3.1. A space X is said to be *SCS* if any subset of X which is semicompact in X is semi-closed.

Remark 3.2. It follows from Remark 2.3, that a space X is *SCS* if and only if X^α is *SCS*.

We recall the following result from [3], it will be helpful to show the next two theorems.

Proposition 3.3. *A space X is extremally disconnected if and only if the intersection of any two semi-open subsets of X is semi-open.*

Theorem 3.4. *Let X be a semi-T_2 extremally disconnected space. Then X is SCS.*

Proof. Let F be a subset of X which is semicompact in X and let $x \notin F$. Then for each $y \in F$ there exist two disjoint semi-open sets U and V containing x and y respectively (as X is semi-T_2). Since F is semicompact in X, there exist $y_1, y_2, \ldots, y_n \in F$ such that $F \subset \bigcup_{i=1}^{n} V_{y_i}$. Let $U = \bigcap_{i=1}^{n} U_{y_i}$. Then U is a semi-open subset of X that contains x and disjoint from F (as X is extremally disconnected using Proposition 3.3). Thus, $x \notin \mathrm{scl}(F)$. Hence, F is semi-closed in X. □

Theorem 3.5. *If X is an SCS space such that every semi-closed subset A of X is semicompact in X, then X is extremally disconnected. In particular, an SCS semicompact space is extremally disconnected.*

Proof. Let $F = A \cup B$, where A and B are semi-closed in X. It follows by assumption that A and B are semicompact in X and thus by Proposition 2.4, F is semicompact in X, but X is *SCS*, so F is semi-closed in X. Hence by Proposition 3.3, X is extremally disconnected. The last part follows by Proposition 2.9. □

Corollary 3.6. *For a semi-T_2 semicompact space, the followings are equivalent:*

(i) *X is SCS.*

(ii) *X is extremally disconnected.*

Observing that a singleton of a space X is semi-open if and only if it is open, the following proposition seems clear.

Proposition 3.7. *If every subset of a space X is semicompact in X, then X is SCS if and only if X is a finite discrete space.*

Theorem 3.8. *Let f be a pre-semi-closed function from a space X onto a space Y such that for each $y \in Y$, $f^{-1}(y)$ is semicompact in X. If X is SCS, then so is Y.*

Proof. Let F be a semicompact set in Y. Then by Proposition 2.11, $f^{-1}(F)$ is semicompact in X, but X is SCS, so $f^{-1}(F)$ is semi-closed in X, but f is a pre-semi-closed surjection, so $F = f(f^{-1}(F))$ is semi-closed. Hence, Y is SCS. □

Theorem 3.9. *Let f be an irresolute one-to-one function from a space X into an SCS space Y. Then X is SCS.*

Proof. Let F be a semicompact set in X. Then it follows from Proposition 2.10(i) that $f(F)$ is semicompact in Y, but Y is SCS, so $f(F)$ is semi-closed in Y. Since f is one-to-one and irresolute, $F = f^{-1}(f(F))$ is semi-closed in X. Hence, X is SCS. □

Lemma 3.10. *A subset A of $\oplus X_\alpha$ is semi-open if and only if $A \cap X_\alpha$ is semi-open in X_α for each α. Thus a subset A of $\oplus X_\alpha$ is semi-closed if and only if $A \cap X_\alpha$ is semi-closed in X_α for each α.*

Proof. Since X_α is open in $\oplus X_\alpha$, it follows that if A is semi-open in $\oplus X_\alpha$, then $A \cap X_\alpha$ is semi-open in $\oplus X_\alpha$ and thus semi-open in X_α for each α. Now suppose that $A \cap X_\alpha$ is semi-open in X_α for each α. Then $A \cap X_\alpha$ is semi-open in $\oplus X_\alpha$ for each α because X_α is open and thus semi-open in $\oplus X_\alpha$. Thus, $A = \cup(A \cap X_\alpha)$ is semi-open in $\oplus X_\alpha$ as the arbitrary union of semi-open sets is semi-open. □

Corollary 3.11. *Being SCS is hereditary with respect to preopen subsets.*

Proof. Let A be a preopen subset of an SCS space X and let B be semicompact in A. Then by Proposition 2.7, B is semicompact in X, but X is SCS, so B is semi-closed in X. By Proposition 1.2, B is semi-closed in A. Hence, A is SCS. □

Corollary 3.12. *$\oplus X_\alpha$ is SCS if and only if X_α is SCS for each α.*

Proof. Necessity. It follows from Corollary 3.11 since X_α is open and thus preopen in $\oplus X_\alpha$.

Sufficiency. Suppose that X_α is an SCS space for each α and let F be a subset of $\oplus X_\alpha$ which is semicompact in $\oplus X_\alpha$. Since X_α is closed and thus semi-closed in $\oplus X_\alpha$, it follows from Proposition 2.9 that $F \cap X_\alpha$ is semicompact in $\oplus X_\alpha$, but X_α is preopen in $\oplus X_\alpha$, so it follows from Proposition 2.7 that $F \cap X_\alpha$ is semicompact in X_α. Since X_α is SCS, $F \cap X_\alpha$ is semi-closed in X_α for each α, thus by Lemma 3.10, F is semi-closed in $\oplus X_\alpha$. Hence, $\oplus X_\alpha$ is SCS. □

Recall that a space X is called s-regular [14] if whenever U is an open subset of X and $x \in U$, there exists a semi-open subset K of X and a semi-closed subset S of X such that $x \in K \subset S \subset U$. We now define a type of regularity which is stronger than s-regularity and weaker than regularity.

Definition 3.13. A space X is called os-regular if whenever U is an open subset of X and $x \in U$, there exists an open subset K of X and a semi-closed subset S of X such that $x \in K \subset S \subset U$.

Theorem 3.14. *If X is an os-regular space in which every point has an SCS neighborhood, then X is SCS.*

Proof. Let F be a subset of X which is semicompact in X and let $x \notin F$. Then by assumption there exists an SCS neighborhood of x. Since being SCS is hereditary with respect to preopen sets (Corollary 3.11), it follows that x has an open SCS neighborhood U. Now since X is os-regular, there exists an open subset K of X and a semi-closed subset S of X such that $x \in K \subset S \subset U$. Since F is semicompact in X and S is a semi-closed subset of X, it follows from Proposition 2.9 that $F \cap S$ is semicompact in X, thus by Proposition 2.5, $F \cap S$ is semicompact in U, but U is SCS, so $F \cap S$ is semi-closed in U, that is, $U \setminus (F \cap S)$ is semi-open in U and thus semi-open in X by Proposition 1.1(ii) as U is open and thus semi-open in X. Thus $K \cap (U \setminus (F \cap S))$ is a semi-open subset of X that contains x and disjoint from F and therefore, $x \notin \mathrm{scl}(F)$. Hence, F is semi-closed in X, and therefore, X is SCS. □

Corollary 3.15. *If X is a regular space in which every point has an SCS neighborhood, then X is SCS.*

Theorem 3.16. *Let X be a space in which every semi-closed subset is semicompact in X, Y be an SCS space. Then any irresolute function f from X into Y is pre-semi-closed. In particular, any irresolute function from a semicompact space X into an SCS space Y is pre-semi-closed.*

Proof. Let F be a semi-closed subset of X. By assumption, F is semicompact in X. Since f is irresolute, it follows by Proposition 2.10 that $f(F)$ is semicompact in Y. Since Y is SCS, it follows that $f(F)$ is semi-closed in Y. The last part follows from Proposition 2.9. □

The following lemma will be helpful to show the next result, the easy proof is omitted.

Lemma 3.17. *(i) The projection function is irresolute.*

(ii) Let $f : X \to Y$ be irresolute and A be an α-open subspace of X. Then the restriction function $f|_A : A \to Y$ is irresolute.

Theorem 3.18. *Let X be an SCS space and Y be any space. If $f : X \to Y$ is a function whose graph G_f is an α-open subspace of $X \times Y$ in which every semi-closed subset is semicompact in G_f, then f is irresolute. In particular, any function having an SCS domain and an α-open, semicompact graph is irresolute.*

Proof. Let $P_X : X \times Y \to X$ and $P_Y : X \times Y \to Y$ be the projection functions. Since G_f is an α-open subspace of $X \times Y$, it follows from Lemma 3.17 that $P_X|_{G_f}$ is irresolute. Thus it follows from Theorem 3.16 that $P_X|_{G_f}$ is pre-semi-closed, that is, $(P_X|_{G_f})^{-1}$ is irresolute. Also, P_Y is irresolute. Thus, $f = P_Y \circ (P_X|_{G_f})^{-1}$ is irresolute. The last part follows from Proposition 2.9. □

4. SLS Spaces

The study of this section is analogous to that of the preceding section, similar proofs are omitted.

Definition 4.1. A space X is said to be SLS if any subset of X which is semi-Lindelöf in X is semi-closed.

Remark 4.2. It follows from Remark 2.3, that a space X is SLS if and only if X^{α} is SLS.

Following Proposition 3.3, we will call a space X ω-extremally disconnected if the countable intersection of semi-open subsets of X is semi-open.

Theorem 4.3. *Let X be a semi-T_2 ω-extremally disconnected. Then X is SLS.*

Theorem 4.4. *If X is an SLS space such that every semi-closed subset A of X is semi-Lindelöf in X, then X is ω-extremally disconnected. In particular, an SLS semi-Lindelöf space is ω-extremally disconnected.*

Corollary 4.5. *For a semi-T_2 semi-Lindelöf space, the followings are equivalent:*

(i) *X is SLS.*
(ii) *X is ω-extremally disconnected.*

Proposition 4.6. *If every subset of a space X is semi-Lindelöf in X, then X is SLS if and only if X is a countable discrete space.*

Theorem 4.7. *Let f be a pre-semi-closed function from a space X onto a space Y such that for each $y \in Y$, $f^{-1}(y)$ is semi-Lindelöf in X. If X is SLS, then so is Y.*

Theorem 4.8. *Let f be an irresolute one-to-one function from a space X into an SLS space Y. Then X is SLS.*

Proposition 4.9. *Being SLS is hereditary with respect to preopen subsets.*

Corollary 4.10. *$\oplus X_\alpha$ is SLS if and only if X_α is SLS for each α.*

Theorem 4.11. *If X is an os-regular space in which every point has an SLS neighborhood, then X is SLS.*

Corollary 4.12. *If X is a regular space in which every point has an SLS neighborhood, then X is SLS.*

Theorem 4.13. *Let X be a space in which every semi-closed subset is semi-Lindelöf in X, Y be an SLS space. Then any irresolute function f from X into Y is pre-semi-closed. In particular, any irresolute function from a semi-Lindelöf space X into an SLS space Y is pre-semi-closed.*

Theorem 4.14. *Let X be an SLS space and Y be any space. If $f : X \to Y$ is a function whose graph G_f is an α-open subspace of $X \times Y$ in which every semi-closed subset is semi-Lindelöf in G_f, then f is irresolute. In particular, any function having an SLS domain and an α-open, semi-Lindelöf graph is irresolute.*

References

[1] N. Levine, “Semi-open sets and semi-continuity in topological spaces,” *The American Mathematical Monthly*, vol. 70, pp. 36–41, 1963.
[2] A. S. Mashhour, M. E. Abd El-Monsef, and S. N. El-Deep, “On precontinuous and weak precontinuous mappings,” *Proceedings of the Mathematical and Physical Society of Egypt*, no. 53, pp. 47–53, 1982.
[3] O. Njastad, “On some classes of nearly open sets,” *Pacific Journal of Mathematics*, vol. 15, pp. 961–970, 1965.
[4] D. S. Janković, “A note on mappings of extremally disconnected spaces,” *Acta Mathematica Hungarica*, vol. 46, no. 1-2, pp. 83–92, 1985.
[5] I. L. Reilly and M. K. Vamanamurthy, “On α-continuity in topological spaces,” *Acta Mathematica Hungarica*, vol. 45, no. 1-2, pp. 27–32, 1985.
[6] C. Dorsett, “Semicompactness, semiseparation axioms, and product spaces,” *Bulletin of the Malaysian Mathematical Sciences Soceity*, vol. 4, no. 1, pp. 21–28, 1981.
[7] M. Ganster, “On covering properties and generalized open sets in topological spaces,” *Mathematical Chronicle*, vol. 19, pp. 27–33, 1990.

[8] S. G. Crossely and S. K. Hilderbrand, "Semitopological properties," *Fundamenta Mathematicae*, vol. 74, pp. 233–254, 1972.
[9] S. N. Maheshwari and R. Prasad, "Some new separation axioms," *Annales de la Societé Scientifique de Bruxelles*, vol. 89, no. 3, pp. 395–402, 1975.
[10] M. H. Stone, "Algebraic characterization of special Boolean rings," *Fundamenta Mathematicae*, vol. 29, pp. 223–302, 1937.
[11] R. Engelking, *General Topology*, vol. 6 of *Sigma Series in Pure Mathematics*, Heldermann, Berlin, Germany, 2nd edition, 1989.
[12] J. Dontchev and M. Ganster, "On covering spaces with semi-regular sets," *Ricerche di Matematica*, vol. 45, no. 1, pp. 229–245, 1996.
[13] M. C. Cueva and J. Dontchev, "On spaces with hereditarily compact α-topologies," *Acta Mathematica Hungarica*, vol. 82, no. 1-2, pp. 121–129, 1999.
[14] S. N. Maheshwari and R. Prasad, "On s-regular spaces," *Glasnik Matematički*, vol. 10(30), no. 2, pp. 347–350, 1975.