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Research Article

On the Uniqueness and Dependence of Positive Periodic Solutions for Delay Differential Systems with Feedback Control

Haitao Li[1] and Yansheng Liu[2]

[1] *School of Control Science and Engineering, Shandong University, Jinan 250061, China*
[2] *Department of Mathematics, Shandong Normal University, Jinan 250014, China*

Correspondence should be addressed to Haitao Li, haitaoli09@gmail.com

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This paper investigates a class of delay differential systems with feedback control. Sufficient conditions are obtained for the existence and uniqueness of the positive periodic solution by utilizing some results from the mixed monotone operator theory. Meanwhile, the dependence of the positive periodic solution on the parameter λ is also studied. Finally, an example together with numerical simulations is worked out to illustrate the main results.

1. Introduction

As is known to all, the periodic environment changes and the unpredictable forces play an important role in many biological and ecological systems. Therefore, several different periodic models with feedback control have been studied by many authors (see [1–10] and references therein). For instance, Gopalsamy and Weng [2] introduced a feedback control variable into the delayed logistic model and discussed the asymptotic behavior of solutions in logistic models with feedback control. Li and Wang [5] investigated the existence and global attractivity of positive periodic solutions for a delay differential system with feedback control. The method they used involved Krasnoselskii's fixed point theorem and estimates of uniform upper and lower bounds of solutions. In a recent work [3], Guo considered the existence of nontrivial periodic solutions for a kind of nonlinear functional differential system with feedback control. By using Leray-Schauder nonlinear alternative, the author obtained several sufficient conditions for the existence of nontrivial solutions. A class of impulsive functional equations with feedback control was studied by Guo and Liu [4], and they presented the existence results of three positive periodic solutions by using Leggett-Williams fixed point theorem.

However, as we know, there are few results on the uniqueness and parameter dependence of the positive periodic solution for delay differential systems with feedback control. Motivated by this fact, this paper is devoted to investigating the uniqueness and parameter dependence of the positive periodic solution for the following nonlinear nonautonomous delay differential system with feedback control:

$$\begin{aligned} \frac{dx}{dt} &= -b(t)x(t) + \lambda f(t, x(t-\tau(t)),\ u(t-\delta(t))), \quad t \in \mathbf{R}, \\ \frac{du}{dt} &= -\eta(t)u(t) + a(t)x(t-\sigma(t)), \end{aligned} \tag{1.1}$$

where $\lambda > 0$ is a parameter, $f(t, x_1, x_2) \in C(\mathbf{R}\times(0,+\infty)\times(0,+\infty) \to (0,+\infty))$, $\tau(t),\ \delta(t),\ \sigma(t) \in C(\mathbf{R}, \mathbf{R})$, and $\eta(t),\ a(t),\ b(t) \in C(\mathbf{R}, (0,+\infty))$. All functions are ω-periodic in t and $\omega > 0$ is a constant.

The main features here are as follows. On one hand, by utilizing the mixed monotone operator theory, the existence and uniqueness of the positive periodic solution of the delay differential system (1.1) are studied in this work. As is known to us, there are few papers to investigate this topic. On the other hand, the dependence of the positive periodic solution on the parameter λ is studied, and some interesting results are obtained.

The rest of this paper is organized as follows. Section 2 presents the existence and uniqueness result of the system (1.1) together with the dependence of the positive periodic solution on the parameter λ. In Section 3, an illustrative example is worked out to support the main results of this work.

2. Main Results

For convenience, let us first list some conditions.

(H1) $f(t, x_1, x_2) \in C(\mathbf{R} \times (0,+\infty) \times (0,+\infty), (0,+\infty))$ is nondecreasing in x_1 and nonincreasing in x_2.

(H2) There exists an $\alpha \in (0,1)$ such that

$$f\left(t, kx_1, k^{-1}x_2\right) \geq k^{\alpha} f(t, x_1, x_2), \quad \forall k \in (0,1),\ t \in \mathbf{R},\ x_1, x_2 \in (0,+\infty). \tag{2.1}$$

Let $C_\omega = \{x \in C(\mathbf{R}, \mathbf{R}) :\ x(t) = x(t+\omega),\ t \in \mathbf{R}\}$. Then, C_ω is a Banach space with norm $\|x\| = \max_{t\in[0,\,\omega]}|x(t)|$. In this paper, we will study the system (1.1) in C_ω.

Denote

$$g(t,s) = \frac{\exp\left\{\int_t^s \eta(r)dr\right\}}{\exp\left\{\int_0^\omega \eta(r)dr\right\} - 1}, \tag{2.2}$$

$$G(t,s) = \frac{\exp\left\{\int_t^s b(r)dr\right\}}{\exp\left\{\int_0^\omega b(r)dr\right\} - 1}. \tag{2.3}$$

Lemma 2.1 (see [5]). *Consider $p \le G(t,s) \le q$, where*

$$p = \frac{\exp\{-\int_0^\omega b(r)dr\}}{\exp\{\int_0^\omega b(r)dr\} - 1}, \qquad q = \frac{\exp\{\int_0^\omega b(r)dr\}}{\exp\{\int_0^\omega b(r)dr\} - 1}. \tag{2.4}$$

Now, we convert the system (1.1) into an operator equation. Define operators Φ and Ψ_λ as follows:

$$\begin{gathered}\Phi x(t) = \int_t^{t+\omega} g(t,s)a(s)x(s-\sigma(s))ds, \quad \forall x \in C_\omega, \\ \Psi_\lambda(x,y)(t) = \lambda \int_t^{t+\omega} G(t,s)f(s, x(s-\tau(s)), \Phi y(s-\delta(s)))\, ds, \quad \forall x,y \in C_\omega,\end{gathered} \tag{2.5}$$

where $g(t,s)$ and $G(t,s)$ are given in (2.2) and (2.3), respectively.

For the sake of using a fixed point theorem on mixed monotone operators, choose a fixed constant $e > 0$. Then, for each $\lambda > 0$, we can choose a proper number $C_\lambda > 1$ such that

$$C_\lambda \ge \max\left\{ \left(\frac{\lambda q \int_0^\omega f(s,e,\Phi(e))ds}{e} \right)^{1/1-\alpha}, \left(\frac{e}{\lambda p \int_0^\omega f(s,e,\Phi(e))ds} \right)^{1/1-\alpha} \right\}, \tag{2.6}$$

where $\Phi(\cdot)$ is given in (2.5). Let

$$P_e(\lambda) = \left\{ x \in C_\omega : C_\lambda^{-1} e \le x(t) \le C_\lambda e \quad \text{on} \quad [0,\omega] \right\}. \tag{2.7}$$

Lemma 2.2. *For any $\lambda > 0$, $x \in P_e(\lambda)$ is a ω-periodic solution of the system* (1.1) *if and only if $x \in P_e(\lambda)$ is a fixed point of the operator equation*

$$x(t) = \Psi_\lambda(x,x)(t), \tag{2.8}$$

where $\Psi_\lambda(\cdot,\cdot)$ is given in (2.5).

Proof. The proof of this lemma is similar to Theorem 2.1 in [5], and thus we omit it. □

Next, we recall some results from the monotone operator theory. The following results are well known (see [11–13], for details).

Definition 2.3 (see [12]). Assume that $T(x,y) : P_e(\lambda) \times P_e(\lambda) \to P_e(\lambda)$. Then, T is called mixed monotone if T is nondecreasing in x and nonincreasing in y; that is, for $x_1, x_2, y_1, y_2 \in P_e(\lambda)$, we have

$$x_1 \le x_2, \quad y_1 \ge y_2 \Longrightarrow T(x_1,y_1) \le T(x_2,y_2). \tag{2.9}$$

Lemma 2.4 (see [12]). *Assume that $T(x,y) : P_e(\lambda)\times P_e(\lambda) \to P_e(\lambda)$ is a mixed monotone operator and there exists $\alpha \in (0,1)$ such that*

$$T\left(kx, k^{-1}y\right) \geq k^{\alpha} T(x,y), \quad \text{for } x, y \in P_e(\lambda),\ k \in (0,1). \tag{2.10}$$

Then, T has a unique fixed point in $P_e(\lambda)$.

Lemma 2.5. *Suppose that (H1) and (H2) hold. Then, $\Psi_\lambda : P_e(\lambda)\times P_e(\lambda) \to P_e(\lambda)$, where $P_e(\lambda)$ is given in* (2.7).

Proof. For any $x, y \in P_e(\lambda)$, we have

$$C_\lambda^{-1} e \leq x(t) \leq C_\lambda e, \quad C_\lambda^{-1} e \leq y(t) \leq C_\lambda e, \quad t \in [0,\omega]. \tag{2.11}$$

This together with (H1), (H2), (2.4), and (2.6) implies that

$$\begin{aligned}
\Psi_\lambda(x,y)(t) &= \lambda \int_t^{t+\omega} G(t,s) f(s, x(s-\tau(s)),\ \Phi y(s-\delta(s)))ds \\
&\leq \lambda q \int_0^{\omega} f\left(s, C_\lambda e, C_\lambda^{-1}\Phi(e)\right)ds \leq \lambda q C_\lambda^{\alpha} \int_0^{\omega} f(s,e,\Phi(e))ds \leq C_\lambda e, \\
\Psi_\lambda(x,y)(t) &= \lambda \int_t^{t+\omega} G(t,s) f(s, x(s-\tau(s)),\ \Phi y(s-\delta(s)))ds \\
&\geq \lambda p \int_0^{\omega} f\left(s, C_\lambda^{-1} e, C_\lambda \Phi(e)\right)ds \geq \lambda p C_\lambda^{-\alpha} \int_0^{\omega} f(s,e,\Phi(e))ds \geq C_\lambda^{-1} e.
\end{aligned} \tag{2.12}$$

Therefore, $\Psi_\lambda : P_e(\lambda)\times P_e(\lambda) \to P_e(\lambda)$. □

Lemma 2.6. *Assume that (H1) and (H2) hold. Then, Ψ_λ is a mixed monotone operator and*

$$\Psi_\lambda\left(kx, k^{-1}y\right) \geq k^{\alpha}\Psi_\lambda(x,y), \quad \text{for } x, y \in P_e(\lambda),\ k \in (0,1). \tag{2.13}$$

Proof. For any $x_1, y_1, x_2, y_2 \in P_e(\lambda)$ with $x_1 \leq x_2, y_1 \geq y_2$, it is easy to see from (H1) that

$$\begin{aligned}
&\Psi_\lambda(x_1,y_1)(t) - \Psi_\lambda(x_2,y_2)(t) \\
&\quad = \lambda \int_t^{t+\omega} G(t,s)\left[f(s, x_1(s-\tau(s)), \Phi y_1(s-\delta(s)))\right. \\
&\qquad\qquad \left. - f(s, x_2(s-\tau(s)), \Phi y_2(s-\delta(s)))\right]ds \leq 0.
\end{aligned} \tag{2.14}$$

Hence, Ψ_λ is a mixed monotone operator.

In addition, for any $x, y \in P_e(\lambda)$ and $k \in (0,1)$, (H2) shows that

$$\begin{aligned}\Psi_\lambda\left(kx, k^{-1}y\right) &= \lambda \int_t^{t+\omega} G(t,s) f\left(s, kx(s-\tau(s)), \Phi k^{-1} y(s-\delta(s))\right) ds \\ &\geq k^\alpha \lambda \int_t^{t+\omega} G(t,s) f(s, x(s-\tau(s)), \Phi y(s-\delta(s))) ds \\ &= k^\alpha \Psi_\lambda(x,y).\end{aligned} \tag{2.15}$$

To sum up, the proof of this lemma is completed. □

Finally, we present the main results of this paper.

Theorem 2.7. *Suppose that (H1) and (H2) hold. Then, for any $\lambda > 0$, the system* (1.1) *has a unique positive ω-periodic solution $x_\lambda(t) \in P_e(\lambda)$.*

Proof. It is easy to see from Lemmas 2.5 and 2.6 that for any $\lambda > 0$, $\Psi_\lambda : P_e(\lambda) \times P_e(\lambda) \to P_e(\lambda)$ is a mixed monotone operator and

$$\Psi_\lambda\left(kx, k^{-1}y\right) \geq k^\alpha \Psi_\lambda(x,y), \quad \text{for } x, y \in P_e(\lambda),\ k \in (0,1). \tag{2.16}$$

Consequently, Lemmas 2.2 and 2.4 imply that the conclusion holds true. □

Theorem 2.8. *Assume that (H1) and (H2) hold. In addition, suppose that $\alpha \in (0, 1/2)$. Then, the unique positive ω-periodic solution of the system* (1.1)*, denoted by $x_\lambda(t)$, satisfies the following properties:*

(i) *$x_\lambda(t)$ is strictly increasing in λ; that is, if $\lambda_1 > \lambda_2 > 0$, then $x_{\lambda_1}(t) > x_{\lambda_2}(t)$, $t \in \mathbf{R}$;*

(ii) *$\lim_{\lambda \to 0^+} \|x_\lambda\| = 0$, and $\lim_{\lambda \to \infty} \|x_\lambda\| = \infty$;*

(iii) *$x_\lambda(t)$ is continuous in λ; that is, if $\lambda \to \lambda_0 > 0$, then $\|x_\lambda - x_{\lambda_0}\| \to 0$.*

Proof. Suppose that $\lambda_1 > \lambda_2 > 0$. Let

$$D = \left\{\gamma > 0 : \gamma^{-1}\left(\lambda_1 \lambda_2^{-1}\right)^{1/1-\alpha} x_{\lambda_2}(t) \geq x_{\lambda_1}(t) \geq \gamma\left(\lambda_1 \lambda_2^{-1}\right)^{1-2\alpha/1-\alpha} x_{\lambda_2}(t) \text{ on } \mathbf{R}\right\}. \tag{2.17}$$

Since $e > 0$, we have $x_{\lambda_1}(t) > 0$ and $x_{\lambda_2}(t) > 0$ for $t \in \mathbf{R}$. Thus

$$\gamma^* := \min\left\{\left(\lambda_1^{-1}\lambda_2\right)^{1-2\alpha/1-\alpha} \min_{t \in \mathbf{R}} \frac{x_{\lambda_1}(t)}{x_{\lambda_2}(t)}, \left(\lambda_1 \lambda_2^{-1}\right)^{1/1-\alpha} \min_{t \in \mathbf{R}} \frac{x_{\lambda_2}(t)}{x_{\lambda_1}(t)}\right\} > 0. \tag{2.18}$$

Obviously, for any γ satisfying $0 < \gamma < \gamma^*$, $\gamma \in D$. Hence, $D \neq \emptyset$.

Define $\overline{\gamma} = \sup D$. Then

$$\overline{\gamma}^{-1}\left(\lambda_1 \lambda_2^{-1}\right)^{1/1-\alpha} x_{\lambda_2}(t) \geq x_{\lambda_1}(t) \geq \overline{\gamma}\left(\lambda_1 \lambda_2^{-1}\right)^{1-2\alpha/1-\alpha} x_{\lambda_2}(t), \quad t \in \mathbf{R}. \tag{2.19}$$

Now let us show that $\overline{\gamma} \geq 1$. In fact, if $0 < \overline{\gamma} < 1$, then (H1) and (H2) imply that

$$\begin{aligned}
&\lambda_1 f(t, x_{\lambda_1}(t-\tau(t)), \Phi x_{\lambda_1}(t-\delta(t))) \\
&\quad \geq \lambda_1 f\left(t, \overline{\gamma}\left(\lambda_1\lambda_2^{-1}\right)^{1-2\alpha/1-\alpha} x_{\lambda_2}(t-\tau(t)), \overline{\gamma}^{-1}\left(\lambda_1\lambda_2^{-1}\right)^{1/1-\alpha} \Phi x_{\lambda_2}(t-\delta(t))\right) \\
&\quad \geq \lambda_1 f\left(t, \overline{\gamma} x_{\lambda_2}(t-\tau(t)), \overline{\gamma}^{-1}\left(\lambda_1\lambda_2^{-1}\right)^{1/1-\alpha} \Phi x_{\lambda_2}(t-\delta(t))\right) \\
&\quad \geq \overline{\gamma}^{\alpha} \lambda_1 f\left(t, x_{\lambda_2}(t-\tau(t)), \left(\lambda_1\lambda_2^{-1}\right)^{1/1-\alpha} \Phi x_{\lambda_2}(t-\delta(t))\right) \\
&\quad \geq \overline{\gamma}^{\alpha} \lambda_1 f\left(t, \left(\lambda_1\lambda_2^{-1}\right)^{-1/1-\alpha} x_{\lambda_2}(t-\tau(t)), \left(\lambda_1\lambda_2^{-1}\right)^{1/1-\alpha} \Phi x_{\lambda_2}(t-\delta(t))\right) \\
&\quad \geq \overline{\gamma}^{\alpha} \lambda_1 \left(\lambda_1\lambda_2^{-1}\right)^{-\alpha/1-\alpha} f(t, x_{\lambda_2}(t-\tau(t)), \Phi x_{\lambda_2}(t-\delta(t))) \\
&\quad = \overline{\gamma}^{\alpha} \left(\lambda_1\lambda_2^{-1}\right)^{1-2\alpha/1-\alpha} \lambda_2 f(t, x_{\lambda_2}(t-\tau(t)), \Phi x_{\lambda_2}(t-\delta(t))), \\
&\lambda_2 f(t, x_{\lambda_2}(t-\tau(t)), \Phi x_{\lambda_2}(t-\delta(t))) \\
&\quad \geq \lambda_2 f\left(t, \overline{\gamma}\left(\lambda_1\lambda_2^{-1}\right)^{-1/1-\alpha} x_{\lambda_1}(t-\tau(t)), \overline{\gamma}^{-1}\left(\lambda_1\lambda_2^{-1}\right)^{-1-2\alpha/1-\alpha} \Phi x_{\lambda_1}(t-\delta(t))\right) \\
&\quad \geq \lambda_2 f\left(t, \overline{\gamma}\left(\lambda_1\lambda_2^{-1}\right)^{-1/1-\alpha} x_{\lambda_1}(t-\tau(t)), \overline{\gamma}^{-1} \Phi x_{\lambda_1}(t-\delta(t))\right) \\
&\quad \geq \overline{\gamma}^{\alpha} \lambda_2 f\left(t, \left(\lambda_1\lambda_2^{-1}\right)^{-1/1-\alpha} x_{\lambda_1}(t-\tau(t)), \Phi x_{\lambda_1}(t-\delta(t))\right) \\
&\quad \geq \overline{\gamma}^{\alpha} \lambda_2 f\left(t, \left(\lambda_1\lambda_2^{-1}\right)^{-1/1-\alpha} x_{\lambda_1}(t-\tau(t)), \left(\lambda_1\lambda_2^{-1}\right)^{1/1-\alpha} \Phi x_{\lambda_1}(t-\delta(t))\right) \\
&\quad \geq \overline{\gamma}^{\alpha} \lambda_2 \left(\lambda_1\lambda_2^{-1}\right)^{-\alpha/1-\alpha} f(t, x_{\lambda_1}(t-\tau(t)), \Phi x_{\lambda_1}(t-\delta(t))) \\
&\quad = \overline{\gamma}^{\alpha} \left(\lambda_1\lambda_2^{-1}\right)^{-1/1-\alpha} \lambda_1 f(t, x_{\lambda_1}(t-\tau(t)), \Phi x_{\lambda_1}(t-\delta(t))).
\end{aligned} \tag{2.20}$$

Therefore,

$$\begin{aligned}
x_{\lambda_1}(t) &= \Psi_{\lambda_1}(x_{\lambda_1}, x_{\lambda_1})(t) \geq \overline{\gamma}^{\alpha}\left(\lambda_1\lambda_2^{-1}\right)^{1-2\alpha/1-\alpha} x_{\lambda_2}(t), \\
x_{\lambda_2}(t) &= \Psi_{\lambda_2}(x_{\lambda_2}, x_{\lambda_2})(t) \geq \overline{\gamma}^{\alpha}\left(\lambda_1\lambda_2^{-1}\right)^{-1/1-\alpha} x_{\lambda_1}(t).
\end{aligned} \tag{2.21}$$

From (2.21), we have

$$\overline{\gamma}^{-\alpha}\left(\lambda_1\lambda_2^{-1}\right)^{1/1-\alpha} x_{\lambda_2}(t) \geq x_{\lambda_1}(t) \geq \overline{\gamma}^{\alpha}\left(\lambda_1\lambda_2^{-1}\right)^{1-2\alpha/1-\alpha} x_{\lambda_2}(t), \quad t \in \mathbf{R}. \tag{2.22}$$

Noticing that $0 < \overline{\gamma} < 1$ and $\alpha \in (0,1)$, one can see $\overline{\gamma}^{\alpha} > \overline{\gamma}$, a contradiction with the definition of $\overline{\gamma}$. Thus, $\overline{\gamma} \geq 1$ and

$$x_{\lambda_1}(t) \geq \overline{\gamma}\left(\lambda_1\lambda_2^{-1}\right)^{1-2\alpha/1-\alpha} x_{\lambda_2}(t) \geq \left(\lambda_1\lambda_2^{-1}\right)^{1-2\alpha/1-\alpha} x_{\lambda_2}(t) > x_{\lambda_2}(t), \quad t \in \mathbf{R}. \tag{2.23}$$

Thus, Conclusion (i) holds.

Next, let us prove Conclusion (ii).

In (2.23), let λ_1 be fixed and $\lambda = \lambda_2$; then

$$x_{\lambda}(t) \leq \left(\lambda\lambda_1^{-1}\right)^{1-2\alpha/1-\alpha} x_{\lambda_1}(t), \quad t \in \mathbf{R}. \tag{2.24}$$

Thus $\|x_\lambda\| \leq (\lambda\lambda_1^{-1})^{1-2\alpha/1-\alpha}\|x_{\lambda_1}\|$, which means $\|x_\lambda\| \to 0$ as $\lambda \to 0$.

Similarly, let λ_2 be fixed and $\lambda = \lambda_1$; then

$$x_{\lambda}(t) \geq \left(\lambda\lambda_2^{-1}\right)^{1-2\alpha/1-\alpha} x_{\lambda_2}(t), \quad t \in \mathbf{R}. \tag{2.25}$$

Therefore, $\|x_\lambda\| \geq (\lambda\lambda_2^{-1})^{1-2\alpha/1-\alpha}\|x_{\lambda_2}\|$, which implies that $\|x_\lambda\| \to \infty$ as $\lambda \to \infty$.

Finally, we prove Conclusion (iii).

For any fixed $\lambda_0 > 0$, let $\lambda > \lambda_0$. Set $\lambda_1 = \lambda_0$ in (2.24); then

$$x_{\lambda}(t) \leq \left(\lambda\lambda_0^{-1}\right)^{1-2\alpha/1-\alpha} x_{\lambda_0}(t), \quad t \in \mathbf{R}, \tag{2.26}$$

which means

$$\|x_\lambda - x_{\lambda_0}\| \leq \left(\left(\lambda\lambda_0^{-1}\right)^{1-2\alpha/1-\alpha} - 1\right)\|x_{\lambda_0}\|. \tag{2.27}$$

As a result, $\|x_\lambda - x_{\lambda_0}\| \to 0$ as $\lambda \to \lambda_0^+$. Similarly, we can show that $\|x_\lambda - x_{\lambda_0}\| \to 0$ as $\lambda \to \lambda_0^-$.

To sum up, the proof of this theorem is completed. □

3. An Illustrative Example

In this section, we give an illustrative example to show how to use our new results.

Example 3.1. Consider the following nonlinear nonautonomous delay differential system with feedback control:

$$\begin{aligned} \frac{dx}{dt} &= -(2+\cos t)x(t) + \lambda f(t, x(t-\tau(t)), u(t-\delta(t))), \quad t \in \mathbf{R}, \\ \frac{du}{dt} &= -(3+\sin t)u(t) + 3x(t-\sigma(t)), \end{aligned} \tag{3.1}$$

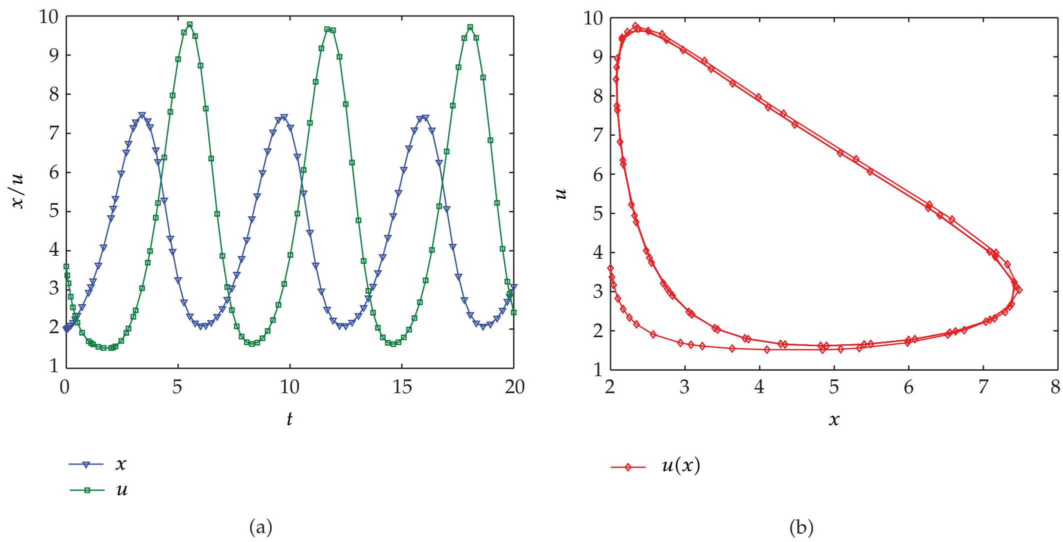

Figure 1: $t-x$, $t-u$ and $x-u$ graphs of Example 3.1 with $\lambda = 2$.

where $\lambda > 0$ is a parameter, $\tau(t), \delta(t), \sigma(t) \in C(\mathbf{R}, \mathbf{R})$ are 2π-periodic in t, and

$$f(t, x_1, x_2) = (2 + \sin t)\sqrt[3]{x_1} + \frac{1}{\sqrt[3]{x_2}}. \quad (3.2)$$

It is easy to see that $f(t, x_1, x_2) \in C(\mathbf{R} \times (0, +\infty) \times (0, +\infty) \to (0, +\infty))$ is 2π-periodic in t. $\eta(t) = 3 + \sin t$, $a(t) = 3$, $b(t) = 2 + \cos t \in C(\mathbf{R}, (0, +\infty))$ are 2π-periodic in t.

Since

$$\begin{aligned} \frac{\partial f(t, x_1, x_2)}{\partial x_1} &= \frac{2 + \sin t}{3x_1^{2/3}} > 0, \quad \forall t \in \mathbf{R},\ x_1, x_2 \in (0, +\infty), \\ \frac{\partial f(t, x_1, x_2)}{\partial x_2} &= -\frac{1}{3x_2^{4/3}} < 0, \quad \forall t \in \mathbf{R},\ x_1, x_2 \in (0, +\infty), \end{aligned} \quad (3.3)$$

we conclude that (H1) is satisfied.

Now, we check (H2). As a matter of fact, for all $t \in \mathbf{R}$, $x_1, x_2 \in (0, +\infty)$, we have

$$f\left(t, kx_1, k^{-1}x_2\right) = \sqrt[3]{k}\left[(2 + \sin t)\sqrt[3]{x_1} + \frac{1}{\sqrt[3]{x_2}}\right] \geq \sqrt[3]{k} f(t, x_1, x_2), \quad (3.4)$$

therefore, (H2) holds.

Hence, Theorem 2.7 shows that for any $\lambda > 0$, the system (3.1) has a unique positive 2π-periodic solution.

Let us set $\lambda = 2$, $\tau(t) = 1$, $\delta(t) = 0.1$, $\sigma(t) = 2$; then, the unique positive 2π-periodic solution of the system (3.1) can be shown in Figure 1.

Next, to illustrate Theorem 2.8, we set $\lambda = 2,\ 2.1,\ 2.2,\ 2.3,\ 2.4$, and 2.5, respectively, and let $\tau(t) = 1$, $\delta(t) = 0.1$, $\sigma(t) = 2$; then the unique positive 2π-periodic solutions of

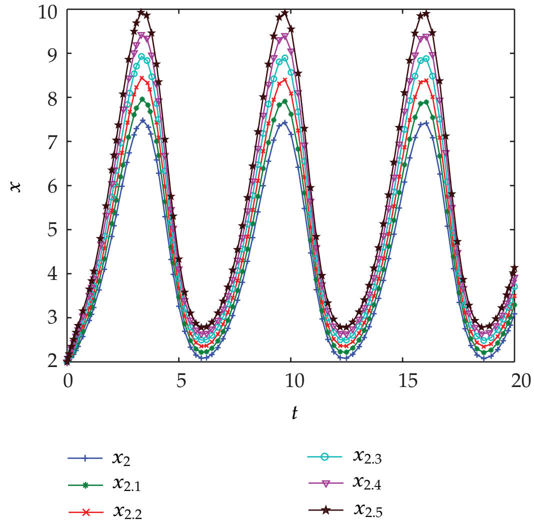

Figure 2: The graphs of x_λ with different λ.

the system (3.1) with these different λ can be shown in Figure 2. From this figure, one can easily see that $x_\lambda(t)$ is strictly increasing in λ.

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