IJMMS 25:6 (2001) 365–372
PII. S0161171201005130
http://ijmms.hindawi.com

ON ALMOST (N,p,q) SUMMABILITY OF CONJUGATE FOURIER SERIES

SHYAM LAL and HARE KRISHNA NIGAM

(Received 9 May 2000)

ABSTRACT. A new theorem on almost generalized Nörlund summability of conjugate series of Fourier series has been established under a very general condition.

2000 Mathematics Subject Classification. Primary 42B05, 42B08.

1. Introduction. Lorentz [3], for the first time in 1948, defined almost convergence of a bounded sequence. It is easy to see that a convergent sequence is almost convergent [4]. The idea of almost convergence led to the formulation of almost generalized Nörlund summability method. Here, almost generalized Nörlund summability method is considered. In 1913, Hardy [1] established (c,α), $\alpha > 0$ summability of the series. Later on in 1948, harmonic summability which is weaker than the summability (c,α), $\alpha > 0$ of the series was discussed by Siddiqi [8]. The generalization of Siddiqi has been obtained by several workers, for example, Singh [9, 10], Iyengar [2], Pati [5], Tripathi [11], Rajagopal [7] for Nörlund mean. But nothing seems to have been done so far in the direction of study of conjugate Fourier series by almost generalized Nörlund summability method. Almost generalized Nörlund summability includes almost Nörlund, Riesz, harmonic and Cesàro as particular cases. In an attempt to make an advance study in this direction, in the present paper, a theorem on almost generalized Nörlund summability of conjugate Fourier series has been obtained.

2. Definitions and notations. Let $\sum a_n$ be an infinite series with $\{S_n\}$ as the sequence of its nth partial sums. Lorentz [3] has given the following definition.

A bounded sequence $\{S_n\}$ is said to be almost convergent to a limit S, if

$$\lim_{n\to\infty} \frac{1}{n+1} \sum_{\nu=m}^{n+m} S_\nu = S, \quad \text{uniformly with respect to } m. \tag{2.1}$$

Let $\{p_n\}$ and $\{q_n\}$ be the two sequences of non-zero real constants such that

$$P_n = p_0 + p_1 + p_2 + \cdots + p_n, \qquad P_{-1} = p_{-1} = 0, \tag{2.2a}$$

$$Q_n = q_0 + q_1 + q_2 + \cdots + q_n, \qquad Q_{-1} = q_{-1} = 0. \tag{2.2b}$$

Given two sequences $\{p_n\}$, $\{q_n\}$, convolution $p*q$ is defined by

$$R_n = (p*q)_n = \sum_{k=0}^{n} p_k q_{n-k}. \tag{2.3}$$

It is familiar and can be easily verified that the operation of convolution is commutative and associative, and

$$(p*1)_n = \sum_{k=0}^{n} p_k. \tag{2.4}$$

The series $\sum a_n$ or the sequence $\{S_n\}$ is said to be almost generalized Nörlund (N,p,q) (Qureshi [6]) summable to S, if

$$t_{n,m} = \frac{1}{R_n}\sum_{\nu=0}^{n} p_{n-\nu}\, q_\nu\, S_{\nu,m} \tag{2.5}$$

tends to S, as $n \to \infty$, uniformly with respect to m, where

$$S_{\nu,m} = \frac{1}{\nu+1}\sum_{k=m}^{\nu+m} S_k. \tag{2.6}$$

Particular cases. (a) Almost (N,p,q) method reduces to almost Nörlund method (N,p_n) if $q_n = 1$ for all n.

(b) Almost (N,p,q) method reduces to almost Riesz method $(\overline{N},q_n)$ if $p_n = 1$ for all n.

(c) In the special case when $p_n = \binom{n+\alpha-1}{\alpha-1}$, $\alpha > 0$, the method (N,p_n) reduces to the well-known method of summability (C,α).

(d) $p_n = (n+1)^{-1}$ of the Nörlund mean is known as harmonic mean and is written as $(N,1/(n+1))$.

Let $f(t)$ be a periodic function with period 2π and integrable in the sense of Lebesgue over an interval $(-\pi,\pi)$.

Let its Fourier series be given by

$$f(t) \sim \frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n\cos nt + b_n \sin nt) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} A_n(t) \tag{2.7}$$

and then the conjugate series of (2.7) is given by

$$\sum_{n=1}^{\infty}(a_n\sin nt - b_n\cos nt) = \sum_{n=1}^{\infty} B_n(t). \tag{2.8}$$

We will use the following notations:

$$\begin{aligned}
\phi(t) &= f(x+t) + f(x-t) - 2f(x),\\
\psi(t) &= f(x+t) - f(x-t),\\
\Phi(t) &= \int_0^t |\phi(u)|\,du,\\
\Psi(t) &= \int_0^t |\psi(u)|\,du,\\
\tau &= \left[\frac{1}{t}\right] = \text{The integral part of } \frac{1}{t},
\end{aligned} \tag{2.9}$$

$$N_{n,m}(t) = \frac{1}{2\pi R_n} \sum_{\nu=0}^{n} p_{n-\nu} q_\nu \frac{\sin(\nu+1)(t/2)\{\cos(\nu+2m+1)(t/2) - \cos(t/2)\}}{(\nu+1)\sin^2(t/2)}, \tag{2.10}$$

$$\overline{N}_{n,m}(t) = \frac{1}{2\pi R_n} \sum_{\nu=0}^{n} p_{n-\nu} q_\nu \frac{\cos(\nu+2m+1)(t/2)\sin(\nu+1)(t/2)}{(\nu+1)\sin^2(t/2)}. \tag{2.11}$$

3. Known theorem. Pati [5] has established the following theorem for Nörlund summability of a Fourier series.

Theorem 3.1. *Let (N, p_n) be a regular Nörlund method defined by a real non-negative monotonic non-increasing sequence of coefficients $\{p_n\}$ such that*

$$P_n = \sum_{\nu=0}^{n} p_\nu \longrightarrow \infty, \quad \text{as } n \longrightarrow \infty, \tag{3.1}$$

and

$$\log n = O(P_n), \quad \text{as } n \longrightarrow \infty, \tag{3.2}$$

then if

$$\Phi(t) = \int_0^t |\phi(u)|\,du = o\left[\frac{t}{P_\tau}\right], \quad \text{as } t \longrightarrow +0, \tag{3.3}$$

the series (2.7) is summable (N, p_n) to $f(x)$ at the point $t = x$.

4. Main theorem. In this paper, we aim to generalize the above result for almost (N,p,q) summability of conjugate Fourier series in the following form.

Theorem 4.1. *Let $\{p_n\}$ and $\{q_n\}$ be the monotonic non-increasing sequences of real constants such that $R_n = \sum_{\nu=0}^{n} p_\nu q_{n-\nu} \to \infty$, as $n \to \infty$. If*

$$\Psi(t) = \int_0^t |\psi(u)|\,du = o\left[\frac{\alpha(1/t)t}{R_{(1/t)}}\right], \quad \text{as } t \longrightarrow +0, \tag{4.1}$$

$$\int_{1/(n+m)}^{1/(n+m)^\delta} \frac{|\psi(t)|}{t^2}\,dt = o(n), \quad \text{as } n \longrightarrow \infty, \tag{4.2}$$

where $0 < \delta < 1/2$, uniformly with respect to m, and $\alpha(t)$ is a positive monotonic non-increasing function of t such that

$$\alpha(n+m)\log(n+m) = O(R_{n+m}), \quad \text{as } n \longrightarrow \infty, \tag{4.3}$$

$$\sum_{\nu=0}^{n} \left(\frac{p_{n-\nu} q_\nu}{(\nu+1)}\right) = O\left(\frac{R_n}{n}\right), \tag{4.4}$$

then the conjugate Fourier series (2.8) is almost (N,p,q) summable to $-(1/2\pi)\int_0^\pi \cot(1/2)t\ \psi(t)dt$ at point $t = x$.

5. Lemmas. For the proof of Theorem 4.1, the following lemmas are required.

Lemma 5.1. *For* $0 < t < 1/(n+m)$, *we have*

$$
\begin{aligned}
&|N_{n,m}(t)| \\
&= \frac{1}{2\pi R_n}\left|\sum_{\nu=0}^{n} p_{n-\nu}q_\nu \frac{\sin(\nu+1)(t/2)\{\cos(\nu+2m+1)(t/2)-\cos(t/2)\}}{(\nu+1)\sin^2(t/2)}\right| \\
&= \frac{1}{2\pi R_n}\left|\sum_{\nu=0}^{n} p_{n-\nu}q_\nu \frac{\sin(\nu+1)(t/2)\{2\sin((\nu+2m+2)/2)(t/2)\sin((\nu+2m)/2)(t/2)\}}{(\nu+1)\sin^2(t/2)}\right| \\
&\le \frac{1}{2\pi R_n}\left|\sum_{\nu=0}^{n} p_{n-\nu}q_\nu \frac{(\nu+1)\sin(t/2)\{2\sin((\nu+2m+2)/2)(t/2)\sin((\nu+2m)/2)(t/2)\}}{(\nu+1)\sin^2(t/2)}\right| \\
&\le \frac{1}{2\pi R_n}\sum_{\nu=0}^{n} p_{n-\nu}q_\nu \frac{2((\nu+2m+2)/2)\,|\sin(t/2)|\,|\sin((\nu+2m)/2)(t/2)|}{|\sin(t/2)|} \\
&= \frac{1}{2\pi R_n}\left\{\sum_{\nu=0}^{n} p_{n-\nu}q_\nu\right\}(n+2m+2) \\
&= O(n+m)\frac{1}{R_n}\sum_{\nu=0}^{n} p_{n-\nu}q_\nu
\end{aligned}
$$

$$|N_{n,m}(t)| = O(n+m) \quad \text{by (4.4).} \tag{5.1}$$

Lemma 5.2. *For* $1/(n+m) < t < \pi$, *we have*

$$\overline{N}_{n,m}(t) = \frac{1}{2\pi R_n}\sum_{\nu=0}^{n} p_{n-\nu}q_\nu \frac{\cos(m+(\nu+1)/2)t\sin((\nu+1)/2)t}{(\nu+1)\sin^2(t/2)},$$

$$
\begin{aligned}
|\overline{N}_{n,m}(t)| &\le \frac{1}{2\pi R_n}\sum_{\nu=0}^{n} p_{n-\nu}q_\nu \frac{\cos(m+(\nu+1)/2)t\sin((\nu+1)/2)t}{(\nu+1)\sin^2(t/2)} \\
&\le \frac{1}{2\pi R_n}\sum_{\nu=0}^{n} \frac{p_{n-\nu}q_\nu}{(\nu+1)}\frac{1}{\sin^2(t/2)} = O\left(\frac{1}{t^2}\right)\frac{1}{R_n}\sum_{\nu=0}^{n}\left(\frac{p_{n-\nu}q_\nu}{(\nu+1)}\right),
\end{aligned}
$$

$$|\overline{N}_{n,m}(t)| = O\left(\frac{1}{t^2 n}\right) \quad \text{by (4.4).} \tag{5.2}$$

Proof of Theorem 4.1. Let $S_k(x)$ denote the nth partial sum of the series (2.8). Then we have

$$
\begin{aligned}
S_k(x) &= \frac{1}{2\pi}\int_0^{\pi}\frac{\cos(k+(1/2))t-\cos(t/2)}{\sin(t/2)}\psi(t)\,dt \\
&= \frac{1}{2\pi}\int_0^{\pi}\frac{\cos(k+(1/2))t}{\sin(t/2)}\psi(t)\,dt - \frac{1}{2\pi}\int_0^{\pi}\cot\left(\frac{t}{2}\right)\psi(t)\,dt.
\end{aligned}
\tag{5.3}
$$

Now, by using (2.6) we get

$$S_{\nu,m} = \frac{1}{\nu+1}\sum_{k=m}^{\nu+m}\left\{\frac{1}{2\pi}\int_0^{\pi}\frac{\cos(k+(1/2))t}{\sin(t/2)}\psi(t)\,dt - \frac{1}{2\pi}\int_0^{\pi}\cot\left(\frac{t}{2}\right)\psi(t)\,dt\right\}, \tag{5.4}$$

so that by using (2.5) we have

$$t_{n,m}=\frac{1}{R_n}\sum_{\nu=0}^{n}p_{n-\nu}q_\nu\left\{\frac{1}{2\pi}\int_0^\pi\sum_{k=m}^{\nu+m}\frac{\cos(k+(1/2))t}{(\nu+1)\sin(t/2)}\psi(t)\,dt-\frac{1}{2\pi}\int_0^\pi\cot\left(\frac{t}{2}\right)\psi(t)\,dt\right\}$$

$$\begin{aligned}
t_{n,m}-&\left(-\frac{1}{2\pi}\int_0^\pi\cot\left(\frac{t}{2}\right)\psi(t)dt\right)\\
&=\frac{1}{R_n}\sum_{\nu=0}^{n}p_{n-\nu}q_\nu\frac{1}{2\pi(\nu+1)}\int_0^\pi\sum_{k=m}^{\nu+m}\frac{\cos(k+(1/2))t}{\sin(t/2)}\psi(t)dt\\
&=\frac{1}{2\pi R_n}\sum_{\nu=0}^{n}p_{n-\nu}q_\nu\int_0^\pi\frac{\sin(\nu+m+1)t-\sin mt}{2(\nu+1)\sin^2(t/2)}\psi(t)\,dt\\
&=\int_0^\pi\left\{\frac{1}{2\pi R_n}\sum_{\nu=0}^{n}p_{n-\nu}q_\nu\frac{\cos(\nu+2m+1)(t/2)\sin(\nu+1)(t/2)}{(\nu+1)\sin^2(t/2)}\right\}\psi(t)\,dt\\
&=\int_0^\pi\overline{N}_{n,m}(t)\psi(t)\,dt\\
&=\left\{\int_0^{1/(n+m)}+\int_{1/(n+m)}^{1/(n+m)^\delta}+\int_{1/(n+m)^\delta}^{\pi}\right\}\overline{N}_{n,m}(t)\psi(t)\,dt=I_1+I_2+I_3.
\end{aligned}\tag{5.5}$$

First we consider,

$$\begin{aligned}
I_1&=\int_0^{1/(n+m)}\overline{N}_{n,m}(t)\psi(t)\,dt\\
&=\int_0^{1/(n+m)}\frac{1}{2\pi R_n}\sum_{\nu=0}^{n}p_{n-\nu}q_\nu\frac{\cos(\nu+2m+1)(t/2)\sin(\nu+1)(t/2)}{(\nu+1)\sin^2(t/2)}\psi(t)\,dt\\
&=\int_0^{1/(n+m)}\frac{1}{2\pi R_n}\sum_{\nu=0}^{n}p_{n-\nu}q_\nu\frac{\sin(\nu+1)(t/2)\{\cos(\nu+2m+1)(t/2)-\cos(t/2)\}}{(\nu+1)\sin^2(t/2)}\psi(t)\,dt\\
&\quad+\int_0^{1/(n+m)}\frac{1}{2\pi R_n}\sum_{\nu=0}^{n}p_{n-\nu}q_\nu\frac{\sin(\nu+1)(t/2)\cot(t/2)}{(\nu+1)\sin(t/2)}\psi(t)\,dt=I_{1.1}+I_{1.2}.
\end{aligned}\tag{5.6}$$

Now

$$\begin{aligned}
|I_{1.1}|&\le\int_0^{1/(n+m)}\frac{1}{2\pi R_n}\left|\sum_{\nu=0}^{n}p_{n-\nu}q_\nu\frac{\sin(\nu+1)(t/2)\{\cos(\nu+2m+1)(t/2)-\cos(t/2)\}}{(\nu+1)\sin^2(t/2)}\right||\psi(t)|\,dt\\
&=\int_0^{1/(n+m)}|N_{n,m}(t)|\,|\psi(t)|\,dt\\
&=O(n+m)\int_0^{1/(n+m)}|\psi(t)|\,dt\quad\text{by Lemma 5.1}\\
&=O(n+m)o\left[\frac{\alpha(n+m)}{(n+m)R_{n+m}}\right]\quad\text{by (4.1)}\\
&=o\left[\frac{\alpha(n+m)}{R_{n+m}}\right]=o\left[\frac{1}{\log(m+n)}\right]\quad\text{by (4.3)}
\end{aligned}$$

$$I_{1.1}=o(1),\quad\text{as } n\longrightarrow\infty,\text{ uniformly with respect to } m.\tag{5.7}$$

Next, for $0 < t \leq 1(n+m)$

$$
\begin{aligned}
|I_{1.2}| &\leq \frac{1}{2\pi R_n} \sum_{\nu=0}^{n} p_{n-\nu} q_\nu \int_0^{1/(n+m)} \frac{\sin(\nu+1)(t/2)\cot(t/2)}{(\nu+1)\sin(t/2)} \psi(t)\,dt \\
&\leq \frac{1}{2\pi R_n} \sum_{\nu=0}^{n} p_{n-\nu} q_\nu \int_0^{1/(n+m)} \frac{(\nu+1)\sin(t/2)\cot(t/2)}{(\nu+1)\sin(t/2)} \psi(t)\,dt \\
&\qquad - \frac{1}{2\pi} \int_0^{1/(n+m)} \cot\left(\frac{t}{2}\right) \psi(t)\,dt
\end{aligned}
$$

since the conjugate function exists, therefore

$$
-\frac{1}{2\pi} \int_0^{1/(n+m)} \cot\left(\frac{t}{2}\right) \psi(t)\,dt = o(1), \quad \text{as } n \longrightarrow \infty, \text{ uniformly with respect to } m. \tag{5.8}
$$

Hence,

$$
I_{1.2} = o(1), \tag{5.9}
$$

thus from (5.6), (5.7), and (5.9)

$$
I_1 = o(1). \tag{5.10}
$$

Now, we take

$$
\begin{aligned}
|I_2| &\leq \int_{1/(n+m)}^{1/(n+m)^\delta} |\overline{N}_{n,m}(t)|\,|\psi(t)|\,dt \\
&= O\int_{1/(n+m)}^{1/(n+m)^\delta} \frac{|\psi(t)|}{t^2 n}\,dt \quad \text{by Lemma 5.2} \\
&= O\left(\frac{1}{n}\right) \int_{1/(n+m)}^{1/(n+m)^\delta} \frac{|\psi(t)|}{t^2}\,dt \\
&= O\left(\frac{1}{n}\right) o(n) \quad \text{by (4.2)} \\
I_2 &= o(1), \quad \text{as } n \longrightarrow \infty, \text{ uniformly with respect to } m.
\end{aligned} \tag{5.11}
$$

Finally, we have

$$
\begin{aligned}
|I_3| &\leq \int_{1/(n+m)^\delta}^{\pi} \frac{1}{2\pi R_n} \sum_{\nu=0}^{n} p_{n-\nu} q_\nu \left| \frac{\cos(\nu+2m+1)(t/2)\sin(\nu+1)(t/2)}{(\nu+1)\sin^2(t/2)} \right| |\psi(t)|\,dt \\
&= \int_{1/(n+m)^\delta}^{\pi} \frac{1}{2\pi R_n} \sum_{\nu=0}^{n} p_{n-\nu} q_\nu \left| \frac{\sin(\nu+m+1)t - \sin mt}{2(\nu+1)\sin^2(t/2)} \right| |\psi(t)|\,dt \\
&= \frac{1}{2\pi R_n} \sum_{\nu=0}^{n} p_{n-\nu} q_\nu \left[\int_{1/(n+m)^\delta}^{\pi} \left| \frac{\sin(\nu+m+1)t}{2(\nu+1)\sin^2(t/2)} \right| |\psi(t)|\,dt \right. \\
&\qquad \left. + \int_{1/(n+m)^\delta}^{\pi} \left| \frac{\sin mt}{2(\nu+1)\sin^2(t/2)} \right| |\psi(t)|\,dt \right] \\
&= I_{3.1} + I_{3.2}.
\end{aligned} \tag{5.12}
$$

Now, by using second mean value theorem, we have

$$|I_{3.1}| \le \frac{1}{2\pi R_n}\sum_{\nu=0}^{n}\frac{p_{n-\nu}q_\nu}{2(\nu+1)}\frac{1}{2\sin^2(1/2(n+m)^\delta)}\int_{1/(n+m)^\delta}^{\epsilon}|\sin(\nu+m+1)t|\,|\psi(t)|\,dt,$$

$$\text{where } \frac{1}{(n+m)^\delta}<\epsilon<\pi,\ 0<\delta<\frac{1}{2}$$

$$= O\left(\frac{1}{n}\right)(n+m)^{2\delta}\left(\frac{1/2(n+m)^\delta}{\sin(1/2(n+m)^\delta)}\right)^2\int_{1/(n+m)^\delta}^{\epsilon}|\psi(t)|\,dt$$

$$I_{3.1}=o(1),\quad \text{as } n\longrightarrow\infty, \text{ uniformly with respect to } m. \tag{5.13}$$

Now,

$$|I_{3.2}| \le \frac{1}{2\pi R_n}\int_{1/(n+m)^\delta}^{\pi}\sum_{\nu=0}^{n}p_{n-\nu}q_\nu\left|\frac{\sin mt}{2(\nu+1)\sin^2(t/2)}\right||\psi(t)|\,dt$$

$$\le \frac{1}{2\sin^2(1/2(n+m)^\delta)}\int_{1/(n+m)^\delta}^{\epsilon}|\psi(t)|\,dt$$

$$I_{3.2}=o(1),\quad \text{as } n\longrightarrow\infty, \text{ uniformly with respect to } m. \tag{5.14}$$

Hence,

$$I_3=o(1),\quad \text{as } n\longrightarrow\infty. \tag{5.15}$$

Now, by combining (5.5), (5.10), (5.11), and (5.15), we have

$$\int_0^{\pi}N_{n,m}(t)\psi(t)\,dt=o(1),\quad \text{as } n\longrightarrow\infty, \text{ uniformly with respect to } m. \tag{5.16}$$

Thus, the theorem is established. □

6. Applications.

In this section, we deduce some corollaries from Theorem 4.1.

Corollary 6.1. *If*

$$\Psi(t)=\int_0^t|\psi(u)|\,du=o\left[\frac{t}{R_{(1/t)}}\right], \tag{6.1}$$

$$\log(n+m)=O(R_{n+m}),\quad \textit{as } n\longrightarrow\infty, \tag{6.2}$$

conditions (4.2) and (4.4) of the main theorem are satisfied, then the conjugate Fourier series is almost (N,p,q) summable to $-(1/2\pi)\int_0^\pi\psi(t)\cot(1/2)t\,dt$.

Corollary 6.2. *If*

$$\Psi(t)=\int_0^t|\psi(u)|\,du=o\left[\frac{t}{\log(1/t)}\right], \tag{6.3}$$

conditions (4.2) and (4.4) of Theorem 4.1 hold, then the conjugate Fourier series is almost (N,p,q) summable to $-(1/2\pi)\int_0^\pi\psi(t)\cot(1/2)t\,dt$ without employing (4.3).

Acknowledgements. The authors are grateful to L. M. Tripathi, Department of Mathematics, B.H.U, Varanasi-221005 for reading the manuscript. The authors are also thankful to A. P. Dwivedi, Head of Department of Mathematics and V. K. Jain, Director, H.B.T.I, Kanpur for their encouragement.

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Shyam Lal: Department of Mathematics, Harcourt Butler Technological Institute, Nawab Ganj, Kanpur 208002 (U.P), India

Hare Krishna Nigam: Department of Mathematics, Harcourt Butler Technological Institute, Nawab Ganj, Kanpur 208002 (U.P), India
E-mail address: harekrishnan@hotmail.com