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UNSTEADY FLOW INDUCED BY VARIABLE SUCTION ON A POROUS DISK ROTATING ECCENTRICALLY WITH A FLUID AT INFINITY

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We study the effect of variable suction or blowing on the flow of an incompressible viscous fluid due to noncoaxial rotations of a porous disk and a fluid at infinity. The inquiries are made about the components of fluid velocity and the shear stress at the disk. It is found that the effect of uniform suction or blowing on the flow is enhanced in the presence of variable suction or blowing.

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1. Introduction. Thornley [5] has studied the unsteady flow developed in an incompressible viscous fluid due to nontorsional oscillations of an infinite rigid plate when both the fluid and the plate are in a state of solid body rotation. A similar problem of magnetohydrodynamic Ekman layer over an infinite rigid nonconducting plate was examined by Gupta [3]. On the other hand, the flow due to noncoaxial rotations of a disk and a fluid at infinity was initiated by Berker [1]. Subsequently, Erdogan [2] constructed solutions of the problem of steady flow due to eccentrically rotations of a porous disk and a fluid at infinity with the same angular velocity both for the cases of uniform suction and blowing at the disk. Of late, Kasiviswanathan and Rao [4] obtained an exact solution for the unsteady flow due to noncoaxial rotations of a porous disk oscillating in its own plane and a fluid at infinity. In the present paper, the flow due to noncoaxial rotations of a porous disk subjected to variable suction or blowing and a fluid at infinity has been investigated. Analytical solutions are obtained both for the components of fluid velocity and the components of shear stress at the disk. Quantitative evolution of the results are also made with a view to examine the effects of variable suction and blowing on the flow. It is found that the effects of uniform suction or blowing on the flow field enhances in presence of variable suction or blowing at the disk.

2. Formulation of the problem. We consider the flow due to a porous disk lying in the xy-plane rotating about the z-axis perpendicular to the disk with uniform angular velocity Ω. The fluid at infinity rotates with the same angular velocity Ω about an axis parallel to the z-axis passing through the point (x, y). The unsteady motion is established in the fluid due to variable suction at the disk. For this motion, the velocity field has the form

$$u = -\Omega y + g(z,t), \qquad v = \Omega x - f(z,t), \qquad w = -V(t), \tag{2.1}$$

where $V(t) > 0$ represents the suction velocity which satisfies the equation of continuity. Introducing (2.1) in Navier-Stokes equations, we get

$$\left(v\frac{\partial^2 g}{\partial z^2}+V(t)\frac{\partial g}{\partial z}-\frac{\partial g}{\partial t}-\Omega f\right)=-\Omega^2 x+\frac{1}{\rho}\frac{\partial p}{\partial x}, \tag{2.2}$$

$$\left(v\frac{\partial^2 f}{\partial z^2}+V(t)\frac{\partial f}{\partial z}-\frac{\partial f}{\partial t}+\Omega g\right)=\Omega^2 y+\frac{1}{\rho}\frac{\partial p}{\partial x}, \tag{2.3}$$

$$\frac{\partial V(t)}{\partial t}=\frac{1}{\rho}\frac{\partial p}{\partial z}. \tag{2.4}$$

We now suppose that the suction velocity normal to the disk oscillates in magnitude and not in direction about a nonzero mean given by

$$V(t)=W_0\left(1+\varepsilon A e^{i\sigma t}\right), \tag{2.5}$$

where W_0 is a positive constant; $\varepsilon > 0$ is small and A is a real positive constant such that $\varepsilon A \ll 1$.

From (2.4) and (2.5), we find that $\partial p/\partial z$ is small and hence can be neglected. This shows that p is independent of z.

Eliminating p from (2.2) and (2.3) by differentiating with respect to z and combining them, we get

$$v\frac{\partial^3 U}{\partial z^3}+V(t)\frac{\partial^2 U}{\partial z^2}-\frac{\partial^2 U}{\partial z\partial t}-i\Omega\frac{\partial U}{\partial z}=0, \tag{2.6}$$

where $U=f+ig$.

Since no unsteady motion other than suction is imposed on the disk, we must have the boundary conditions for $U(z,t)$ as

$$U(z,t)=0 \quad \text{at } z=0, \qquad U(z,t)=\Omega\left(x_1+iy_1\right) \quad \text{at } z=\infty,\ t>0. \tag{2.7}$$

In addition to these, we assume that the solutions are bounded at infinity.

Again, from (2.5), we assumed that

$$U(z,t)=F_0(z)+\varepsilon F_1(z)e^{i\sigma t}. \tag{2.8}$$

Substituting (2.8) and (2.5) in (2.3), comparing harmonic terms and neglecting coefficient of ε^2, we get

$$v\frac{d^3 F_0}{dz^3}+W_0\frac{d^2 F_0}{dz^2}-i\Omega\frac{dF_0}{dz}=0, \tag{2.9}$$

$$v\frac{d^3 F_1}{dz^3}+W_0\frac{d^2 F_1}{dz^2}-i(\Omega+\sigma)\frac{dF_1}{dz}=-W_0 A\frac{d^2 F_0}{dz^2}, \tag{2.10}$$

with

$$F_0(0)=0,\qquad F_1(0)=0,$$
$$F_0(\infty)=\Omega(x_1+iy_1),\qquad F_1(\infty)=0. \tag{2.11}$$

3. Solution of the problem. We introduce $\xi=\sqrt{\Omega/2\nu}z$, $S=W_0/2\sqrt{\Omega\nu}$, and $n=\sqrt{1+\sigma/\Omega}$ in (2.10) and (2.11) to obtain

$$\frac{d^3F_0}{d\xi^3}+2\sqrt{2}S\frac{d^2F_0}{d\xi^2}-2i\frac{dF_0}{d\xi}=0, \tag{3.1}$$

$$\frac{d^3F_1}{d\xi^3}+2\sqrt{2}S\frac{d^2F_1}{d\xi^2}-2in^2\frac{dF_1}{d\xi}=-2^{3/2}\varepsilon A\frac{d^2F_0}{d\xi^2}, \tag{3.2}$$

subject to conditions (2.11).

On solving (3.1) and (3.2) subject to (2.11), we get

$$\begin{aligned}\frac{f}{\Omega}&=x_1\left(1-e^{-\alpha_0\xi}\cos\beta_0\xi\right)-y_1e^{-\alpha_0\xi}\sin\beta_0\xi\\&\quad+\frac{\varepsilon 2^{3/2}AS}{P^2+Q^2}\Big[(x_1L-y_1M)\left[e^{-\alpha_0\xi}\cos(\beta_0\xi-\sigma t)-e^{-\alpha_1\xi}\cos(\beta_1\xi-\sigma t)\right]\\&\qquad+(y_1L+x_1M)\left[e^{-\alpha_0\xi}\sin(\beta_0\xi-\sigma t)-e^{-\alpha_1\xi}\sin(\beta_1\xi-\sigma t)\right]\Big],\end{aligned} \tag{3.3}$$

$$\begin{aligned}\frac{g}{\Omega}&=y_1\left(1-e^{-\alpha_0\xi}\cos\beta_0\xi\right)+x_1e^{-\alpha_0\xi}\sin\beta_0\xi\\&\quad+\frac{\varepsilon 2^{3/2}AS}{P^2+Q^2}\Big[(y_1L+x_1M)\left[e^{-\alpha_0\xi}\cos(\beta_0\xi-\sigma t)-e^{-\alpha_1\xi}\cos(\beta_1\xi-\sigma t)\right]\\&\qquad-(x_1L-y_1M)\left[e^{-\alpha_0\xi}\sin(\beta_0\xi-\sigma t)-e^{-\alpha_1\xi}\sin(\beta_1\xi-\sigma t)\right]\Big],\end{aligned} \tag{3.4}$$

where $\alpha_0=\sqrt{2}S+\gamma_0$, $\gamma_0=[\sqrt{S^4+1}+S^2]^{1/2}$, $\beta_0=[\sqrt{S^4+1}-S^2]^{1/2}$, $\alpha_1=\sqrt{2}S+\gamma_1$, $\gamma_1=[\sqrt{S^4+n^4}+S^2]^{1/2}$, $\beta_1=[\sqrt{S^4+n^4}-S^2]^{1/2}$, $L=\alpha_0P-\beta_0Q$, $M=\beta_0P-\alpha_0Q$, $P=(\alpha_1-\alpha_0)[\alpha_1+\alpha_0-2\sqrt{2}S]-(\beta_1^2-\beta_0^2)$, $Q=(\beta_1-\beta_0)[\alpha_1+\alpha_0-2\sqrt{2}S]-(\alpha_1-\alpha_0)(\beta_1+\beta_0)$.

In particular, when $A=0$, the general results (3.3) and (3.4) reduce to

$$\frac{f}{\Omega}=x_1\left(1-e^{-\alpha_0\xi}\cos\beta_0\xi\right)-y_1e^{-\alpha_0\xi}\sin\beta_0\xi, \tag{3.5}$$

$$\frac{g}{\Omega}=y_1\left(1-e^{-\alpha_0\xi}\sin\beta_0\xi\right)+x_1e^{-\alpha_0\xi}\sin\beta_0\xi. \tag{3.6}$$

These results coincide with the nonoscillating part of the results [4, (11), (12)] and describe the flow in absence of variable suction at the disk.

Again, on putting $x_1=0$ and $y_1=l$ in (3.3) and (3.4), and replacing $-f$ by g and g by f, we get

$$\begin{aligned}\frac{f}{\Omega l}&=\left(1-e^{-\alpha_0\xi}\cos\beta_0\xi\right)+\frac{\varepsilon 2^{3/2}AS}{P^2+Q^2}\Big\{L\left[e^{-\alpha_0\xi}\cos(\beta_0\xi-\sigma t)-e^{-\alpha_1\xi}\cos(\beta_1\xi-\sigma t)\right]\\&\qquad+M\left[e^{-\alpha_0\xi}\sin(\beta_0\xi-\sigma t)-e^{-\alpha_1\xi}\sin(\beta_1\xi-\sigma t)\right]\Big\},\end{aligned} \tag{3.7}$$

$$\frac{g}{\Omega l} = e^{-\alpha_0\xi}\sin\beta_0\xi + \frac{\varepsilon 2^{3/2}AS}{P^2+Q^2}\Big\{M\big[e^{-\alpha_0\xi}\cos(\beta_0\xi-\sigma t) - e^{-\alpha_1\xi}\cos(\beta_1\xi-\sigma t)\big] - L\big[e^{-\alpha_0\xi}\sin(\beta_0\xi-\sigma t) - e^{-\alpha_1\xi}\sin(\beta_1\xi-\sigma t)\big]\Big\}, \tag{3.8}$$

which are exactly the same as those given in [2] when $A = 0$. Thus, the effect of variable suction at the disk introduces a transient part depending on ε, A, and σ superposed on the steady solution corresponding to uniform suction at the disk. The case of $S = 0$ corresponds to impermeable case and recovers the solution for steady Ekman layer on the disk. For the flow very near to the porous disk, we have, from (3.7) and (3.8),

$$\frac{f}{\Omega l} = \alpha_0\xi + \frac{\varepsilon 2^{3/2}AS}{P^2+Q^2}\Big[L\{(\alpha_1-\alpha_0)\cos\sigma t - (\beta_1-\beta_0)\sin\sigma t\} - M\{(\beta_1-\beta_0)\cos\sigma t + (\alpha_1-\alpha_0)\sin\sigma t\}\Big], \tag{3.9}$$

$$\frac{g}{\Omega l} = \beta_0\xi + \frac{\varepsilon 2^{3/2}AS}{P^2+Q^2}\Big[M\{(\alpha_1-\alpha_0)\cos\sigma t - (\beta_1-\beta_0)\sin\sigma t\} - L\{(\beta_1-\beta_0)\cos\sigma t + (\alpha_1-\alpha_0)\sin\sigma t\}\Big]. \tag{3.10}$$

Consequently, the inclination of the fluid velocity vector to y-axis near $z = 0$ becomes

$$\theta = \tan^{-1}\left(\frac{C}{D}\right), \tag{3.11}$$

where $C = \beta_0(P^2+Q^2) + \varepsilon 2^{3/2}AS[M\{(\alpha_1-\alpha_0)\cos\sigma t - (\beta_1-\beta_0)\sin\sigma t\} - L\{(\beta_1-\beta_0)\cos\sigma t + (\alpha_1-\alpha_0)\sin\sigma t\}]$, $D = \alpha_0(P^2+Q^2) + \varepsilon 2^{3/2}AS[L\{(\alpha_1-\alpha_0)\cos\sigma t - (\beta_1-\beta_0)\sin\sigma t\} - M\{(\beta_1-\beta_0)\cos\sigma t + (\alpha_1-\alpha_0)\sin\sigma t\}]$. When $S = 0$, $\theta = 45^\circ$ and when $S \neq 0$ but $A = 0$, $\theta = \tan^{-1}(\beta_0/\alpha_0) < 45^\circ$.

In the case $\sigma t = \pi/2$ and $S \neq 0$, $A \neq 0$, the inclination of the fluid velocity to y-axis near $z = 0$ will be

$$\tan^{-1}\left(\frac{\beta_0(P^2+Q^2) - \varepsilon 2^{3/2}AS[L(\alpha_1-\alpha_0) + M(\beta_1-\beta_0)]}{\alpha_0(P^2+Q^2) - \varepsilon 2^{3/2}AS[M(\alpha_1-\alpha_0) + L(\beta_1-\beta_0)]}\right) \tag{3.12}$$

which indicates a further reduction in the value of the inclination of the fluid velocity compared with its value in the case of uniform suction.

The variations of $f(\xi)$ and $g(\xi)$ corresponding to (3.7) and (3.8) for various values of the suction parameter S, the magnitude of fluctuation of suction velocity A, and the frequency of fluctuation of suction velocity n are illustrated in Figures 3.1, 3.2, 3.3, 3.4, 3.5, and 3.6.

For the case of blowing, $S < 0$ and the components of fluid velocity in presence of variable blowing at the disk can be obtained easily from (3.7) on replacing S by $-\lambda$, where $\lambda > 0$. The results in the case of blowing are represented in Figures 3.7, 3.8, 3.9, 3.10, 3.11, and 3.12.

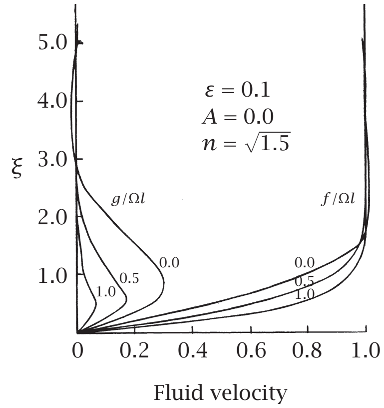

FIGURE 3.1. Variations of $f/\Omega l$ and $g/\Omega l$ for different values of suction parameter S in absence of variable suction A.

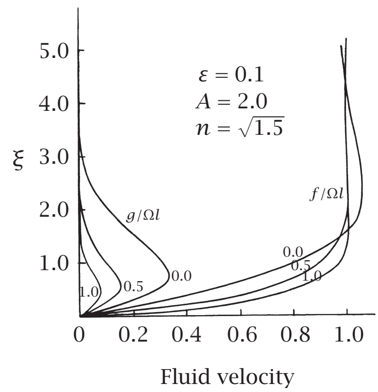

FIGURE 3.2. Variations of $f/\Omega l$ and $g/\Omega l$ for different values of suction parameter S in presence of variable suction A.

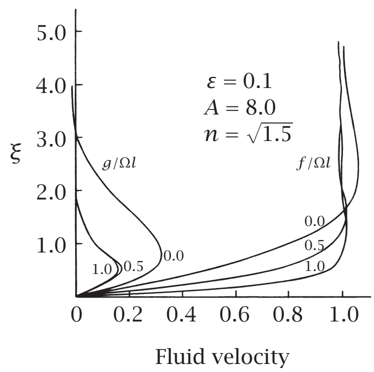

FIGURE 3.3. Variations of $f/\Omega l$ and $g/\Omega l$ for different values of suction parameter S in presence of variable suction A.

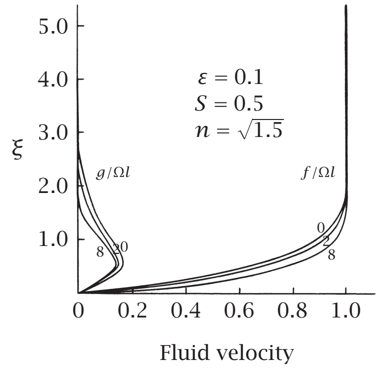

FIGURE 3.4. Variations of $f/\Omega l$ and $g/\Omega l$ for different values of A, the magnitude of fluctuations in suction velocity.

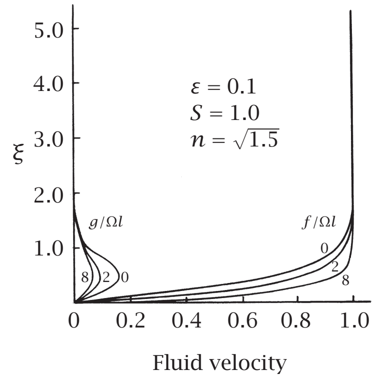

FIGURE 3.5. Variations of $f/\Omega l$ and $g/\Omega l$ for different values of A, the magnitude of fluctuation in suction velocity.

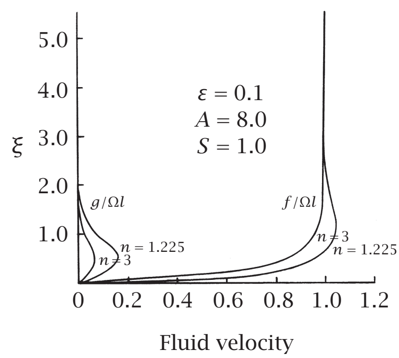

FIGURE 3.6. Variations of $f/\Omega l$ and $g/\Omega l$ with frequency n of the fluctuation in suction velocity.

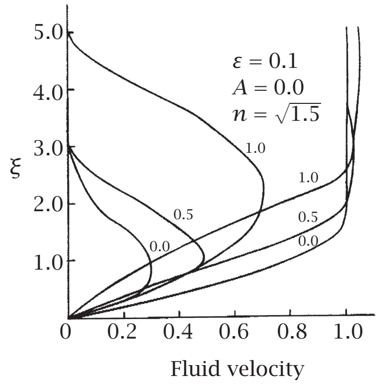

FIGURE 3.7. Variations of $f/\Omega l$ and $g/\Omega l$ for different values of the blowing parameter and in absence of A, the magnitude of the fluctuation in blowing velocity.

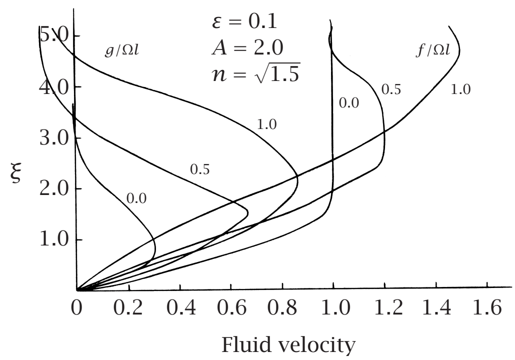

FIGURE 3.8. Variations of $f/\Omega l$ and $g/\Omega l$ for different values of the blowing parameter and in presence of A, the magnitude of the fluctuation in blowing velocity.

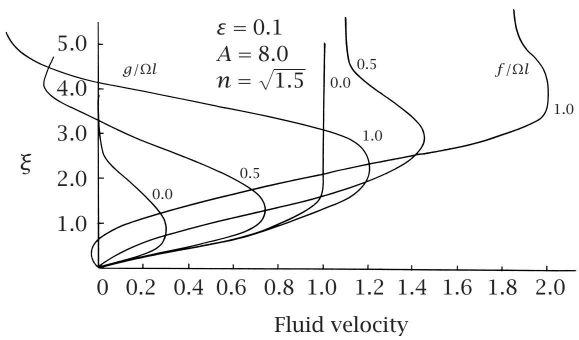

FIGURE 3.9. Variations of $f/\Omega l$ and $g/\Omega l$ for different values of the blowing parameter and in presence of A, the magnitude of fluctuation in blowing velocity.

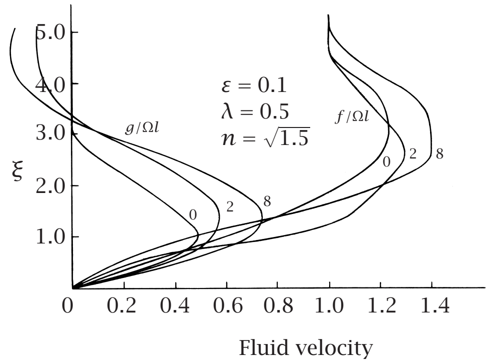

FIGURE 3.10. Variations of $f/\Omega l$ and $g/\Omega l$ for different values of A, the magnitude of fluctuations in blowing velocity.

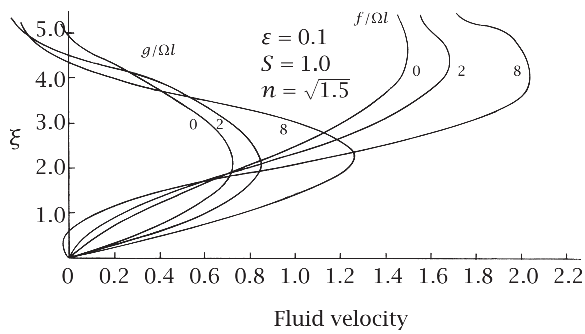

FIGURE 3.11. Variations of $f/\Omega l$ and $g/\Omega l$ for different values of A, the magnitude of fluctuations in blowing velocity.

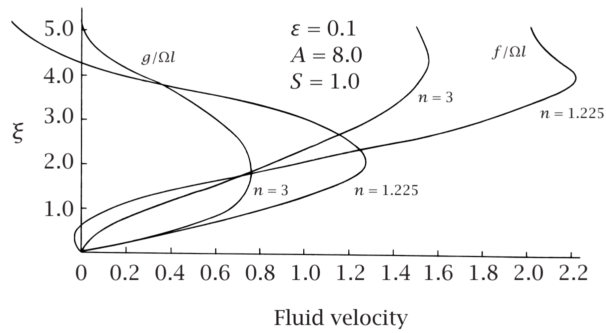

FIGURE 3.12. Variation of $f/\Omega l$ and $g/\Omega l$ with n, the frequency of fluctuations of the blowing velocity.

Finally, the components of the shear stress at the disk $z = 0$ corresponding to the fluid velocity given by (3.7) and (3.8) can be obtained as

$$\begin{aligned} P_{x_0} + iP_{y_0} &= \frac{\tau_{x_0} + i\tau_{y_0}}{(\mu\Omega^3\rho l^2/2)^{1/2}} \\ &= \left(\frac{\partial}{\partial\xi}\left(\frac{f}{\Omega l}\right) + i\frac{\partial}{\partial\xi}\left(\frac{g}{\Omega l}\right)\right)_{\xi=0} \\ &= \alpha_0 + \frac{\varepsilon AS2^{3/2}}{P^2+Q^2}\Big\{[L(\alpha_1-\alpha_0) - M(\beta_1-\beta_0)]\cos\sigma t \\ &\qquad\qquad - [M(\alpha_1-\alpha_0) + L(\beta_1-\beta_0)]\sin\sigma t\Big\} \\ &\quad + i\Big\{\beta_0 + \frac{\varepsilon AS2^{3/2}}{P^2+Q^2}[L(\beta_1-\beta_0) + M(\alpha_1-\alpha_0)]\cos\sigma t \\ &\qquad\qquad + [L(\alpha_1-\alpha_0) - M(\beta_1-\beta_0)]\sin\sigma t\Big\}, \end{aligned} \tag{3.13}$$

which, when $\sigma t = \pi/2$, yields

$$P_{x_0} + iP_{y_0} = \alpha_0 - R[M(\alpha_1-\alpha_0) + L(\beta_1-\beta_0)] + i\{\beta_0 + R[L(\alpha_1-\alpha_0) - M(\beta_1-\beta_0)]\}, \tag{3.14}$$

where $R = \varepsilon AS2^{3/2}/P^2pQ^2$.

The components of shear stress at the disk in the presence of variable blowing can also be found similarly.

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