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NONLINEAR DIFFRACTION OF WATER WAVES BY OFFSHORE STRUCTURES

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ABSTRACT. This paper is concerned with a variational formulation of a non-axisymmetric water wave problem. The full set of equations of motion for the problem in cylindrical polar coordinates is derived. This is followed by a review of the current knowledge on analytical theories and numerical treatments of nonlinear diffraction of water waves by offshore cylindrical structures. A brief discussion is made on water waves incident on a circular harbor with a narrow gap. Special emphasis is given to the resonance phenomenon associated with this problem. A new theoretical analysis is also presented to estimate the wave forces on large conical structures. Second-order (nonlinear) effects are included in the calculation of the wave forces on the conical structures. A list of important references is also given.

KEY WORDS AND PHRASES. Variational principle for non-axisymmetric water waves, nonlinear diffraction of water waves, incident and reflected waves, wave forces, waves incident on harbors, and Helmholtz resonance.
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1. INTRODUCTION. Diffraction of water waves by offshore structures or by natural boundaries is of considerable interest in ocean engineering. Due to the tremendous need and growth of ocean exploration and extraction of wave energy from oceans, it is becoming increasingly important to study the wave forces on the offshore structures or natural boundaries. Current methods of calculating wave forces on the offshore structures and/or harbors are very useful for building such structures that are used for exploration of oil and gas from the ocean floor.

In the theory of diffraction, it is important to distinguish between small and large structures (of typical dimension b) in comparison with the characteristic wavelength $(2\pi/k)$ and the wave amplitude a. Physically, when a/b is small and kb is large (the characteristic dimension b of the body is large compared with k^{-1}, k is the wavenumber) the body becomes efficient as a generator of dipole wave radiation, and the wave force on it becomes more resistive in nature. This means that flow separation becomes insignificant while diffraction is dominant. In other words, the body radiates a very large amount of scattered (or reflected) wave energy. On the other hand, for small kb (the characteristic dimension b of the body is small compared with k^{-1}) and a/b is large $(a/b \geq O(1))$, the body radiates a very small amount of scattered wave energy. This corresponds to a case of a rigid lid on the ocean inhibiting wave scattering, that is, diffraction is insignificant.

Historically, Havelock [1] gave the linearized diffraction theory for small amplitude water waves in a deep ocean. Based upon this work, MacCamy and Fuchs [2] extended the theory for a fluid of finite depth. These authors successfully used the linearized theory for calculation of wave loading on a vertical circular cylinder extending from a horizontal ocean floor to above the free surface of water. Subsequently, several authors including Mogridge and Jamieson [3], Mei [4], Hogben et. al. [5], Garrison [6-7] obtained analytical solutions of the linearized diffraction problems for simple geometrical configurations. However, the linearized theory has limited applications since it is only applicable to water waves of small steepness. In reality, ocean waves are inherently nonlinear and often irregular in nature. Hence, water waves of large amplitude are of special interest in estimating wave forces on offshore structure or harbors.

In recent years, there has been considerable interest in the study of the hydrodynamic forces that ocean waves often exert on offshore structures, natural boundaries and harbors of various geometrical shapes. Historically, the wave loading estimation for offshore structures was based upon the classical work of Morison et. al. [8] or on the linear diffraction theory of water waves due to Havelock [1] and MacCamy and Fuchs [2]. Morison's formula was generally used to calculate wave forces on solid structures in oceans. According to Debnath and Rahman [9], Morison's equation expresses the total drag D as a sum of the inertia force, $\rho\, C_M\, V\, U$ associated with the irrotational flow component, and the viscous drag force, $\frac{1}{2}\rho\, C_D\, A\, U^2$ related to the vortex-flow component of the fluid flows under the assumption that the incident wave field is not significantly affected by the presence of the structures. Mathematically, the Morison equation is

$$D = \rho\, C_M\, V\, \dot{U} + \frac{1}{2}\rho\, C_D\, A\, U^2 \tag{1.1}$$

where ρ is the fluid density, $C_M = (1 + \frac{M_a}{\rho V})$ is the Morison (or inertial) coefficient, M_a is the added mass, V is the volumetric displacement of the body, U is the fluctuating fluid velocity along the horizontal direction, A is the projected frontal area of the wake vortex, and C_D is the drag coefficient. For cases of the flow past a cylinder or a sphere, these coefficients can be determined relatively simply from the potential flow theory. It is also assumed that inertia and viscous drag forces acting on the solid structure in an unsteady flow are independent in the sense that there is no interaction between them.

There are several characteristic features of the Morison equation. One deals with the nature of the inertia force which is linear in velocity U. The other includes the nonlinear factor U^2 in the viscous drag force term. It is generally believed that all nonlinear effects in experimental data are associated with drag forces. However, for real ocean waves with solid structures, there is a significant nonlinear force associated with the irrotational component of the fluid flow because of the large amplitude of ocean waves. These waves are of special interest in the wave loading estimation. It is important to include all significant nonlinear effects associated with the nonlinear free surface boundaryconditions in the irrotational flow component of the wave loading on the structures. However, the effect of large amplitude waves on offshore structures of small mean diameter may be insignificant, but it is no longer true as the diameter increases in relation to the wavelength of the indident wave field. Consequently the Morison equation is no longer applicable, and diffraction theory must be reformulated. It is necessary to distinguish between the structures of small and large diameters, the diameter being compared to the characteristic wavelength and amplitude of the wave.

Several studies have shown that the Morison formula is fairly satisfactory. However, several difficulties in using it in the design and construction of offshore structures have been reported in the literature. These are concerned with the drag force which has relativly large scale effects. The shortage of reliable full-scale drag data in ocean waves is another problem. There is another significant question whether a linear theory of the irrotational flow response is appropriate at all to water wave motions with a free surface. Despite these difficulties and short-comings, the use of the Morison equation had extensively been documented in the past literature throughplentiful data for determining the coefficients C_M and C_D.

Several recent studies indicate that the second-order theories for the diffraction of nonliner water waves by offshore structures provide an accurate estimate of the linearized analysis together with corrections approximating to the effect of finite wave amplitude.

This paper is concerned with a variational formulation of a non-axisymmetric water wave problem. The full set of equations of motion for the problem in cylindrical polar coordinates is derived. This is followed by a review of the current knowledge on analytical theories and numerical treatments of nonlinear diffraction of water waves by offshore cylindrical structures. A brief discussion is made on water waves incident on a circular harbor with a narrow gap. Special emphasis is given to the resonance phenomenon associated with the problem. A new theoretical analysis is also presented to estimate the wave forces on large conical structures. Second-order (nonlinear) effects are included in the calculation of the wave forces on the conical structures.

2. VARITIONAL PRINCIPLE FOR NON-AXISYMMETRIC NONLINEAR WATER WAVES.

We consider an inviscid irrotational non-axisymmetric fluid flow of constant density ρ subjected to a gravitational field g acting in the negative z-axis which is directed vertically downward. The fluid with a free surface $z = \eta\ (r, \theta, t)$ is confined in a region $0 \leq r < \infty$, $0 < z < \eta$, $-\pi \leq \theta < \pi$. There exists a velocity potential $\phi(r,\theta,z)$ such that the fluid velocity is given by $\underline{u} = -\nabla\phi = -(\phi_r, \frac{1}{r}\phi_\theta, \phi_z)$, and the potential is lying in between $z = 0$ and $z = \eta\ (r,\theta, t)$. Then the variational principle is

$$\delta I = \delta \int_D \int L\, d\underline{x}\, dt = 0 \tag{2.1}$$

where D is an arbitrary region in the $(\underline{x}, t)$ space, and the Lagrangian L is

$$L \equiv \int_0^{\eta(r,\theta,t)} [\phi_t + \frac{1}{2} (\nabla\phi)^2 + gz] \, dz, \tag{2.2}$$

and $\phi(r,\theta,z,t)$, $\eta(r,\theta,t)$ are allowed to vary subject to the restrictions $\delta\phi = 0$, $\delta\eta = 0$ on the boundary ∂D of D.

According to the standard procedure of the calculus of variations, result (2.1) yields

$$0 = \delta I = \int_D\int \{[\phi_t + \frac{1}{2} (\nabla\phi)^2 + gz]_{z=\eta} \, \delta\eta$$

$$+ \int_0^\eta (\delta\phi_t + \phi_r \, \delta\phi_r + \frac{1}{r^2} \phi_\theta \, \delta\phi_\theta + \phi_z \, \delta\phi_z) \, dz\} \, r \, dr d\theta \, dt$$

Integrating the z-integral by parts along with r and θ-integrals, it turns out that

$$0 = \delta I = \int_D\int [\phi_t + \frac{1}{2} (\nabla\phi)^2 + gz]_{z=\eta} \, \delta\eta$$

$$+ \int_D\int \{[\frac{\partial}{\partial t} \int_0^\eta \delta\phi \, dz - (\eta_t \, \delta\phi)_{z=\eta}]$$

$$+ [\frac{\partial}{\partial r} \int_0^\eta \phi_r \, \delta\phi \, dz - \int_0^\eta (\phi_{rr} + \frac{1}{r} \phi_r) \, \delta\phi \, dz - (\eta_r\phi_r\delta\phi)_{z=\eta}]$$

$$+ [\frac{\partial}{\partial\theta} \int_0^\eta \frac{1}{r^2} \phi_\theta\delta\phi \, dz - \int_0^\eta \frac{1}{r^2} \phi_{\theta\theta} \, \delta\phi \, dz - (\frac{1}{r^2} \eta_\theta\phi_\theta\delta\phi)_{z=\eta}]$$

$$+ [(\phi_z\delta\phi)_{z=\eta} - (\phi_z\delta\phi)_{z=0} - \int_0^\eta \phi_{zz}\delta\phi \, dz]\} \, d\underline{x} \, dt$$

In view of the fact that the first z-integral in each of the square brackets vanishes on the boundary ∂D, we obtain

$$0 = \delta I = \int_D\int \{[\phi_t + \frac{1}{2} (\nabla\phi)^2 + gz]_{z=\eta}\delta\eta + [(-\eta_t - \eta_r\phi_r - \frac{1}{r^2} \eta_\theta\phi_\theta + \phi_z) \, \delta\phi]_{z=\eta}$$

$$- \int_0^\eta [\phi_{rr} + \frac{1}{r} \phi_r + \frac{1}{r^2} \phi_{\theta\theta} + \phi_{zz}] \, \delta\phi \, dz - [\phi_z\delta\phi]_{z=0}\} \, d\underline{x} \, dt.$$

We first choose $\delta\eta = 0$, $[\delta\phi]_{z=0} = [\delta\phi]_{z=\eta} = 0$; since $\delta\phi$ is arbitrary, we derive

$$\phi_{rr} + \frac{1}{r} \phi_r + \frac{1}{r^2} \phi_{\theta\theta} + \phi_{zz} = 0, \; 0 < z < \eta \; (r,\theta,t), \tag{2.3}$$

Then, since $\delta\eta$, $[\delta\phi]_{z=0}$ and $[\delta\phi]_{z=\eta}$ can be given arbitrary independent values, we deduce

$$\phi_t + \frac{1}{2}(\nabla\phi)^2 + gz = 0, \qquad z = \eta\ (r,\theta,t) \tag{2.4}$$

$$\eta_t + \eta_r\phi_r + \frac{1}{r^2}\eta_\theta\phi_\theta - \phi_z = 0, \qquad z = \eta\ (r,\theta,t) \tag{2.5}$$

$$\phi_z = 0, \qquad z = 0 \tag{2.6}$$

Evidently, the Laplace equation (2.3), two free-surface conditions (2.4)-(2.5) and the bottom boundary conditon (2.6) constitute the non-axisymmetric water wave equations in cylindrical polar coordinates. This set of equations has also been used by several authors including Debnath [10], Mohanti [11] and Mondal [12], for the initial value investigation of linearized axisymmetric water wave problems. The elegance of the variational formulation is that within its framework, the treatment of both linear and nonlinear problems become identical

3. DIFFRACTION OF NONLINEAR WATER WAVES IN AN OCEAN BY CYLINDERS.

Several authors including Charkrabarti [13], Lighthill [14], Debnath and Rahman [9], Rahman and his collaborators [15-18], Hunt and Baddour [19], Hunt and Williams [20], Sabuncu and Gören [21]. Demirbilek and Gaston [22] have made an investigation of the theory of nonlinear diffraction of water waves in a liquid of finite and infinite depth by a circular cylinder. These authors obtained some interesting theoretical and numerical results. We first discuss the basic formulation of the problem and indicate how the problem can be solved by a perturbation method.

We formulate a nonlinear diffraction problem in an irrotational incompressible fluid of finite depth h. We consider a large rigid vertical cylinder of radius b which is acted on by a train of two-dimensional, periodic progressive waves of amplitude a propagating in the positive x direction as shown in Figure 1. In the absence of the wave, the water depth is h and in the presence of the wave the free surface elevation is η above the mean surface level.

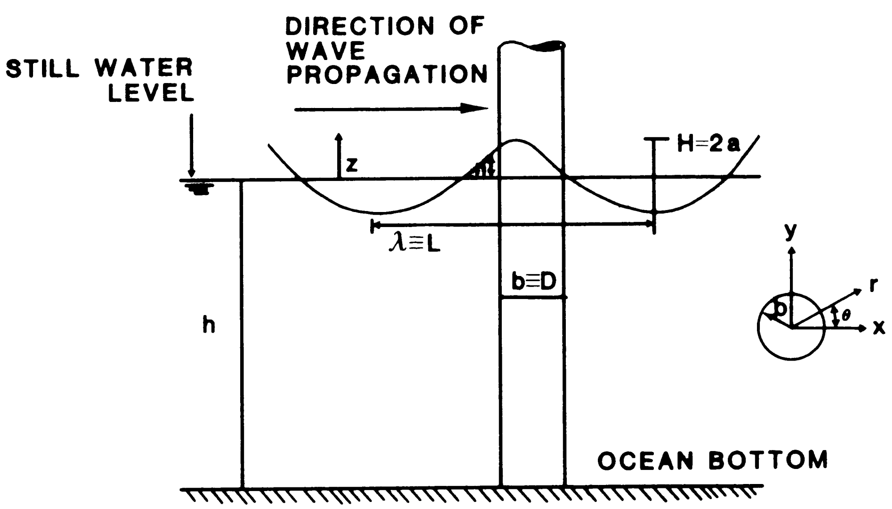

FIG. 1. Schematic diagram of a cylindrical structure in waves

In cylindrical polar coordinates (r,θ,z) with the z-axis vertically upwards from the origin at the mean free surface, the governing equation, free surface and boundary conditions at the rigid bottom and the body surface are given by

$$\nabla^2\phi(r,\theta,z,t) \equiv \phi_{rr} + \frac{1}{r}\phi_r + \frac{1}{r^2}\phi_{\theta\theta} + \phi_{zz} = 0, \; b<r<\infty, \; -\pi<\theta<\pi, \; -h<z<\eta, \tag{3.1}$$

$$\phi_t + g\eta + \frac{1}{2}(\phi_r^{\,2} + \frac{1}{r^2}\phi_\theta^{\,2} + \phi_z^{\,2}) = 0, \; z=\eta, \; r \geq b, \tag{3.2}$$

$$\eta_t + \phi_r\eta_r + \frac{1}{r^2}\phi_\theta\eta_\theta - \phi_z = 0, \; z=\eta, \; r\geq b, \tag{3.3}$$

$$\phi_z = 0 \quad \text{on} \quad z = -h \tag{3.4}$$

$$\phi_r = 0 \quad \text{on} \quad r = b, \; -h\leq z<\eta, \tag{3.5}$$

where ϕ is the velocity potential and g is the acceleration due to gravity.

Finally, the radiation conditon is

$$\lim_{kr\to\infty} (kr)^{1/2}\left[\left(\frac{\partial}{\partial r} \pm ik\right)\phi_R\right] = 0 \tag{3.6}$$

where $k = 2\pi/\lambda$ is the wavenumber of the reflected (or scattered) wave, $\phi = \phi_I + \phi_R$ is the total potential, ϕ_I and ϕ_R represent the incident and reflected potentials respectively.

We apply the Stokes expansion of the unknown functions ϕ and η in the form

$$\phi = \sum_{n=1}^{\infty} \varepsilon^n \phi_n, \quad \eta = \sum_{n=1}^{\infty} \varepsilon^n \eta_n \tag{3.7ab}$$

where ε is a small parameter of the order of the wave steepness

For any given n the sum of only the first n terms of the series (3.7ab) may be considered as the nth-order approximation to the solutions of the problem governed by (3.1)-(3.6). The nth-order approximation is the solution subject to the neglect of terms ε^m when $m > n$.

We next carry out a Taylor-series expansion of the nonlinear boundary conditions (3.1)-(3.2) about $z = 0$, substitute (3.7ab) into (3.1)-(3.2) and equate powers of ε. Equating the first powers of ε leads to the following equations with the linearized boundary conditions on $z = 0$, valid for all r,θ and t:

$$\nabla^2\phi_1 (r,\theta,z,t) = 0 \tag{3.8}$$

$$\phi_{1t} + g\eta_1 = 0, \quad \text{for} \quad z = 0, \; r \geq b \tag{3.9}$$

$$\eta_{1t} - \phi_{1z} = 0, \quad \text{for} \quad z = 0, \; r \geq b \tag{3.10}$$

$$\phi_{1z} = 0, \quad \text{for} \quad z = -h \tag{3.11}$$

$$\phi_{1r} = 0, \quad \text{for} \quad r = b \tag{3.12}$$

$$\lim_{kr\to\infty} (kr)^{1/2} [(\frac{\partial}{\partial r} \pm ik) \phi_{1R}] = 0 \tag{3.13}$$

where $\phi_1 = \phi_{1I} + \phi_{1R}$ representing the total potential as the sum of the first order incident potential, ϕ_{1I} and the first-order reflected potential.

Similarly, equating the second powers of ε leads to the following system of equations that express the second-order terms ϕ_2 and η_2 as functions of ϕ_1 and η_1:

$$\nabla^2\phi_2 = 0 \tag{3.14}$$

$$g\eta_2 + \phi_{2t} + \eta_1\phi_{1tz} + \frac{1}{2} (\phi_{1r}{}^2 + \frac{1}{r}\phi_{1\theta}{}^2 + \phi_{1z}{}^2) = 0, \; z = 0, \; r \geq b, \tag{3.15}$$

$$\eta_{2t} + \phi_{1r}\eta_{1r} + \frac{1}{r^2}\phi_{1\theta}\eta_{1\theta} - (\phi_{2z} + \eta_1\phi_{1zz}) = 0, \; z = 0, \; r \geq b, \tag{3.16}$$

$$\phi_{2z} = 0 \qquad \text{for} \qquad z = -h, \tag{3.17}$$

$$\phi_{2r} = 0 \qquad \text{for} \qquad r = b, \tag{3.18}$$

with the radiation condition

$$\lim_{k_2 r\to\infty} (k_2 r)^{1/2} [(\frac{\partial}{\partial r} \pm ik_2)(\phi_2 - \phi_{2I})] = 0 \tag{3.19}$$

where k_2 is the wavenumber corresponding to second-order wave theory, and ϕ_{2I} is the second-order term of the incident potential ϕ_I.

The function η_1 can be eliminated from (3.9)-(3.10) to derive

$$\phi_{1tt} + g\phi_{1z} = 0, \quad \text{for} \quad z = 0, \; r \geq b \tag{3.20}$$

Similarly, η_2 can be eliminated from equations (3.15) and (3.16) to obtain

$$\phi_{2tt} + g\phi_{2z} = -\eta_1 \frac{\partial}{\partial z} [\phi_{1tt} + g\phi_{1z}] - \frac{\partial}{\partial t} [\phi_{1r}{}^2 + (\frac{1}{r}\phi_{1\theta})^2 + \phi_z{}^2], \text{ for } z=o, \; r\geq b \tag{3.21}$$

The pressure $p(r,\theta,z,t)$ can be determined from the Bernoulli equation

$$\frac{p}{\rho} + gz + \phi_t + \frac{1}{2} [\phi_r{}^2 + (\frac{1}{r}\phi_\theta)^2 + \phi_z{}^2] = 0 \tag{3.22}$$

Substituting ϕ as a power series in ε into (3.22), we can express p as

$$p = -\rho gz - \varepsilon\rho\phi_{1t} - \varepsilon^2\rho \{\phi_{2t} + \frac{1}{2} [\phi_{1r}{}^2 + (\frac{1}{r}\phi_{1\theta})^2 + \phi_{1z}{}^2]\} + 0\;(\varepsilon^3) \tag{3.23}$$

The total horizonatal force is

$$F_x = \int_0^{2\pi} \int_{-h}^{\eta} [p]_{r=b} (-b \cos\theta)\, dz d\theta \tag{3.24}$$

where η is given by the perturbation expansion (3.7b).

Substituting (3.23) into (3.24) and expressing the z-integral as the sum of

$\int_{-h}^{0} + \int_{0}^{\eta}$, we obtain

$$F_x = b\rho \int_0^{2\pi} \Bigg\{ \int_{-h}^{0} [gz + \varepsilon\phi_{1t} + \varepsilon^2 \{\phi_{2t} + \frac{1}{2}(\phi_{1z}{}^2 + \frac{1}{b^2}\phi_{1\theta}{}^2)\}]dz$$

$$+ \int_0^{\varepsilon\eta_1 + \varepsilon^2\eta_2} [gz + \varepsilon^2 \phi_{1t} + \varepsilon^2 \{\phi_{2t} + \frac{1}{2}(\phi_{1z}^2 + \frac{1}{b^2}\phi_{1\theta}{}^2)\}]_{r=b} dz \Bigg\} \cos\theta\, d\theta \tag{3.25}$$

It is noted that condition (3.12) is used to derive (3.25). It is clear from (3.25) that the integral of gz, the hydrostatic term, up to $z=0$ contain no $\cos\theta$ term and hence may be neglected. Also, the upper limit of the z-integral of the second-order terms may be taken at $z=0$ in place of $z = \varepsilon\eta_1 + \varepsilon^2\eta_2$ which would only introduce higher-order terms ε^3, etc. Thus F_x may be written as

$$F_x = \varepsilon F_{x1} + \varepsilon^2 F_{x2} \tag{3.26}$$

where the first-order contribution is

$$\varepsilon F_{x1} = b\rho \int_0^{2\pi} [\int_{-h}^{0} (\varepsilon\phi_{1t})_{r=b}\, dz] \cos\theta\, d\theta \tag{3.27}$$

and the second-order contribution is

$$\varepsilon^2 F_{x2} = b\rho \int_0^{2\pi} [\int_0^{\varepsilon\eta_1} \{gz + \varepsilon\phi_{1t}\}_{r=b}\, dz$$

$$+ \varepsilon^2 \int_{-h}^{0} \{\phi_{2t} + \frac{1}{2}(\phi_{1z}{}^2 + \frac{1}{b^2}\phi_{1\theta}{}^2)\}\, dz]_{r=b} \cos\theta\, d\theta \tag{3.28}$$

4. FIRST-ORDER WAVE POTENTIAL AND FREE SURFACE ELEVATION FUNCTION.

MacCamy and Fuchs [2] solved the system (3.8)-(3.13) and obtained the first-order solution which can be expressed in the complex form

$$\phi_1 = \frac{\omega\cosh k(z+h)}{k^2 \sinh kh} e^{-i\omega t} \sum_{m=0}^{\infty} i^m \delta_m A_m(kr) \cos m\theta, \tag{4.1}$$

$$\eta_1 = \frac{e^{-i\omega t}}{k} \sum_{m=0}^{\infty} i^{m+1} \delta_m A_m(kr) \cos m\theta, \tag{4.2}$$

where

$$\delta_m = \begin{cases} 1, & \text{when } m = 0 \\ 2, & \text{when } m \neq 0 \end{cases}, \tag{4.3}$$

$$A_m(kr) = J_m(kr) - \frac{J_m'(kb)}{H_m^{(1)\prime}(kb)} H_m^{(1)}(kr), \tag{4.4}$$

and the frequency ω and wavenumber k satisfy the dispersion relation

$$\omega^2 = gk \tanh kh, \tag{4.5}$$

$H_m^{(1)}$ is the mth-order Hankel function of the first kind defined by

$$H_m^{(1)}(kr) = J_m(kr) + i\, Y_m(kr) \tag{4.6}$$

in which $J_m(kr)$ and $Y_m(kr)$ are the Bessel functions of the first and the second kind respectively, $J_m'(x)$ denotes the first derivative.

It is noted that result (4.2) represents the complex form of a plane wave of amplitude k^{-1} propagating in the x-direction and represented by the terms whose radial dependence is given by $J_m(kr)$, together with a reflected component described by the terms whose radial dependence is given by $H_m^{(1)}(kr)$. The first order solution $\varepsilon\phi_1$ includes the complex form of an incident plane wave of amplitude ε.

5. SECOND-ORDER WAVE POTENTIAL AND FREE-SURFACE ELEVATION FUNCTION.

With known values for ϕ_1 and η_1 given in (4.2) and (4.3), equation (3.2) assumes the following form for $z = 0$ and $r \geq b$:

$$\phi_{2tt} + g\phi_{2z} = \frac{g\omega}{2k} e^{-2i\omega t} \sum_{m=0}^{\infty} B_m(kr) \cos m\theta, \tag{5.1}$$

where

$$\begin{aligned}\sum_{m=0}^{\infty} B_m(kr) \cos m\theta = {} & \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \delta_m \delta_n i^{m+n-1} A_{mn} \left[\cos (m+n)\theta + \cos(m-n)\theta\right] \\ & + \frac{2 \coth kh}{k^2 r^2} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \delta_m \delta_n i^{m+n-1} \times \\ & \times A_m A_n (mn) \left[\cos (m-n)\theta - \cos (m+n)\theta\right]\end{aligned} \tag{5.2}$$

with

$$A_{mn} = (3 \tanh kh - \coth kh) A_m A_n + 2 \coth kh\, A_m' A_n', \tag{5.3}$$

and a prime in (5.3) denotes differentiation with respect to kr.

The form of (5.1) indicates that the general solution of (3.14) with (3.17)-(3.18) can be written as

$$\phi_2 = \frac{\omega e^{-2i\omega t}}{2k^2} \sum_{m=0}^{\infty} [\int_0^{\infty} D_m(k_2)\, A_m\, (rk_2) \cosh k_2(z+h)\, dk_2] \cos m\theta, \tag{5.4}$$

where k_2 denotes the wavenumber of a second-order wave taking only continuous values in $(0, \infty)$.

We next substitute (5.4) into (5.1) to obtain a relation between D_m and B_m in the form

$$\frac{1}{k}\int_0^{\infty} [k_2 \sinh k_2 h - 4k \tanh kh \cosh k_2 h]\, A_m(rk_2)\, D_m(k_2)\, dk_2 = B_m(kr) \tag{5.5}$$

where $B_m(r)$ can be obtained by equating similar terms of the Fourier Series (5.2).

Equation (3.15) gives the second-order free surface elevation η_2 in the form

$$\eta_2 = \frac{-1}{g} [\frac{\partial \phi_2}{\partial t} + \eta_1 \frac{\partial^2 \phi_1}{\partial t \partial z} + \frac{1}{2} \{(\frac{\partial \phi_1}{\partial r})^2 + (\frac{\partial \phi_1}{r\partial \theta})^2 + (\frac{\partial \phi_1}{\partial z})^2 \}], \; z = 0, \; r \geq b, \tag{5.6}$$

where η_1, ϕ_1 and ϕ_2 are determined earlier.

In order to compute the total horizontal force on the cylinder as given by (3.26) combined with (3.27)-(3.28), it is necessary to solve (5.5) for the case of $m=1$.

A tedious, but straight forward, algebraic manipulation gives the value of the complex quantity $B_1(r)$ as

$$B_1(r) = 8 \sum_{m=0}^{\infty} (-1)^{m+1} [A_{m,m+1} + \frac{2 \coth kh}{k^2 r^2} m(m+1)\, A_m\, A_{m+1}] \tag{5.7}$$

Since the first-order problem described by (3.8)-(3.13) is linear, its real physical solution is given by $\frac{1}{2}(\phi_1 + \phi_1^*)$ where ϕ_1^* is the complex conjugate of ϕ_1. However, since (3.21) is nonlinear in ϕ_1, it is not possible to express the solution of (3.21) as $\frac{1}{2}(\phi_2 + \phi_2^*)$. We next discuss physical meaningful second-order solution for ϕ_2.

We first write a real solution $\frac{1}{2}(\phi_1 + \phi_1^*)$ from (4.1) in the form

$$\phi_1 = \frac{\omega \cosh k(z+h)}{2k^2 \sinh kh} \sum_{m=-\infty}^{\infty} \delta_m\, i^m\, e^{-i\omega_m t} A_m(kr) \cos m\theta, \tag{5.8}$$

where δ_m is defined by (4.3) and ω_m is given by

$$\omega_m = \begin{bmatrix} \omega, & m > 0 + \\ -\omega, & m \leq 0 - \end{bmatrix} \tag{5.9ab}$$

and the function $A_m(kr)$ is defined by (4.3) for positive values of m, and by the relation

$$A_{-m}(kr) = A_m^*(kr) \tag{5.10}$$

for negative values of m. The summation in (5.8) includes both $m = 0+$ and $m = 0-$ in order to incorporate $e^{-i\omega t} A_o(kr)$ and $e^{i\omega t} A_o^*(kr)$.

The corresponding real solution for η_1 is given by

$$\eta_1(r,\theta,t) = \frac{1}{2k} \sum_{m=-\infty}^{\infty} \delta_m \gamma_m i^{m+1} e^{-i\omega_m t} A_m(kr) \cos m\theta \tag{5.11}$$

where

$$\gamma_m = \begin{bmatrix} 1, & m \geq 0+ \\ -1, & m \leq 0- \end{bmatrix} \tag{5.12ab}$$

Equation (3.21) then takes the following form which is similar to (5.1):

$$\frac{\partial^2 \phi_2}{\partial t^2} + g \frac{\partial \phi_2}{\partial z} = \frac{g\omega}{4k} [e^{-2i\omega t} \sum_{m=0}^{\infty} B_m(kr) \cos m\theta + c.c] \tag{5.13}$$

where c.c. stands for the complex conjugate, and

$$e^{-2i\omega t} \sum_{m=0}^{\infty} B_m(kr) \cos m\theta + e^{2i\omega t} \sum_{m=0}^{\infty} B_m^*(kr) \cos m\theta$$

$$= \frac{1}{4} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta_m \delta_n i^{m+n+1} e^{-i(\omega_m+\omega_n)t} A_{mn} [\cos(m+n)\theta + \cos(m-n)\theta]$$

$$+ \frac{1}{4} \frac{\coth kh}{(kr)^2} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta_m \delta_n i^{m+n+1} mn\, \omega^{-1} (\omega_m + \omega_n) e^{-i(\omega_m+\omega_n)t} A_m A_n \times$$

$$\times [\cos(m+n)\theta - \cos(m-n)\theta] \tag{5.14}$$

where A_{mn} is given by

$$A_{mn} = [\gamma_m (\tanh kh - \coth kh) + \frac{1}{\omega} (\omega_m + \omega_n) \tanh kh] A_m A_n$$

$$+ \frac{1}{\omega} (\omega_m + \omega_n) \coth kh\, A_m' A_n' \tag{5.15}$$

It is noted that for $m, n > 0$ this expression is identical with (5.3) and hence the definition of A_{mn} is consistent with the previous definition. Furthermore, $A_{-m,-n} = A_{m,m}^*$. It can be verified that the double series in (5.14) contains terms that are independent of time t and hence correspond to standing waves. However, it can readily be checked that these terms add up to zero and hence there are no standing waves in the solution.

Finally, the solution for ϕ_2 satisfying the required boundary conditions has the form

$$\phi_2 = \frac{\omega e^{-2i\omega t}}{4k^2} \sum_{m=0}^{\infty} [\int_0^{\infty} D_m(k_2)\, A_m(rk_2) \cosh k_2\,(z+h)\, dk_2] \cos m\theta$$

$$+ \frac{\omega e^{+2i\omega t}}{4k^2} \sum_{m=0}^{\infty} [\int_0^{\infty} D_m^{*}(k_2)\, A_m^{*}(rk_2) \cosh k_2\,(z+h)\, dk_2] \cos m\theta, \tag{5.16}$$

This result is similar to that of (5.4), and $D_m(k_2)$ is the solution of (5.5) with $B_m(kr)$ given in (5.14) which is analogous to (5.2).

6. RESULTANT HORIZONTAL FORCES ON THE CYLINDER.

For a diffracted wave whose first-order potential is of the form (5.8), the hydrodynamic pressure evaluated at the cylinder $r = b$ depends on ϕ_1, ϕ_2, etc. The first-order horizontal force on the cylinder is obtained from (3.27) in the form

$$F_{x1} = \frac{4\rho g}{k^3} \frac{\tanh kh}{|H_1^{(1)'}(kb)|} \cos(\omega t - \alpha_1) \tag{6.1}$$

where

$$\alpha_1 = \tan^{-1} \left[\frac{J_1'(kb)}{Y_1'(kb)}\right] \tag{6.2}$$

This result was obtained earlier by other researchers including MacCamy and Fuchs [2], Lighthill [14] and Rahman [17]. In the limit $kh \to \infty$, (6.1) corresponds to the result for deep water waves which are in agreement with Lighthill [14], and Hunt and Baddour [19].

We next summarize below the second-order contribution to the total horizontal force given by (3.28). We first put values of ϕ_1 and η_1 from (5.8) and (5.11) into (3.28) and then evaluate the z-integral to obtain the coefficient of $\cos\theta$ in (3.28), apart from the $\partial\phi_2/\partial t$ term, in the form

$$\frac{g\varepsilon^2}{16k^2} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \delta_m \delta_n R_m R_n \Big[\{(3 - \frac{2kh}{\sinh 2kh}) \cos[-2\omega t + \alpha_m + \alpha_n + \frac{\pi}{2}(m+n)]$$

$$- (1 + \frac{2kh}{\sinh 2kh}) \cos(\alpha_m - \alpha_n + \frac{\pi}{2}(m-n))\}\,[\cos(m+n)\theta + \cos(m-n)\theta]$$

$$- \frac{mn}{b^2k^2}(1 + \frac{2kh}{\sinh 2kh})\{\cos(-2\omega t + \alpha_m + \alpha_n + \frac{\pi}{2}(m+n))$$

$$+ \cos(\alpha_m - \alpha_n + \frac{\pi}{2}(m-n))\}\{\cos(m+n)\theta - \cos(m-n)\theta\}\Big] \tag{6.3}$$

where the Wronskian property of Bessel functions yields

$$A_m(kb) = 2i\,[\pi kb\, H_m^{(1)'}(kb)]^{-1} = R_m e^{i\alpha_m}, \tag{6.4}$$

with

$$R_m = \left(\frac{2}{\pi kb}\right)\left[J_m'^2(kb) + Y_m'^2(kb)\right]^{-1/2}, \quad \alpha_m = \tan^{-1}\left[\frac{J_m'(kb)}{Y_m'(kb)}\right], \tag{6.5ab}$$

In view of the subsequent θ-integration described in (3.28), we need only the coefficient of $\cos\theta$ in the double series (6.3) and hence obtain the following

$$-\frac{2g}{k^2}\left(\frac{\varepsilon}{\pi kb}\right)^2 \sum_{s=0}^{\infty} \left\{\left(1 - \frac{s(s+1)}{b^2k^2}\right)\left(1 + \frac{2kh}{\sinh 2kh}\right) E_s\right.$$

$$\left. + (-1)^s \left[\left(3 - \frac{2kh}{\sinh 2kh}\right) + \frac{s(s+1)}{b^2k^2}\left(1 + \frac{2kh}{\sinh 2kh}\right)\right]\left[C_s \cos 2\omega t - S_s \sin 2\omega t\right]\right. \tag{6.6}$$

where

$$[E_s, C_s, S_s] = \frac{1}{M_s}\left[\left(J_s'Y_{s+1}' - J_{s+1}'Y_s'\right), \left(Y_s'J_{s+1}' + Y_{s+1}'J_s'\right), \left(Y_s'Y_{s+1}' - J_s'J_{s+1}'\right)\right], \tag{6.7abc}$$

with $\quad M_s = \left(J_s'^2 + Y_s'^2\right)\left(J_{s+1}'^2 + Y_{s+1}'^2\right) \qquad (6.8)$

and the argument of the Bessel functions involved in (6.6)-(6.7abc) is kb.

The part of $\frac{\partial \phi_2}{\partial t}$ proportional to $\cos\theta$, given by (5.16), contributes to the z-integral in (3.28) the term

$$\frac{g \tanh kh}{k\pi b} e^{-2i\omega t} \int_0^{\infty} \frac{D_1(k_2) \sinh k_2 h}{k_2^2 H^{(1)'}(k_2 b)} dk_2 + \text{c.c.} \tag{6.9}$$

where $D_1(k_2)$ is related to $B_1(kb)$ through (5.5).

Combining (6.3) and (6.9), integrating with respect to θ, the result can be expressed as the sum of steady and oscillatory components:

$$F_{x2} = F_{x2}^S + F_{x2}^O, \tag{6.10}$$

where

$$F_{x2}^S = -\frac{2b\rho g}{\pi k^2}\left(\frac{1}{kb}\right)^2 \sum_{s=0}^{\infty}\left\{\left[1 - \frac{s(s+1)}{b^2k^2}\right]\left[1 + \frac{2kh}{\sinh 2kh}\right] E_s\right\}, \tag{6.11}$$

and

$$F_{x2}^O = \left[\frac{\rho g \tanh kh}{k} e^{-2i\omega t} \int_0^{\infty} \frac{D_1(k_2)\sinh k_2 h}{k_2^2 H_1^{(1)'}(kb_2)} dk_2 + \text{c.c.}\right]$$

$$-\frac{2b\rho g}{\pi k^2}\left(\frac{1}{kb}\right)^2 \sum_{s=0}^{\infty} (-1)^s \left[\left(3 - \frac{2kh}{\sinh 2kh}\right)\left(1 + \frac{2kh}{\sinh 2kh}\right)\left(\frac{s(s+1)}{b^2k^2}\right)\right] \times$$

$$\times \left(C_s \cos 2\omega t - S_s \sin 2\omega t\right), \tag{6.12}$$

These expressions for F^S_{x2} and F^0_{x2} correspond to results (6.1), (6.2) and (6.3) obtained by Hunt and Williams [20] for the diffraction of nonlinear progressive waves in shallow water. The second-order contributions to the total horizontal force F_x on the vertical cylinder each consist of two components, a steady component together with an oscillatory term having twice the frequency of the first-order term. Hunt and Williams calculated the maximum value of F_x for various values of wave steepness, water depths and cylinder diameters. The maximum value of F_x is found to be significantly higher than that predicted by the linear diffraction theory. The second-order effects are found to be greatest in shallow water for slender cylinders, but in deep water they are greatest for cylinders of larger diameter. These predictions are supported by some existing experimental results which, for finite wave steepness, shows an increase over the linear solution.

In order to establish the fact that (6.12) represents the oscillatory component of the second-order solution, it is necessary to evaluate the complex form of $D_1(k_2)$ from the integral equation (5.5) for $m=1$ and in conjunction with (5.7). Following the analyses of Hunt and Baddour [19] and Hunt and Williams [20], we obtain from (5.5) and (5.7) that

$$D_1(k_2) = \frac{kk_2 \int_b^\infty A_1(k_2 r)\, B_1(r) dr}{k_2 \sinh k_2 h - 4k \tanh kh \cosh k_2 h}, \tag{6.13}$$

which can be written as

$$\frac{D_1(k_2) \sinh k_2 h}{k_2^{\,2}\, H_1^{(1)'}(k_2 b)} = \frac{k \int_b^\infty A_1(k_2 r)\, B_1(r) dr}{k_2\left(k_2 - \frac{4k \tanh kh}{\tanh k_2 h}\right) H_1^{(1)'}(k_2 b)}, \tag{6.14}$$

The nondimensional form of (6.14) is given by

$$G(bk_2) = \frac{D_1(bk_2) \sinh k_2 h}{(bk_2)^2\, H_1^{(1)'}(bk_2)} = \frac{\int_{kb}^\infty A_1(bk_2)\, B_1(kr)\, d(kr)}{(bk_2)\,[bk_2 - \frac{4\, bk \tanh kh}{\tanh k_2 h}]\, H_1^{(1)'}(bk_2)} \tag{6.15}$$

It is noted that the function $G_2(bk_2)$ is analytic near and at $k_2 = 0$. However, it is singular when k_2 is a root of the equation

$$k_2 \tanh k_2 h = 4\, k \tanh kh \tag{6.16}$$

Clearly, $k_2 = 4k$ for deep water case and $k_2 = 2k$ for shallow water problem. The root k_2 of (6.16) lies between $2k$ and $4k$ and hence may be regarded as corresponding to an ocean of intermediate depth. An argument similar to that of Griffith [23] shows that the integrand in the integral $\int_0^\infty G(k_2)\, dk_2$ is singular at $k_2 = 4k$ for a particular deep water wave, and at $k_2 = 2k$ for a particular shallow water wave.

The non-dimensional forms of the first-order and the second-order forces can be expressed as

$$\frac{F_{x1}}{\rho g D^3} = \left[\frac{\tanh kh}{2(kb)^3}\right]\left[\frac{\cos(\omega t - \alpha_1)}{\left|H_1^{(1)'}(kb)\right|}\right] \tag{6.17}$$

$$\frac{F_{x2}}{\rho g D^3} = \left[\frac{\tanh kh\, e^{-2i\omega t}}{8\,(kb)} \int_0^\infty G(k_2)\, dk_2 + \text{c.c.}\right]$$

$$- \frac{1}{4\pi (kb)^4} \sum_{s=0}^{\infty} (-1)^s \left[\left(3 - \frac{2kh}{\sinh 2kh}\right) + \frac{s(s+1)}{b^2k^2}\left(1 + \frac{2kh}{\sinh 2kh}\right)\right] \times$$

$$\times \left(C_s \cos 2\omega t - S_s \sin 2\omega t\right)$$

$$- \frac{1}{4\pi (kb)^4} \sum_{s=0}^{\infty} \left[\left(1 - \frac{s(s+1)}{b^2k^2}\right)\left(1 + \frac{2kh}{\sinh 2kh}\right) E_s\right] \tag{6.18}$$

where D is the diameter of the cylinder.

Thus the total horizontal force in nondimensional form is expressed as

$$F = F_1 + F_2 \tag{6.19}$$

where

$$F_1 = C_M \frac{(\pi/8)(H/L)}{D/L} \tanh kh \cos(\omega t - \alpha_1), \tag{6.20}$$

$$F_2 = \left[\frac{(\pi/8)(H/L)^2}{(D/L)} \tanh kh\, e^{-2i\omega t} \int G(k_2)\, dk_2 + \text{c.c.}\right]$$

$$- \frac{(H/L)^2}{4\,(D/L)(kb)^3} \sum_{s=0}^{\infty} (-1)^s \left\{\left(3 - \frac{2kh}{\sinh 2kh}\right) + \frac{s(s+1)}{b^2k^2}\left(1 + \frac{2kh}{\sin 2kh}\right)\right.$$

$$\times \left(C_s \cos 2\omega t - S_s \sin 2\omega t\right)$$

$$- \frac{(H/L)^2}{4(D/L)(kb)^3} \sum_{s=0}^{\infty} \left\{\left[1 - \frac{s(s+1)}{b^2k^2}\right]\left(1 + \frac{2kh}{\sin 2kh}\right) E_s\right\} \tag{6.21}$$

where $H = 2a$ is the total waveheight and L is the wavelength of the basic wave, and C_M is defined to be the Morison coefficient due to linearized theory and is given by

$$C_M = 4/\left[\pi (kb)^2 \left|H_1^{(1)'}(kb)\right|\right] \tag{6.22}$$

In ocean engineering problems, wave forces on the structures depend essentially on three dimensionless parameters H/L, D/L and h/L. However, Hunt and Williams have pointed out that many experimental studies of wave forces have been published with such a variation of parameters that precise experimental verification is not possible. Recent findings of Rahman and Heaps have been compared with experimental data collected by Mogridge and Jamieson [3]. An agreement between theory and experiment is quite satisfactory as shown in Figures 2 - 4. Another comparison is made in Fig. 5 with the experimental data due to Raman and Venkatanarasaiah [24]. The second-order results of Rahman and Heaps [17] seem to compare well with these experimental data. In Fig. 6, both the first-order and the second-order solutions are compared with force measurements of Chakrabarti [13] which are generally found to be closer to the second-order theory.

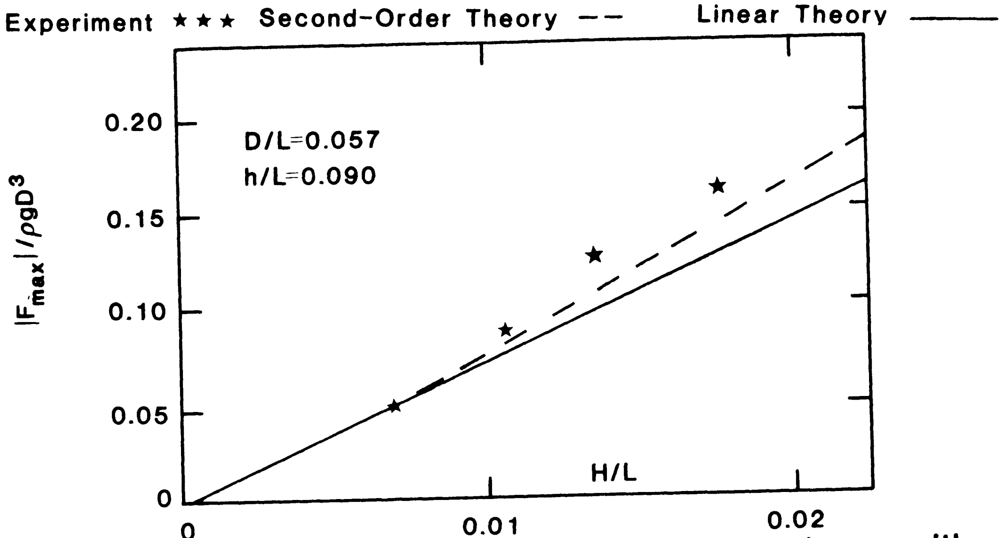

FIG. 2. Comparison of linear and second-order wave forces with experimental data of Mogridge and Jamieson [3].

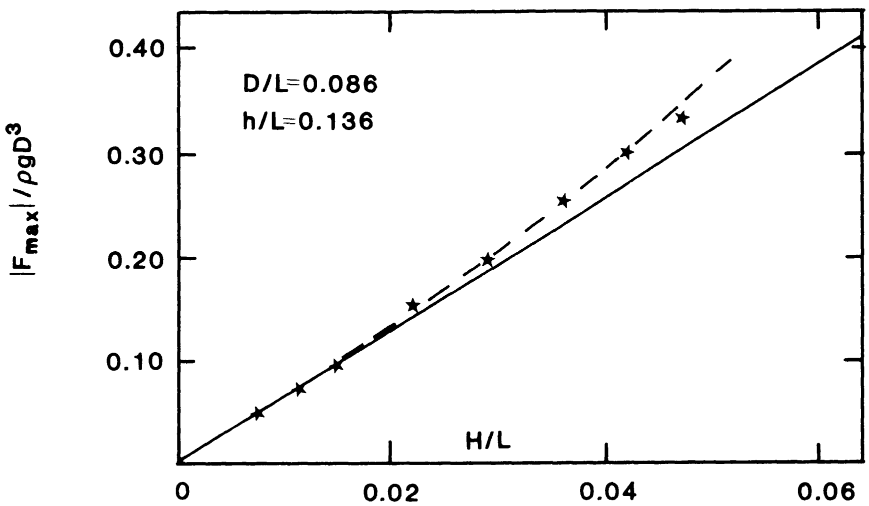

FIG. 3. Comparison of linear and second-order wave forces with experimental data of Mogridge and Jamieson [3].

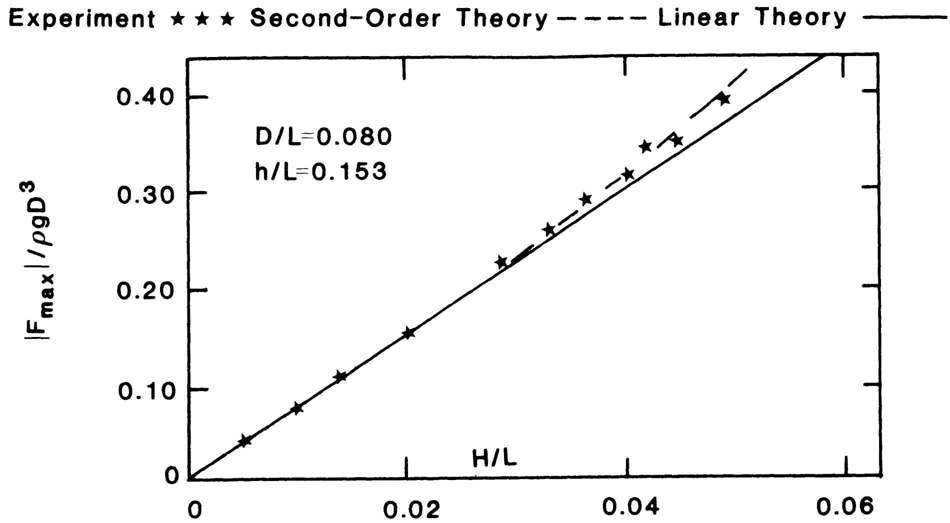

FIG.4 Comparison of linear and second-order wave forces with experimental data of Mogridge and Jamieson [3].

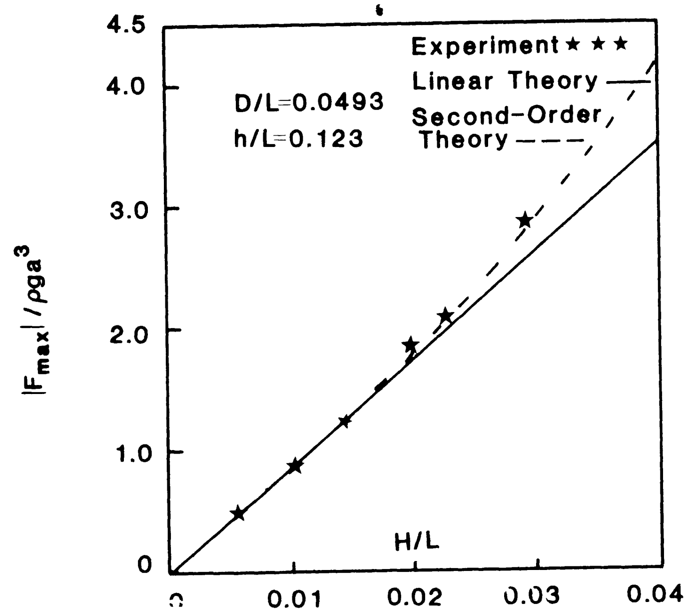

FIG. 5. Comparison of linear and second-order wave forces with experimental data of Raman and Venkatanarasaiah [24].

A final comment on the singular nature of $G(k_2)$ is in order. For a cylindrical structure, the wavenumber k_2 of the second-order wave theory must not coincide with the root of the equation (6.16) unless the corresponding integral in (6.15) vanishes. Otherwise, the structure will experience a resonant response at the wavenumber k_2. This kind of resonance is predicted by the second-order diffraction theory but not by the linear wave theory. In real situations involving ocean waves, such nonlinear resonant phenomenon is frequently observed. Hence the correct values of the wave forces on the offshore structures cannot be predicted by the linear wave theory.

According to Rahman and Heaps' analysis, the cylindrical structure will experience a resonance when $k_2 = 4k$ for the case of deep water waves, and when $k_2 = 2k$ for the case of shallow water waves. Obviously, there is a need for modification of the existing theories in order to obtain a meaningful solution at the resonant wave-number. A partial answer to the resonant behavior related to the shallow water case has been given by Rahman [25].

Recently, Sabuncu and Goren [21] have studied the problem of nonlinear diffraction of a progressive wave in finite deep water, incident on a fixed circular dock. This study shows that the second-order contribution to the horizontal force is also highly significant. Their numerical results for the vertical and horizontal wave forces on the dock are in excellent agreement with those of others. Demirbilek and Gaston [22] have also reported some improvements on the existing results concerning the nonlinear wave loading on a vertical circular cylinder. In spite of various analytical and numerical treatments of the problems, further study is desirable in order to resolve certain discrepancies of the predicted results.

Finally, we close this section by citing a somewhat related problem of waves incident on harbors. A recent study of Burrows [26] on linear waves incident on a circular harbor with a narrow gap demonstrates that the wave amplitude inside the harbor is significantly affected by the frequency of the incident waves. At certain

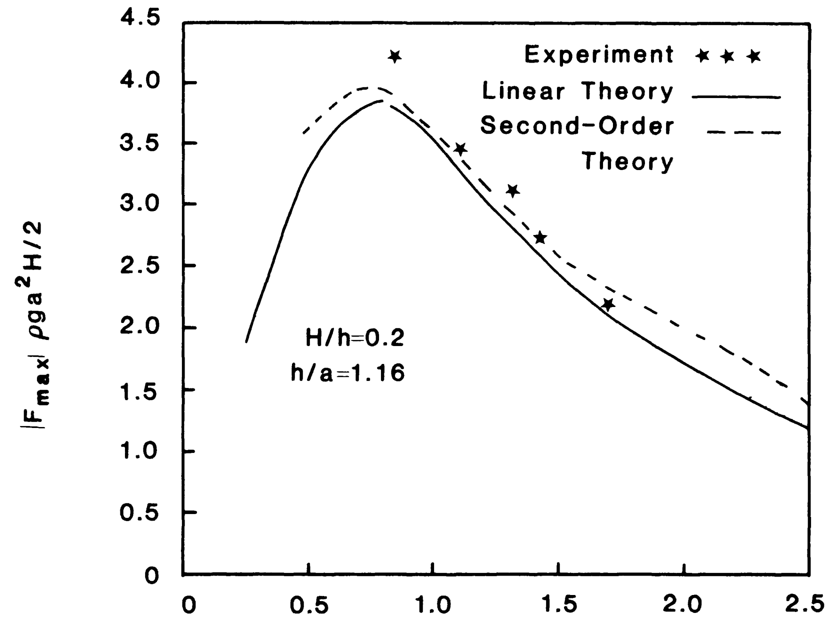

FIG. 6. Comparison of linear and second-order wave forces with experimental data of Chakrabarti [13].

frequencies the harbor acts as a resonator and the wave amplitude becomes very large. If the harbor is closed and the damping neglected, the free-wave motion is the superposition of normal modes of standing waves with a discrete spectrum of characteristic frequencies. With a circular harbor with a narrow opening, a resonance occurs whenever the frequency of the incident waves approaches to a characteristic frequency of the closed harbor. A resonance of a different kind is given by the so called Helmholtz mode when the oscillatory motion inside the harbor is much slower than each of the normal modes. Burrows determined the steady-state response of the harbor with a narrow gap of angular width 2α to an incident wave of a single frequency under the assumption of small width compared to the wavelength. The response is a function of frequency and has a large value (a resonance) at the frequency of the Helmholtz mode and also near the characteristic frequencies of the closed harbor. The actual nature of the response near these frequencies depends on 2α. It is shown that the peak value at each resonance increases as α decreases, that is the harbor paradox for a single incident wave frequency. However, the increase is slow. The peak width also depends on α, and decreases as α decreases, but the decrease for the Helmholtz mode is less than for the higher modes.

Some authors including Lee [27] gave a numerical treatment of the resonance problem inside the harbor. In approximate calculations it is assumed that the total flow through the gap will effectively determine the flow near the resonant frequency. This is correct near the Helmholtz resonance, but incorrect near the higher resonances where the through-flow is small. Most of the work on the subject of Helmholtz resonance was based on the linear theory. The question remains whether or not the circular harbor is a Helmholtz resonator for nonlinear water waves.

7. NONLINEAR WAVE DIFFRACTION CAUSED BY LARGE CONICAL STRUCTURES.

We consider a rigid conical structure in waves as depicted in Figure 7. With reference to this figure, the equation of the cone may be given by $r = (b-z)\tan\alpha$ where b is the distance between the vertex of the cone and undisturbed water surface, α is the semi-vertical angle of the cone and r is the radial distance of the cylindrical coordinates (r,θ,z). The fluid occupies the space $(b-z)\tan\alpha \le r < \infty$, $-\pi \le \theta \le \pi$, $-h \le z \le \eta(r,\theta,t)$, where h is the height of the undisturbed free surface from the ocean-bed and $\eta(r,\theta,t)$ is the vertical elevation of the free surface.

The governing partial differential equation for the velocity potential $\phi = \phi(r,\theta,z,t)$ is

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \tag{7.1}$$

within the region $(b-z)\tan\alpha \le r < \infty$, $-\pi < \theta < \pi$, $-h \le z \le \eta$.

The free surface conditions are

$$\frac{\partial\phi}{\partial t} + g\eta + \frac{1}{2}\left[\left(\frac{\partial\phi}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial\phi}{\partial\theta}\right)^2 + \left(\frac{\partial\phi}{\partial z}\right)^2\right] = 0 \tag{7.2}$$

for $z = \eta$ and $(b-z)\tan\alpha \le r$;

$$\frac{\partial\eta}{\partial t} + \frac{\partial\phi}{\partial r}\cdot\frac{\partial\eta}{\partial r} + \frac{1}{r^2}\frac{\partial\phi}{\partial\theta}\cdot\frac{\partial\eta}{\partial\theta} = \frac{\partial\phi}{\partial z} \tag{7.3}$$

for $z = \eta$ and $(b-z)\tan\alpha \le r$.

The boundary condition at the ocean-bed is

$$\frac{\partial\phi}{\partial z} = 0 \qquad \text{at} \qquad z = -h \tag{7.4}$$

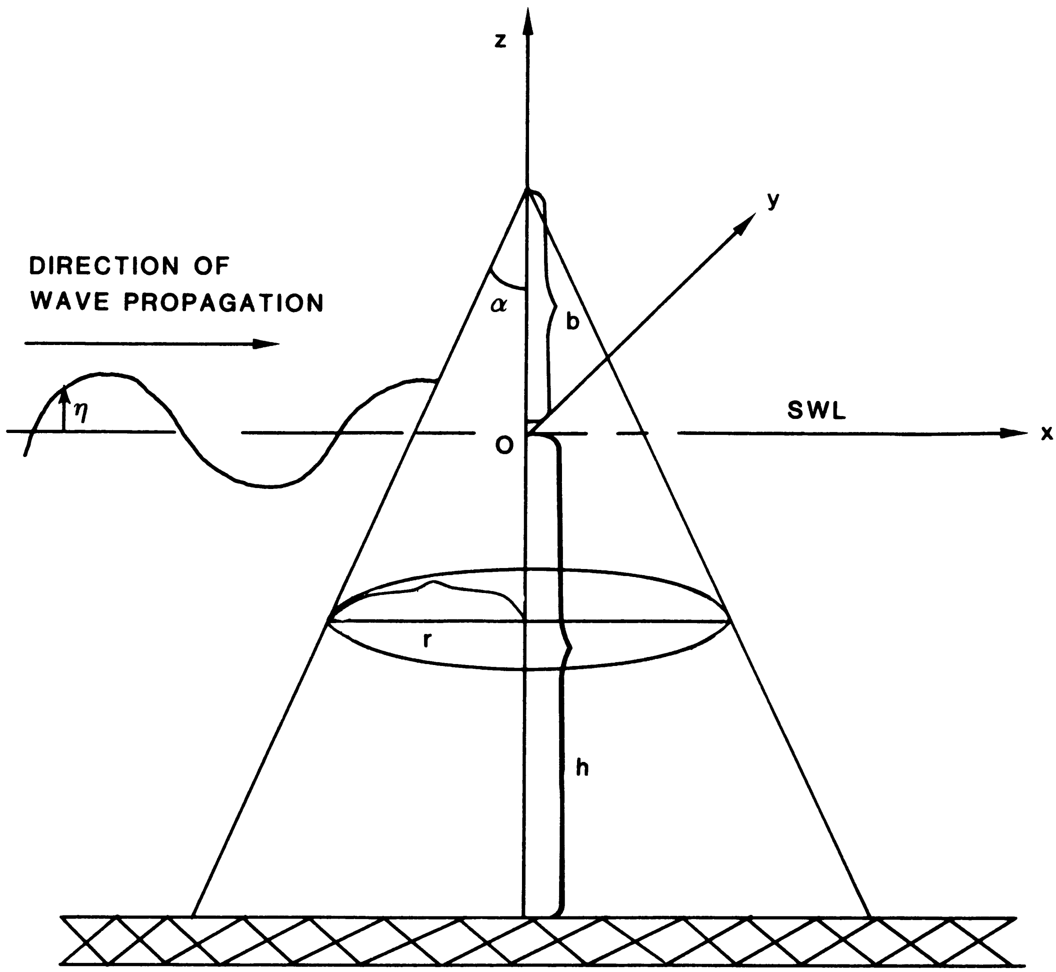

FIG.7. Schematic Diagram of a Conical Structure in waves.

The boundary condition on the body surface is

$$\frac{\partial \phi}{\partial n} = \frac{\partial \phi}{\partial r} \cos \alpha + \frac{\partial \phi}{\partial z} \sin \alpha = 0 \tag{7.5}$$

at $r = (b-z) \tan \alpha$, $-h \leq z \leq \eta$ where n is the distance normal to the body surface. There is another boundary condition which is needed for the unique solution of this boundary value problem. This condition is known as the Sommefeld radiation condition which is discussed by Stoker [28]. This is briefly deduced as follows:

The velocity potential ϕ may be expressed as

$$\phi(r,\theta,z,t) = \mathrm{Re}[\Phi(r,\theta,z)\, e^{i\omega t}] \tag{7.6}$$

where Re stands for the real part and ω is the frequency. We assume

$$\Phi(r,\theta,z) = \Phi_I + \Phi_S$$

such that $\phi_I = \mathrm{Re}[\Phi_I\, e^{-i\omega t}]$, $\phi_S = \mathrm{Re}[\Phi_S\, e^{-i\omega t}]$

Therefore, $\phi = \phi_I + \phi_S$ (7.7)

where ϕ_I and ϕ_S are the incident wave and scattered wave potentials respectively. Then the radiation condition is written as

$$\lim_{r\to\infty} \sqrt{r}\ \left(\frac{\partial \Phi_S}{\partial r} - i\, k\, \Phi_S\right) = 0 \tag{7.8}$$

This condition may be generally satisfied when Φ_S takes an asymptotic form proportional to $(r)^{-1/2}\exp(-ikr)$. Here k is a wave number.

The linear incident wave potential ϕ_I may be obtained from the solution of the Laplace's equation,

$$\nabla^2\phi_I = 0 = \frac{\partial^2\phi_I}{\partial x^2} + \frac{\partial^2\phi_I}{\partial y^2} + \frac{\partial^2\phi_I}{\partial z^2} \tag{7.9}$$

subject to the linear boundary conditions (Sarpkaya and Isaacson [29])

$$\phi_I(x,y,z,t) = \mathrm{Re}[\Phi_I(x,y,z)\, e^{-i\omega t}]$$

where

$$\Phi_I = C\,\frac{\cosh k(z+h)}{\cosh kh}\exp(i(kx\cos\gamma + ky\sin\gamma)), \tag{7.10}$$

and $C = -\left(\frac{H\,g\,i}{2\omega}\right)$, γ is the direction of propogation of the incident wave in the x-y plane.

The famous dispersion relation for water waves is

$$\omega^2 = gk\tanh kh \tag{7.11}$$

Using this relation, we find

$$\exp(i(kx\cos\gamma + ky\sin\gamma))$$

$$= \exp(i(kr\cos(\theta-\gamma))$$

$$= \sum_{m=0}^{\infty} \beta_m\, J_m\,(kr)\cos m(\theta-\gamma)$$

where $\beta_0 = 1$, and $\beta_m = \delta_m \exp(i_m\,\pi/2)$, $\delta_0 = 1$, $m=0$; $\delta_m = 2$, $m \geq 1$.

The incident wave expression (7.10) can be written as

$$\Phi_I = C \frac{\cosh k(z+h)}{\cosh kh} \sum_{m=0}^{\infty} \beta_m J_m (kr) \cos m(\theta-\gamma) \tag{7.12}$$

We are now in a position to construct the scattered potential Φ_S which is given by

$$\Phi_S = C \frac{\cosh k(z+h)}{\cosh kh} \sum_{m=0}^{\infty} \beta_m B_m H_m^{(1)} (kr) \cos m(\theta-\gamma) \tag{7.13}$$

where B_m is a constant.

It can be easily verified that (7.13) satisfies the radiation condition (7.8). The surface boundary condition (7.5) gives that

$$\frac{\partial \Phi_S}{\partial n} = - \frac{\partial \Phi_I}{\partial n} \qquad \text{at} \quad r = (b-z) \tan \alpha \tag{7.14}$$

$$\frac{\partial \Phi_I}{\partial n} = \frac{\partial \Phi_I}{\partial r} \cos \alpha + \frac{\partial \Phi_I}{\partial z} \sin \alpha$$

$$= \frac{kC}{\cosh kh} \sum_{m=0}^{\infty} \beta_m [J_m'(kr) \cosh k(z+h) \cos \alpha$$

$$+ J_m(kr) \sinh k(z+h) \sin \alpha] \cos m(\theta-\gamma) \tag{7.15}$$

where γ is the angle made by unit normal with the radial distance r. Similarly, we get

$$\frac{\partial \Phi_S}{\partial n} = \frac{\partial \Phi_S}{\partial r} \cos \alpha + \frac{\partial \Phi_S}{\partial z} \sin \alpha$$

$$= \frac{kC}{\cosh kh} \sum_{m=0}^{\infty} \beta_m B_m [H_m^{(1)'}(kr) \cosh k(z+h) \cos \alpha$$

$$+ H_m^{(1)} (kr) \sinh k(z+h) \sin \alpha] \cos m(\theta-\gamma) \tag{7.16}$$

Comparing the coefficients of $\beta_m \cos m(\theta-\gamma)$, using the conditions (7.14), we obtain

$$J_m' (kr) + J_m (kr) \tanh k(z+h) \tan \alpha$$

$$= - B_m [H_m^{(1)'} (kr) + H_m^{(1)}(kr) \tanh k(z+h) \tan \alpha] \tag{7.17}$$

$$\text{at} \quad r = (b-z) \tan \alpha, \; - h \leq z \leq \eta.$$

It is to be noted from (7.17) that the constant B_m turns out to be a function of z instead of a constant. In order to overcome this difficulty, we estimate the constant B_m by taking the depth average value, which is obtained by integrating both sides of (7.17) with respect to z from $z = -h$ to $z = 0$, such that

$$B_m = \frac{-\int_{-h}^{0} [J_m'(k(b-z)\tan \alpha) + J_m(k(b-z)\tan \alpha)\tanh k(z+h)\tan \alpha] \, dz}{\int_{-h}^{0} [H_m^{(1)'} (k(b-z)\tan \alpha) + H_m^{(1)}(k(b-z)\tan \alpha)\tanh k(z+h)\tan \alpha] \, dz}, \tag{7.18}$$

where $m = 0,1,2,\ldots.$

Once the scattered potential Φ_S is determined, we can formulate the wave forces on the structures. Therefore, the total complex potential Φ may be written as

$$\Phi_\ell(x,y,z) = C\,\frac{\cosh k(z+h)}{\cosh kh}\sum_{m=0}^{\infty}\beta_m\,[J_m(kr) + B_m\,H_m^{(1)}(kr)]\,\cos m(\theta-\gamma) \tag{7.19}$$

The formulation of the wave forces is given in the next section.

8. WAVE FORCES FORMULATION.

Lighthill [14] demonstrated that second order wave forces on arbitrary shaped structures may be determined from the knowledge of linear velocity potential alone. The exact calculation of second order forces on right circular cylinders has been obtained by Debnath and Rahman [9] using the Lighthill's technique. The total potential has been obtained in the following form:

$$F = F_\ell + F_d + F_w + F_q \tag{8.1}$$

where F_ℓ is the linear force, F_d is the second order dynamic force, F_w is the second order waterline force, and F_q is the quadratic force. These force components are all functions of the linear diffraction potential ϕ_ℓ. They may be obtained using the following formulas:

The linear force is

$$F_\ell = \int_S \left(-\rho\,\frac{\partial\phi_\ell}{\partial t}\right) n_x\,dS \tag{8.2}$$

which can be subsequently written as

$$F_\ell = \mathrm{Re}\,[-i\rho\omega e^{-i\omega t}\int_S \Phi_\ell\, n_x\,dS] \tag{8.3}$$

where $\qquad \Phi_\ell = \Phi_I + \Phi_S.$

The second order dynamic force is

$$F_d = -\frac{1}{2}\rho\int_S (\nabla\phi_\ell)^2\, n_x\,dS \tag{8.4}$$

Making use of the identity $s_1\,s_2 = \mathrm{Re}\,[\frac{1}{2}z_1\,z_2\,e^{-2i\omega t} + \frac{1}{2}z_1\,z_2^*]$
where, $s_1 = \mathrm{Re}\,[z_1\,e^{-i\omega t}]$, $s_2 = \mathrm{Re}\,[z_2\,e^{-i\omega t}]$ and the asterick denotes the complex conjugate, we can write

$$F_d = -\frac{1}{4}\rho\,\mathrm{Re}\,[e^{-2i\omega t}\int_S (\nabla\Phi_\ell)\,n_x\,dS] - \frac{1}{4}\rho\int_S |\nabla\Phi_\ell|^2\,n_x\,dS. \tag{8.5}$$

The waterline force is

$$F_w = \int_{z=0} (\rho/2g)\left(\frac{\partial\phi_\ell}{\partial t}\right)^2 dy \tag{8.6}$$

which can be subsequently written as

$$F_w = -\frac{1}{4}(\rho\omega^2/g)\ \mathrm{Re}\ [\int_{z=0} e^{-2i\omega t}\ (\Phi_\ell)^2\ dy] + \frac{1}{4}(\rho\omega^2/g)\int_{z=0} |\Phi_\ell|^2\ dy. \tag{8.7}$$

Making reference to Rahman [15] the quadratic force may be written as

$$F_q = \mathrm{Re}\ [(-2\rho\omega^2/g\sigma\ e^{-2i\omega t}\int_{z=0}(\frac{\partial\psi}{\partial z})_{z=0}\ \{(\nabla\Phi_\ell)^2$$

$$-\frac{1}{2g}\Phi_\ell\ (-\omega^2\frac{\partial\Phi_\ell}{\partial z} + g\frac{\partial^2\Phi_\ell}{\partial z^2})\}\ dx\ dy] \tag{8.8}$$

where $\sigma = 4\omega^2/g = 4k\tanh kh$ and ψ is the complex time independent potential generated by the structure surging at a frequency of 2ω.

The vertical particle velocity $(\frac{\partial\psi}{\partial z})$ on $z = 0$ may be written in a series form for finite water depth

$$(\frac{\partial\psi}{\partial z})_{z=0} = [\sum_{j=1}^{\infty}\{-\frac{2K_1(m_j r)}{K_1'(m_j a)}\frac{\sin^2(m_j h)}{(m_j h + \sin m_j h\cos m_j h)}\}$$

$$+\frac{2H_1^{(2)}(\nu r)}{H_1^{(2)'}(\nu a)}\frac{\sinh^2\nu h}{(\nu h + \sinh\nu h\cosh\nu h)}]\cos\theta \tag{8.9}$$

where $(4\omega^2/g) = -m_j\tan m_j h$ for $j = 1,2,\ldots,\infty$ and $\sigma = \nu\tanh\nu h = 4k\tanh kh$. Expression (8.9) is valid only for right circular cylindrical structures. The wave drift forces on the structures may be obtained from the equations (8.5) and (8.7) collecting the steady state components of the forces F_d and F_w. Thus the drift forces on the structure is

$$F_{drift} = -\frac{1}{4}\rho\int_S |\nabla\Phi_\ell|^2\ n_x dS + \frac{1}{4}(\rho\omega^2/g)\int_{z=0}|\Phi_\ell|^2\ dy. \tag{8.10}$$

9. CALCULATION OF WAVE FORCES.

The total wave forces may be obtained from the formulas (8.3), (8.5), (8.7) and (8.8). The linear resultant force can be obtained from (8.3) and is given by

$$F_\ell = \mathrm{Re}\ [-i\rho\omega\ e^{-i\omega t}\int_S \Phi_\ell\ n_x\ dS]$$

$$= \mathrm{Re}\ [-i\rho\omega\ e^{-i\omega t}\int_{\theta=0}^{2\pi}\int_{z=-h}^{0}\Phi_\ell\ \{(b-z)\tan\alpha\ dz\}(-\cos\theta)d\theta]$$

$$= \mathrm{Re}\ [\frac{2\pi\rho\omega C\tan\alpha}{\cosh kh}\cos\gamma\ e^{-i\omega t}\int_{z=-h}^{0}(b-z)\cosh k(z+h)A_1(k(b-z)\tan\alpha)dz] \tag{9.1}$$

where $A_m(kr) = J_m(kr) + B_m H_m^{(1)}(kr)$, $m = 1,2,3,\ldots.$

Therefore, the horizontal and vertical forces can be obtained respectively as

$$F_{\ell X} = F_\ell \cos\alpha, \quad F_{\ell Z} = F_\ell \sin\alpha \tag{9.2ab}$$

The resultant dynamic force can be calculated from (8.5) and is given by

$$F_d = -\frac{\rho}{4} \operatorname{Re}\left[e^{-2i\omega t}\int_S (\nabla\Phi_\ell)^2 \underset{\sim}{n}\, dS\right] - \frac{\rho}{4}\int_S |\nabla\Phi_\ell|^2 \underset{\sim}{n}\, dS$$

$$= -\frac{\rho}{4}\operatorname{Re}\left[e^{-2i\omega t}\int_{\theta=0}^{2\pi}\int_{z=-h}^{0} (\nabla\Phi_\ell)^2((b-z)\tan\alpha)dz\,(-\cos\theta)d\theta\right]$$

$$-\frac{\rho}{4}\int_0^{2\pi}\int_{-h}^{0} |\nabla\Phi_\ell|^2((b-z)\tan\alpha)dz\,(-\cos\theta)\,d\theta \tag{9.3}$$

After extensive algebraic calculations, the dynamic force can be written as

$$F_d = \left(\frac{\rho\pi\cos\gamma}{4}\right)\operatorname{Re}\Big[e^{-2i\omega t}\Big\{\sum_{\ell=0}^{\infty}\int_{-h}^{0}(b-z)\tan\alpha\,\frac{4C^2}{\cosh^2 kh}\Big(\frac{\cosh^2 k(z+h)}{(b-z)^2\tan^2\alpha}\,\ell(\ell+1)$$

$$+ k^2\sec^2\alpha\,\sinh^2 k(z+h)\Big)\Big(e^{-(2\ell+1)\frac{\pi i}{2}} A_\ell A_{\ell+1}\Big)\,dz\Big\}\Big]$$

$$+\left(\frac{\rho\pi\cos\gamma}{4}\right)\sum_{\ell=0}^{\infty}\int_{-h}^{0}\frac{2(b-z)\tan\alpha\,|C|^2}{\cosh^2 kh}\times$$

$$\times\left(\frac{\cosh^2 k(z+h)}{(b-z)^2\tan^2\alpha}\,\ell(\ell+1) + k^2\sec^2\alpha\,\sinh^2 k(z+h)\right)\left(e^{\frac{\pi i}{2}} A_\ell A^*_{\ell+1} + e^{-\frac{\pi i}{2}} A^*_\ell A_{\ell+1}\right)dz \tag{9.4}$$

Therefore, the horizontal and vertical dynamic forces can be obtained respectively as

$$F_{dX} = F_d\cos\alpha, \quad F_{dZ} = F_d\sin\alpha \tag{9.5ab}$$

The resultant waterline force can be obtained from (8.7) and is given by

$$F_w = -\left(\frac{\rho\omega^2}{4g}\right)\operatorname{Re}\left[e^{-2i\omega t}\int_{z=0}(\Phi_\ell)^2 dy\right] + \frac{\rho\omega^2}{4g}\int_{z=0}|\Phi_\ell|^2 dy$$

$$= -\left(\frac{\rho\omega^2}{4g}\right)\operatorname{Re}\left[e^{-2i\omega t}(-4\pi C^2\cos\gamma)(b\tan\alpha)\sum_{\ell=0}^{\infty}A_\ell A_{\ell+1}\,e^{-(2\ell+1)\frac{\pi i}{2}}\right]$$

$$+\left(\frac{\rho\omega^2}{4g}\right)(-2\pi|C|^2\cos\gamma)(b\tan\alpha)\sum_{\ell=0}^{\infty}\left(e^{\frac{\pi i}{2}}A_\ell A^*_{\ell+1} + e^{-\frac{\pi i}{2}}A^*_\ell A_{\ell+1}\right) \tag{9.6}$$

Therefore, the horizontal and vertical forces can be obtained respectively as

$$F_{wX} = F_w\cos\alpha, \quad F_{wZ} = F_w\sin\alpha \tag{9.7ab}$$

Thus the drift force as defined by (8.10) can be obtained as follows:

$$F_{drift} = -\frac{\rho}{4}\int_{\theta=0}^{2\pi}\int_{z=-h}^{0} |\nabla\Phi_\ell|^2 \,((b-z)\tan\alpha)dz\,(-\cos\theta)d\theta$$

$$+ \frac{\rho\omega^2}{4g}\int_{\theta=0}^{2\pi} |\Phi_\ell|^2_{z=0}\ b\tan\alpha\ (-\cos\theta)d\theta$$

$$= (\frac{\rho\pi\cos\gamma}{4})\sum_{\ell=0}^{\infty}\int_{-h}^{0}\frac{2|C|^2(b-z)\tan\alpha}{\cosh^2 kh}\times$$

$$\times\ (\ell(\ell+1)\frac{\cosh^2 k(z+h)}{(b-z)^2\tan^2\alpha} + k^2\sec^2\alpha\sinh^2 k(z+h))\times$$

$$\times\ (e^{\frac{\pi i}{2}} A_\ell A^*_{\ell+1} + e^{-\frac{\pi i}{2}} A^*_\ell A_{\ell+1})dz$$

$$-\ (\frac{\rho\omega^2}{4g})(2\pi|C|^2\cos\gamma)(b\tan\alpha)\sum_{\ell=0}^{\infty}(e^{\frac{\pi i}{2}} A_\ell A^*_{\ell+1} + e^{-\frac{\pi i}{2}} A^*_\ell A_{\ell+1}) \quad (9.8)$$

Therefore, the horizontal and vertical drift forces may be respectively obtained as

$$(F_{drift})_X = (F_{drift})\cos\alpha,\ (F_{drift})_Z = (F_{drift})\sin\alpha \quad (9.9ab)$$

The resultant quadratic force can be obtained from the formula (8.8) and is given by

$$F_q = \mathrm{Re}[-\frac{2\rho\omega^2}{g\sigma}e^{-2i\omega t}\int_{z=0}(\frac{\partial\Psi}{\partial z})_{z=0}\{(\nabla\Phi_\ell)^2 - \frac{1}{2g}\Phi_\ell(-\omega^2\frac{\partial\Phi_\ell}{\partial z} + g\frac{\partial\Phi_\ell^2}{\partial z^2})dx\,dy] \quad (9.10)$$

After extensive algebraic calculations, the quadratic force can be written as

$$F_q = \mathrm{Re}[-\frac{2\rho\omega^2}{gK}e^{-2i\omega t}\{\int_{r=b\tan\alpha}^{\infty}(rdr)\times$$

$$\times\int_{\theta=0}^{2\pi}(\frac{\partial\Psi}{\partial z})_{z=0}[4C^2k^2\cos(\theta-\gamma)\sum_{\ell=0}^{\infty}\{(\frac{\ell(\ell+1)}{k^2r^2} + \tanh^2 kh)\times$$

$$\times\ A_\ell A_{\ell+1} + A'_\ell A'_{\ell+1}\}\,e^{-(2\ell+1)\frac{\pi i}{2}} + 2C^2k^2(\tanh^2 kh-1)\cos(\theta-\gamma)\times$$

$$\times\sum_{\ell=0}^{\infty}A_\ell A_{\ell+1}e^{-(2\ell+1)\frac{\pi i}{2}}]d\theta] \quad (9.11)$$

After simplifying, (9.11) reduces to

$$F_q = \mathrm{Re}[-\frac{\rho}{2}e^{-2i\omega t}\int_{r=b\tan\alpha}^{\infty}(rdr)\{\int_{\theta=0}^{2\pi}(\frac{\partial\Psi}{\partial z})_{z=0}\cos(\theta-\gamma)\times$$

$$\times\ 2C^2k^2\sum_{\ell=0}^{\infty}[\{\frac{\ell(\ell+1)}{k^2r^2} + 3\tanh^2 kh-1\}A_\ell A_{\ell+1} + 2A'_\ell A'_{\ell+1}]\,e^{-(2\ell+1)\frac{\pi i}{2}}d\theta\}$$

Therefore, the horizontal and vertical quadratic forces may be respectively obtained as

$$F_{qX} = F_q \cos \alpha, \qquad F_{qZ} = F_q \sin \alpha \tag{9.12ab}$$

10. CONCLUDING REMARKS.

Second order nonlinear effects are included in the derivation of the wave forces on the large conical structures. The second order theory is consistent because it satisfies all the necessary boundary conditions including the radiation condition. Theoretical expressions for the wave forces have been obtained; the linear forces could be improved by adding to it the second order contributions namely, dynamic, waterline and quadratic forces. It would be of considerable value if the theoretical results presented in this paper could be checked experimentally under laboratory conditions. Plans are made in future research to check the accuracy of the predicted results with the experimental measurements.

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