

Research Article

Starlikeness Properties of a New Integral Operator for Meromorphic Functions

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We define here an integral operator $\mathcal{L}_{\gamma_1, \dots, \gamma_n}$ for meromorphic functions in the punctured open unit disk. Several starlikeness conditions for the integral operator $\mathcal{L}_{\gamma_1, \dots, \gamma_n}$ are derived.

1. Introduction

Let Σ denotes the class of functions of the form

$$f(z) = \frac{1}{z} + \sum_{n=0}^{\infty} a_n z^n, \quad (1.1)$$

which are analytic in the punctured open unit disk

$$\mathbb{U}^* = \{z \in \mathbb{C} : 0 < |z| < 1\} = \mathbb{U} \setminus \{0\}, \quad (1.2)$$

where \mathbb{U} is the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$.

We say that a function $f \in \Sigma$ is meromorphic starlike of order α ($0 \leq \alpha < 1$), and belongs to the class $\Sigma^*(\alpha)$, if it satisfies the inequality

$$-\Re\left(\frac{zf'(z)}{f(z)}\right) > \alpha. \quad (1.3)$$

A function $f \in \Sigma$ is a meromorphic convex function of order α ($0 \leq \alpha < 1$), if f satisfies the following inequality

$$-\Re\left(1 + \frac{zf''(z)}{f'(z)}\right) > \alpha, \quad (1.4)$$

and we denote this class by $\Sigma_k(\alpha)$.

Analogous to the integral operator defined by Breaz et al. [1] on the normalized analytic functions, we now define the following integral operator on the space meromorphic functions in the class Σ .

Definition 1.1. Let $n \in \mathbb{N}$, $\gamma_i > 0$, $i \in \{1, 2, 3, \dots, n\}$. We define the integral operator $\mathcal{L}_{\gamma_1, \dots, \gamma_n}(f_1, f_2, \dots, f_n) : \Sigma^n \rightarrow \Sigma$ by

$$\mathcal{L}_{\gamma_1, \dots, \gamma_n}(f_1, \dots, f_n)(z) = \frac{1}{z^2} \int_0^z \left(-u^2 f_1'(u)\right)^{\gamma_1} \cdots \left(-u^2 f_n'(u)\right)^{\gamma_n} du. \quad (1.5)$$

For the sake of simplicity, from now on we will write $\mathcal{L}_{\gamma_1, \dots, \gamma_n}(z)$ instead of $\mathcal{L}_{\gamma_1, \dots, \gamma_n}(f_1, \dots, f_n)(z)$.

By $\Sigma_{k_p}(\beta)$ ($-1 \leq \beta < 1$), we denote the class of functions $f \in \Sigma$ such that

$$\left| \frac{zf''(z)}{f'(z)} + 2 \right| < -\Re\left(\frac{zf''(z)}{f'(z)} + \beta\right) - 1. \quad (1.6)$$

In order to derive our main results, we have to recall here the following preliminary results.

Lemma 1.2 (see [2]). *Suppose that the function $\Psi : \mathbb{C}^2 \rightarrow \mathbb{C}$ satisfies the following condition:*

$$\Re\{\Psi(is, t)\} \leq 0, \quad \left(s, t \in \mathbb{R}; \quad t \leq \frac{-(1+s^2)}{2}\right). \quad (1.7)$$

If the function $p(z) = 1 + p_1z + \dots$ is analytic in \mathbb{U} and

$$\Re\{\Psi(p(z), zp'(z))\} > 0, \quad (z \in \mathbb{U}), \quad (1.8)$$

then,

$$\Re\{p(z)\} > 0 \quad (z \in \mathbb{U}). \quad (1.9)$$

Proposition 1.3 (see [3]). *If $f \in \Sigma$ satisfying*

$$-\Re\left\{\frac{z(zf''(z) + 3f'(z))}{zf'(z) + 2f(z)}\right\} > \alpha, \quad 0 \leq \alpha < 1, \quad (1.10)$$

$$\left|\frac{zf'(z)}{f(z)} + 1\right| < 1,$$

then,

$$-\Re \left\{ \frac{zf'(z)}{f(z)} \right\} > \alpha. \quad (1.11)$$

2. Starlikeness of the Operator $\mathcal{H}_{\gamma_1, \dots, \gamma_n}(z)$

In this section, we investigate sufficient conditions for the integral operator $\mathcal{H}_{\gamma_1, \dots, \gamma_n}(z)$ which is defined in Definition 1.1, to be in the class $\Sigma^*(\alpha)$, $0 \leq \alpha < 1$.

Theorem 2.1. Let $f_i \in \Sigma$, $\gamma_i > 0$ for all $i \in \{1, \dots, n\}$. If

$$-\Re \left(\frac{zf_i''(z)}{f_i'(z)} \right) > \frac{-1}{n\gamma_i} + 2, \quad (2.1)$$

then $\mathcal{H}_{\gamma_1, \dots, \gamma_n}(z)$ belongs to $\Sigma^*(0)$.

Proof. On successive differentiation of $\mathcal{H}_{\gamma_1, \dots, \gamma_n}(z)$, which is defined in (1.5), we get

$$\begin{aligned} 2z\mathcal{H}_{\gamma_1, \dots, \gamma_n}(z) + z^2\mathcal{H}'_{\gamma_1, \dots, \gamma_n}(z) &= \left(-z^2f_1'(z)\right)^{\gamma_1} \cdots \left(-z^2f_n'(z)\right)^{\gamma_n}, \\ z^2\mathcal{H}''_{\gamma_1, \dots, \gamma_n}(z) + 4z\mathcal{H}'_{\gamma_1, \dots, \gamma_n}(z) + 2\mathcal{H}_{\gamma_1, \dots, \gamma_n}(z) & \\ &= \sum_{i=1}^n \gamma_i \left(-z^2f_i'(z)\right)^{\gamma_i-1} \left(-z^2f_i''(z) - 2zf_i'(z)\right) \prod_{j=1, j \neq i}^n \left(-z^2f_j'(z)\right)^{\gamma_j}. \end{aligned} \quad (2.2)$$

Then from (2.2), we obtain

$$\frac{z^2\mathcal{H}''_{\gamma_1, \dots, \gamma_n}(z) + 4z\mathcal{H}'_{\gamma_1, \dots, \gamma_n}(z) + 2\mathcal{H}_{\gamma_1, \dots, \gamma_n}(z)}{z^2\mathcal{H}'_{\gamma_1, \dots, \gamma_n}(z) + 2z\mathcal{H}_{\gamma_1, \dots, \gamma_n}(z)} = \sum_{i=1}^n \gamma_i \left(\frac{zf_i''(z)}{f_i'(z)} + \frac{2}{z} \right). \quad (2.3)$$

By multiplying (2.3) with z yield,

$$\frac{z^2\mathcal{H}''_{\gamma_1, \dots, \gamma_n}(z) + 4z\mathcal{H}'_{\gamma_1, \dots, \gamma_n}(z) + 2\mathcal{H}_{\gamma_1, \dots, \gamma_n}(z)}{z\mathcal{H}'_{\gamma_1, \dots, \gamma_n}(z) + 2\mathcal{H}_{\gamma_1, \dots, \gamma_n}(z)} = \sum_{i=1}^n \gamma_i \left(\frac{zf_i''(z)}{f_i'(z)} + 2 \right). \quad (2.4)$$

That is equivalent to

$$\left\{ \frac{z \left(z\mathcal{H}''_{\gamma_1, \dots, \gamma_n}(z) + 3\mathcal{H}'_{\gamma_1, \dots, \gamma_n}(z) \right)}{z\mathcal{H}'_{\gamma_1, \dots, \gamma_n}(z) + 2\mathcal{H}_{\gamma_1, \dots, \gamma_n}(z)} \right\} + 1 = \sum_{i=1}^n \gamma_i \left(\frac{zf_i''(z)}{f_i'(z)} + 2 \right). \quad (2.5)$$

Or

$$-\left\{ \frac{z \left(z \mathcal{H}'_{\gamma_1, \dots, \gamma_n}(z) + 3 \mathcal{H}'_{\gamma_1, \dots, \gamma_n}(z) \right)}{z \mathcal{H}'_{\gamma_1, \dots, \gamma_n}(z) + 2 \mathcal{H}_{\gamma_1, \dots, \gamma_n}(z)} \right\} = \sum_{i=1}^n \gamma_i \left(-\frac{z f_i''(z)}{f_i'(z)} \right) - 2 \sum_{i=1}^n \gamma_i + 1. \quad (2.6)$$

We can write the left-hand side of (2.6), as the following:

$$\begin{aligned} & \frac{-\left(z \mathcal{H}'_{\gamma_1, \dots, \gamma_n}(z) / \mathcal{H}_{\gamma_1, \dots, \gamma_n}(z) \right) \left(\left(z \mathcal{H}''_{\gamma_1, \dots, \gamma_n}(z) / \mathcal{H}'_{\gamma_1, \dots, \gamma_n}(z) \right) + 3 \right)}{\left(z \mathcal{H}'_{\gamma_1, \dots, \gamma_n}(z) / \mathcal{H}_{\gamma_1, \dots, \gamma_n}(z) \right) + 2} \\ &= \sum_{i=1}^n \gamma_i \left(-\frac{z f_i''(z)}{f_i'(z)} \right) - 2 \sum_{i=1}^n \gamma_i + 1. \end{aligned} \quad (2.7)$$

We define the regular function p in \mathbb{U} by

$$p(z) = -\frac{z \mathcal{H}'_{\gamma_1, \dots, \gamma_n}(z)}{\mathcal{H}_{\gamma_1, \dots, \gamma_n}(z)}, \quad (2.8)$$

and $p(0) = 1$. Differentiating $p(z)$ logarithmically, we obtain

$$-p(z) + \frac{z p'(z)}{p(z)} = 1 + \frac{z \mathcal{H}''_{\gamma_1, \dots, \gamma_n}(z)}{\mathcal{H}'_{\gamma_1, \dots, \gamma_n}(z)}. \quad (2.9)$$

From (2.7), (2.8), and (2.9), we obtain

$$p(z) + \frac{z p'(z)}{-p(z) + 2} = \sum_{i=1}^n \gamma_i \left(-\frac{z f_i''(z)}{f_i'(z)} \right) - 2 \sum_{i=1}^n \gamma_i + 1. \quad (2.10)$$

Let us put

$$\Psi(u, v) = u + \frac{v}{-u + 2}. \quad (2.11)$$

From (2.1), (2.10), and (2.11), we obtain

$$\begin{aligned} \Re \{ \Psi(p(z), z p'(z)) \} &= \gamma_1 \left(-\Re \frac{z f_1''(z)}{f_1'(z)} \right) + \dots + \left(-\Re \frac{z f_n''(z)}{f_n'(z)} \right) - 2(\gamma_1 + \dots + \gamma_n) + 1 \\ &> \gamma_1 \left(\frac{-1}{n \gamma_1} + 2 \right) + \dots + \gamma_n \left(\frac{-1}{n \gamma_n} + 2 \right) - 2(\gamma_1 + \dots + \gamma_n) + 1 = 0. \end{aligned} \quad (2.12)$$

Now, we proceed to show that

$$\Re\{\Psi(is, t)\} \leq 0, \quad \left(s, t \in \mathbb{R}; t \leq \frac{-(1+s^2)}{2}\right). \quad (2.13)$$

Indeed, from (2.11), we have

$$\Re\{\Psi(is, t)\} = \Re\left\{is + \frac{t}{-is+2}\right\} = \frac{2t}{4+s^2} \leq -\frac{1+s^2}{4+s^2} < 0. \quad (2.14)$$

Thus, from (2.12), (2.14), and by using Lemma 1.2, we conclude that $\Re\{p(z)\} > 0$, and so

$$-\Re\left\{\frac{z\mathcal{H}'_{\gamma_1, \dots, \gamma_n}(z)}{\mathcal{H}_{\gamma_1, \dots, \gamma_n}(z)}\right\} > 0 \quad (2.15)$$

that is, $\mathcal{H}_{\gamma_1, \dots, \gamma_n}(z)$ is starlike of order 0. \square

Theorem 2.2. For $i \in \{1, \dots, n\}$, let $\gamma_i > 0$ and $f_i \in \Sigma_k(\alpha_i)$ ($0 \leq \alpha_i < 1$). If $0 < \sum_{i=1}^n \gamma_i(1 - \alpha_i) \leq 1$, $\mathcal{H}_{\gamma_1, \dots, \gamma_n}(z)$ be the integral operator given by (1.5) and

$$\left|\frac{z\mathcal{H}'_{\gamma_1, \dots, \gamma_n}(z)}{\mathcal{H}_{\gamma_1, \dots, \gamma_n}(z)} + 1\right| < 1. \quad (2.16)$$

Then $\mathcal{H}_{\gamma_1, \dots, \gamma_n}(z)$ belong to $\Sigma^*(\mu)$, where $\mu = 1 - \sum_{i=1}^n \gamma_i(1 - \alpha_i)$.

Proof. Following the same steps as in Theorem 2.1, we obtain

$$-\left\{\frac{z\left(z\mathcal{H}''_{\gamma_1, \dots, \gamma_n}(z) + 3\mathcal{H}'_{\gamma_1, \dots, \gamma_n}(z)\right)}{z\mathcal{H}'_{\gamma_1, \dots, \gamma_n}(z) + 2\mathcal{H}_{\gamma_1, \dots, \gamma_n}(z)}\right\} = \sum_{i=1}^n \gamma_i \left\{-\left(\frac{zf_i''(z)}{f_i'(z)} + 1\right)\right\} + 1 - \sum_{i=1}^n \gamma_i. \quad (2.17)$$

Taking the real part of both terms of the last expression, we have

$$-\Re\left\{\frac{z\left(z\mathcal{H}''_{\gamma_1, \dots, \gamma_n}(z) + 3\mathcal{H}'_{\gamma_1, \dots, \gamma_n}(z)\right)}{z\mathcal{H}'_{\gamma_1, \dots, \gamma_n}(z) + 2\mathcal{H}_{\gamma_1, \dots, \gamma_n}(z)}\right\} = \sum_{i=1}^n \gamma_i \left\{-\Re\left(\frac{zf_i''(z)}{f_i'(z)} + 1\right)\right\} + 1 - \sum_{i=1}^n \gamma_i. \quad (2.18)$$

Since $f_i \in \Sigma_k(\alpha_i)$, for $i \in \{1, \dots, n\}$, we receive

$$-\Re \left\{ \frac{z \left(z \mathcal{L}''_{\gamma_1, \dots, \gamma_n}(z) + 3 \mathcal{L}'_{\gamma_1, \dots, \gamma_n}(z) \right)}{z \mathcal{L}'_{\gamma_1, \dots, \gamma_n}(z) + 2 \mathcal{L}_{\gamma_1, \dots, \gamma_n}(z)} \right\} > \sum_{i=1}^n \gamma_i \alpha_i + 1 - \sum_{i=1}^n \gamma_i. \quad (2.19)$$

Therefore,

$$-\Re \left\{ \frac{z \left(z \mathcal{L}''_{\gamma_1, \dots, \gamma_n}(z) + 3 \mathcal{L}'_{\gamma_1, \dots, \gamma_n}(z) \right)}{z \mathcal{L}'_{\gamma_1, \dots, \gamma_n}(z) + 2 \mathcal{L}_{\gamma_1, \dots, \gamma_n}(z)} \right\} > 1 - \sum_{i=1}^n \gamma_i (1 - \alpha_i). \quad (2.20)$$

Using (2.16), (2.20), and applying Proposition 1.3, we get $\mathcal{L}_{\gamma_1, \dots, \gamma_n}(z) \in \Sigma^*(\mu)$, where $\mu = 1 - \sum_{i=1}^n \gamma_i (1 - \alpha_i)$. \square

Letting $\alpha_i = \alpha$, $i \in \{1, \dots, n\}$ in Theorem 2.2, we get the following.

Corollary 2.3. For $i \in \{1, \dots, n\}$, let $\gamma_i > 0$ and $f_i \in \Sigma_k(\alpha)$ ($0 \leq \alpha < 1$). If

$$0 < \sum_{i=1}^n \gamma_i \leq \frac{1}{1 - \alpha}, \quad (2.21)$$

$\mathcal{L}_{\gamma_1, \dots, \gamma_n}$ be the integral operator given by (1.5) and

$$\left| \frac{z \mathcal{L}'_{\gamma_1, \dots, \gamma_n}(z)}{\mathcal{L}_{\gamma_1, \dots, \gamma_n}(z)} + 1 \right| < 1. \quad (2.22)$$

Then $\mathcal{L}_{\gamma_1, \dots, \gamma_n}(z)$ is starlike of order $1 - (1 - \alpha) \sum_{i=1}^n \gamma_i$.

Theorem 2.4. For $i \in \{1, \dots, n\}$, let $\gamma_i > 0$ and $f_i \in \Sigma_{k_p}(\beta_i)$ ($-1 \leq \beta_i < 1$). If

$$0 < \sum_{i=1}^n \gamma_i (1 - \beta_i) \leq 1, \quad (2.23)$$

$\mathcal{L}_{\gamma_1, \dots, \gamma_n}(z)$ be the integral operator given by (1.5) and

$$\left| \frac{z \mathcal{L}'_{\gamma_1, \dots, \gamma_n}(z)}{\mathcal{L}_{\gamma_1, \dots, \gamma_n}(z)} + 1 \right| < 1. \quad (2.24)$$

Then $\mathcal{L}_{\gamma_1, \dots, \gamma_n}(z)$ is starlike of order $1 - \sum_{i=1}^n \gamma_i (1 - \beta_i)$.

Proof. Following the same steps as in Theorem 2.1, we obtain

$$\begin{aligned}
 -\left\{ \frac{z \left(z \mathcal{L}''_{\gamma_1, \dots, \gamma_n}(z) + 3 \mathcal{L}'_{\gamma_1, \dots, \gamma_n}(z) \right)}{z \mathcal{L}'_{\gamma_1, \dots, \gamma_n}(z) + 2 \mathcal{L}_{\gamma_1, \dots, \gamma_n}(z)} \right\} &= -\sum_{i=1}^n \gamma_i \left(\frac{z f_i''(z)}{f_i'(z)} + 2 \right) + 1 \\
 &= \sum_{i=1}^n \gamma_i \left\{ -\left(\frac{z f_i''(z)}{f_i'(z)} + \beta_i \right) - 1 \right\} + 1 - \sum_{i=1}^n \gamma_i + \sum_{i=1}^n \gamma_i \beta_i \\
 &= \sum_{i=1}^n \gamma_i \left\{ -\left(\frac{z f_i''(z)}{f_i'(z)} + \beta_i \right) - 1 \right\} + 1 - \sum_{i=1}^n \gamma_i (1 - \beta_i).
 \end{aligned} \tag{2.25}$$

We calculate the real part from both terms of the above equality and obtain

$$\begin{aligned}
 -\Re \left\{ \frac{z \left(z \mathcal{L}''_{\gamma_1, \dots, \gamma_n}(z) + 3 \mathcal{L}'_{\gamma_1, \dots, \gamma_n}(z) \right)}{z \mathcal{L}'_{\gamma_1, \dots, \gamma_n}(z) + 2 \mathcal{L}_{\gamma_1, \dots, \gamma_n}(z)} \right\} & \\
 &= \sum_{i=1}^n \gamma_i \left\{ -\Re \left(\frac{z f_i''(z)}{f_i'(z)} + \beta_i \right) - 1 \right\} + 1 - \sum_{i=1}^n \gamma_i (1 - \beta_i).
 \end{aligned} \tag{2.26}$$

Since $f_i \in \Sigma_{k_p}(\beta_i)$ for all $i \in \{1, \dots, n\}$, the above relation then yields

$$\begin{aligned}
 -\Re \left\{ \frac{z \left(z \mathcal{L}''_{\gamma_1, \dots, \gamma_n}(z) + 3 \mathcal{L}'_{\gamma_1, \dots, \gamma_n}(z) \right)}{z \mathcal{L}'_{\gamma_1, \dots, \gamma_n}(z) + 2 \mathcal{L}_{\gamma_1, \dots, \gamma_n}(z)} \right\} & \\
 &> \sum_{i=1}^n \gamma_i \left| \frac{z f_i''(z)}{f_i'(z)} + 2 \right| + 1 - \sum_{i=1}^n \gamma_i (1 - \beta_i).
 \end{aligned} \tag{2.27}$$

Because $\sum_{i=1}^n \gamma_i |z f_i''(z) / f_i'(z) + 2| \geq 0$, we obtain that

$$-\Re \left\{ \frac{z \left(z \mathcal{L}''_{\gamma_1, \dots, \gamma_n}(z) + 3 \mathcal{L}'_{\gamma_1, \dots, \gamma_n}(z) \right)}{z \mathcal{L}'_{\gamma_1, \dots, \gamma_n}(z) + 2 \mathcal{L}_{\gamma_1, \dots, \gamma_n}(z)} \right\} > 1 - \sum_{i=1}^n \gamma_i (1 - \beta_i). \tag{2.28}$$

Using (2.24), (2.28) and applying Proposition 1.3, we get $\mathcal{L}_{\gamma_1, \dots, \gamma_n}(z)$ is a starlike function of order $1 - \sum_{i=1}^n \gamma_i (1 - \beta_i)$. \square

Letting $\beta_i = \beta, i \in \{1, \dots, n\}$ in Theorem 2.4, we get the following.

Corollary 2.5. For $i \in \{1, \dots, n\}$, let $\gamma_i > 0$ and $f_i \in \Sigma_{k_p}(\beta)$ ($-1 \leq \beta < 1$). If

$$0 < \sum_{i=1}^n \gamma_i \leq \frac{1}{1 - \beta}, \tag{2.29}$$

$\mathcal{H}_{\gamma_1, \dots, \gamma_n}$ be the integral operator given by (1.5) and

$$\left| \frac{z \mathcal{H}'_{\gamma_1, \dots, \gamma_n}(z)}{\mathcal{H}_{\gamma_1, \dots, \gamma_n}(z)} + 1 \right| < 1. \quad (2.30)$$

Then $\mathcal{H}_{\gamma_1, \dots, \gamma_n}(z)$ is starlike of order $1 - (1 - \beta) \sum_{i=1}^n \gamma_i$.

Letting $n = 1$, $\gamma_1 = \gamma$ and $f_1 = f$ in Corollary 2.5, we get the following.

Corollary 2.6. Let $\gamma > 0$, and $f \in \Sigma_{k_p}(\beta)$ ($-1 \leq \beta < 1$). If

$$0 < \gamma \leq \frac{1}{1 - \beta}, \quad (2.31)$$

$\mathcal{H}_\gamma(z)$ be the integral operator,

$$\mathcal{H}_\gamma(z) = \frac{1}{z^2} \int_0^z \left(-u^2 f'(u) \right)^\gamma du, \quad (2.32)$$

$$\left| \frac{z \mathcal{H}'_\gamma(z)}{\mathcal{H}_\gamma(z)} + 1 \right| < 1.$$

Then $\mathcal{H}_\gamma(z)$ is starlike of order $1 - (1 - \beta)\gamma$.

Other work related to integral operator for different studies can also be found in [4–6].

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