

## Research Article

# Natural Convection in an Inclined Porous Cavity with Spatial Sidewall Temperature Variations

**M. S. Selamat,<sup>1</sup> R. Roslan,<sup>2</sup> and I. Hashim<sup>3</sup>**

<sup>1</sup> Department of Mathematics & Statistics, Faculty of Computer & Mathematical Sciences, Universiti Teknologi MARA Pahang, 62400 Bandar Jengka, Malaysia

<sup>2</sup> Faculty of Science, Technology and Human Development, Universiti Tun Hussein Onn Malaysia, Batu Pahat, Johor, 86400 Parit Raja, Malaysia

<sup>3</sup> School of Mathematical Sciences and Solar Energy Research Institute, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, Selangor, 43600 Bangi, Malaysia

Correspondence should be addressed to I. Hashim, [ishak.h@ukm.my](mailto:ishak.h@ukm.my)

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The natural convection in an inclined porous square cavity is investigated numerically. The left wall is assumed to have spatial sinusoidal temperature variations about a constant mean value, while the right wall is cooled. The horizontal walls are considered adiabatic. A finite difference method is used to solve numerically the nondimensional governing equations. The effects of the inclination angle of the cavity, the amplitude and wave numbers of the heated sidewall temperature variation on the natural convection in the cavity are studied. The maximum average Nusselt number occurs at different wave number. It also found that the inclination could influence the Nusselt number.

## 1. Introduction

Convective heat transfer has attracted significant attention due to wide applications in engineering such as operation of solar collectors, cooling systems in electronics equipments, insulations of buildings and so forth. Many studies with application to the above research areas may be found in the book by Nield and Bejan [1].

The problem of natural convection in an enclosure has been studied extensively by many researchers such as Weber [2], Bejan [3], Bradean et al. [4], Goyeau et al. [5], Guo and Bathe [6] and Saeid and Pop [7]. Saeid and Mohamad [8] studied numerically the natural convection in a porous cavity with spatial sidewall temperature variation. They found that the average Nusselt number is dependent on the amplitude and the wave number of the spatial sinusoidal temperature. Saeid and Yaacob [9] investigated the natural convection in

a square cavity filled with a pure air with a nonuniform side wall temperature. They found that the average Nusselt number along the hot wall varies sinusoidally based on the hot wall temperature.

Most of the studies on natural convection are devoted to the classical Rayleigh-Benard model (hot bottom wall and cold top wall) or to the case of rectangular or square cavity with one vertical wall heated and the opposite one cooled. However, in some engineering applications, enclosures are inclined to the direction of gravity. Hence, the flow structure and the heat transfer within the enclosure are modified by the components of buoyancy forced. The effects of inclination on natural convection in an enclosure have been discussed by several investigators. For example Hart [10] studied the stability of the flow in an inclined box. Holst and Aziz [11] and Ozoe et al. [12] studied three- and two-dimensional natural convection in porous media, respectively. A good review on the study the inclination of natural convection can be seen in Yang [13]. Rasoul and Prinos [14] studied the effect of the inclination angle on steady natural convection in a square cavity for the Raleigh number ranging from  $10^3$  to  $10^6$  and the Prandtl number from 0.02 to 4000. Baytaşç [15] investigated entropy generation distribution according to inclination angle for saturated porous cavity by using the second law of thermodynamics. Kalabin et al. [16] investigated the influence of inclination angle and oscillation frequency on heat transfer through the square enclosure. Meanwhile, Chamkha and Al-Mudhaf [17] studied the double-diffusive natural convection in inclined porous cavities with the presence of temperature-dependent heat generation or absorption. They concluded that the heat and mass transfer and the flow characteristics inside the cavity are strongly dependent on the buoyancy ratio, inclination angle, and the heated generations or absorption effect. A numerical study has been carried out by Wang et al. [18] for the natural convection heat transfer in an inclined porous cavity with time-periodic boundary condition. They found that, if the inclination angle is maintained at a fixed value and the oscillating approaches infinity, the oscillating temperature on a sidewall has a little effect on the temperature near the opposite sidewall.

In this paper we study the natural convection in a porous cavity with a non-uniform hot wall temperature and a uniform cold wall temperature. The heated wall is assumed to have spatial sinusoidal temperature variations about a constant mean value which is higher than the cold sidewall temperature. This work extends in particular the work of Saeid and Mohamad [8] to the more general setup of an inclined cavity.

## 2. Mathematical Formulation

The heat transfer by natural convection across porous media is considered as shown in Figure 1. The top and bottom horizontal walls are adiabatic, and the right sidewall is maintained at a constant cold temperature  $T_c$ . The temperature of opposing sidewall is assumed to have spatial sinusoidal temperature variations about a constant mean value which is higher than the cold sidewall temperature.

Applying Darcy's flow model and the Boussinesq approximation, the governing equations are [8]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = \frac{g\beta K}{\nu} \left( \frac{\partial T}{\partial y} \cos \phi - \frac{\partial T}{\partial x} \sin \phi \right),$$

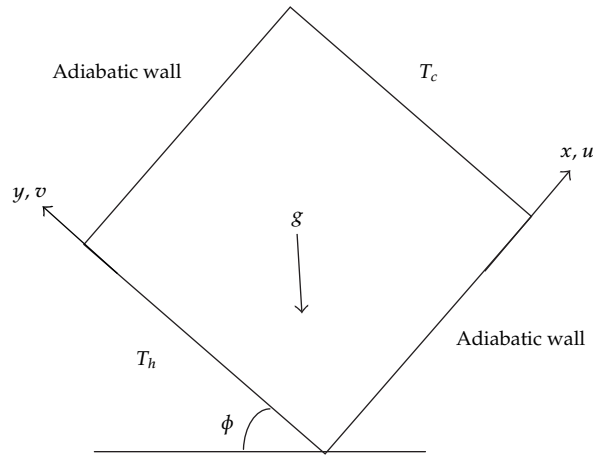


Figure 1: Schematic diagram of the physical model and coordinate system.

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right), \quad (2.1)$$

where  $u$  and  $v$  are the velocity components along the  $x$ - and  $y$ -axes, respectively,  $T$  is the fluid temperature,  $g$  is the gravitational acceleration,  $K$  is the permeability of the porous medium,  $\alpha$  is the effective thermal diffusivity,  $\beta$  is the coefficient of thermal expansion, and  $\nu$  is the kinematic viscosity of the fluid. The temperature of the hot wall is assumed to have a sinusoidal variation about a minimum value of  $\bar{T}_h$  in the form

$$T_h(y) = \bar{T}_h + \varepsilon (\bar{T}_h - T_c) \left[ 1 - \cos\left(\frac{2\pi\kappa y}{L}\right) \right], \quad (2.2)$$

where  $L$  is the cavity height/width,  $\kappa$  is the wave number, and  $\varepsilon$  is the nondimensional amplitude.

Equations (2) are subject to the following boundary conditions:

$$\begin{aligned} u(0, y) = v(0, y) = 0, & \quad T(0, y) = T_h(y), \\ u(L, y) = v(L, y) = 0, & \quad T(L, y) = T_c, \\ u(x, 0) = v(x, 0) = 0, & \quad \frac{\partial T(x, 0)}{\partial y} = 0, \\ u(x, L) = v(x, L) = 0, & \quad \frac{\partial T(x, L)}{\partial y} = 0. \end{aligned} \quad (2.3)$$

Using the stream functions defined by  $u = \partial\psi/\partial y$  and  $v = -\partial\psi/\partial x$  and nondimensional variables

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad \theta = \frac{T - T_0}{\bar{T}_h - T_c}, \quad \Psi = \frac{\psi}{\alpha}, \quad (2.4)$$

where  $T_0 = (\bar{T}_h + T_c)/2$ , the governing equations (2) and boundary conditions (2.3) can be written in dimensionless forms:

$$\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = \text{Ra} \left( \frac{\partial \theta}{\partial Y} \cos \phi - \frac{\partial \theta}{\partial X} \sin \phi \right), \quad (2.5)$$

$$\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} = \frac{\partial \Psi}{\partial Y} \frac{\partial \theta}{\partial X} - \frac{\partial \Psi}{\partial X} \frac{\partial \theta}{\partial Y}, \quad (2.6)$$

$$\Psi(0, Y) = 0, \quad \theta(0, Y) = 0.5 + \varepsilon[1 - \cos(2\pi\kappa Y)], \quad (2.7)$$

$$\Psi(1, Y) = 0, \quad \theta(1, Y) = -0.5, \quad (2.8)$$

$$\Psi(X, 0) = 0, \quad \frac{\partial \theta(X, 0)}{\partial Y} = 0, \quad (2.9)$$

$$\Psi(X, 1) = 0, \quad \frac{\partial \theta(X, 1)}{\partial Y} = 0, \quad (2.10)$$

where Ra is the Rayleigh number defined as

$$\text{Ra} = \frac{g\beta K(\bar{T}_h - T_c)L}{\nu\alpha}. \quad (2.11)$$

The local Nusselt numbers along the hot and cold wall are given, respectively, by

$$\text{Nu}_h = \frac{hY}{\lambda} = \left( -\frac{\partial \theta}{\partial X} \right)_{X=0}, \quad (2.12)$$

$$\text{Nu}_c = \frac{hY}{\lambda} = \left( -\frac{\partial \theta}{\partial X} \right)_{X=1},$$

where  $h$  is the heat transfer coefficient and  $\lambda$  is the thermal conductivity of the porous media.

**Table 1:** Comparison of the average Nusselt number,  $\overline{Nu}$ , against some previous results for isothermal vertical walls in a square cavity in the case  $\phi = 90^\circ$ .

References	Mesh size	Ra = 100		Ra = 1000	
		$\overline{Nu}_h$	$\overline{Nu}_c$	$\overline{Nu}_h$	$\overline{Nu}_c$
[3]		4.200		15.800	
[5]		3.110		13.470	
[8]	(20 × 20)	3.081	3.081	12.890	12.890
[8]	(40 × 40)	3.099	3.099	13.431	13.431
[8]	(80 × 80)	3.108	3.107	13.531	13.531
[8]	(160 × 160)	3.110	3.110	13.592	13.592
Present work	(20 × 20)	3.005	3.005	9.865	9.864
Present work	(40 × 40)	3.080	3.080	12.157	12.154
Present work	(80 × 80)	3.100	3.100	13.180	13.178
Present work	(160 × 160)	3.106	3.106	13.498	13.494

The average Nusselt number is defined as

$$\overline{Nu}_i = \frac{hL}{\lambda} = \int_0^1 Nu \, dY, \quad (2.13)$$

where  $Nu_i$  is  $Nu_h$  or  $Nu_c$ , respectively.

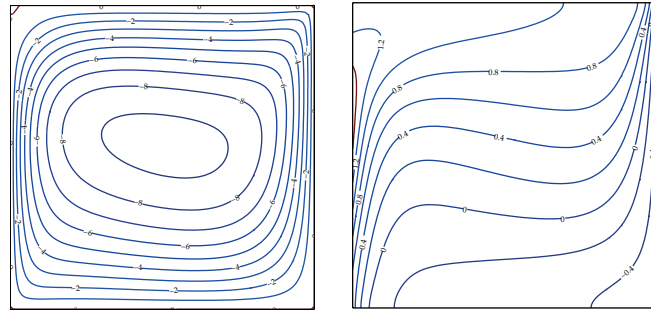
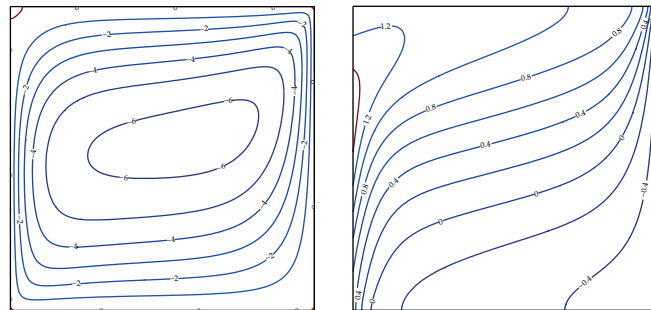
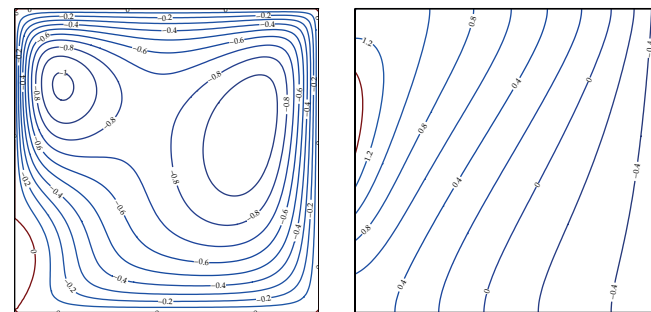
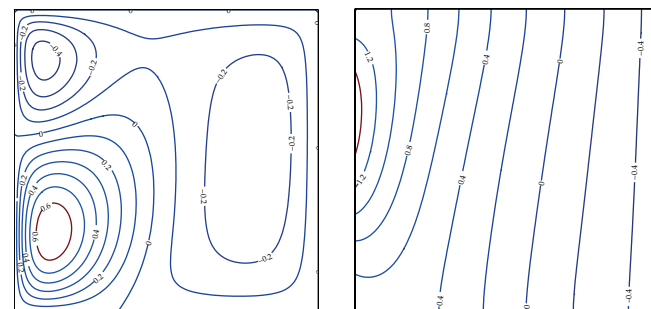
### 3. Numerical Scheme

The coupled system of (2.5) and (2.6) subject to boundary conditions (2.7)–(2.10) is solved numerically using a finite difference method. The central difference method was applied for discretizing the equations. The resulting algebraic equations were solved by using the Gauss-Seidel iteration with a relaxation method. The unknowns  $\Psi$  and  $\theta$  are calculated until the following convergence criterium is fulfilled:

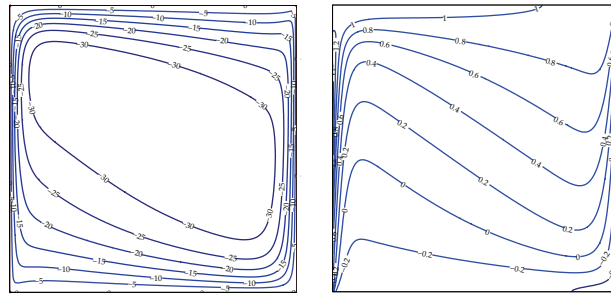
$$\frac{\sum_{i,j} |\zeta_{i,j}^{n+1} - \zeta_{i,j}^n|}{\sum_{i,j} |\zeta_{i,j}^{n+1}|} \leq \epsilon, \quad (3.1)$$

where  $\zeta$  is either  $\Psi$  or  $\theta$ ,  $n$  represents the iteration number, and  $\epsilon$  is the convergence criterium. In this study, the convergence criterium was set at  $\epsilon = 10^{-6}$ .

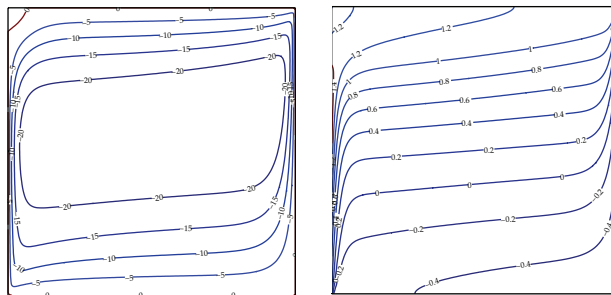
The numerical code was validated in a square porous cavity with constant isothermal vertical walls ( $\varepsilon = 0$ ) using different mesh sizes,  $20 \times 20$ ,  $40 \times 40$ ,  $80 \times 80$ , and  $160 \times 160$ . The average Nusselt numbers along the hot wall and cold wall of the cavity are calculated and compared with the results by different authors for  $Ra = 100$  and  $Ra = 1000$  as shown in Table 1. From Table 1, the error between  $\overline{Nu}_c$  and  $\overline{Nu}_h$  is found less than 0.004 percent for  $Ra = 100$ , and it is less than 0.03 percent for  $Ra = 1000$  which reflects the accuracy of the present results. Also, Table 1 shows the good agreement between our result and the existing results for a porous square enclosure for  $\phi = 90^\circ$ .

(a)  $\phi = 50^\circ$ (b)  $\phi = 90^\circ$ (c)  $\phi = 150^\circ$ (d)  $\phi = 170^\circ$ 

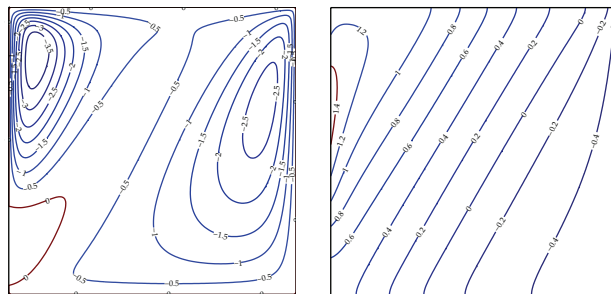
**Figure 2:** Effects of inclination angle  $\phi$  on streamlines (left) and isotherms (right) for  $\varepsilon = 0.5$ ,  $\kappa = 0.75$ , and  $Ra = 100$ .



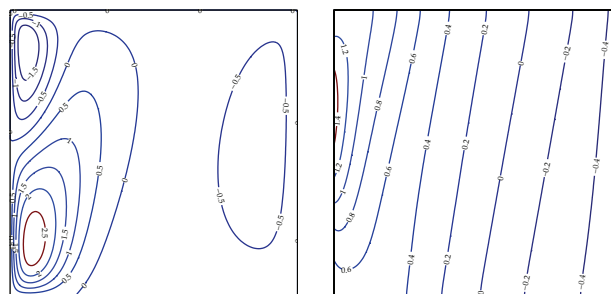
(a)  $\phi = 50^\circ$



(b)  $\phi = 90^\circ$

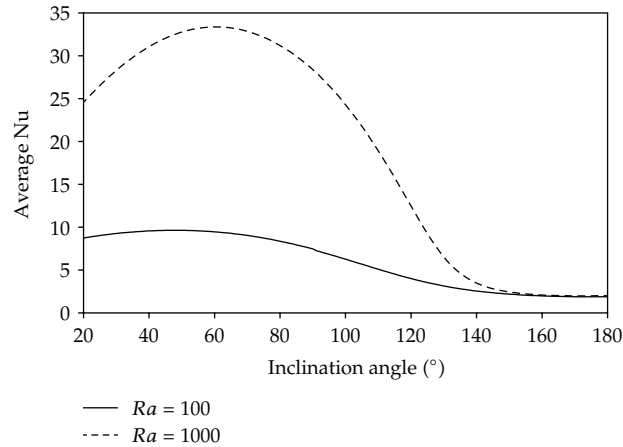


(c)  $\phi = 150^\circ$

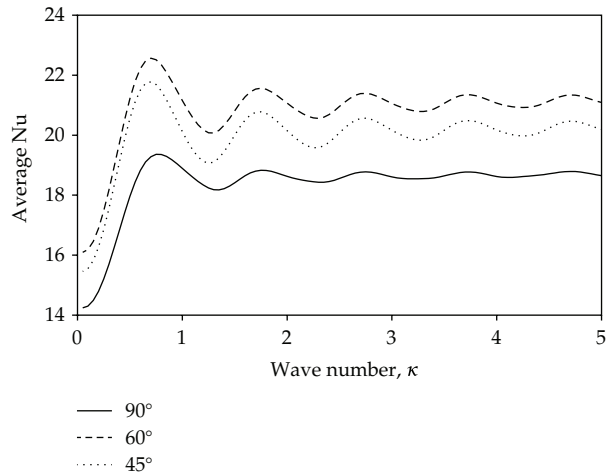


(d)  $\phi = 170^\circ$

**Figure 3:** Effects of inclination angle  $\phi$  on streamlines (left) and isotherms (right) for  $\varepsilon = 0.5$ ,  $\kappa = 0.75$ , and  $Ra = 1000$ .



**Figure 4:** Average Nusselt number versus inclination angle.



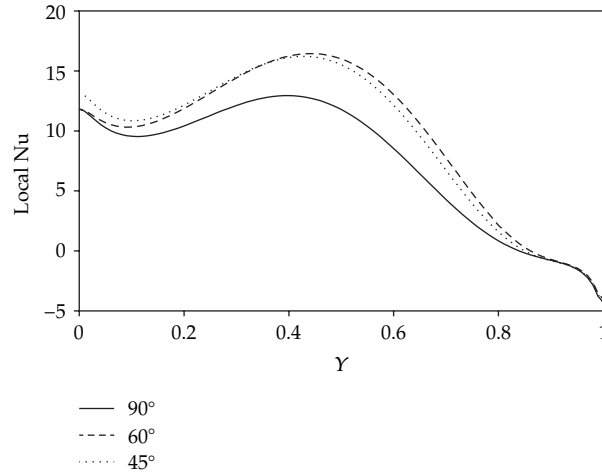
**Figure 5:** Variation of the Nusselt number with the wavenumber for the case  $\varepsilon = 0.2$  and  $Ra = 1000$ . The solid line for the case  $\phi = 90^\circ$  is the recomputed results of Saeid and Mohamad [8].

A grid independence test has been performed in the case of maximum amplitude ( $\varepsilon = 1$ ) and maximum wavenumber ( $\kappa = 5$ ) for  $Ra = 1000$ . The  $80 \times 80$  mesh gives  $\overline{Nu}_h = 40.7623$  and  $\overline{Nu}_c = 41.0717$ , and the  $120 \times 120$  mesh gives  $\overline{Nu}_h = 42.9346$  and  $\overline{Nu}_c = 43.7658$ . Meanwhile, the  $100 \times 100$  mesh gives  $\overline{Nu}_h = 42.0375$  and  $\overline{Nu}_c = 42.6741$ . Therefore, the  $100 \times 100$  mesh will give grid-independent solution for our study of sidewall temperature variations and thus has been chosen in all the calculations in this paper.

#### 4. Results and Discussion

Investigation is carried out for the case  $Ra = 100$  and  $Ra = 1000$  with  $\varepsilon = 0.5$  and  $\kappa = 0.75$  for selected inclination angles,  $\phi$  between  $0^\circ$  and  $180^\circ$ . The effects of inclination angle on the flow patterns and temperature fields are presented as streamline and isotherms in Figures





**Figure 6:** Variation of the local Nusselt number along the hot wall for  $\varepsilon = 0.5$ ,  $\kappa = 0.75$  and  $Ra = 1000$ . The solid line for the case  $\phi = 90^\circ$  is the recomputed results of Saeid and Mohamad [8].

2 and 3. At  $\phi = 0^\circ$ , the hot wall is horizontal and at the bottom. As the cavity is inclined, the gravity components started to assist and accelerate the flow motion until the maximum Nusselt number is reached at  $\phi = 50^\circ$  for  $Ra = 100$  and  $\phi = 60^\circ$  for  $Ra = 1000$ . For  $\phi$  close to  $180^\circ$ , two and three cells are formed along the hot walls as shown in Figures 2(c) and 2(d) and Figures 3(c) and 3(d), indicating that the fluid from hot wall and cold wall rotates back to the same wall. For  $\phi = 170^\circ$ , the isotherms far from the hot wall are almost perpendicular to the gravitational vector and the gradients are relatively small, implying the small value in the Nusselt number along the walls.

The influence of the inclination angles on the average Nusselt number is demonstrated in Figure 4. It is clear that the maximum average Nusselt number is attained at about  $\phi = 50^\circ$  for  $Ra = 100$  and  $\phi = 60^\circ$  for  $Ra = 1000$ . Beyond that angle, the Nusselt number decreases until it reaches the condition where the Nusselt number has its minimum point or close to the pure conduction value. In general, the Nusselt number increases with increasing in  $Ra$ .

Figure 5 shows the variations of the average Nusselt number along the hot wall for  $Ra = 1000$  and  $\varepsilon = 0.2$ . It can be seen from Figure 5 that the average Nusselt number varies spatially with increasing the wave number. Also, we found that the maximum average Nusselt numbers occur at  $\kappa = 0.6, 0.65$ , and  $0.75$  for  $\phi = 45^\circ, 60^\circ$ , and  $90^\circ$ , respectively.

As comparison with the work by Saied and Mohamad [8], we plotted the variation of the local Nusselt number along the hot wall for  $Ra = 1000$ ,  $\varepsilon = 0.5$ , and  $\kappa = 0.75$  in Figure 6. It is found that the value of local Nusselt numbers became negative near the upper portion of the hot wall which means that heat transfer occurred from the higher temperature parts to the lower temperature parts of the hot wall.

## 5. Conclusion

In this work, the natural convection in an inclined porous cavity has been investigated numerically using a finite difference approach. The effects of the inclination angle of the cavity are investigated. The numerical results indicate that the maximum natural convection

is dependent on the inclination angle, where the maximum Nusselt number occurs at different wave numbers for different inclination angle.

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