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Research Article

The Complex Network Synchronization via Chaos Control Nodes

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We investigate chaos control nodes of the complex network synchronization. The structure of the coupling functions between the connected nodes is obtained based on the chaos control method and Lyapunov stability theory. Moreover a complex network with nodes of the new unified Loren-Chen-Lü system, Coullet system, Chee-Lee system, and the New system is taken as an example; numerical simulations are used to verify the effectiveness of the method.

1. Introduction

Since the most famous random graph model was proposed by Erdös and Rényi [1], the complex network has attracted much attention in many fields of research, such as biology, physics, computer networks, the World Wide Web (WWW) [2], and so on. Network synchronization has obvious advantages, it has great application value in practice. Therefore, Atay et al. [3] studied synchronization of complex network when delays exist among the nodes; Motter et al. [4] studied the influence of coupling strength on the synchronizing ability of a complex network; Timme et al. [5] studied the web synchronization law of pulse-coupled dynamical systems; Checco et al. [6] studied the synchronization of random web. Lü et al. [7] constructed general complex dynamical networks and studied the synchronization; Lu and Chen [8] studied synchronization analysis of linearly coupled networks of discrete time systems; Han and Lu [9] studied the changes of synchronization ability of coupled networks from ring networks to chain networks; He and Yang [10] studied adaptive synchronization in nonlinearly coupled dynamical networks; Hung et al. [11] studied globally generalized synchronization in scale-free networks.

Recently, network synchronization has been an important part of the dynamic study of complex network. It has aroused great interest of scholars both domestically and abroad to build weighted network models and study the characteristics of them. Lü et al. [7] studied chaos synchronization of general complex dynamic networks. Gao et al. [12] realized the adaptive synchronization of complex network. Pei et al. [13] made statistical analysis of a class of real networks; Barrat et al. [14] presented a weighted network model, and studied its dynamic character; Atay et al. [15] studied the synchronization of a complex network with time delay. Hung et al. [11] realized the generalized synchronization of a scale-free network. Qin and Yu [16] achieved the synchronization of the star-network of hyperchaotic Rossler systems. In addition to the above researches, much other work [17, 18] in the field have been done, and complex network synchronization become a focus of attention, such as a random network synchronization, small-world network synchronization, scale-free network synchronization, and so on. However, universal synchronization methods of weighted network still need further exploration and study.

The motivation in this paper lies in the complex network synchronization and chaos control importance. Network synchronization is one of the most practical and valuable issues. A synchronization of network means the situation in which the output of all nodes in the study of the complex network is consistent with any given external input signal under a certain condition. The working mechanism of a single chaotic system to track any given external input signal is relatively interesting

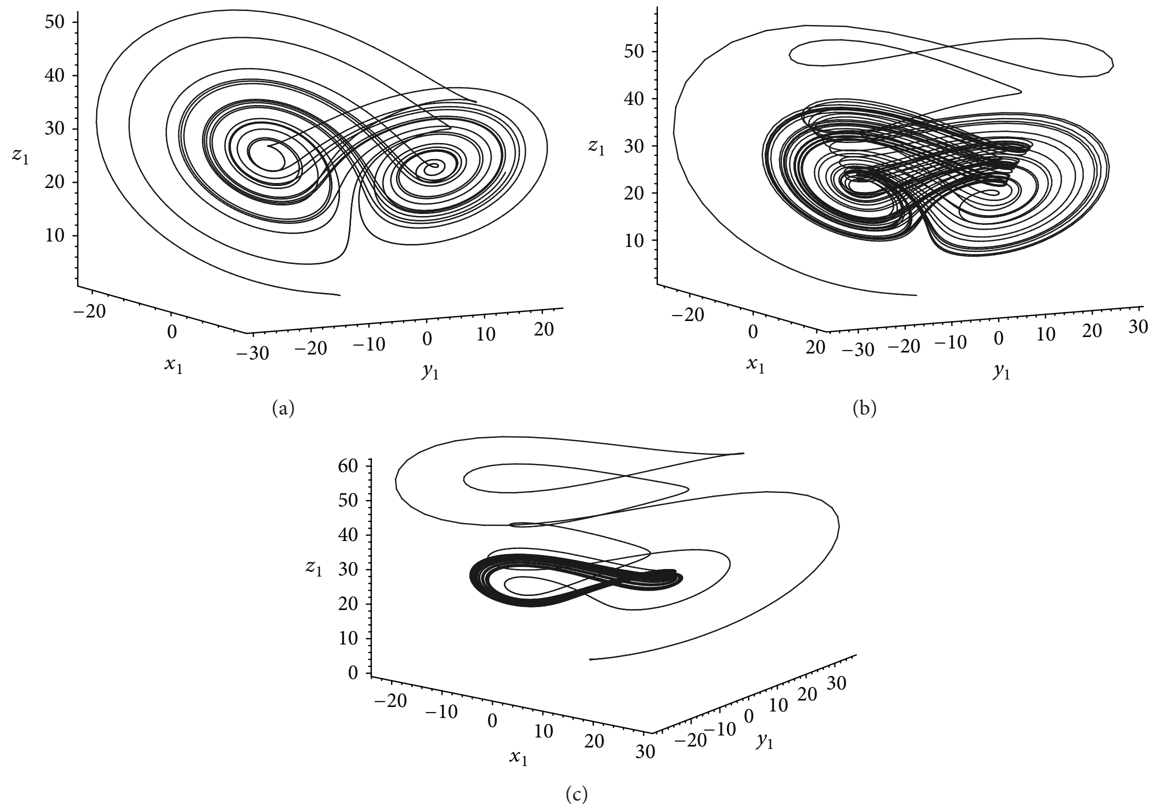

FIGURE 1: Generlized chaotic attractors: (a) Lorenz, (b) Lü, (c) Chen.

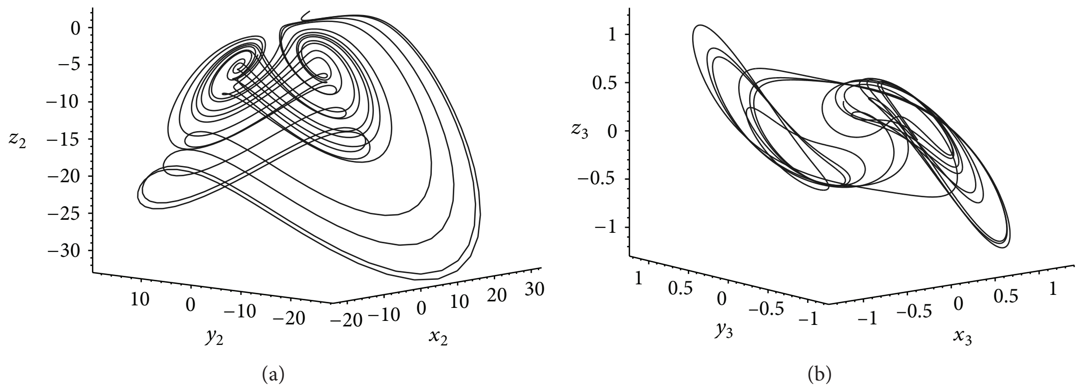

FIGURE 2: The chaos attractor of new chaotic system (a) and the Coullet system (b).

and significance. Numerical simulations are used to verify the effectiveness of the proposed techniques.

It is organized as follows. Firstly, the theory and the method are presented in Section 2. Then, different order chaotic systems are adopted as the nodes of this complex network; the structure of the coupling functions among the connected nodes is obtained based on Lyapunov stability theory. With the help of symbolic computation, the temporal evolution of variables and node interaction of the dynamic equation are discussed and simulated by computer in Section 3. The theoretical analysis and the numerical simulations show that this is a universal method and the number of the nodes does not affect the stability of the whole network. Finally, the conclusions are given in Section 4.

2. Theory and Method

We summarized the main steps for the complex network synchronization based on Lü et al. [19–23] and Chen et al. [24–27], as follows.

Step 1. Assume the state of node i is x_i, where $x_i = (x_{i1}, x_{i2}, \ldots, x_{in})^T$ and $x_{i1}, x_{i2}, \ldots, x_{in} \epsilon R^n$, then the dynamic

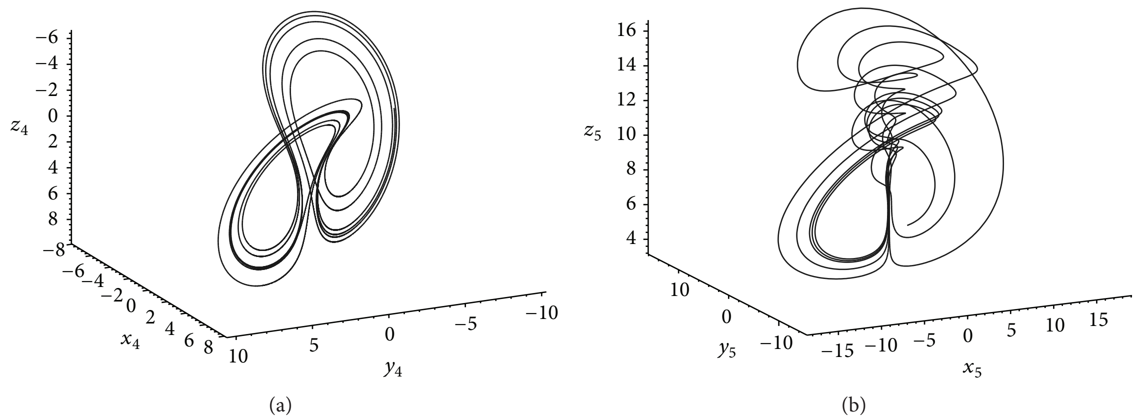

Figure 3: The chaos attractor of (14) system (a) and Chee-Lee system (b).

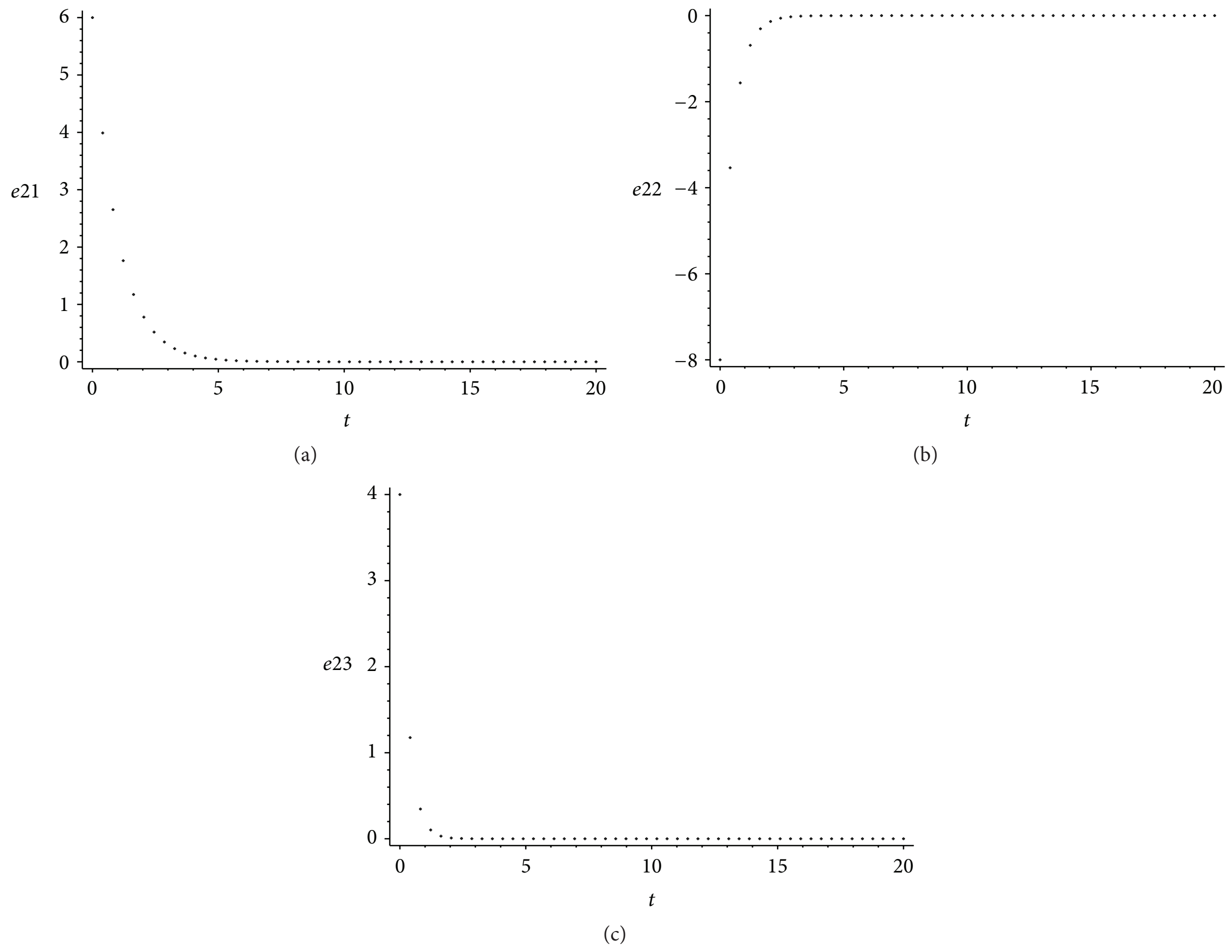

Figure 4: Network synchronization errors between node 1 and node 2.

function for node i without coupling can be described as

$$\dot{x}_i = f_i(x_i) = L_i(x_i) + S_i(x_i), \quad (1)$$

where $L_i(x_i) = (A_i - B_i)x_i$, A_i is the linear coefficient matrix of the system, and B_i is the coefficient matrix of control gain.

Step 2. Considering the coupling of network, the dynamic function for node i can be described as

$$\begin{aligned}\dot{x}_i &= f_i(x_i) + g_i(x_1, x_2, \ldots, x_m) \\ &= (A_i - B_i)x_i + S_i(x_i) + g_i(x_1, x_2, \ldots, x_m),\end{aligned} \quad (2)$$

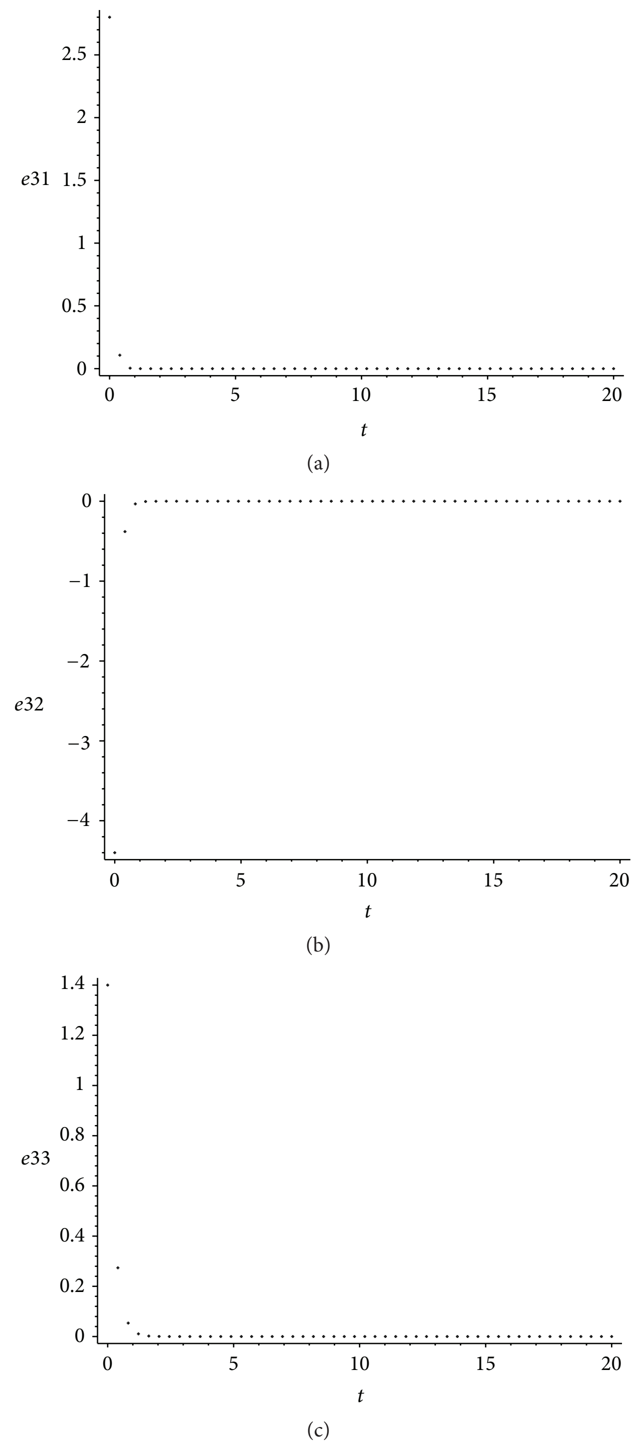

Figure 5: Network synchronization errors between node 2 and node 3.

where $g_i(x_1, x_2, \ldots, x_n)$ denotes the coupling function. The proportional scale for node i is α_i, then we will have $\alpha_i = \operatorname{diag}(\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{in})$.

Step 3. The errors between the state variables of the network are defined as

$$e_i = \alpha_i x_i + \alpha_{i+1} x_{i+1}. \tag{3}$$

We obtain from (2) and (3)

$$\begin{aligned} \dot{e}_i &= \alpha_i \dot{x}_i + \alpha_{i+1} \dot{x}_{i+1} \\ &= \alpha_i \left[\left(A_i - B_i \right) x_i + S_i \left(x_i \right) + g_i \right] \\ &\quad + \alpha_{i+1} \left[\left(A_{i+1} - B_{i+1} \right) x_{i+1} + S_{i+1} \left(x_{i+1} \right) + g_{i+1} \right] \\ &= \left(A_i - B_i \right) e_i + S_i \alpha_i + \alpha_i g_i + \alpha_{i+1} \left(A_{i+1} - B_{i+1} \right) x_{i+1} \\ &\quad + \alpha_{i+1} S_{i+1} + \alpha_{i+1} g_{i+1} - \left(A_i - B_i \right) \alpha_{i+1} x_{i+1} \\ &= \left(A_i - B_i \right) e_i + \left[\left(A_{i+1} - B_{i+1} \right) - \left(A_i - B_i \right) \right] \alpha_{i+1} x_{i+1} \\ &\quad + \Delta S_i \left(x_i, x_{i+1} \right) + \Delta g_i \left(x_i, x_{i+1} \right) \\ &= \left(A_i - B_i - K_i \right) e_i + \left[\left(A_{i+1} - B_{i+1} \right) - \left(A_i - B_i \right) \right] \alpha_{i+1} x_{i+1}, \end{aligned} \tag{4}$$

where $\Delta S_i(x_i, x_{i+1}) = \alpha_i S_i + \alpha_{i+1} S_{i+1}$, $\Delta g_i(x_i, x_{i+1}) = \alpha_i g_i + \alpha_{i+1} g_{i+1} + K_i e_i$.

Step 4. Choose $\Delta g_i(x_i, x_{i+1}) = -\Delta S_i(x_i, x_{i+1}) + K_i e_i$, then we will have

$$\alpha_j g_j = -\alpha_1 g_1 - \alpha_1 S_1 \left(x_1 \right) - \alpha_j S_j \left(x_j \right) + k \left(\alpha_1 x_1 + \alpha_j x_j \right), \quad \left(j = 2, 3, \ldots, m \right). \tag{5}$$

If we choose node 1 as target node, then the coupling function of node j $(j = 2, 3, m)$ can be described as

$$g_j = -\frac{\alpha_1}{\alpha_j} g_1 - \frac{\alpha_1}{\alpha_j} S_1 \left(x_1 \right) - S_j \left(x_j \right) + k \left(\frac{\alpha_1}{\alpha_j} x_1 + x_j \right). \tag{6}$$

In order to realize the synchronization of the complex network, if we choose other node as target node, the coupling functions of all nodes can also be obtained.

Step 5. Constructing the Lyapunov function according to the weighted complex network with different nodes,

$$V = \frac{1}{2} \sum_{i=1}^{m-1} e_i^2, \tag{7}$$

and considering (3) and (4), we can obtain the derivative form of V as

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{m-1} e_i \dot{e}_i \\ &= \sum \left\{ \left(A_i - B_i - K_i \right) e_i \right. \\ &\qquad \left. + \left[\left(A_{i+1} - B_{i+1} \right) - \left(A_i - B_i \right) \right] \alpha_{i+1} x_{i+1} \right\} e_i. \end{aligned} \tag{8}$$

From (8), we can easily see that if

$$A_i \leq B_i + K_i, \qquad A_{i+1} = B_{i+1} + A_i - B_i. \tag{9}$$

Then

$$\dot{V} \leq 0. \tag{10}$$

According to Lyapunov stability theory [20] and the weighted complex network [19, 20, 28], the synchronization of the complex network can be realized.

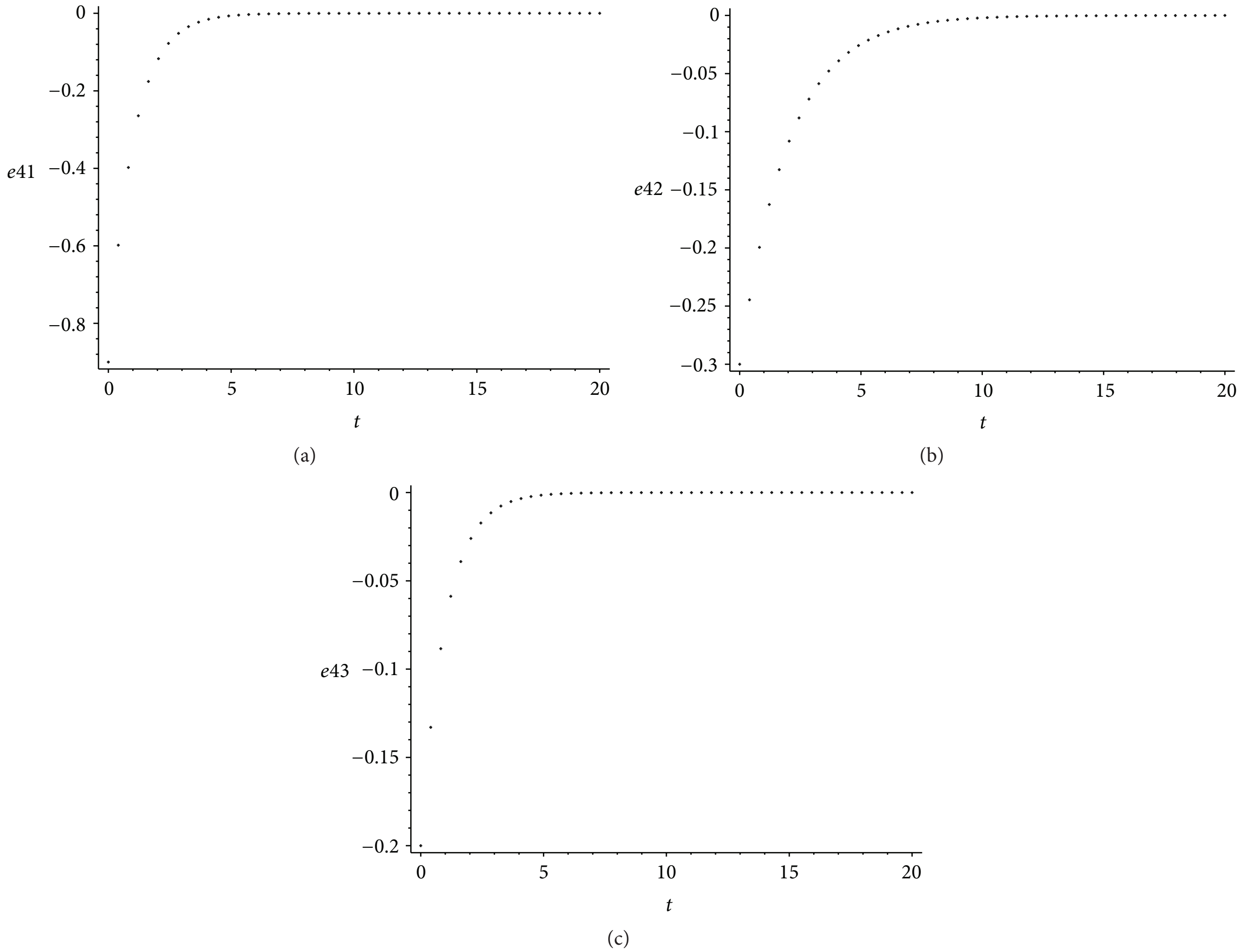

Figure 6: Network synchronization errors between node 3 and node 4.

3. Application of Chaos Control Method to the Network Synchronization

The unified chaotic system, the New system, Coullet system, and Chee-Lee system are taken as nodes of the network to show the synchronization mechanism mentioned above. Simulation is made as the number of the nodes is $m = 5$.

The dynamic equation of the unified chaotic system [21, 29] is presented as follows:

$$\begin{aligned} \dot{x}_1 &= (25\alpha + 10)(y_1 - x_1), \\ \dot{y}_1 &= (28 - 35\alpha)x_1 - x_1 z_1 + (29\alpha - 1)y_1, \\ \dot{z}_1 &= x_1 y_1 - \frac{8+\alpha}{3} z_1, \end{aligned} \tag{11}$$

where $\alpha \in [-0.0016, 1.15]$. When $-0.0016 < \alpha < 0.8$, LCL system (11) belongs to the generalized Lorenz system [30]; when $\alpha = 0.8$, LCL system (11) belongs to the generalized Lü system [23]; when $0.8 < \alpha < 1.15$ LCL system (11) belongs to the generalized Chen system [31]. Figures 1(a)–1(c) display these chaotic attractors of the generalized Lorenz system ($\alpha = 0.2$), the generalized Lü system ($\alpha = 0.8$), and the generalized Chen system ($\alpha = 1.12$), respectively.

The new chaotic system of three-dimensional quadratic autonomous ordinary differential equations [22], which can display the chaotic attractors:

$$\begin{aligned} \dot{x}_2 &= -\frac{a_2 b_2}{a_2 + b_2} x_2 - y_2 z_2 + c_2, \\ \dot{y}_2 &= a_2 y_2 + x_2 z_2, \\ \dot{z}_2 &= b_2 z_2 + x_2 y_2, \end{aligned} \tag{12}$$

where a_2, b_2, and c_2 are real constants; and x, y, and z are status variables. This system is found to be chaotic in a wide parameter range and has many interesting complex dynamical characteristics. The system is chaotic for the parameters $a_2 = -10$, $b_2 = -4$, and $\|c\| = 19.2$; it displays the chaotic attractor as shown in Figure 2(a). Detailed dynamic properties of this system can be found in [22].

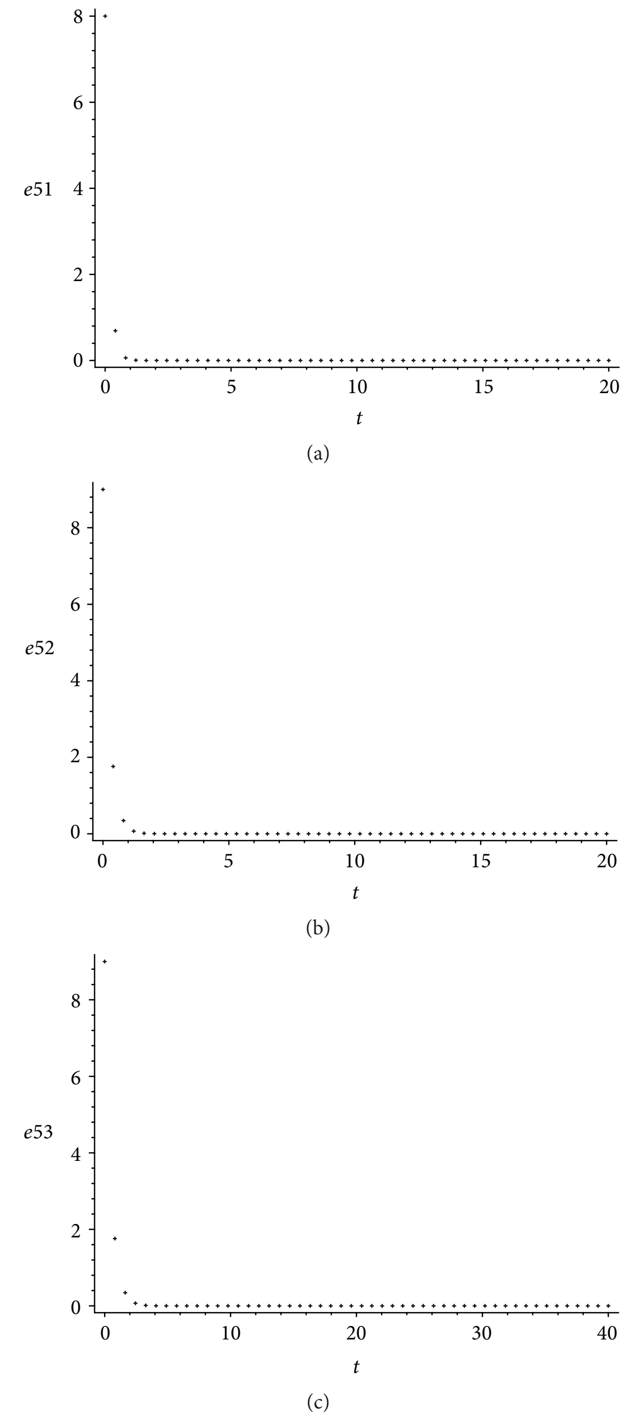

FIGURE 7: Network synchronization errors between node 4 and node 5.

The dynamic equation of Coullet system [19] is described as follows:

$$\begin{aligned}\dot{x}_3 &= y_3,\\ \dot{y}_3 &= z_3,\\ \dot{z}_3 &= a_3 z_3 + b_3 y_3 + c_3 x_3 + x_3^3,\end{aligned} \tag{13}$$

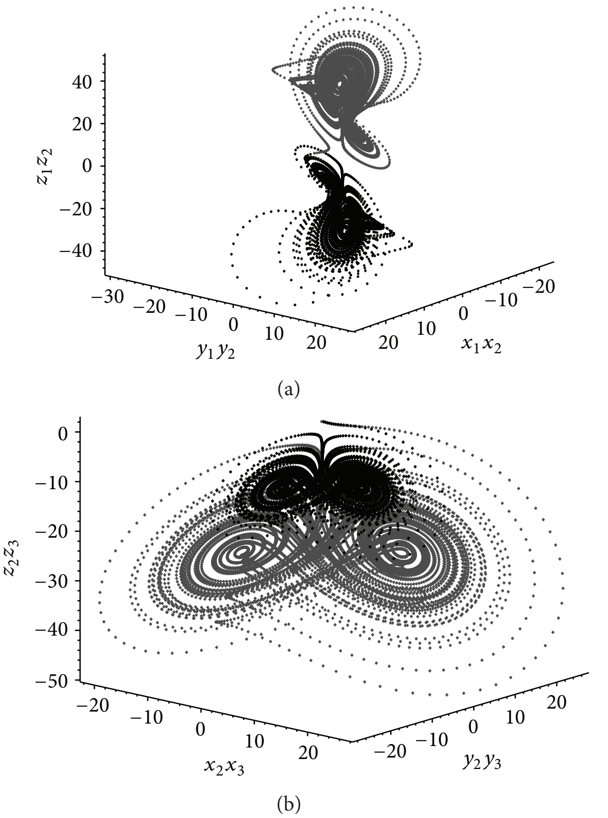

FIGURE 8: The chaos attractors between node 1 and node 2 (a) and between node 2 and node 3 (b).

where a_3, b_3, and c_3 are parameters of the system. When the parameters are given as $a_3 = 0.45$, $b_3 = 1.1$, and $c_3 = 0.8$, the chaos attractor of (13) is shown in Figure 2(b).

The dynamic equation of new system [23] is described as follows:

$$\begin{aligned}\dot{x}_4 &= d\left(y_4 - x_4\right),\\ \dot{y}_4 &= -y_4 + x_4 z_4,\\ \dot{z}_4 &= e - y_4 x_4 - f z_4,\end{aligned} \tag{14}$$

where d, e, and f are parameters of the system. When the parameters are given as $d = 5$, $e = 16$, and $f = 1$, the chaos attractor of (14) is shown in Figure 3(a).

The dynamic equation of the Chen-Lee system [32] is described as follows:

$$\begin{aligned}\dot{x}_5 &= -y_5 z_5 + a_5 x_5,\\ \dot{y}_5 &= x_5 z_5 + b_5 y_5,\\ \dot{z}_5 &= \frac{1}{3} x_5 y_5 + c_5 z_5,\end{aligned} \tag{15}$$

where $a = 5$, $b = -10$, and $c = -0.38$, (15) is a chaotic system. Figure 3(b) displays the chaotic attractors in (x_1, y_1, z_1)-space. It is easy to see that it admits a symmetry (x_5, y_5, z_5). That is, the system (15) is symmetrical about the three coordinate axes x_5, y_5, and z_5, respectively.

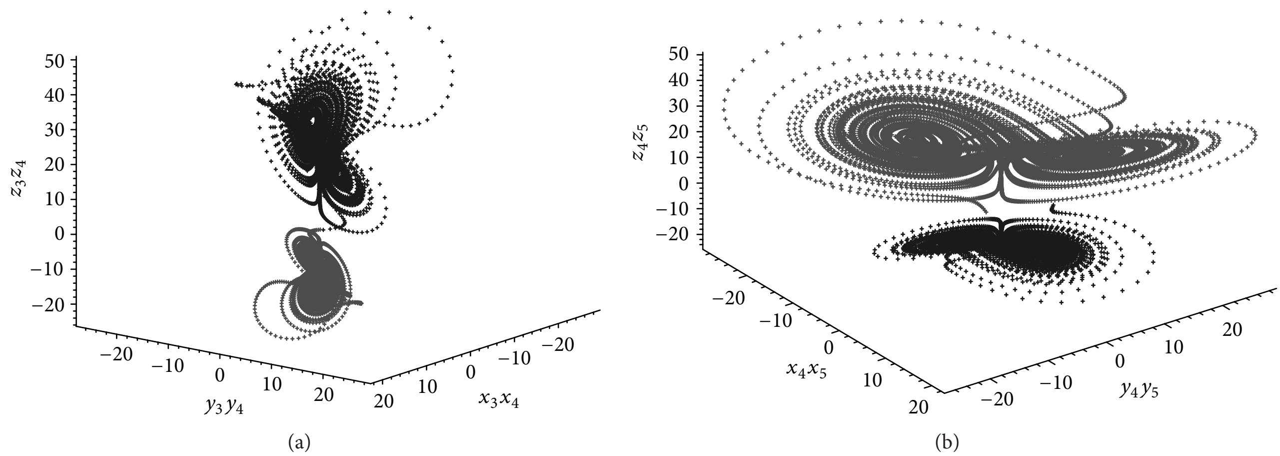

FIGURE 9: The chaos attractors between node 3 and node 4 (a) and between node 4 and node 5 (b).

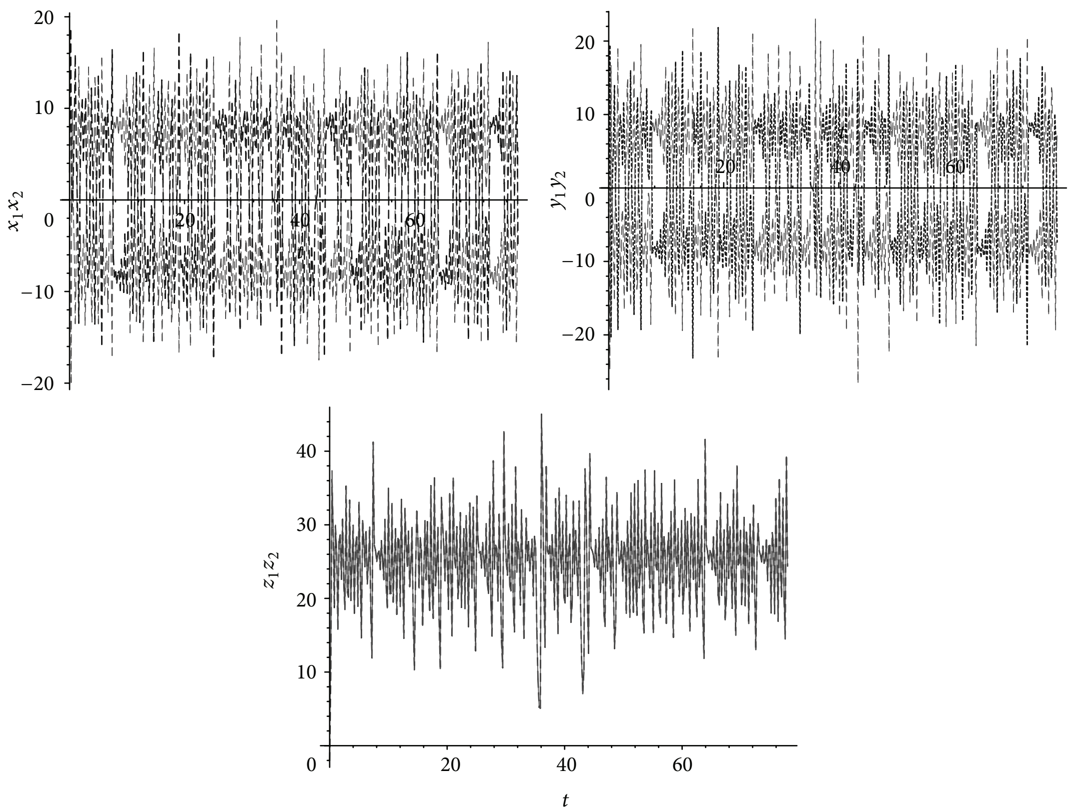

FIGURE 10: The temporal evolution of variables between node 1 ("$\cdots$") and node 2 ("—").

Choose the unified chaotic system (16) (node 1) as a target system, that is, coupling function $g_{1i} = 0$, then the dynamic functions of node 2, node 3, node 4, and node 5 with coupling can be described, respectively, as

$$\begin{aligned} \dot{x}_1 &= (25\alpha + 10)(y_1 - x_1), \\ \dot{y}_1 &= (28 - 35\alpha)x_1 - x_1z_1 + (29\alpha - 1)y_1, \\ \dot{z}_1 &= x_1y_1 - \frac{8+\alpha}{3}z_1, \end{aligned} \tag{16}$$

$$\begin{aligned} \dot{x}_2 &= -\frac{a_2b_2}{a_2+b_2}x_2 - y_2z_2 + c_2 + g_{21}, \\ \dot{y}_2 &= a_2y_2 + x_2z_2 + g_{22}, \\ \dot{z}_2 &= b_2z_2 + x_2y_2 + g_{23}, \end{aligned} \tag{17}$$

$$\begin{aligned} \dot{x}_3 &= y_3 + g_{31}, \\ \dot{y}_3 &= z_3 + g_{32}, \\ \dot{z}_3 &= a_3z_3 + b_3y_3 + c_3x_3 + x_3^3 + g_{33}, \end{aligned} \tag{18}$$

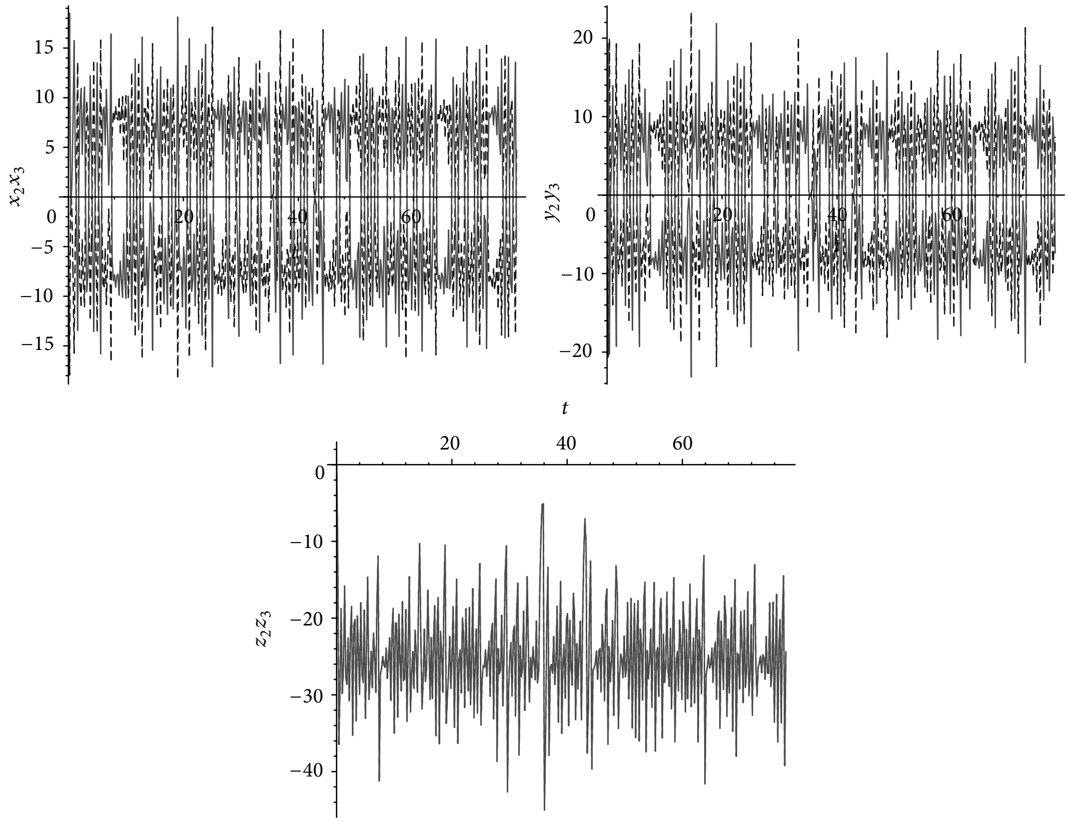

Figure 11: The temporal evolution of variables between node 2 ("···") and node 3 ("—").

$$\begin{aligned}\dot{x}_4 &= d\left(y_4 - x_4\right) + g_{41},\\ \dot{y}_4 &= -y_4 + x_4 z_4 + g_{42},\\ \dot{z}_4 &= e - y_4 x_4 - f z_4 + g_{43},\end{aligned} \tag{19}$$

$$\begin{aligned}\dot{x}_5 &= -y_5 z_5 + a_5 x_5 + g_{51},\\ \dot{y}_5 &= x_5 z_5 + b_5 y_5 + g_{52},\\ \dot{z}_5 &= \frac{1}{3} x_5 y_5 + c_5 z_5 + g_{53},\end{aligned} \tag{20}$$

where g_j $(j = 2, 3, 4, 5)$ fulfills (6):

$$g_2 = \begin{pmatrix} g_{21} \\ g_{22} \\ g_{23} \end{pmatrix} = \begin{pmatrix} -15y_1 + 14x_1 - \dfrac{27}{7}x_2 - 1 \\ -21x_1 - 6.8y_1 + 8y_2 \\ -0.267z_1 + z_2 \end{pmatrix} + \begin{pmatrix} y_2 z_2 \\ x_1 z_1 - x_2 z_2 \\ -x_1 y_1 - x_2 y_2 \end{pmatrix},$$

$$g_3 = \begin{pmatrix} g_{31} \\ g_{32} \\ g_{33} \end{pmatrix} = \begin{pmatrix} -7.5y_1 + 7x_1 + \dfrac{7}{2}x_2 - y_3 - 8x_1 \\ -10.5x_1 - 3.4y_1 + 2y_2 - z_3 + 6y_3 \\ -0.13z_1 + \dfrac{1}{2}z_2 - 3.55z_3 + 1.1y_3 - 0.8x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ \dfrac{1}{2}x_1 z_1 \\ -\dfrac{1}{2}x_1 y_1 + x_3^3 \end{pmatrix},$$

$$g_4 = \begin{pmatrix} g_{41} \\ g_{42} \\ g_{43} \end{pmatrix} = \begin{pmatrix} 15y_1 - 14x_1 - 7x_2 + 14x_3 - 5y_4 + 4x_4 \\ 21x_1 + 6.8y_1 - 4y_2 + 11y_3 + 0.5y_4 \\ 0.267z_1 - 16 - z_2 + 6z_3 \end{pmatrix} + \begin{pmatrix} 0 \\ -x_1 z_1 - x_4 z_4 \\ x_1 y_1 + x_4 y_4 \end{pmatrix},$$

$$g_5 = \begin{pmatrix} g_{51} \\ g_{52} \\ g_{53} \end{pmatrix}$$

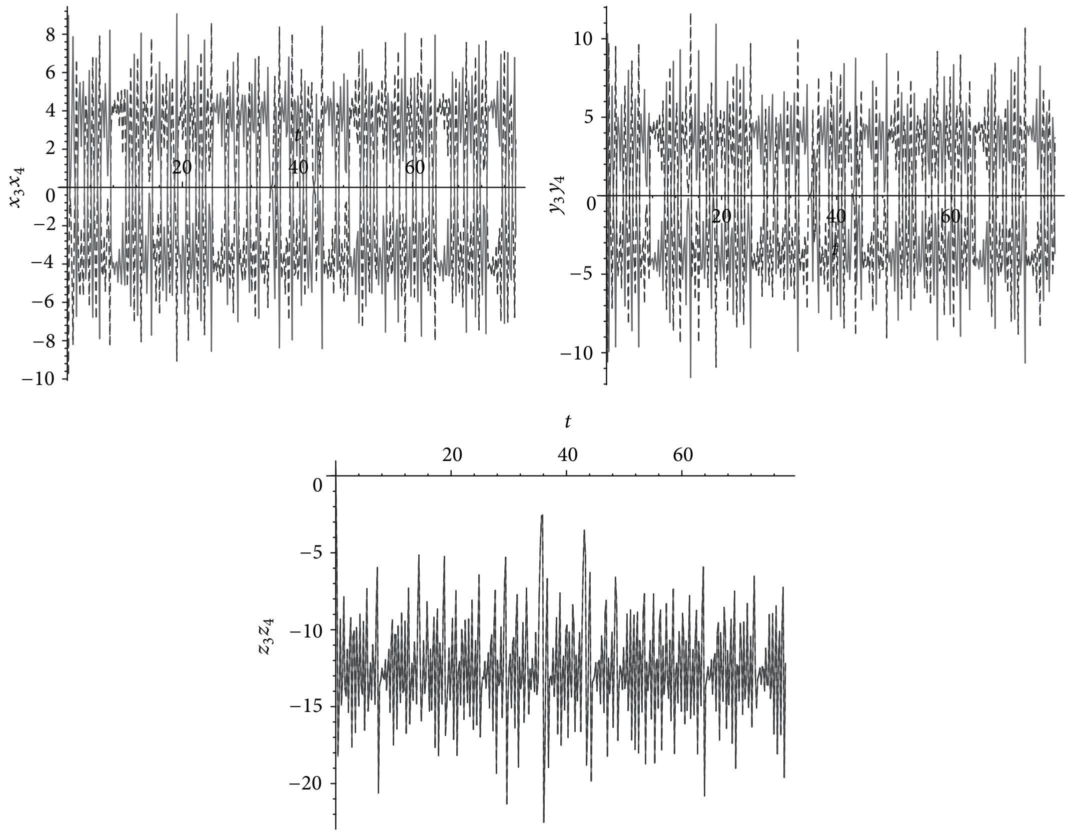

Figure 12: The temporal evolution of variables between node 3 ("⋯") and node 4 ("—").

$$= \begin{pmatrix} -7.5y_1 + 7x_1 + 3.5x_2 - 7x_3 - 2.5x_4 - 11x_5 \\ -10.5x_1 - 3.4y_1 + 2y_2 - 5.5y_3 - 1.75y_4 + 6y_5 \\ -0.133z_1 + 0.5z_2 - 3z_3 - 0.5z_4 - 1.62z_5 \end{pmatrix} + \begin{pmatrix} z_5 y_5 \\ 0.5x_1 z_1 - x_5 z_5 \\ -0.5x_1 y_1 - 0.33x_5 y_5 \end{pmatrix}. \quad (21)$$

The relative weight of the coupling strength between the nodes of the network is arbitrarily taken as $K = [-1, -2, -3, 4, 3, 2, -2, -1, -2, -3, -2, -1]$, $\alpha_i = [1, 1, 1, -2, -2, -2, 0.5, 0.5, 0.5, 2, 2, 2]$, then the network synchronization error between system (16) and system (17) will be $e_{21} = x_1 + x_2$, $e_{22} = y_1 + y_2$, and $e_{23} = z_1 + z_2$ and the network synchronization error between system (17) and system (18) will be $e_{31} = x_2 - 2x_3$, $e_{32} = y_2 - 2y_3$, and $e_{33} = z_2 - 2z_3$, and the network synchronization error between system (18) and system (19) will be $e_{41} = x_3 - 0.5x_4$, $e_{42} = y_3 - 0.5y_3$, and $e_{43} = z_3 - 0.5z_4$, and the network synchronization error between system (19) and system (20) will be $e_{51} = x_4 + 2x_5$, $e_{52} = y_4 + 2y_5$, and $e_{53} = z_4 + 2z_5$. The simulations are shown in Figures 4, 5, 6, and 7.

For further details, the dynamic attractors for each node with coupling network synchronization are shown in Figures 8 and 9. The network synchronization temporal evolution of variables is shown in Figures 10, 11, 12, and 13. From Figures 8 and 9, we can see that the network synchronization has been realized on the required proportional scale.

4. Summary and Discussion

In this paper, the complex network synchronization is investigated. With the help of symbolic computation, different order chaotic systems are adopted as the nodes of this complex network; the structure of the coupling functions among the connected nodes is obtained based on Lyapunov stability theory. Being of network and physical interests, the temporal evolution of variables and node interaction of the dynamic equation are discussed and simulated by computer. This method has universal significance for network synchronization, and the weight value of the coupling strength between the nodes and the number of the nodes does not affect synchronization of the whole network.

Acknowledgments

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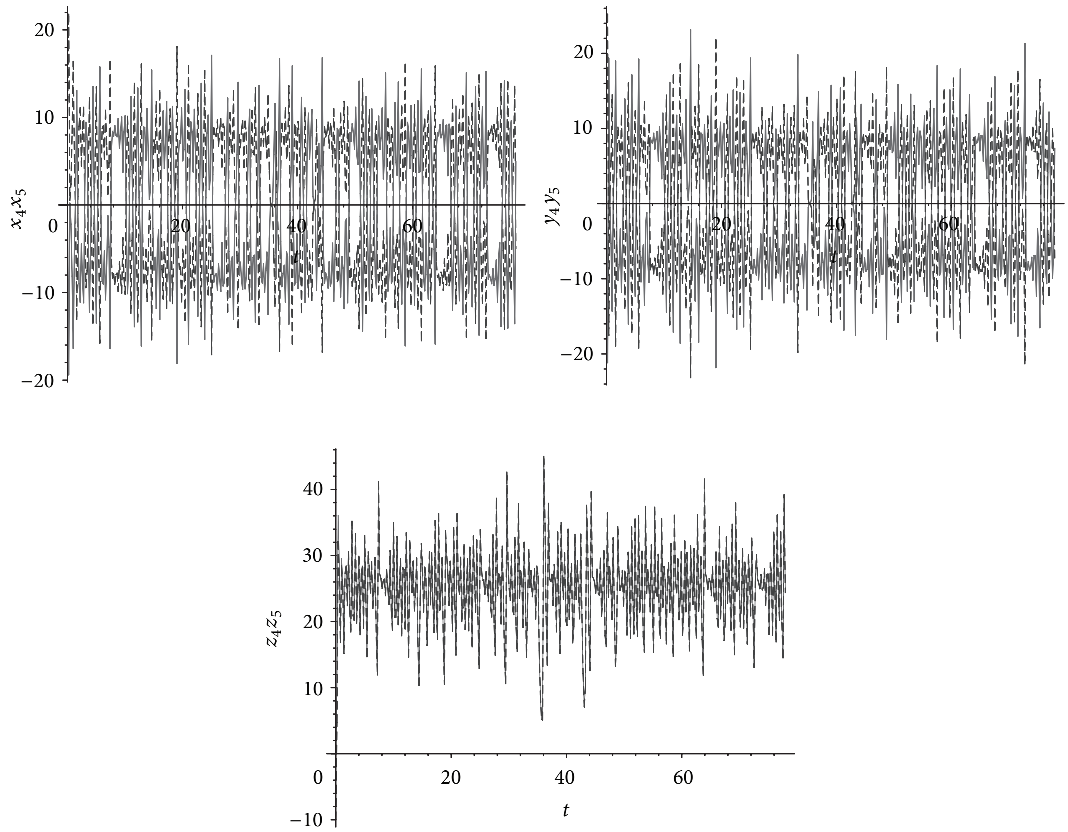

Figure 13: The temporal evolution of variables between node 4 ("⋯") and node 5 ("—").

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