

## Research Article

# Adaptive Synchronization and Antisynchronization of a Hyperchaotic Complex Chen System with Unknown Parameters Based on Passive Control

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This paper investigates the synchronization and antisynchronization problems of a hyperchaotic complex Chen system with unknown parameters based on the properties of a passive system. The essential conditions are derived under which the synchronization or antisynchronization error dynamical system could be equivalent to a passive system and be globally asymptotically stabilized at a zero equilibrium point via smooth state feedback. Corresponding parameter estimation update laws are obtained to estimate the unknown parameters as well. Numerical simulations verify the effectiveness of the theoretical analysis.

## 1. Introduction

Hyperchaos [1] is generally characterized as a chaotic attractor with more than one positive Lyapunov exponent and has richer dynamical behaviors than chaos. Over the past three decades, hyperchaotic systems with real variables have been investigated extensively [2–5]. Since Fowler et al. [6] introduced a complex Lorenz model to generalize the real Lorenz model in 1982, chaotic and hyperchaotic complex systems have attracted increasing attention to the systems with complex variables which can be used to describe the physics of a detuned laser, rotating fluids, disk dynamos, electronic circuits, and particle beam dynamics in high energy accelerators [7]. When applying the complex systems in communications, the complex variables will double the number of variables and can increase the content and security of the transmitted information.

In recent years, chaos synchronization has attracted increasing attention among scientists due to its potential applications in the fields of secure communications; optical, chemical, physical, and biological systems; neural networks; and so forth [8, 9]. Several types of synchronization have been investigated on complex chaotic and hyperchaotic systems

including complete synchronization [7], antisynchronization [10, 11], phase and antiphase synchronization [12], lag and antilag synchronization [13, 14], hybrid projective synchronization [15], and modified function projective synchronization [16]. Among the abovementioned synchronization phenomena, the most widely investigated one is complete synchronization (synchronization for short hereafter), which implies that the differences of state variables of synchronized systems starting from different initial values converge to zero eventually. On the other hand, antisynchronization is another interesting phenomenon, which is characterized by the vanishing of the sum of the relevant state variables of synchronized systems. When applying antisynchronization to communication systems, the security and secrecy of communication can be strengthened while transmitting digital signals by the transform between synchronization and antisynchronization continuously.

Recently, many researchers have begun to give their attention to the concept of passivity of nonlinear systems. The passivity theory is considered to be an alternative tool for analyzing the stability of nonlinear systems. The main idea of passivity theory is that the passive properties of a system can keep the system internally stable. In order to make a system

stable, one can design a controller which renders the closed-loop system passive with the help of passivity theory. For the past decade, the passivity theory has played an important role in designing an asymptotically stabilizing controller for control and synchronization of chaotic and hyperchaotic systems with real variables [17–20]. For complex nonlinear systems, only Mahmoud et al. [21] applied the passive control to investigate the control of  $n$ -dimensional chaotic complex nonlinear systems. In this paper, we apply the passive control to investigate the synchronization and antisynchronization problems of the newly reported hyperchaotic complex Chen system [22].

This paper is organized as follows. Section 2 introduces the passive control theory and a new hyperchaotic complex Chen system. Sections 3 and 4 investigate the synchronization and antisynchronization problems of two identical hyperchaotic complex Chen systems with unknown parameters, respectively. In Section 5, two numerical examples are provided to illustrate the analytical results. Finally, conclusions are given in Section 6.

## 2. The Passive Control Theory and a New Hyperchaotic Complex Chen System

Consider the following nonlinear affine system:

$$\begin{aligned}\dot{x} &= f(x) + g(x)u, \\ w &= h(x),\end{aligned}\quad (1)$$

where  $x \in R^n$  is the state variable and  $u \in R^m$  and  $w \in R^m$  are input and output values, respectively.  $f(x)$  and  $g(x)$  are smooth vector fields,  $f(0) = 0$ , and  $h(x)$  is a smooth mapping.

*Definition 1* (see [23]). System (1) is a minimum phase system if  $L_g h(0)$  is nonsingular and  $x = 0$  is one of the asymptotically stabilized equilibrium points of  $f(x)$ .

*Definition 2* (see [24]). System (1) is passive if there exists a real constant  $\beta$  such that for all  $t \geq 0$ , the following inequality holds:

$$\int_0^t u^T(\tau) w(\tau) d\tau \geq \beta, \quad (2)$$

or there exists a  $\rho \geq 0$  and a real constant  $\beta$  such that

$$\int_0^t u^T(\tau) w(\tau) d\tau + \beta \geq \int_0^t \rho w^T(\tau) w(\tau) d\tau. \quad (3)$$

If system (1) has relative degree  $[1, \dots, 1]$  at  $x = 0$  (i.e.,  $L_g h(0)$  is nonsingular) and the distribution spanned by the vector field  $g_1(x), \dots, g_m(x)$  is innovative, then it can be represented as the following normal form:

$$\begin{aligned}\dot{z} &= f_0(z) + p(z, w)w, \\ \dot{w} &= b(z, w) + a(z, w)u,\end{aligned}\quad (4)$$

where  $a(z, w)$  is nonsingular for any  $(z, w)$ .

**Theorem 3** (see [24]). *Suppose the system (4) is passive with a storage function  $V$ , which is positive-definite, and the system (4) is locally zero-state detectable. Let  $\phi$  be a smooth function such that  $\phi(0) = 0$  and  $w^T \phi(w) > 0$  for each nonzero  $w$ . Then the control law  $u = -\phi(w)$  asymptotically stabilizes the equilibrium of system (4).*

Recently, the authors [22] introduced six different versions of the hyperchaotic complex Chen system by adding a state feedback controller to the chaotic complex Chen system [25], and the dynamics of the six hyperchaotic complex Chen systems were studied in detail in [22]. In this paper, we apply the passive control to investigate the synchronization and antisynchronization problems of the following form of hyperchaotic complex Chen system:

$$\begin{aligned}\dot{x} &= \alpha(y - x) + w, \\ \dot{y} &= (\gamma - \alpha)x - xz + \gamma y, \\ \dot{z} &= \frac{1}{2}(\bar{x}y + x\bar{y}) - \beta z + w, \\ \dot{w} &= \frac{1}{2}(\bar{x}y + x\bar{y}) - dw,\end{aligned}\quad (5)$$

where  $\alpha, \beta, \gamma$ , and  $d$  are positive real parameters,  $x = u_1 + iu_2$  and  $y = u_3 + iu_4$  are complex functions,  $i = \sqrt{-1}$ ,  $z = u_5$ , and  $w = u_6$ .  $u_j$  ( $j = 1, 2, 3, 4, 5, 6$ ) are real functions. The overbar represents complex conjugate function.

## 3. Synchronization of the Hyperchaotic Complex Chen System with Unknown Parameters

In this section, we study the synchronization problem of the hyperchaotic complex Chen system (5) with unknown parameters using the technique of passive control. We consider system (5) as the drive system, and the response system is described by

$$\begin{aligned}\dot{x}_1 &= \alpha(y_1 - x_1) + w_1, \\ \dot{y}_1 &= (\gamma - \alpha)x_1 - x_1 z_1 + \gamma y_1 + \mu_1 + i\mu_2, \\ \dot{z}_1 &= \frac{1}{2}(\bar{x}_1 y_1 + x_1 \bar{y}_1) - \beta z_1 + w_1 + \mu_3, \\ \dot{w}_1 &= \frac{1}{2}(\bar{x}_1 y_1 + x_1 \bar{y}_1) - dw_1 + \mu_4,\end{aligned}\quad (6)$$

where  $x_1 = v_1 + iv_2$ ,  $y_1 = v_3 + iv_4$ , and  $z_1 = v_5$ ,  $w_1 = v_6$ .  $v_j$  ( $j = 1, 2, 3, 4, 5, 6$ ) are real functions, and  $\mu_1, \mu_2, \mu_3$ , and  $\mu_4$  are real control functions to be determined to achieve synchronization.

By subtracting the drive system (5) from the response system (6), we obtain the following error dynamical system

$$\begin{aligned}\dot{e}_1 + i\dot{e}_2 &= \alpha(y_1 - x_1) + w_1 - (\alpha(y - x) + w), \\ \dot{e}_3 + i\dot{e}_4 &= (\gamma - \alpha)x_1 - x_1z_1 + \gamma y_1 \\ &\quad - ((\gamma - \alpha)x - xz + \gamma y) + \mu_1 + i\mu_2, \\ \dot{e}_5 &= \frac{1}{2}(\bar{x}_1y_1 + x_1\bar{y}_1) - \beta z_1 + w_1 \\ &\quad - \left(\frac{1}{2}(\bar{x}y + x\bar{y}) - \beta z + w\right) + \mu_3, \\ \dot{e}_6 &= \frac{1}{2}(\bar{x}_1y_1 + x_1\bar{y}_1) - dw_1 - \left(\frac{1}{2}(\bar{x}y + x\bar{y}) - dw\right) + \mu_4,\end{aligned}\quad (7)$$

where  $e_j = v_j - u_j$  ( $j = 1, 2, 3, 4, 5, 6$ ) are error states and  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $d$  are unknown parameters.

Separating the real and imaginary parts of error dynamical system (7), we obtain the following real system:

$$\begin{aligned}\dot{e}_1 &= \alpha(e_3 - e_1) + e_6, \\ \dot{e}_2 &= \alpha(e_4 - e_2), \\ \dot{e}_3 &= (\gamma - \alpha)e_1 + \gamma e_3 - e_1e_5 - e_1u_5 - u_1e_5 + \mu_1, \\ \dot{e}_4 &= (\gamma - \alpha)e_2 + \gamma e_4 - e_2e_5 - e_2u_5 - u_2e_5 + \mu_2, \\ \dot{e}_5 &= e_1e_3 + e_1u_3 + u_1e_3 + e_2e_4 + e_2u_4 + u_2e_4 - \beta e_5 + e_6 + \mu_3, \\ \dot{e}_6 &= e_1e_3 + e_1u_3 + u_1e_3 + e_2e_4 + e_2u_4 + u_2e_4 - de_6 + \mu_4.\end{aligned}\quad (8)$$

Let  $z_1 = e_1$ ,  $z_2 = e_2$ ,  $y_1 = e_3$ ,  $y_2 = e_4$ ,  $y_3 = e_5$ , and  $y_4 = e_6$ ; then the error dynamical system (8) can be rewritten as

$$\begin{aligned}\dot{z}_1 &= \alpha(y_1 - z_1) + y_4, \\ \dot{z}_2 &= \alpha(y_2 - z_2), \\ \dot{y}_1 &= (\gamma - \alpha)z_1 + \gamma y_1 - z_1y_3 - z_1u_5 - u_1y_3 + \mu_1, \\ \dot{y}_2 &= (\gamma - \alpha)z_2 + \gamma y_2 - z_2y_3 - z_2u_5 - u_2y_3 + \mu_2, \\ \dot{y}_3 &= z_1y_1 + z_1u_3 + u_1y_1 + z_2y_2 + z_2u_4 + u_2y_2 - \beta y_3 \\ &\quad + y_4 + \mu_3, \\ \dot{y}_4 &= z_1y_1 + z_1u_3 + u_1y_1 + z_2y_2 + z_2u_4 + u_2y_2 - dy_4 + \mu_4,\end{aligned}\quad (9)$$

which is the following normal form:

$$\begin{aligned}\dot{z} &= f_0(z) + p(z, y)y, \\ \dot{y} &= b(z, y) + a(z, y)\mu,\end{aligned}\quad (10)$$

where  $z = [z_1, z_2]^T$ ,  $y = [y_1, y_2, y_3, y_4]^T$ ,  $\mu = [\mu_1, \mu_2, \mu_3, \mu_4]^T$  and

$$\begin{aligned}f_0(z) &= [-\alpha z_1, -\alpha z_2]^T, \quad p(z, y) = \begin{bmatrix} \alpha & 0 & 0 & 1 \\ 0 & \alpha & 0 & 0 \end{bmatrix}, \\ a(z, y) &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\ b(z, y) &= \begin{bmatrix} (\gamma - \alpha)z_1 + \gamma y_1 - z_1y_3 - z_1u_5 - u_1y_3 \\ (\gamma - \alpha)z_2 + \gamma y_2 - z_2y_3 - z_2u_5 - u_2y_3 \\ z_1y_1 + z_1u_3 + u_1y_1 + z_2y_2 + z_2u_4 + u_2y_2 - \beta y_3 + y_4 \\ z_1y_1 + z_1u_3 + u_1y_1 + z_2y_2 + z_2u_4 + u_2y_2 - dy_4 \end{bmatrix}.\end{aligned}\quad (11)$$

Then we can arrive at the following result.

**Theorem 4.** *If the passive controllers are designed as*

$$\begin{aligned}\mu_1 &= -\tilde{\gamma}z_1 - \tilde{\gamma}y_1 + z_1y_3 + z_1u_5 + u_1y_3 - k_1y_1 + v_1, \\ \mu_2 &= -\tilde{\gamma}z_2 - \tilde{\gamma}y_2 + z_2y_3 + z_2u_5 + u_2y_3 - k_2y_2 + v_2, \\ \mu_3 &= -z_1y_1 - z_1u_3 - u_1y_1 - z_2y_2 - z_2u_4 - u_2y_2 + \tilde{\beta}y_3 \\ &\quad - y_4 - k_3y_3 + v_3, \\ \mu_4 &= -z_1y_1 - z_1u_3 - u_1y_1 - z_2y_2 - z_2u_4 - u_2y_2 + \tilde{d}y_4 - z_1 \\ &\quad - k_4y_4 + v_4\end{aligned}\quad (12)$$

and the parameter estimation update laws as

$$\dot{\tilde{\beta}} = -\gamma_3^2,$$

$$\dot{\tilde{\gamma}} = z_1y_1 + y_1^2 + z_2y_2 + y_2^2, \quad (13)$$

$$\dot{\tilde{d}} = -\gamma_4^2,$$

where  $k = [k_1, k_2, k_3, k_4]^T$  is a positive constant vector,  $v = [v_1, v_2, v_3, v_4]^T$  is an external signal vector which is connected with the reference input, and  $\tilde{\beta}$ ,  $\tilde{\gamma}$ , and  $\tilde{d}$  are estimated values of the unknown parameters  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $d$ , respectively, then the error dynamical system (8) will be rendered passive and will be asymptotically stable at its equilibrium  $(0, 0, 0, 0)$  and the two systems (5) and (6) starting from different initial values will be synchronized.

*Proof.* Construct the following storage function:

$$\begin{aligned}V(z, y) &= W(z) + \frac{1}{2}y^T y + \frac{1}{2}(\tilde{\alpha} - \alpha)^2 + \frac{1}{2}(\tilde{\beta} - \beta)^2 \\ &\quad + \frac{1}{2}(\tilde{\gamma} - \gamma)^2 + \frac{1}{2}(\tilde{d} - d)^2,\end{aligned}\quad (14)$$

where  $W(z) = (1/2)(z_1^2 + z_2^2)$  is a Lyapunov function of  $f_0(z)$ .

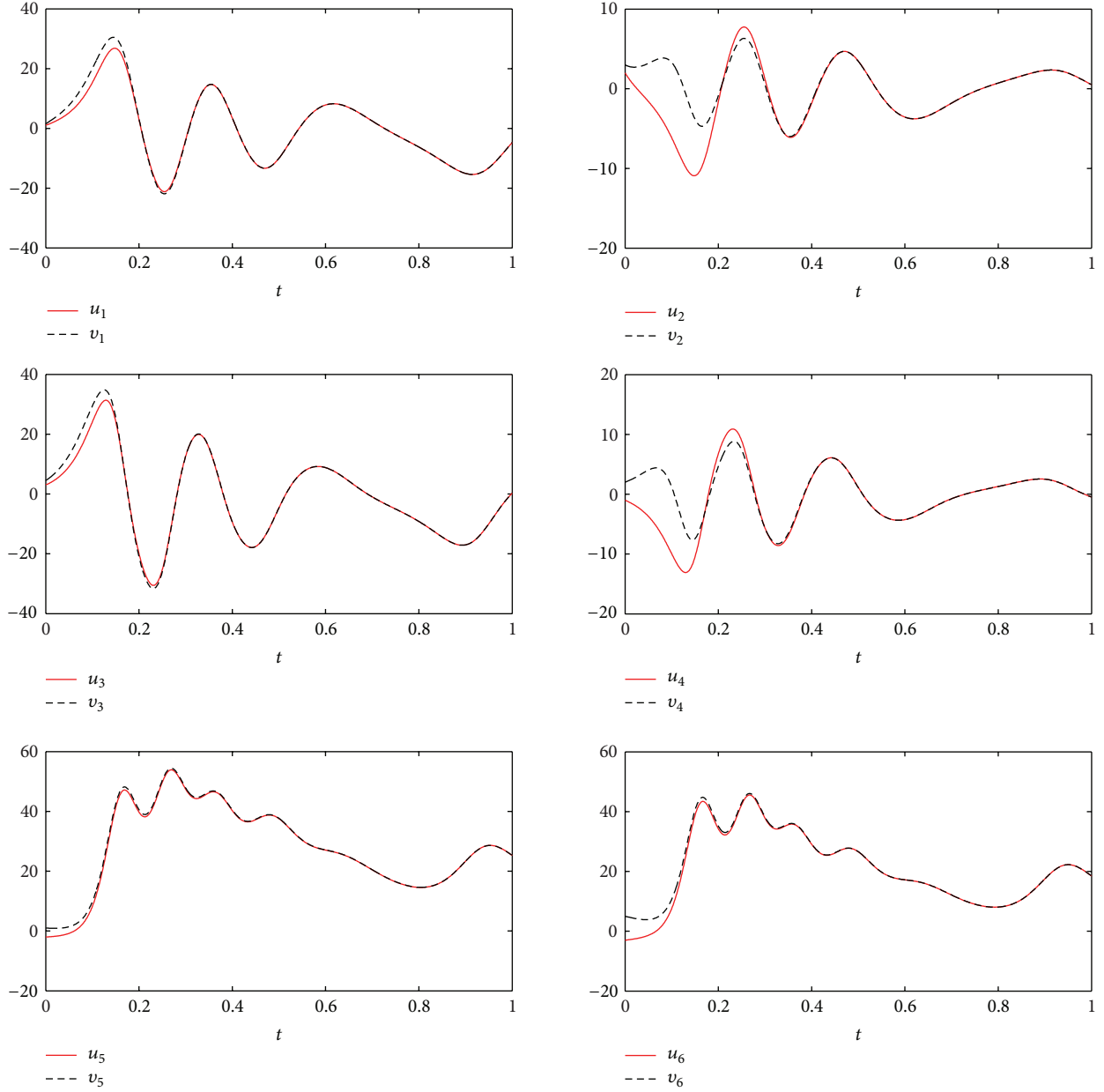


FIGURE 1: The time response of states for the drive system (5) and the response system (6) with controllers (12) and parameter estimation update laws (13).

The zero dynamics of system (10) describe the internal dynamics, which are consistent with the external constraint  $y = 0$ ; that is,  $\dot{z} = f_0(z)$ .

Differentiating  $W(z)$  with respect to  $t$ , we have

$$\frac{d}{dt}W(z) = \frac{\partial W(z)}{\partial z} f_0(z) = -\alpha z_1^2 - \alpha z_2^2 \leq 0. \quad (15)$$

Then  $f_0(z)$  is globally asymptotically stable; that is, the zero dynamics of system (8) is Lyapunov stable. In the light of Definition 1, system (8) is a minimum phase system.

Furthermore, taking the time derivative of  $V(z, y)$  along the trajectory of the error dynamical system (8) yields

$$\begin{aligned} \frac{d}{dt}V(z, y) &= \frac{\partial W(z)}{\partial z} \dot{z} + y^T \dot{y} + (\bar{\alpha} - \alpha) \dot{\bar{\alpha}} + (\bar{\beta} - \beta) \dot{\bar{\beta}} \\ &\quad + (\bar{\gamma} - \gamma) \dot{\bar{\gamma}} + (\bar{d} - d) \dot{\bar{d}} \\ &= \frac{\partial W(z)}{\partial z} f_0(z) + \frac{\partial W(z)}{\partial z} p(z, y) y \\ &\quad + y^T b(z, y) + y^T a(z, y) \mu + (\bar{\alpha} - \alpha) \dot{\bar{\alpha}} \\ &\quad + (\bar{\beta} - \beta) \dot{\bar{\beta}} + (\bar{\gamma} - \gamma) \dot{\bar{\gamma}} + (\bar{d} - d) \dot{\bar{d}}. \end{aligned} \quad (16)$$

Since the error dynamical system (8) is a minimum phase system; that is,  $((\partial W(z))/\partial z)f_0(z) \leq 0$ , then (15) becomes

$$\begin{aligned} \frac{d}{dt}V(z, y) \leq & \frac{\partial W(z)}{\partial z} p(z, y) y + y^T b(z, y) + y^T a(z, y) \mu \\ & + (\tilde{\alpha} - \alpha) \dot{\tilde{\alpha}} + (\tilde{\beta} - \beta) \dot{\tilde{\beta}} + (\tilde{\gamma} - \gamma) \dot{\tilde{\gamma}} \\ & + (\tilde{d} - d) \dot{\tilde{d}}. \end{aligned} \quad (17)$$

Substituting (12) and (13) into (17) yields

$$\frac{d}{dt}V(z, y) \leq -ky^T y + v^T y. \quad (18)$$

Then, taking integration on both sides of (18), we get

$$\begin{aligned} V(z, y) - V(z_0, y_0) \leq & - \int_0^t ky^T(\tau) y(\tau) d\tau \\ & + \int_0^t v^T(\tau) y(\tau) d\tau. \end{aligned} \quad (19)$$

For  $V(z, y) \geq 0$ , let  $V(z_0, y_0) = u$ ; then the above inequality can be rewritten as

$$\begin{aligned} \int_0^t v^T(\tau) y(\tau) d\tau + u \geq & \int_0^t ky^T(\tau) y(\tau) d\tau + V(z, y) \\ \geq & \int_0^t ky^T(\tau) y(\tau) d\tau. \end{aligned} \quad (20)$$

Letting  $v = [v_1, v_2, v_3, v_4]^T = [0, 0, 0, 0]^T$ , in light of Definition 2 and Theorem 3, system (8) will be stabilized at its equilibrium  $(0, 0, 0, 0)$  with the controllers (12) and the parameter estimation update laws (13); that is, the drive system (5) and response system (6) with different initial conditions will be synchronized with each other asymptotically.

This completes the proof.  $\square$

*Remark 5.* The controllers (12) are only related to the parameters  $\beta$ ,  $\gamma$ , and  $d$ , and so we do not need to estimate the parameter  $\alpha$ .

#### 4. Antisynchronization of the Hyperchaotic Complex Chen System with Unknown Parameters

To investigate the antisynchronization of the hyperchaotic complex Chen system, we need to add system (6) to system (5) and obtain the following antisynchronization error dynamical system:

$$\dot{e}_1 + i\dot{e}_2 = \alpha(y_1 - x_1) + w_1 + (\alpha(y - x) + w),$$

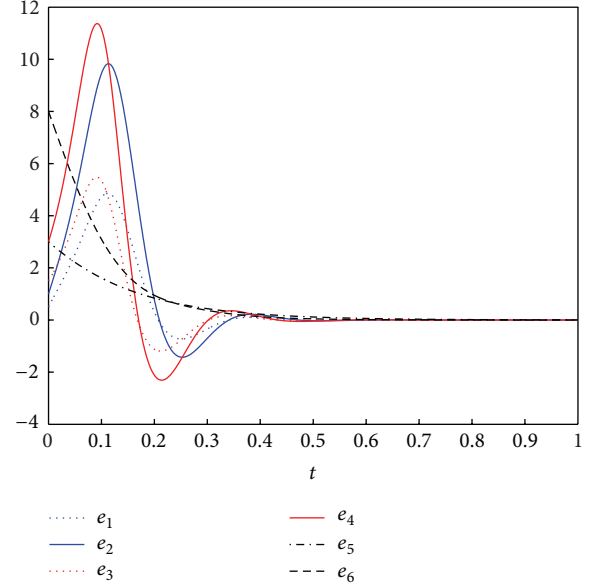


FIGURE 2: The time response of synchronization error states  $e_j$  ( $j = 1, 2, 3, 4, 5, 6$ ) for the error dynamical system (8).

$$\begin{aligned} \dot{e}_3 + i\dot{e}_4 = & (\gamma - \alpha) x_1 - x_1 z_1 + \gamma y_1 + ((\gamma - \alpha) x - xz + \gamma y) \\ & + \mu_1 + i\mu_2, \end{aligned}$$

$$\begin{aligned} \dot{e}_5 = & \frac{1}{2} (\bar{x}_1 y_1 + x_1 \bar{y}_1) - \beta z_1 + w_1 \\ & + \left( \frac{1}{2} (\bar{x} y + x \bar{y}) - \beta z + w \right) + \mu_3, \end{aligned}$$

$$\dot{e}_6 = \frac{1}{2} (\bar{x}_1 y_1 + x_1 \bar{y}_1) - d w_1 + \left( \frac{1}{2} (\bar{x} y + x \bar{y}) - d w \right) + \mu_4, \quad (21)$$

where  $e_j = v_j + u_j$  ( $j = 1, 2, 3, 4, 5, 6$ ) are antisynchronization error states and  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $d$  are unknown parameters. By separating the real and imaginary parts of the antisynchronization error dynamical system (21), we get the following real system:

$$\dot{e}_1 = \alpha(e_3 - e_1) + e_6,$$

$$\dot{e}_2 = \alpha(e_4 - e_2),$$

$$\dot{e}_3 = (\gamma - \alpha) e_1 + \gamma e_3 - e_1 e_5 + e_1 u_5 + u_1 e_5 - 2u_1 u_5 + \mu_1,$$

$$\dot{e}_4 = (\gamma - \alpha) e_2 + \gamma e_4 - e_2 e_5 + e_2 u_5 + u_2 e_5 - 2u_2 u_5 + \mu_2,$$

$$\begin{aligned} \dot{e}_5 = & e_1 e_3 - e_1 u_3 - u_1 e_3 + 2u_1 u_3 + e_2 e_4 - e_2 u_4 - u_2 e_4 \\ & + 2u_2 u_4 - \beta e_5 + e_6 + \mu_3, \end{aligned}$$

$$\begin{aligned} \dot{e}_6 = & e_1 e_3 - e_1 u_3 - u_1 e_3 + 2u_1 u_3 + e_2 e_4 - e_2 u_4 - u_2 e_4 \\ & + 2u_2 u_4 - d e_6 + \mu_4. \end{aligned} \quad (22)$$

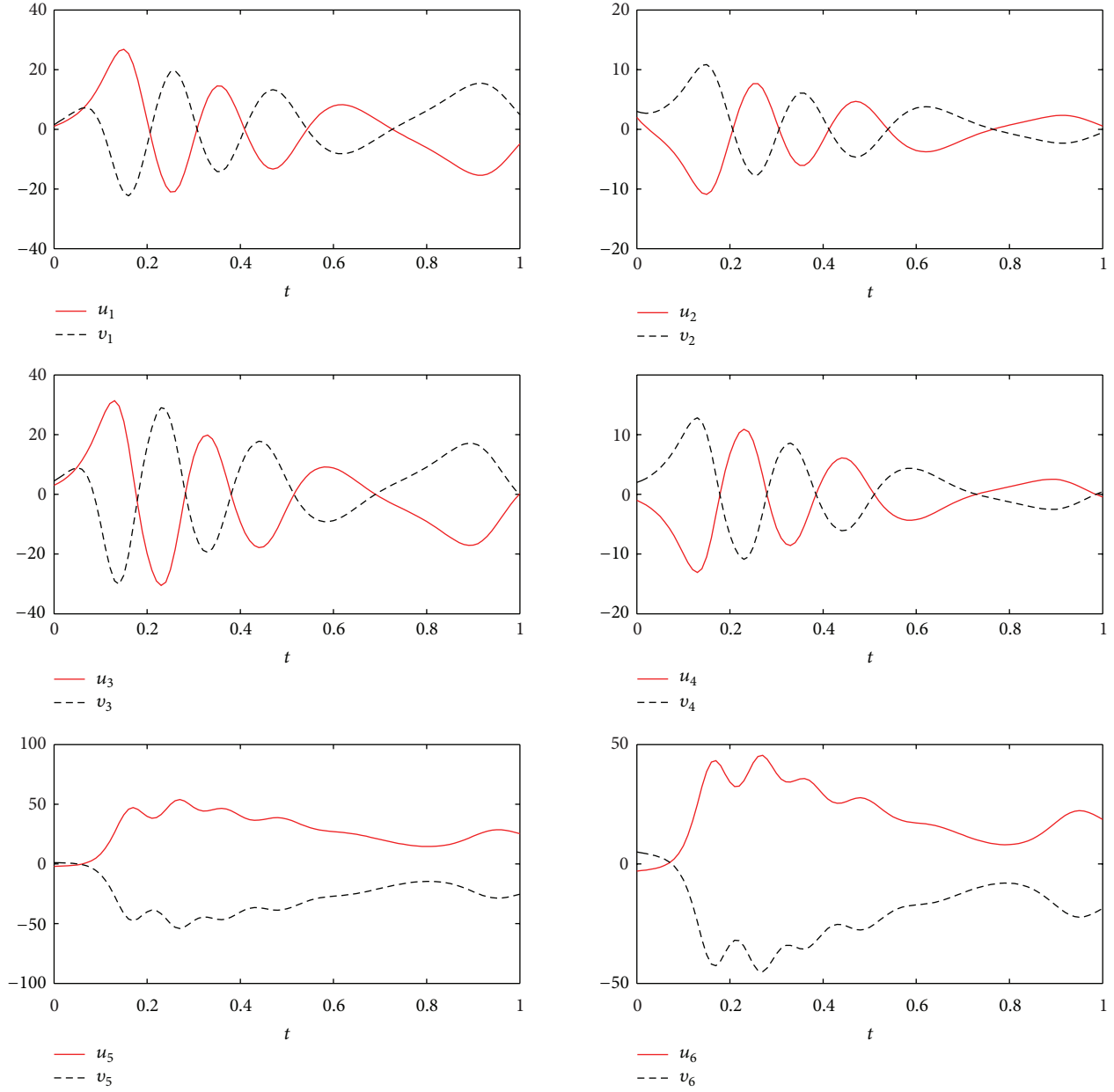


FIGURE 3: The time response of states for the drive system (5) and the response system (6) with controllers (24) and parameter estimation update laws (25).

Let  $z_1 = e_1$ ,  $z_2 = e_2$ ,  $y_1 = e_3$ ,  $y_2 = e_4$ ,  $y_3 = e_5$ , and  $y_4 = e_6$ ; then the error dynamical system (22) can be rewritten as

$$\dot{z}_1 = \alpha(y_1 - z_1) + y_4,$$

$$\dot{z}_2 = \alpha(y_2 - z_2),$$

$$\dot{y}_1 = (\gamma - \alpha)z_1 + \gamma y_1 - z_1 y_3 + z_1 u_5 + u_1 y_3 - 2u_1 u_5 + \mu_1,$$

$$\dot{y}_2 = (\gamma - \alpha)z_2 + \gamma y_2 - z_2 y_3 + z_2 u_5 + u_2 y_3 - 2u_2 u_5 + \mu_2,$$

$$\begin{aligned} \dot{y}_3 = & z_1 y_1 - z_1 u_3 - u_1 y_1 + 2u_1 u_3 + z_2 y_2 - z_2 u_4 - u_2 y_2 \\ & + 2u_2 u_4 - \beta y_3 + y_4 + \mu_3, \end{aligned}$$

$$\begin{aligned} \dot{y}_4 = & z_1 y_1 - z_1 u_3 - u_1 y_1 + 2u_1 u_3 + z_2 y_2 - z_2 u_4 - u_2 y_2 \\ & + 2u_2 u_4 - d y_4 + \mu_4. \end{aligned} \quad (23)$$

Thus, we can establish the following theorem.

**Theorem 6.** *If the passive controllers are designed as*

$$\mu_1 = -\tilde{\gamma} z_1 - \tilde{\gamma} y_1 + z_1 y_3 - z_1 u_5 - u_1 y_3 + 2u_1 u_5 - k_1 y_1 + v_1,$$

$$\mu_2 = -\tilde{\gamma} z_2 - \tilde{\gamma} y_2 + z_2 y_3 - z_2 u_5 - u_2 y_3 + 2u_2 u_5 - k_2 y_2 + v_2,$$

$$\begin{aligned} \mu_3 = & -z_1 y_1 + z_1 u_3 + u_1 y_1 - 2u_1 u_3 - z_2 y_2 + z_2 u_4 + u_2 y_2 \\ & - 2u_2 u_4 + \tilde{\beta} y_3 - y_4 - k_3 y_3 + v_3, \end{aligned}$$

$$\begin{aligned} \mu_4 = & -z_1 y_1 + z_1 u_3 + u_1 y_1 - 2u_1 u_3 - z_2 y_2 + z_2 u_4 + u_2 y_2 \\ & - 2u_2 u_4 + \tilde{d} y_4 - z_1 - k_4 y_4 + \nu_4 \end{aligned} \quad (24)$$

and the parameter estimation update laws as

$$\begin{aligned} \tilde{\beta} &= -y_3^2, \\ \tilde{\gamma} &= z_1 y_1 + y_1^2 + z_2 y_2 + y_2^2, \\ \tilde{d} &= -y_4^2, \end{aligned} \quad (25)$$

where  $k = [k_1, k_2, k_3, k_4]^T$  is a positive constant vector,  $\nu = [\nu_1, \nu_2, \nu_3, \nu_4]^T$  is an external signal vector which is connected with the reference input, and  $\tilde{\beta}$ ,  $\tilde{\gamma}$ , and  $\tilde{d}$  are estimated values of the unknown parameters  $\beta$ ,  $\gamma$ , and  $d$ , respectively, then the antisynchronization error dynamical system (22) will be rendered passive and will be asymptotically stable at its equilibrium  $(0, 0, 0, 0)$  and the two systems (5) and (6) starting from different initial values will be antisynchronized.

*Proof.* The proof of Theorem 6 is similar to that of Theorem 4, so it is omitted here.  $\square$

### 5. Numerical Simulations

In this section, we perform two numerical simulations to demonstrate the effectiveness of the above synchronization and antisynchronization schemes. In the following numerical simulations, the fourth-order Runge-Kutta method is used to solve the systems with time step size 0.001. The system parameters are selected as  $\alpha = 32$ ,  $\beta = 5$ ,  $\gamma = 25$ , and  $d = 6$  so that the hyperchaotic complex Chen system (5) exhibits hyperchaos.

*Example 7.* For the synchronization of the hyperchaotic complex Chen system, we consider the drive system (5) and the response system (6) with the controllers (12) and update laws (13). The initial values for the drive system (5) and response system (6) are given as  $(x(0), y(0), z(0), w(0)) = (1 + 2i, 3 - i, -2, -3)$  and  $(x_1(0), y_1(0), z_1(0), w_1(0)) = (1.5 + 3i, 4.5 + 2i, 1, 5)$ ; thus  $(u_1, u_2, u_3, u_4, u_5, u_6) = (1, 2, 3, -1, -2, -3)$  and  $(\nu_1, \nu_2, \nu_3, \nu_4, \nu_5, \nu_6) = (1.5, 3, 4.5, 2, 1, 5)$ , respectively. The initial errors are  $(e_1, e_2, e_3, e_4, e_5, e_6) = (0.5, 1, 1.5, 3, 3, 8)$ . And the initial values of the parameter estimation update laws are  $\tilde{\beta}(0) = \tilde{\gamma}(0) = \tilde{d}(0) = 0.1$ . We choose  $k_1 = k_2 = k_3 = k_4 = 1$  and  $\nu_1 = \nu_2 = \nu_3 = \nu_4 = 0$ . Figure 1 shows the time response of states determined by the drive system (5) and the response system (6) with the controllers (12) and the parameter estimation update laws (13). Figure 2 shows the time response of error states for the error dynamical system (8). From Figures 1 and 2, we can see that the two systems (5) and (6) starting from different initial conditions synchronize with each other immediately and the trajectories of the error dynamical system (8) are asymptotically stabilized at the equilibrium point  $O(0, 0, 0, 0)$ .

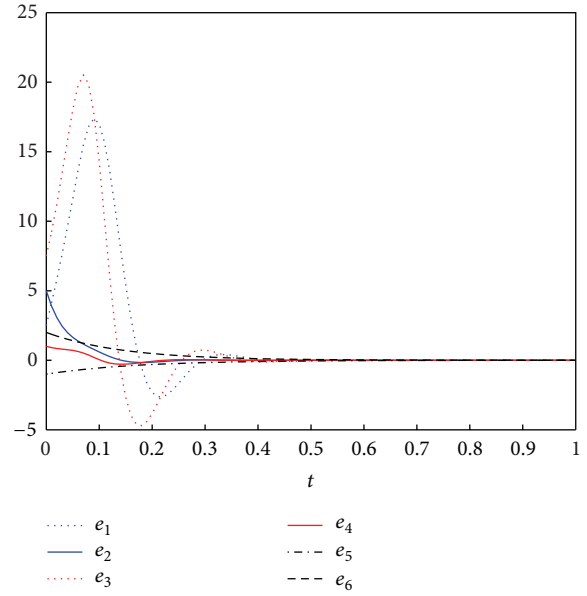


FIGURE 4: The time response of antisynchronization error states  $e_j$  ( $j = 1, 2, 3, 4, 5, 6$ ) for the error dynamical system (22).

*Example 8.* For the antisynchronization of the hyperchaotic complex Chen system, we also consider the drive system (5) and the response system (6) but with the controllers (24) and update laws (25). The initial values for the drive system (5) and response system (6) are also given as  $(x(0), y(0), z(0), w(0)) = (1 + 2i, 3 - i, -2, -3)$  and  $(x_1(0), y_1(0), z_1(0), w_1(0)) = (1.5 + 3i, 4.5 + 2i, 1, 5)$ ; thus  $(u_1, u_2, u_3, u_4, u_5, u_6) = (1, 2, 3, -1, -2, -3)$  and  $(\nu_1, \nu_2, \nu_3, \nu_4, \nu_5, \nu_6) = (1.5, 3, 4.5, 2, 1, 5)$ , respectively. But the initial errors are  $(e_1, e_2, e_3, e_4, e_5, e_6) = (2.5, 5, 7.5, 1, -1, 2)$ . And the initial values of the parameter estimation update laws are  $\tilde{\beta}(0) = \tilde{\gamma}(0) = \tilde{d}(0) = 0.1$ . We choose  $k_1 = k_2 = k_3 = k_4 = 1$  and  $\nu_1 = \nu_2 = \nu_3 = \nu_4 = 0$ . Figure 3 shows the time response of states determined by the drive system (5) and the response system (6) with the controllers (24) and the parameter estimation update laws (25). The time response of error states for the antisynchronization error dynamical system (22) is shown in Figure 4. From Figures 3 and 4, we can see that the two systems starting from different initial conditions antisynchronize with each other immediately and the trajectories of the antisynchronization error dynamical system (22) are asymptotically stabilized at the zero equilibrium.

### 6. Conclusions

Hyperchaotic systems with real variables have been investigated extensively over the past three decades. But hyperchaotic complex systems have attracted increasing attention due to the fact that they have much wider applications. So we investigate the synchronization and antisynchronization problems of a hyperchaotic complex Chen system by applying the passive control technique. Based on the fact that once a



system is passive, there exists a control law that makes the passive system stable, then the passivity-based controller can be proposed to asymptotically stabilize the error dynamical system. Then corresponding passive controllers and update laws of the parameters are proposed to achieve synchronization and antisynchronization between two hyperchaotic complex Chen systems with different initial conditions, respectively. Furthermore, this work can be extended to achieve synchronization and antisynchronization of other versions of the hyperchaotic complex Chen system, even other types of hyperchaotic complex systems, such as Lorenz system [26] and Lü system [27].

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