Classroom Note

Radioactivity Half-Lives Considered as Data

T. P. HUTCHINSON phutchin@bunyip.bhs.mq.edu.au
School of Behavioural Sciences, Macquarie University, Sydney, N.S.W. 2109, Australia

Abstract. Half-lives of radioactive nuclides range over more than 20 orders of magnitude. It is striking that, nevertheless, statistical laws may be discovered in these numbers: a log-normal distribution provides a good description.

Keywords: Half-Life (Radioactivity), Log-Normal Distribution, Transformation of Data

The 11 numbers in the first column of Table 1 cover an unusually wide range, some 25 orders of magnitude. They provide quite a dramatic example of the effectiveness of data transformation, one that I think will be unfamiliar to most statisticians.

The numbers in the first column of Table 1 are half-lives of a certain set of radioactive nuclides. I will now give a few sentences of introduction to radioactivity. One process of radioactive decay is the emission of an α particle. This reduces the mass number of the nucleus (that is, the total number of protons and neutrons) by 4. Another process is by the emission of a β particle. This does not change the mass number of the nucleus. Consequently, there are four series of radioactive nuclides — with mass numbers that respectively have remainders 0, 1, 2, and 3 when divided by 4. In the first column of Table 1 are the half-lives of α-emitting nuclides in the first of these series, starting from ${}^{252}_{100}\mathrm{Fm}$ (an isotope of Fermium

Table 1. Half-lives of nuclides in a certain decay series.

Half-life (sec)	log(half-life)	Rank	$z(\frac{\text{rank}}{12})$
9.14×10^{4}	4.96	4	−0.431
2.89×10^{7}	7.46	6	0.000
5.72×10^{8}	8.76	8	0.431
2.06×10^{11}	11.31	9	0.674
7.38×10^{14}	14.87	10	0.967
4.42×10^{17}	17.65	11	1.383
6.04×10^{7}	7.78	7	0.210
3.16×10^{5}	5.50	5	−0.210
55.6	1.75	3	−0.674
0.145	−0.84	2	−0.967
2.98×10^{-7}	−6.53	1	−1.383

Figure 1. Box-and-whisker plot of log(half-life). (The "box" extends from the lower quartile to the upper quartile, and thus accounts for the middle 50% of observations, and the "whiskers" extend out as far as the smallest and the largest observations. There being 11 observations, the quartiles have been taken to be the 3rd and the 9th smallest.)

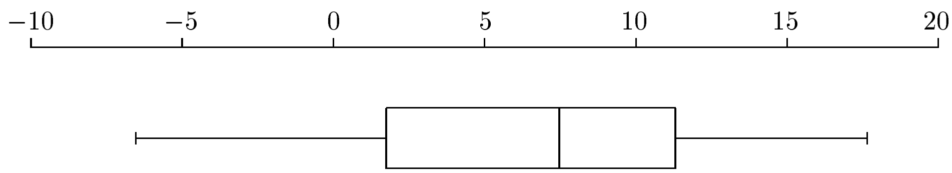
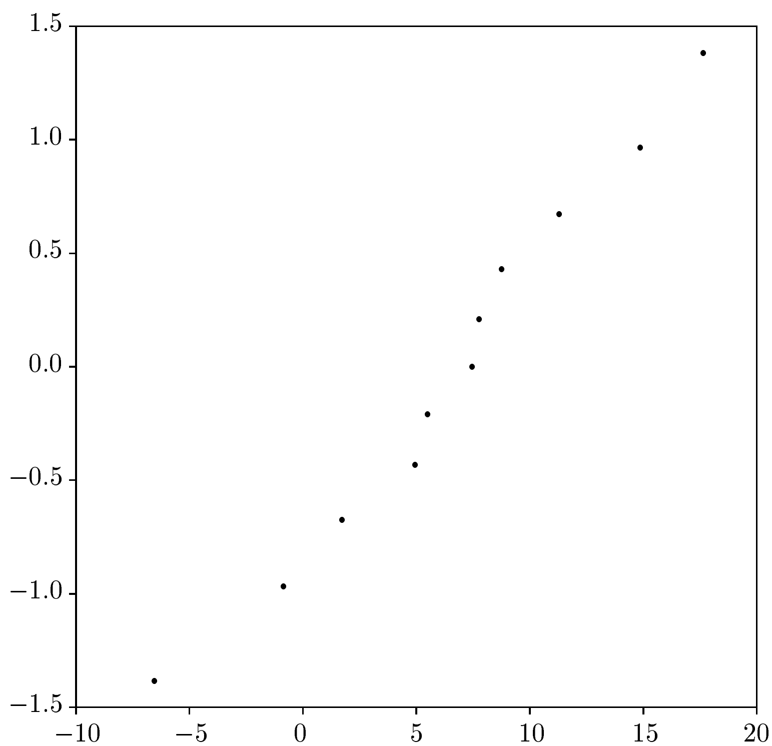


Figure 2. Cumulative distribution (plotted as the z-transformation) of log(half-life).



that has a half-life of about 1 day), and proceeding by way of ${}^{232}_{90}$Th (an isotope of Thorium that has a half-life of about 10^{10} years), to end with the stable ${}^{208}_{82}$Pb (an isotope of Lead). There are some β-emitters in the series, which have been omitted from this table. It should also be mentioned that one might question exactly what nuclides should be included in this series. For example: (i) The series is usually considered to start with ${}^{232}_{90}$Th; I have taken it back as far as ${}^{252}_{100}$Fm, but not as far as ${}^{256}_{102}$No. (ii) Some nuclides decay by both α-emission and β-emission. However, details like these are unimportant for present purposes.

It is difficult to get to grips with a set of numbers that vary so much from each other, and it might seem a lost cause to try to discover any regularity or law applicable to them. But we shall see that this is overly pessimistic. Column 2 of Table 1 shows the logarithms of the half-lives. If one did not know these were logarithms to base 10, one would consider them and their box-and-whisker plot (Figure 1) to be very ordinary-looking.

Can the log-half-lives be described by some well-known statistical distribution, such as the normal? A standard way of proceeding is to plot z against log(half-life), as this will be approximately linear in such a case; here, z is defined via $\Phi(z) =$ cumulative proportion of observations, where Φ is the cumulative normal integral. Column 3 of Table 1 shows the rank of the observations, from smallest to largest, and column 4 shows the value of z that corresponds to rank/12. (It is common to use $i/(n+1)$, not i/n, as the "plotting position" of the ith smallest observation out of a total of n.) Figure 2 shows that this plot is indeed approximately linear; thus we can conclude that the half-lives have approximately a log-normal distribution. (The Weibull distribution, with the same general shape as the log-normal, is also a good fit.)

A common disadvantage of transforming data is that there is a loss of interpretability — in this case, time is a familiar variable, but it is by no means obvious what the meaning of log(time) is. Happily, in this example there is an interpretation of log(time), in terms of a theory of nuclear decay by α-emission. This theory postulates the quantum-mechanical tunnelling of an α-particle through the Coulomb barrier formed by the attractive strong-interaction forces and the repulsive electrostatic forces. See, for example, [1] (pp. 115–119) or [2] (pp. 551–553). The energy of the α-particles emitted by a given nuclide is just as characteristic of the decay process as the half-life is, and the theory referred to predicts a relation between these two things; this relationship will be close in the sense of there being a high correlation, and strong in the sense that a small change in energy leads to a large change in half-life. The relationship is one of linearity between log(half-life) and energy$^{-1/2}$. (And some reasons are known why the relationship is not exact.)

So, instead of the distribution of half-life, let us look at the distribution of energy$^{-1/2}$. Table 2 shows the values of this. The log-normal distribution of half-life that we had in Table 1, extending over 25 orders of magnitude, is reflecting a normal distribution of energy$^{-1/2}$ that extends over a fraction of an order of magnitude, see Figure 3. (Incidentally, I make no claim that it is only the $-\frac{1}{2}$ power of disintegration energy that is approximately normally distributed — this is true for

Figure 3. Cumulative distribution (plotted as the z-transformation) of energy$^{-1/2}$.

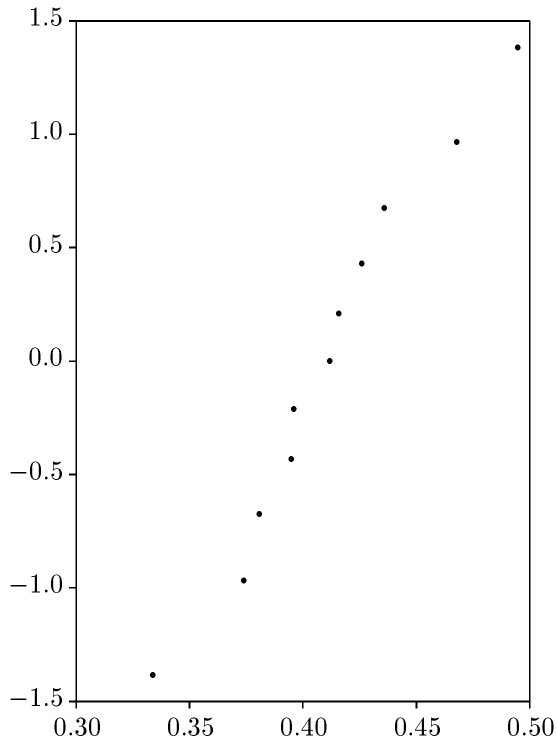


Figure 4. Scatterplot of log(half-life) vs. energy$^{-1/2}$.

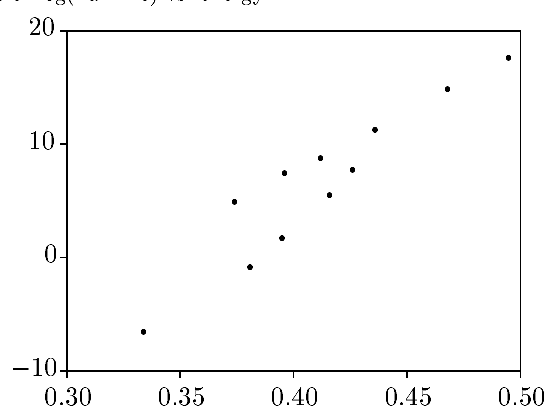


Table 2. Disintegration energies of the nuclides in Table 1.

Disintegration energy (MeV)	Energy$^{-1/2}$	Rank	$z(\frac{\text{rank}}{12})$
7.154	0.374	2	−0.967
6.369	0.396	5	−0.210
5.902	0.412	6	0.000
5.255	0.436	9	0.674
4.569	0.468	10	0.967
4.081	0.495	11	1.383
5.520	0.426	8	0.431
5.789	0.416	7	0.210
6.404	0.395	4	−0.431
6.906	0.381	3	−0.674
8.953	0.334	1	−1.383

several other powers, also.) For this set of nuclides, the scatterplot of energy$^{-1/2}$ and log(half-life) is shown as Figure 4; the correlation is 0.94.

I need hardly say that there is much more that could be done with such data as this — different sets of nuclides could be considered, and distributions other than the log-normal could be used as the standard. However, I think there is a fundamental limitation on this line of work, which is that it is usual to think of half-lives and disintegration energies as constants, not as realisations of some random process. It would therefore be misguided to ask why they should exhibit particular statistical features (such as a log-normal or a Weibull distribution), or imagine a stochastic mechanism by which they are generated. Presuming that this is right, it would be meaningless to test, for example, whether the half-lives in the four disintegration series (these were mentioned in the second paragraph of this paper) are from the same distribution or not — there is no population and no procedure of random sampling.

References

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2. W. Loveland. Radioactivity. In G. L. Trigg (Editor), *Encyclopedia of Applied Physics. Volume 15*, pp. 547–563. VCH Publishers, New York, 1996.