CONTROLLABILITY OF QUASILINEAR STOCHASTIC EVOLUTION EQUATIONS IN HILBERT SPACES

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Controllability of the quasilinear stochastic evolution equation is studied using semigroup theory and a stochastic version of the well known fixed point theorem. An application to stochastic partial differential equations is given.
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1. Introduction

The fixed point technique for finite and infinite dimensional Banach spaces is widely used and is one of several methods with which to examine the controllability of non-linear systems. The controllability of classical nonlinear systems is examined by means of the Schaefer theorem (Anichini [2]) and the Schauder theorem (Yamamoto [11]). Several authors have extended finite dimensional controllability results to infinite dimensional controllability represented by the evolution equations with bounded and unbounded operators in Banach spaces (for example, see Balachandran et. al [4] and Dauer and Balasubramaniam [5]).

Semigroup theory gives a unified treatment of a wide class of stochastic parabolic, hyperbolic and functional differential equations. Much effort has been devoted to the study of the controllability of such evolution equations (Rabah and Karrakchou [9]). Controllability of nonlinear stochastic systems has been a well known problem and frequently discussed in the literature (eg. Astrom [3], Wonham [10], and Zabczyk [12]). Stochastic control theory is a stochastic generalization of classical control theory. The purpose of this paper is to consider the controllability of quasilinear stochastic systems represented by the evolution equations with bounded linear operators in Hilbert spaces. The Banach fixed point theorem is employed to get suitable controllability conditions. The considered system is an abstract formulation of the stochastic partial differential equations discussed in Fuhrman [6].

2. Preliminaries

Consider the stochastic equation

$$\frac{dx(t)}{dt} + Ax(t) = (Bu(t) + f(x(t)) + \sigma(x)\frac{dw}{dt}, \ x(0) = x_0, t \in J = [0, t_1]. \tag{1}$$

Here A is the infinitesimal generator of an analytic semigroup $T(t)$, $t \geq 0$ in a Hilbert space X. State variable $x(t)$ takes on values in Hilbert spaces X and F. F is assumed to be separable. The control function u is given in $L^2(J; U)$, a Hilbert space of admissible control functions with U a Hilbert space. B is a bounded linear operator from U into X. Let $(\Omega, \mathfrak{F}, \mathfrak{F}_t, t \geq 0, P)$ be a complete probability space furnished with a complete family of right continuous increasing sigma algebras $\{\mathfrak{F}_t\}$ satisfying $\mathfrak{F}_t \subset \mathfrak{F}$ for $t \geq 0$. The process $\{w(t), t \geq 0\}$ is an F-valued $\mathfrak{F}_t$-adapted Brownian motion with $P\{w(0) = 0\} = 1$, and x_0 is an X-valued $\mathfrak{F}_0$ measurable random variable. For any Banach space K, let $L_2(\Omega, K)$ denote the space of strongly measurable K-valued square integrable random variables equipped with the norm topology $\|\xi\|_{L_2(\Omega, K)} = \{E\|\xi\|_K^2\}^{1/2}$ where E stands for integration with respect to the probability measure P. $L_2(\Omega, K)$ is a Hilbert space if K is Hilbert space. For convenience we shall denote $L_2^0(\Omega, K)$ as the class of K-valued $\mathfrak{F}_0$ measurable square integrable random variables. Let $M(J, K)$ denote the space of $\mathfrak{F}_t$-adapted stochastic processes defined on J. $M(J, K)$ takes on values in K, has square integrable norms, and is continuous in t on J in the mean square sense. This is a Banach space with respect to the norm topology

$$\|\xi\|_{M(J, K)} = \left\{\sup_{t \in J} E\|\xi(t)\|_K^2\right\}^{1/2}, \ \xi \in M(J, K). \tag{2}$$

Assume the following conditions:

(i)   For $0 \leq \alpha < 1/2$, $X_\alpha = [D(A^\alpha)]$ is a Banach space with respect to the graph topology induced by the graph norm given by

$$\|\xi\|_\alpha = \|A^\alpha\xi\| + \|\xi\| \text{ for } \xi \in D(A^\alpha). \tag{3}$$

(ii)   For the function f from X_α to X, there exists a constant $C > 0$ such that

$$\|f(\xi) - f(\zeta)\|_X \leq C\|\xi - \zeta\|_\alpha \text{ and}$$

$$\|f(\zeta)\|_X \leq C\{1 + \|\zeta\|_\alpha\} \text{ for all } \xi, \zeta \in X_\alpha. \tag{4}$$

(iii)   For functions σ from X_α to $L(F, X)$, there exists a constant $C > 0$ such that

$$\|\sigma(\xi) - \sigma(\zeta)\|_{L(F, X)} \leq C\|\xi - \zeta\|_\alpha \text{ and}$$

$$\|\sigma(\zeta)\|_{L(F, X)} \leq C\{1 + \|\zeta\|_\alpha\}. \tag{5}$$

(iv)   The linear operator W from U into X which is defined by

$$Wu = \int_0^{t_1} T(t_1 - s)Bu(s)ds \qquad (6)$$

has an invertible operator W^{-1} defined on $X\backslash \ker W$ (See Jousis and Wexler [7]); there exist positive constants N_1, N_2 such that

$$\| B \|^2 \leq N_1 \text{ and } \| W^{-1} \|^2 \leq N_2.$$

Then for every $x_0 \in L_2^0(\Omega, X_\alpha)$, w an F-valued $\mathfrak{F}_t$-adapted Brownian motion having a nuclear covariance operator $Q \in L_n^+(F)$, and conditions (i)-(iii), there exists a unique solution $x \in M(J, X_\alpha)$ of equation (1) which satisfies the following stochastic integral equation (see Ahmed [1])

$$x(t) = T(t)x_0 + \int_0^t T(t-s)[(Bu)(s) + f(x(s))]ds + \int_0^t T(t-s)\sigma(x(s))dw(s) \qquad (7)$$

Definition: The stochastic system (1) is *controllable*, if for some control $u(t)$ on J, the solution of (1) such that $x(t_0) = x_0$ and $x(t_1) = x_1$, where x_1 and t_1 are the state and preassigned terminal time respectively. If the system is controllable for all x_0 at $t = t_0$ and for all x_1 at $t = t_1$, the system is called *completely controllable* on J.

3. Main Result

Theorem 3.1: *Suppose the conditions (i)-(iv) are satisfied, then system (1) is completely controllable on J.*

Proof: Using the hypothesis, define the control

$$u(t) = W^{-1}[x_1 - T(t_1)x_0 - \int_0^{t_1} T(t_1 - s)f(x(s))ds$$

$$- \int_0^{t_1} T(t_1 - s)\sigma(x(s))dw(s)](t). \qquad (8)$$

It will be shown that, when using this control, the operator defined by

$$(\Phi x)(t) = T(t)x_0 + \int_0^t T(t-\mu)BW^{-1}[x_1 - T(t_1)x_0$$

$$- \int_0^{t_1} T(t_1 - s)f(x(s))ds - \int_0^{t_1} T(t_1 - s)\sigma(x(s))dw(s)](\mu)d\mu \qquad (9)$$

$$+ \int_0^t T(t-s)f(x(s))ds + \int_0^t T(s-s)\sigma(x(s))dw(s)$$

has a fixed point. This fixed point is a solution of equation (1). Clearly $(\Phi x)(0) = x_0$, which means that the control u steers the quasilinear differential system from initial state x_0 to state x_1 in time t_1 provided a fixed point of the nonlinear operator Φ can be obtained.

First we show that Φ maps $M(J, X_\alpha)$ into $M(J, X_\alpha)$. Without loss of generality, we assume that $0 \in \rho(A)$. If $0 \notin \rho(A)$, add the term γI to A which gives $A_\gamma = A + \gamma I$ and $0 \in \rho(A_\gamma)$. This simplifies the graph norm to $\| \zeta \|_\alpha = \| A^\alpha \zeta \|$ for $\zeta \in D(A^\alpha)$. Since $T(t)$, $t \geq 0$ is an analytic semigroup and A^α is a closed operator, there exist numbers $C_1 \geq 1$ and C_α such that

$$\sup_{t \in J} \| T(t) \|^2_{L(X)} \leq C_1 \text{ and } \| A^\alpha T(t) \|_{L(X)} \leq C_\alpha t^{-\alpha}, \text{ for } t > 0. \qquad (10)$$

Further, $|a + b + c|^2 \leq 9(|a|^2 + |b|^2 + |c|^2)$ for any real numbers a, b, c. Hence, for $x \in M(J, X_\alpha)$ with $x(0) = x_0$, we have

$$\| (\Phi x)(t) \|^2 \leq 9 \sup_{t \in J} E(\| T(t) x_0 \|^2_\alpha) + 9 E \| \int_0^t T(t - \mu) B W^{-1} [x_1 - T(t_1) x_0$$

$$- \int_0^{t_1} T(t_1 - s) f(x(s)) ds - \int_0^{t_1} T(t_1 - s) \sigma(x(s)) dw(s)](\mu) \|^2_\alpha d\mu$$

$$+ 9 E \| \int_0^t T(t - s) f(x(s)) ds \|^2_\alpha + 9 T_r Q \int_0^t E(\| A^\alpha T(t - s) \sigma(x(s)) \|^2_{L(F, X)}) ds$$

$$\leq 9 \sup_{t \in J} E(\| A^\alpha T(t) x_0 \|^2_X) + 9 N_1 N_2 \int_0^t \| A^\alpha T(t_1 - \mu) \|^2_{L(X)} d\mu$$

$$\times [E \| x_1 \|^2_\alpha + E \| A^\alpha T(t_1) x_0 \|^2_X) + (\int_0^{t_1} \| A^\alpha T(t_1 - s) \|^2_{L(X)} ds) E \int_0^{t_1} \| f(x(s)) \|^2_X ds$$

$$+ E \int_0^{t_1} \| A^\alpha T(t_1 - s) \sigma(x(s)) dw(s) \|^2_X]$$

$$+ 9 (\int_0^t \| A^\alpha T(t - s) \|^2_{L(X)} ds) E \int_0^t \| f(x(s)) \|^2_X ds$$

$$+ 9 T_r Q \int_0^t E(\| A^\alpha T(t - s) \sigma(x(s)) \|^2_{L(F, X)}) ds$$

$$\leq 9 C_1 E(\| x_0 \|^2_\alpha) + (9 N_1 N_2 C^2_\alpha)/(1 - 2\alpha) t_1^{1 - 2\alpha}$$

$$\times [E \parallel x_1 \parallel_\alpha^2 + C_1 E(\parallel x_0 \parallel_\alpha^2) + \{[C_\alpha C]^2/(1-2\alpha)\} t_1^{2(1-\alpha)} \{1 + \sup_{0 \leq s \leq t} E \parallel x(s) \parallel_\alpha^2\}$$

$$+ T_r Q\{[2(C_\alpha C)^2]/(1-2\alpha)\} t_1^{(1-2\alpha)} \{1 + \sup_{0 \leq s \leq t} E \parallel x(s) \parallel_\alpha^2\}]$$

$$+ 9\{[C_\alpha C]^2/(1-2\alpha)\} t^{2(1-\alpha)} \{1 + \sup_{0 \leq s \leq t} E \parallel x(s) \parallel_\alpha^2\}$$

$$+ 9 T_r Q\{[2(C_\alpha C)^2]/(1-2\alpha)\} t_1^{(1-2\alpha)} \{1 + \sup_{0 \leq s \leq t} E \parallel x(s) \parallel_\alpha^2\}$$

$$\leq 9 C_1 E(\parallel x_0 \parallel_\alpha^2) + (9 N_1 N_2 C_\alpha^2)/(1-2\alpha) t_1^{1-2\alpha}$$

$$\times [E(\parallel x_1 \parallel_\alpha^2) + C_1 E(\parallel x_0 \parallel_\alpha^2) + \{[C_\alpha C]^2/(1-2\alpha)\} t_1^{2(1-\alpha)} \{1 + \parallel x \parallel_{M(I,X_\alpha)}^2\}$$

$$+ 2 T_r Q\{(C_\alpha C)^2/(1-2\alpha)\} t_1^{(1-2\alpha)} \{1 + \parallel x \parallel_{M(I,X_\alpha)}^2\}]$$

$$+ 9[C_\alpha C]^2/(1-2\alpha)\} t_1^{2(1-\alpha)} \{1 + \parallel x \parallel_{M(I,X_\alpha)}^2\}$$

$$+ 18 T_r Q\{(C_\alpha C)^2/(1-2\alpha)\} t_1^{(1-2\alpha)} \{1 + \parallel x \parallel_{M(I,X_\alpha)}^2\}$$

$$\leq 9 C_1 E(\parallel x_0 \parallel_\alpha^2) + (9 N_1 N_2 C_\alpha^2)/(1-2\alpha) t_1^{1-2\alpha}$$

$$\times [E(\parallel x_1 \parallel_\alpha^2) + C_1 E(\parallel x_0 \parallel_\alpha^2) + \eta(T_r Q)] + 9\eta(T_r Q) \tag{11}$$

where $T_r Q$ represents the trace of the operator Q and

$$\eta(T_r Q) = \{[C_\alpha C]^2/(1-2\alpha)\}\{t_1 + 2 T_r Q\} t_1^{(1-2\alpha)} \{1 + \parallel x \parallel_{M(I,X_\alpha)}^2\}. \tag{12}$$

Hence

$$\sup_{t \in J} \parallel (\Phi x)(t) \parallel_\alpha^2 < \infty \text{ for } x \in M(J, X_\alpha), t \in J.$$

To complete the proof, it remains to be shown that $\Phi \in C((0, t_1), L_2(\Omega, X\alpha))$. Let $t \in (0, t_1)$, $h > 0$ and $t + h \in J$. For analytic semigroups, there exists a constant $\gamma_\beta > 0$ such that

$$\parallel (T(h) - I)\xi \parallel_X \leq \gamma_\beta h^\beta \parallel A^\beta \xi \parallel_X \text{ for all } \xi \in D(A^\beta)$$

and for all $\beta \geq 0$ and $\zeta \in X$, $T(t)\zeta \in D(A^\beta)$ for $t > 0$ (see Pazy [8], Theorem 6.13). Thus for $t > 0$, the closedness of A^α, the fact that $T(t)$ commutes with A^α on $D(A^\alpha)$, and by choosing $\beta > 0$ such that $0 \leq \alpha + \beta \leq 1/2$,

$$E\{\parallel (\Phi x)(t+h) - (\Phi x)(t) \parallel_\alpha^2\}$$

$$\leq 9 E(\parallel (T(h) - I)T(t) A^\alpha x_0 \parallel_\alpha^2)$$

$$+ 9E \| \int_0^t (T(h)-I)A^\alpha T(t-\mu)BW^{-1}[x_1 - T(t_1)x_0 - \int_0^{t_1} T(t_1-s)f(x(s))ds$$

$$- \int_0^{t_1} T(t_1-s)\sigma(x(s))dw(s)](\mu) \|_\alpha^2 d\mu$$

$$+ 9E \| \int_t^{t+h} A^\alpha T(t+h-\mu)BW^{-1}[x_1 - T(t_1)x_0 - \int_0^{t_1} T(t_1-s)f(x(s))ds$$

$$- \int_0^{t_1} T(t_1-s)\sigma(x(s))dw(s)](\mu) \|_\alpha^2 d\mu$$

$$+ 9E \| \int_0^t (T(h)-I)A^\alpha T(t-s)f(x(s))ds \|_\alpha^2 + 9E \| \int_t^{t+h} A^\alpha T(t+h-s)f(x(s))ds \|_\alpha^2$$

$$+ 9E \| \int_0^t (T(h)-I)A^\alpha T(t-s)\sigma(x(s))dw(s) \|_{L(F,X)}^2$$

$$+ 9E \| \int_t^{t+h} A^\alpha T(t+h-s)\sigma(x(s))dw(s) \|_{L(F,X)}^2$$

$$\leq 9\gamma_\beta^2 h^{2\beta} \| A^\beta T(t) \|^2 E \| A^\alpha x_0 \|_0^2 + 9N_1 N_2 \gamma_\beta^2 C_{\alpha+\beta}^2 h^{2\beta} \int_0^t [1/(t-\mu)]^{2(\alpha+\beta)} d\mu$$

$$\times [E(\| x_1 \|_\alpha^2) + C_1 E(\| x_0 \|_\alpha^2) + \{[C_\alpha C]^2/(1-2\alpha)\}\{t_1 + 2T_r Q\}t_1^{1-2\alpha}$$

$$\times \{1 + \sup_{0 \leq s \leq t} E \| x(s) \|_\alpha^2\}] + 9N_1 N_2 C_\alpha^2 \int_t^{t+h} [1/(t+h-\mu)]^{2\alpha} d\mu$$

$$\times [E(\| x_1 \|_\alpha^2) + C_1 E(\| x_0 \|_\alpha^2) + \{[C_\alpha C]^2/(1-2\alpha)\}\{t_1 + 2T_r Q\}t_1^{1-2\alpha}$$

$$\times \{1 + \sup_{0 \leq s \leq t} E \| x(s) \|_\alpha^2\}] + 9\gamma_\beta^2 C_{\alpha+\beta}^2 h^{2\beta} E(\int_0^{t_1} [1/(t-s)]^{2(\alpha+\beta)} \| f(x(s)) \|_\alpha^2)ds$$

$$+ 9C_\alpha^2 E \| \int_t^{t+h} [1/(t+h-s)]^{2\alpha} \| f(x(s)) \|_\alpha^2 ds$$

$$+ 9T_r Q \gamma_\beta^2 C_{\alpha+\beta}^2 h^{2\beta} E(\int_0^t [1/(t-s)]^{2(\alpha+\beta)} \| \sigma((s)) \|_{L(F,X)}^2)ds$$

$$+ 9T_r Q C_\alpha^2 E(\int_t^{t+h} [1/(t+h-s)]^{2\alpha} \| \sigma(x(s)) \|_{L(F,X)}^2)ds$$

$$\leq 9((\gamma_\beta C_\beta)/t^\beta)^2 h^{2\beta} E \parallel x_0 \parallel_\alpha^2 + 9N_1 N_2 \{ \frac{((\gamma_\beta C_\alpha + \beta)/t^\beta)^2}{(1 - 2\alpha - 2\beta)} h^{2\beta} t^{1 - 2\alpha - 2\beta}$$

$$+ C_\alpha^2/(1 - 2\alpha) h^{2(1 - \alpha)} \} \{ E \parallel x_1 \parallel_\alpha^2 + C_1 E \parallel x_0 \parallel_\alpha^2 + \eta(T_r Q) \}$$

$$+ 9 \frac{((\gamma_\beta CC_\alpha + \beta)/t^\beta)^2}{(1 - 2\alpha - 2\beta)} h^{2\beta} t^{2(1 - \alpha - \beta)} (1 + \sup_{s \in J} E \parallel x(s) \parallel_\alpha^2)$$

$$+ 9(CC_\alpha)^2 \frac{h^{2(1 - \alpha)}}{(1 - 2\alpha)} (1 + \sup_{s \in J} E \parallel x(s) \parallel_\alpha^2)$$

$$+ \frac{9T_r Q((\gamma_\beta CC_\alpha + \beta)/t^\beta)^2}{(1 - 2\alpha - 2\beta)} h^{2\beta} t^{2(1 - \alpha - \beta)} (1 + \sup_{s \in J} E \parallel x(s) \parallel_\alpha^2)$$

$$+ \frac{9T_r Q(CC_\alpha)^2}{(1 - 2\alpha)} h^{1 - 2\alpha} (1 + \sup_{s \in J} E \parallel x(s) \parallel_\alpha^2) \tag{13}$$

for $t \in (0, t_1)$. Thus letting $h \to 0$, the desired continuity is followed and hence Φ maps $M(J, X_\alpha)$ into itself.

Now we prove that for sufficiently small t_1, defining the interval J is a contraction in $M(J, X_\alpha)$. Indeed for $x, y \in M(J, X_\alpha)$ satisfying $x(0) = y(0) = x_0$, it can easily be seen that

$$\sup_{t \in J} E \parallel (\Phi x)(t) - (\Phi y)(t) \parallel^2 \leq K_\alpha \sup_{t \in J} E \parallel x(t) - y(t) \parallel_\alpha^2 \tag{14}$$

where

$$K_\alpha = (9N_1 N_2 C_\alpha^2)/(1 - 2\alpha) \{ [C_\alpha C]^2/(1 - 2\alpha) \} (t_1 + 2T_r Q) t_1^{2(1 - 2\alpha)}$$

$$+ 9\{ [C_\alpha C]^2/(1 - 2\alpha) \} \{ t_1 + 2T_r Q \} t_1^{(1 - 2\alpha)}.$$

Thus for sufficiently small $t_1, K_\alpha < 1$ and Φ is a contraction in S, and by the Banach fixed point theorem, Φ has a unique fixed point $x \in M(J, X_\alpha)$. Any fixed point of Φ is a solution of (1) on J satisfying $(\Phi x)(t) = x(t) \in X$ for all x_0 and t_1. Thus, the system (1) is completely controllable on J.

4. Example

Consider the following nonlinear stochastic partial differential equation of the form

$$\partial_t x(t, \xi) = \frac{\partial^2}{\partial \xi^2} x(t, \xi) dt + f(\xi, \langle x(t, \cdot), h_1 \rangle, \ldots, \langle x(t, \cdot), h_n \rangle) dt$$

$$+ (Bu)(t) dt + \sum_{k=1}^\infty \lambda_k \sin(k\xi) d\beta_k(t), \xi \in (0, \pi), t > 0$$

$$x(t, 0) = x(t, \pi) = 0, \ t > 0$$

$$x(0, \cdot) = x_0(\cdot) \in X = H = L^2(0, \pi). \tag{15}$$

Here

(1)   Let $\mathrm{dom}\, A = H^2(0, \pi) \cap H_0^1(0, \pi)$ and $(A\phi)\xi = \frac{\partial^2}{\partial\xi^2}\phi(\xi),\ \xi \in (0, \pi) \in \mathrm{dom}\, A$,

and let B be a bounded linear operator from the control space U into H.

(2)   Define the function $F \colon H \to H$ by choosing $h_1, h_2, \ldots, h_n \in H$ and a function $f \colon [0, \pi] \times R^n \to R,\ (\xi, y_1, y_2, \ldots, y_n) \to f(\xi, y_1, y_2, \ldots, y_n)$. Set

$$(Fg)(\xi) = f(\xi, \langle g, h_1 \rangle, \langle g, h_2 \rangle, \ldots, \langle g, h_n \rangle), \xi \in [0, \pi], g \in H,$$

where $\langle \cdot, \cdot \rangle$ is the usual scalar product in H.

(3)   Also for $y_1, y_2, \ldots, y_n \in J$ and $i = 1, 2, \ldots, n$, assume

$$f, \frac{\partial f}{\partial \xi}, \frac{\partial f}{\partial y_i}, \frac{\partial^2 f}{\partial \xi \partial y_i}$$

are bounded and continuous on $[0, \pi] \times R^n$ such that

$$f(0, y_1, y_2, \ldots, y_n) = f(\pi, y_1, y_2, \ldots, y_n) = 0$$

and

$$[\partial f / \partial y_i](0, y_1, y_2, \ldots, y_n) = [\partial f / \partial y_i](\pi, y_1, y_2, \ldots, y_n) = 0.$$

(4)   The functions $e_k(\xi) = \sqrt{2/\pi}\sin k\xi,\ \xi \in (0, \pi)$ form an orthonormal basis of H consisting of eigenvectors of A which corresponds to the eigenvalues $\alpha_k = -k^2,\ k = 1, 2, \ldots$, etc. $\beta_k(t)$ are standard real independent Wiener processes. Take a sequence of numbers $\{\lambda_k\}$ and define the operator Q by setting $Qe_k = \lambda_k e_k,\ k = 1, 2, \ldots$. Assume that $\lambda_k > 0,\ \sup_k \lambda_k < \infty$, $\sup_k k\lambda_k^{-1/2}e^{-tk^2} < \infty$, for $t > 0$.

Then (15) has an abstract formulation of the following nonlinear stochastic equation in a Hilbert space with a constant, but possibly degenerate, diffusion term

$$\frac{dx(t)}{dt} = Ax(t) + f(x(t)) + (Bu)(t) + Q^{1/2}\frac{dw(t)}{dt},\ t \geq 0. \tag{16}$$

$$x(0) = x_0 \in H.$$

The linear operator A is the infinitesimal generator of a strongly continuous semigroup $e^{At}, t \geq 0$ in H, Q is a continuous linear self adjoint nonnegative operator in H, and the operators defined by

$$Q_t x = \int_0^t e^{sA} Q e^{sA^*} x_0 ds,\ x_0 \in H, t \in J \tag{17}$$

are trace class. Further B is a bounded linear operator which maps from the control space U into H, function $f \colon H \to H$ is Lipschitz continuous and $w(t),\ t \geq 0$ is a cylindrical Wiener process in H. Then (16) has the following unique solution (see, Fuhrman [6]),

$$x(t) = e^{tA}x_0 + \int_0^t e^{(t-s)A}[(Bu)(s) + f(x(s))]ds + \int_0^t e^{(t-s)A}Q^{1/2}dw(s). \qquad (18)$$

Hence by Theorem 3.1, for $T(t) = e^{tA}$, the system (1) is completely controllable on J.

Acknowledgments

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