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Research Article

L^p Solutions of BSDEs with Stochastic Lipschitz Condition

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We are concerned with the solutions of a special class of backward stochastic differential equations which are driven by a Brownian motion, where the uniform Lipschitz continuity is replaced by a stochastic one. We prove the existence and uniqueness of the solution in L^p with $p > 1$.

1. Introduction

In this paper, we study backward stochastic differential equations (BSDEs for short) of the form

$$-dY_t = f(t,Y_t,Z_t)\,dt - Z_t\,dW_t, \qquad Y_\tau = \xi, \tag{1.1}$$

where τ is a bounded stopping time for the filtration $\mathfrak{F}$.

Since the first result about the solutions in L^2 was obtained by Pardoux and Peng [1], some related results have been generalized. Moreover, for mathematical interest, many people have studied the results of existence and uniqueness in L^p. Let us mention that when the generator is uniformly Lipschitz continuous, a result of El Karoui et al. [2] provides the existence of a solution when the data ξ and $\{f(t,0,0)\}_{t\in[0,T]}$ are in L^p for $p \in (1,\infty)$. But in many applications, Lipschitz condition is too restrictive to be assumed. Consequently, we are interested in replacing the Lipschitz condition with a weaker one and we always assume that τ is bounded. In this field, in [3], Briand and Carmona have discussed the L^p solutions for BSDEs with polynomial growth generators and then in [4], Briand et al. generalized the result.

Now let us mention that the pricing problem of an American claim is equivalent to solving the BSDE

$$dY_t = [r(t)Y_t + \theta(t)Z_t]dt + Z_t dW_t, \qquad Y_\tau = \xi, \tag{1.2}$$

where $r(t)$ is the interest rate and $\theta(t)$ is the risk premium vector. In general, both of them may be unbounded, therefore the results mentioned above may be invalid.

In this paper, we try to get the existence and uniqueness result of L^p $(p > 1)$ solutions for BSDEs with stochastic Lipschitz condition, which was introduced by El Karoui and Huang [5]. We have to mention that Bender and Kohlman have discussed BSDEs with stochastic Lipschitz condition and by strengthening the integrability conditions on the generator and the terminal value, they got a wellposedness result in L^2 in [6]. We also strengthen the integrability conditions both on the data (ξ, f) and on the solutions, but we do not use the contraction mapping theorem which plays a key role in [6] any longer. Instead, just like the work in [4], we construct a sequence of special BSDEs which have unique solutions in L^2, and then prove that the sequence of their solutions converge in L^p. However, now it is not constants $(|f(t,y,z) - f(t,y,z')| \leq \mu|z - z'|$ and $\langle y - y', f(t,y,z) - f(t,y',z)\rangle \leq \lambda|y - y'|^2)$, but processes $(|f(t,y,z) - f(t,y',z')| \leq \mu(t)|y - y'| + \gamma(t)|z - z'|)$ that control the generator. On the other hand, noting that the maturity of an American claim is bounded in general, we assume the stopping time is bounded in this paper.

The paper is organized as follows. In Section 2, we introduce the assumptions, some notations including some spaces, which are different from the standard spaces used before. In Section 3, some useful a priori estimates are given. The main result of this paper, an existence and uniqueness theorem in L^p, is obtained in Section 4.

2. Preliminaries

2.1. Definition and notations. First of all, $W = \{W_t\}_{t\geq 0}$ is a standard Brownian motion with values in $\mathbb{R}^d$ defined on some complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$. $\mathfrak{F} = \{\mathcal{F}_t\}_{t\geq 0}$ augmented by all $\mathbb{P}$-null-sets is the natural filtration of W, which satisfies the usual conditions.

For convenience in writing and reading, we always consider the space L^{2p} where $p > 1/2$ instead of the space L^p where $p > 1$.

The standard inner product of $\mathbb{R}^m$ is denoted by $\langle\cdot,\cdot\rangle$, the Euclidean norm by $|\cdot|$. A norm on $\mathbb{R}^{m\times d}$ is defined by $\sqrt{\operatorname{tr}(ZZ^*)}$, we will denote this norm by $|\cdot|$ too.

We study the following BSDE:

$$-dY_t = f(t, Y_t, Z_t)dt - Z_t dW_t, \qquad Y_\tau = \xi, \tag{2.1}$$

where τ is a stopping time for filtration $\mathfrak{F}$.

Now we can introduce the appropriate spaces.

Let a be a nonnegative $\mathfrak{F}$-adapted process, we define the increasing continuous process A by

$$A_t = \int_0^t a_s^2 ds. \tag{2.2}$$

For $p > 1/2$ and $\beta > 3$, we set

$$\begin{aligned}
\mathcal{M}^{2p}(\beta, a, \tau, \mathbb{R}^n) &= \Big\{ Y \text{ is progressively measurable; } Y_t \in \mathbb{R}^n; \\
&\qquad \|Y\|_{\mathcal{M}^{2p}}^{2p} = E\Big[\Big(\int_0^\tau e^{\beta A_s} |Y_s|^2 ds\Big)^p\Big] < \infty \Big\}; \\
\mathcal{N}^{2p}(\beta, a, \tau, \mathbb{R}^n) &= \Big\{ Y \text{ is progressively measurable; } Y_t \in \mathbb{R}^n; \\
&\qquad \|Y\|_{\mathcal{N}^{2p}}^{2p} = E\Big[\sup_{0 \le t \le \tau} e^{p\beta A_t} |Y_t|^{2p}\Big] < \infty \Big\}; \\
\mathcal{N}^{2p,a}(\beta, a, \tau, \mathbb{R}^n) &= \Big\{ Y \text{ is progressively measurable; } Y_t \in \mathbb{R}^n; \\
&\qquad \|Y\|_{\mathcal{N}^{2p,a}}^{2p} = E\Big[\int_0^\tau a_s^2 e^{p\beta A_s} |Y_s|^{2p} ds\Big] < \infty \Big\}.
\end{aligned} \tag{2.3}$$

Consequently,

$$\mathcal{B}^{2p}(\beta, a, \tau) = \left(\mathcal{N}^{2p}(\beta, a, \tau, \mathbb{R}^m) \cap \mathcal{N}^{2p,a}(\beta, a, \tau, \mathbb{R}^m)\right) \times \mathcal{M}^{2p}(\beta, a, \tau, \mathbb{R}^{m\times d}) \tag{2.4}$$

is a Banach space with the norm

$$\|(Y,Z)\|_{\mathcal{B}^{2p}}^{2p} = \|Y\|_{\mathcal{N}^{2p}}^{2p} + \|Y\|_{\mathcal{N}^{2p,a}}^{2p} + \|Z\|_{\mathcal{M}^{2p}}^{2p}. \tag{2.5}$$

Now we illustrate what we mean by a solution of the BSDE (2.1) in this paper.

Definition 2.1. A solution of the BSDE (2.1) is a pair of progressively measurable processes (Y,Z) with values in $\mathbb{R}^m \times \mathbb{R}^{m\times d}$ such that on the set $\{t \ge \tau\}$, $Y_t = \xi$ and $Z_t = 0$, $\mathbb{P}$-a.s., $t \mapsto Z_t$ belongs to $L^2_{\text{loc}}(0,\tau)$, $t \mapsto f(t, Y_t, Z_t)$ belongs to $L^1_{\text{loc}}(0,\tau)$, and for all $t \in (0,\tau)$, $\mathbb{P}$-a.s.,

$$Y_t = \xi + \int_{t\wedge\tau}^{\tau} f(s, Y_s, Z_s) ds - \int_{t\wedge\tau}^{\tau} Z_s dW_s. \tag{2.6}$$

Moreover, let $\beta > 0$ and let a be an $\mathfrak{F}$-adapted process, a solution (Y,Z) is said to be an (a,β)-solution of the BSDE (2.1) if $\mathbb{P}$-a.s., $t \mapsto e^{(1/2)\beta A_t}\mathbf{1}_{t\le\tau} f(t, Y_t, Z_t)$ and $t \mapsto a_t^2 e^{(1/2)\beta A_t}\mathbf{1}_{t\le\tau} Y_t$ belong to $L^1_{\text{loc}}(0,\infty)$, $t \mapsto e^{(1/2)\beta A_t}\mathbf{1}_{t\le\tau} Z_t$ belongs to $L^2_{\text{loc}}(0,\infty)$.

For $2p > 1$, a solution is said to be an L^{2p} solution if we have, moreover, $(Y,Z) \in \mathcal{B}^{2p}(\beta, a, \tau)$.

2.2. Assumptions on data (ξ, f). Now we make the following assumptions. For $\beta > 0$,

(A1) τ is a stopping time for the filtration $\mathfrak{F}$ and $\mathbb{P}$-a.s., $\tau \le T < \infty$, where T is a positive constant;

(A2) there are two nonnegative $\mathfrak{F}$-adapted processes $\mu(t)$ and $\gamma(t)$ such that $\forall (y,z,y',z') \in \mathbb{R}^m \times \mathbb{R}^{m\times d} \times \mathbb{R}^m \times \mathbb{R}^{m\times d}$,

$$\left| f(t,y,z) - f(t,y',z') \right| \le \mu(t)|y-y'| + \gamma(t)|z-z'|; \tag{2.7}$$

(A3) $\exists \varepsilon > 0$, $a_t^2 = \mu(t) + \gamma^2(t) \ge \varepsilon$;

(A4) $f(t,0,0)/a_t \in \mathcal{M}^{2p}(\beta, a, \tau, \mathbb{R}^m)$;

(A5) the $\mathbb{R}^k$-valued $\mathcal{F}_\tau$-measurable vector ξ satisfies

$$E\left[e^{p\beta A_\tau}|\xi|^{2p}\right] < \infty; \tag{2.8}$$

(A6) let $L < \infty$ be a positive constant such that

$$\text{for } p \ge 1,\ A_\tau < \infty,\ \mathbb{P}\text{-a.s.}; \quad \text{for } p \in \left(\frac{1}{2}, 1\right),\ A_\tau < L,\ \mathbb{P}\text{-a.s.} \tag{2.9}$$

We refer to (A2) as the stochastic Lipschitz condition.

Lemma 2.2. *For $2p > 1$, if $(Y,Z) \in \mathcal{B}^{2p}(\beta, a, \tau)$ and (A2), (A3), (A4), (A6) hold, then $t \mapsto e^{(1/2)\beta A_t}\mathbf{1}_{t\le\tau} f(t, Y_t, Z_t)$ and $t \mapsto a_t^2 e^{(1/2)\beta A_t}\mathbf{1}_{t\le\tau} Y_t$ belong to $L^1_{\text{loc}}(0,\infty)$, $t \mapsto e^{(1/2)\beta A_t}\mathbf{1}_{t\le\tau} Z_t$ belongs to $L^2_{\text{loc}}(0,\infty)$.*

Proof. It is obvious that $t \mapsto e^{(1/2)\beta A_t}\mathbf{1}_{t\le\tau} Z_t$ belongs to $L^2_{\text{loc}}(0,\infty)$

On the other hand, for $p \in (1/2, 1]$, we have

$$\begin{aligned}\int_0^\tau a_s^2 e^{\beta A_s}\left|Y_s\right|^2 ds &= \int_0^\tau \left(e^{(1-p)\beta A_s}\left|Y_s\right|^{2-2p}\right)\left(a_s^2 e^{p\beta A_s}\left|Y_s\right|^{2p}\right) ds \\ &\le \left(\sup_{0\le t\le\tau} e^{(1-p)\beta A_t}\left|Y_t\right|^{2-2p}\right)\left(\int_0^\tau a_s^2 e^{p\beta A_s}\left|Y_s\right|^{2p} ds\right) < \infty.\end{aligned} \tag{2.10}$$

For $p > 1$, we have

$$\begin{aligned}\int_0^\tau a_s^2 e^{\beta A_s}\left|Y_s\right|^2 ds &= \int_0^\tau \left(a_s^{(2p-2)/p}\right)\left(a_s^{(2/p)} e^{\beta A_s}\left|Y_s\right|^2\right) ds \\ &\le \left(\int_0^\tau a_s^2 ds\right)^{(p-1)/p}\left(\int_0^\tau a_s^2 e^{p\beta A_s}\left|Y_s\right|^{2p} ds\right)^{1/p} < \infty.\end{aligned} \tag{2.11}$$

Now we get that

$$\int_0^\tau a_s^2 e^{\beta A_s}\left|Y_s\right|^2 ds < \infty, \tag{2.12}$$

it follows that

$$\int_0^\tau a_s^2 e^{(1/2)\beta A_s}|Y_s|\,ds \le \left(\int_0^\tau a_s^2 ds\right)^{1/2}\left(\int_0^\tau a_s^2 e^{\beta A_s}|Y_s|^2 ds\right)^{1/2} < \infty. \tag{2.13}$$

From the assumption on f, we obtain that

$$\begin{aligned}
&\int_0^\tau e^{(1/2)\beta A_s}|f(s,Y_s,Z_s)|\,ds \\
&\quad\le \int_0^\tau e^{(1/2)\beta A_s}\big(|f(s,0,0)| + \mu(s)|Y_s| + \gamma(s)|Z_s|\big)ds \\
&\quad\le \left(\int_0^\tau a_s^2 ds\right)^{1/2}\left(\int_0^\tau e^{\beta A_s}\left|\frac{f(s,0,0)}{a_s}\right|^2 ds\right)^{1/2} + \int_0^\tau a_s^2 e^{(1/2)\beta A_s}|Y_s|\,ds \\
&\qquad + \left(\int_0^\tau a_s^2 ds\right)^{1/2}\left(\int_0^\tau e^{\beta A_s}|Z_s|^2 ds\right)^{1/2} < \infty,
\end{aligned} \tag{2.14}$$

the second inequality follows from the fact that $a_t^2 = \mu(t) + \gamma^2(t)$. □

3. A priori estimates

The goal of this section is to study some estimates concerning solutions to the BSDE (2.1). In what follows, we always assume that $2p > 1$.

Firstly, we recall the result of Bender and Kohlmann [6, Theorem 3].

Theorem 3.1. *For $p = 1$, let (A1), (A2), (A3), (A4), and (A5) hold for a sufficient large β. There is a unique pair (Y,Z) in $\mathcal{B}^2(\beta,a,\tau)$ satisfying (2.1).*

Since Theorem 3.1 demands that β is large enough, we can always assume that

$$\beta > \left(\frac{2}{2p-1} \vee 3\right). \tag{3.1}$$

Moreover, letting (A6) holds, by Lemma 2.2, the unique pair (Y,Z) in Theorem 3.1 is an (a,β)-solution of BSDE (2.1). Now we give a basic estimate concerning the solution.

Lemma 3.2. *For $p > 1$ and $\beta > (2/(2p-1) \vee 3)$, assume that (A1), (A2), (A3) hold, let $(Y,Z) \in \mathcal{B}^2(\beta,a,\tau)$ be a solution of BSDE (2.1) and assume that $\mathbb{P}$-a.s.,*

$$\sup_{0\le t\le\tau} e^{(1/2)\beta A_t}\left|\frac{f(t,0,0)}{a_t}\right| \le n, \qquad e^{(1/2)\beta A_\tau}|\xi| \le n, \tag{3.2}$$

then

$$Y \in \mathcal{N}^{2p}(\beta,a,\tau,\mathbb{R}^m) \cap \mathcal{N}^{2p,a}(\beta,a,\tau,\mathbb{R}^m). \tag{3.3}$$

For $p \in (1/2,1)$, moreover, assuming that (A6) holds, then one can reach the same conclusion as the case where $p > 1$.

Proof. Applying Itô's formula to $e^{\beta A_{t\wedge\tau}}|Y_t|^2$, we obtain

$$\begin{aligned}
& e^{\beta A_{t\wedge\tau}}|Y_{t\wedge\tau}|^2+\int_{t\wedge\tau}^{\tau}e^{\beta A_s}\left(|Z_s|^2+\beta a_s^2|Y_s|^2\right)ds \\
&\quad = e^{\beta A_\tau}|\xi|^2+2\int_{t\wedge\tau}^{\tau}e^{\beta A_s}\langle Y_s,f(s,Y_s,Z_s)\rangle ds-2\int_{t\wedge\tau}^{\tau}e^{\beta A_s}\langle Y_s,Z_s dW_s\rangle \\
&\quad \le n^2+\int_{t\wedge\tau}^{\tau}e^{\beta A_s}\left(2|Y_s||f(s,0,0)|+2\mu(s)|Y_s|^2+2\gamma(s)|Y_s||Z_s|\right)ds \\
&\qquad -2\int_{t\wedge\tau}^{\tau}e^{\beta A_s}\langle Y_s,Z_s dW_s\rangle \\
&\quad \le n^2+\int_{t\wedge\tau}^{\tau}e^{\beta A_s}\left(a_s^2|Y_s|^2+\left|\frac{f(s,0,0)}{a_s}\right|^2+\left(2\mu(s)+\gamma^2(s)\right)|Y_s|^2\right)ds \\
&\qquad +\int_{t\wedge\tau}^{\tau}e^{\beta A_s}|Z_s|^2 ds-2\int_{t\wedge\tau}^{\tau}e^{\beta A_s}\langle Y_s,Z_s dW_s\rangle \\
&\quad \le n^2+\int_{t\wedge\tau}^{\tau}e^{\beta A_s}\left(3a_s^2|Y_s|^2+\left|\frac{f(s,0,0)}{a_s}\right|^2\right)ds+\int_{t\wedge\tau}^{\tau}e^{\beta A_s}|Z_s|^2 ds \\
&\qquad -2\int_{t\wedge\tau}^{\tau}e^{\beta A_s}\langle Y_s,Z_s dW_s\rangle.
\end{aligned} \tag{3.4}$$

Thus, it follows that

$$e^{\beta A_{t\wedge\tau}}|Y_{t\wedge\tau}|^2+\int_{t\wedge\tau}^{\tau}(\beta-3)a_s^2e^{\beta A_s}|Y_s|^2 ds\le n^2+n^2T-2\int_{t\wedge\tau}^{\tau}e^{\beta A_s}\langle Y_s,Z_s dW_s\rangle. \tag{3.5}$$

Noting that $\{\int_0^{t\wedge\tau}e^{\beta A_s}\langle Y_s,Z_s dW_s\rangle\}_{t\ge 0}$ is a martingale and taking the conditional expectation with respect to $\mathcal{F}_{t\wedge\tau}$, we have

$$e^{\beta A_{t\wedge\tau}}|Y_{t\wedge\tau}|^2+E\left[\int_{t\wedge\tau}^{\tau}(\beta-3)a_s^2e^{\beta A_s}|Y_s|^2 ds\mid \mathcal{F}_{t\wedge\tau}\right]\le n^2+n^2T. \tag{3.6}$$

Thus, we can conclude that

$$\sup_{0\le t\le\tau}e^{p\beta A_t}|Y_t|^{2p}\le\left(n^2+n^2T\right)^p. \tag{3.7}$$

For $p>1$, we have

$$\begin{aligned}
E\left[\int_0^{\tau}a_s^2e^{p\beta A_s}|Y_s|^{2p}ds\right] &= E\left[\int_0^{\tau}\left(e^{(p-1)\beta A_s}|Y_s|^{2p-2}\right)\left(a_s^2e^{\beta A_s}|Y_s|^2\right)ds\right] \\
&\le E\left[\left(\sup_{0\le t\le\tau}e^{(p-1)\beta A_t}|Y_t|^{2p-2}\right)\left(\int_0^{\tau}a_s^2e^{\beta A_s}|Y_s|^2 ds\right)\right]<\infty,
\end{aligned} \tag{3.8}$$

the last inequality follows from estimates (3.7). Now we have proved the first result.

For $p \in (1/2,1)$, by (A6), we have

$$E\left[\int_0^\tau a_s^2 e^{p\beta A_s}|Y_s|^{2p}ds\right] = E\left[\int_0^\tau (a_s^{2-2p})(a_s^{2p}e^{p\beta A_s}|Y_s|^{2p})ds\right] \\ \leq L^{1-p}\left(E\left[\int_0^\tau a_s^2 e^{\beta A_s}|Y_s|^2 ds\right]\right)^p < \infty, \tag{3.9}$$

the second result follows. □

Now we show how to control the process Z in terms of the data and Y.

Lemma 3.3. *For $2p > 1$ and $\beta > (2/(2p-1) \vee 3)$, let the assumption (A2), (A3), (A4), (A6) hold and let (Y,Z) be an (a,β)-solution of BSDE (2.1). Moreover, assume that $Y \in \mathcal{N}^{2p}(\beta,a,\tau,\mathbb{R}^m) \cap \mathcal{N}^{2p,a}(\beta,a,\tau,\mathbb{R}^m)$, then $Z \in \mathcal{M}^{2p}(\beta,a,\tau,\mathbb{R}^{m\times d})$ and there exists a constant C_p depending only on p such that*

$$\|Z\|_{\mathcal{M}^{2p}}^{2p} \leq C_p\left(\|Y\|_{\mathcal{N}^{2p}}^{2p} + \left\|\frac{f(t,0,0)}{a_t}\right\|_{\mathcal{M}^{2p}}^{2p}\right). \tag{3.10}$$

Proof. For each integer $n \geq 1$, let us introduce the stopping time

$$\tau_n = \inf\left\{t \in [0,\tau] \mid \int_0^t e^{\beta A_s}|Z_s|^2 ds \geq n\right\} \wedge \tau. \tag{3.11}$$

Applying Itô's formula to $e^{\beta A_{t\wedge\tau}}|Y_{t\wedge\tau}|^2$, we obtain that

$$|Y_0|^2 + \int_0^{\tau_n} e^{\beta A_s}|Z_s|^2 ds + \int_0^{\tau_n} \beta a^2(s) e^{\beta A_s}|Y_s|^2 ds \\ = e^{\beta A_{\tau_n}}|Y_{\tau_n}|^2 + 2\int_0^{\tau_n} e^{\beta A_s}\langle Y_s, f(s,Y_s,Z_s)\rangle ds - 2\int_0^{\tau_n} e^{\beta A_s}\langle Y_s, Z_s dW_s\rangle. \tag{3.12}$$

However, from the assumption on f, we get that

$$2|\langle y, f(t,y,z)\rangle| \leq 2|y||f(t,0,0)| + 2\mu(t)|y|^2 + 2\gamma(t)|y||z| \\ \leq \left|\frac{f(t,0,0)}{a_t}\right|^2 + a_t^2|y|^2 + 2\mu(t)|y|^2 + 2\gamma^2(t)|y|^2 + \frac{1}{2}|z|^2 \\ = \left|\frac{f(t,0,0)}{a_t}\right|^2 + 3a_t^2|y|^2 + \frac{1}{2}|z|^2. \tag{3.13}$$

Thanks to the estimate (2.12) in last section, since $\tau_n \leq \tau$ and $\beta > 3$, it follows that

$$\frac{1}{2}\int_0^{\tau_n} e^{\beta A_s}|Z_s|^2 ds \\ \leq \sup_{0\leq t\leq\tau} e^{\beta A_t}|Y_t|^2 + \int_0^\tau e^{\beta A_s}\left|\frac{f(s,0,0)}{a_s}\right|^2 ds + 2\left|\int_0^{\tau_n} e^{\beta A_s}\langle Y_s, Z_s dW_s\rangle\right|. \tag{3.14}$$

Thus,

$$\left(\int_0^{\tau_n} e^{\beta A_s}|Z_s|^2 ds\right)^p \leq c_p\left[\sup_{0\leq t\leq\tau} e^{p\beta A_t}|Y_t|^{2p}+\left(\int_0^{\tau} e^{\beta A_s}\left|\frac{f(s,0,0)}{a_s}\right|^2 ds\right)^p+\left|\int_0^{\tau_n} e^{\beta A_s}\langle Y_s,Z_s dW_s\rangle\right|^p\right]. \tag{3.15}$$

By the Burkholder-Davis-Gundy (BDG) inequality, we get

$$\begin{aligned} c_pE\left[\left|\int_0^{\tau_n} e^{\beta A_s}\langle Y_s,Z_s dW_s\rangle\right|^p\right] &\leq d_pE\left[\left(\int_0^{\tau_n} e^{2\beta A_s}|Y_s|^2|Z_s|^2 ds\right)^{p/2}\right] \\ &\leq d_pE\left[\left(\sup_{0\leq t\leq\tau} e^{(1/2)p\beta A_t}|Y_t|^p\right)\left(\int_0^{\tau_n} e^{\beta A_s}|Z_s|^2 ds\right)^{p/2}\right] \\ &\leq d_pE\left[\sup_{0\leq t\leq\tau} e^{p\beta A_t}|Y_t|^{2p}\right]+\frac{1}{2}E\left[\left(\int_0^{\tau_n} e^{\beta A_s}|Z_s|^2 ds\right)^p\right], \end{aligned} \tag{3.16}$$

where we use the notation d_p for a constant depending on p and whose value could be changing from line to line. Combining this with the estimate of $(\int_0^{\tau_n} e^{\beta A_s}|Z_s|^2 ds)^p$, we get, for each $n>1$,

$$\begin{aligned} E\left[\left(\int_0^{\tau_n} e^{\beta A_s}|Z_s|^2 ds\right)^p\right] &\leq C_p\left(E\left[\sup_{0\leq t\leq\tau} e^{p\beta A_t}|Y_t|^{2p}\right]+E\left[\left(\int_0^{\tau} e^{\beta A_s}\left|\frac{f(s,0,0)}{a_s}\right|^2 ds\right)^p\right]\right) \\ &= C_p\left(\|Y\|_{\mathcal{N}^{2p}}^{2p}+\left\|\frac{f(t,0,0)}{a_t}\right\|_{\mathcal{M}^{2p}}^{2p}\right)<\infty. \end{aligned} \tag{3.17}$$

Letting $n\to\infty$ and using Fatou's lemma, we get that

$$E\left[\left(\int_0^{\tau} e^{\beta A_s}|Z_s|^2 ds\right)^p\right] \leq C_p\left(\|Y\|_{\mathcal{N}^{2p}}^{2p}+\left\|\frac{f(t,0,0)}{a_t}\right\|_{\mathcal{M}^{2p}}^{2p}\right)<\infty. \tag{3.18}$$

So we obtain the result and finish the proof. □

After estimating $\|Z\|_{\mathcal{M}^{2p}}^{2p}$, the next ones we want to estimate are $\|Y\|_{\mathcal{N}^{2p}}^{2p}$ and $\|Y\|_{\mathcal{N}^{2p,a}}^{2p}$. To this end, we recall [4, Corollary 2.3].

Lemma 3.4. *If (Y,Z) is a solution of BSDE (2.1), $2p>1$, $c(p)=p[(2p-1)\wedge 1]$, and $0\le t\le u\le T$, then*

$$\begin{aligned}&|Y_t|^{2p}+c(p)\int_t^u |Y_s|^{2p-2}\mathbf{1}_{Y_s\neq 0}|Z_s|^2ds\\&\quad\le |Y_u|^{2p}+2p\int_t^u |Y_s|^{2p-1}\langle \hat{Y}_s,f(s,Y_s,Z_s)\rangle ds-2p\int_t^u |Y_s|^{2p-1}\langle \hat{Y}_s,Z_s dW_s\rangle,\end{aligned}\tag{3.19}$$

where $\hat{x}=(x/|x|)\mathbf{1}_{x\neq 0}$.

An immediate consequence of Lemma 3.4 is the following result.

Corollary 3.5. *If (Y,Z) is an (a,β)-solution of the BSDE (2.1), $2p>1$, $c(p)=p[(2p-1)\wedge 1]$, and $0\le t\le u\le T$, then*

$$\begin{aligned}&e^{p\beta A_{t\wedge\tau}}|Y_{t\wedge\tau}|^{2p}+c(p)\int_{t\wedge\tau}^{\tau} e^{p\beta A_s}|Y_s|^{2p-2}\mathbf{1}_{Y_s\neq 0}|Z_s|^2ds+\int_{t\wedge\tau}^{\tau} p\beta a_s^2 e^{p\beta A_s}|Y_s|^{2p}ds\\&\quad\le e^{p\beta A_\tau}|\xi|^{2p}+2p\int_{t\wedge\tau}^{\tau} e^{p\beta A_s}|Y_s|^{2p-1}\langle \hat{Y}_s,f(s,Y_s,Z_s)\rangle ds\\&\qquad-2p\int_{t\wedge\tau}^{\tau} e^{p\beta A_s}|Y_s|^{2p-1}\langle \hat{Y}_s,Z_s dW_s\rangle.\end{aligned}\tag{3.20}$$

Proof. Applying Itô's formula to $e^{(1/2)\beta A_t}Y_t$ and letting

$$\overline{Y}_t=e^{(1/2)\beta A_t}Y_t,\qquad \overline{Z}_t=e^{(1/2)\beta A_t}Z_t,\tag{3.21}$$

we get

$$-d\overline{Y}_t=\overline{f}(t,\overline{Y}_t,\overline{Z}_t)dt-\overline{Z}_t dW_t,\qquad \overline{Y}_\tau=\overline{\xi},\tag{3.22}$$

where

$$\overline{\xi}=e^{(1/2)\beta A_\tau}\xi,\qquad \overline{f}(t,y,z)=e^{(1/2)\beta A_t}f(t,e^{-(1/2)\beta A_t}y,e^{-(1/2)\beta A_t}z)-\frac{1}{2}\beta a_t^2 y.\tag{3.23}$$

By Definition 2.1 and Lemma 3.4, we can get the result. □

Now we can give the estimates of $\|Y\|_{\mathcal{N}^{2p}}^{2p}$ and $\|Y\|_{\mathcal{N}^{2p,a}}^{2p}$.

Proposition 3.6. *For $\beta>(2/(2p-1)\vee 3)$, let the assumption (A2), (A3), (A4), (A5), (A6) hold and let (Y,Z) be an (a,β)-solution of BSDE (2.1). Moreover, assume that $Y\in\mathcal{N}^{2p}(\beta,a,\tau,\mathbb{R}^m)\cap\mathcal{N}^{2p,a}(\beta,a,\tau,\mathbb{R}^m)$. Then, for $p>1$, there exists a constant $C_{p,\beta}$ depending only on p and β such that*

$$\|Y\|_{\mathcal{N}^{2p}}^{2p}+\|Y\|_{\mathcal{N}^{2p,a}}^{2p}+\|Z\|_{\mathcal{M}^{2p}}^{2p}\le C_{p,\beta}\left(E[e^{p\beta A_\tau}|\xi|^{2p}]+\left\|\frac{f(t,0,0)}{a_t}\right\|_{\mathcal{M}^{2p}}^{2p}\right);\tag{3.24}$$

for $p\in(1/2,1)$, the estimate (3.24) still holds where the constant $C_{p,\beta}$ is replaced by another constant $C_{p,\beta,L}$ depending only on p, β and L.

Proof. Because of the result of Lemma 3.3, we only need to prove

$$\|Y\|_{\mathcal{N}^{2p}}^{2p}+\|Y\|_{\mathcal{N}^{2p,a}}^{2p}\leq C_{p,\beta}\left(E[e^{p\beta A_\tau}|\xi|^{2p}]+\left\|\frac{f(t,0,0)}{a_t}\right\|_{\mathcal{M}^{2p}}^{2p}\right). \tag{3.25}$$

By Corollary 3.5, we get that

$$\begin{aligned}
&e^{p\beta A_{t\wedge\tau}}|Y_{t\wedge\tau}|^{2p}+c(p)\int_{t\wedge\tau}^{\tau}e^{p\beta A_s}|Y_s|^{2p-2}\mathbf{1}_{Y_s\neq 0}|Z_s|^2ds+\int_{t\wedge\tau}^{\tau}p\beta a_s^2e^{p\beta A_s}|Y_s|^{2p}ds\\
&\quad\leq e^{p\beta A_\tau}|\xi|^{2p}+2p\int_{t\wedge\tau}^{\tau}e^{p\beta A_s}|Y_s|^{2p-1}\langle\hat{Y}_s,f(s,Y_s,Z_s)\rangle ds\\
&\qquad-2p\int_{t\wedge\tau}^{\tau}e^{p\beta A_s}|Y_s|^{2p-1}\langle\hat{Y}_s,Z_sdW_s\rangle\\
&\quad\leq e^{p\beta A_\tau}|\xi|^{2p}+2p\int_{t\wedge\tau}^{\tau}e^{p\beta A_s}(|Y_s|^{2p-1}|f(s,0,0)|+\mu(s)|Y_s|^{2p})ds\\
&\qquad+2p\int_{t\wedge\tau}^{\tau}e^{p\beta A_s}\gamma(s)|Y_s|^{2p-1}|Z_s|ds-2p\int_{t\wedge\tau}^{\tau}e^{p\beta A_s}|Y_s|^{2p-1}\langle\hat{Y}_s,Z_sdW_s\rangle\\
&\quad\leq e^{p\beta A_\tau}|\xi|^{2p}+2p\int_{t\wedge\tau}^{\tau}e^{p\beta A_s}\left(|Y_s|^{2p-1}|f(s,0,0)|+\frac{1}{(2p-1)\wedge 1}a_s^2|Y_s|^{2p}\right)ds\\
&\qquad+\frac{c(p)}{2}\int_{t\wedge\tau}^{\tau}e^{p\beta A_s}|Y_s|^{2p-2}\mathbf{1}_{Y_s\neq 0}|Z_s|^2ds-2p\int_{t\wedge\tau}^{\tau}e^{p\beta A_s}|Y_s|^{2p-1}\langle\hat{Y}_s,Z_sdW_s\rangle.
\end{aligned} \tag{3.26}$$

The last inequality follows from the following one:

$$\begin{aligned}
2pe^{p\beta A_s}\gamma(s)|Y_s|^{2p-1}|Z_s|&=2\left(pe^{(p/2)\beta A_s}\gamma(s)|Y_s|^{p}\right)\left(e^{(p/2)\beta A_s}|Y_s|^{p-1}\mathbf{1}_{Y_s\neq 0}|Z_s|\right)\\
&\leq\frac{2p}{(2p-1)\wedge 1}e^{p\beta A_s}\gamma^2(s)|Y_s|^{2p}+\frac{c(p)}{2}e^{p\beta A_s}|Y_s|^{2p-2}\mathbf{1}_{Y_s\neq 0}|Z_s|^2.
\end{aligned} \tag{3.27}$$

Letting

$$X=e^{p\beta A_\tau}|\xi|^{2p}+2p\int_0^{\tau}e^{p\beta A_s}|Y_s|^{2p-1}|f(s,0,0)|ds, \tag{3.28}$$

since $\beta>(2/(2p-1)\vee 3)\geq 2/((2p-1)\wedge 1)$, we get the inequality

$$\begin{aligned}
&e^{p\beta A_{t\wedge\tau}}|Y_{t\wedge\tau}|^{2p}+\frac{c(p)}{2}\int_{t\wedge\tau}^{\tau}e^{p\beta A_s}|Y_s|^{2p-2}\mathbf{1}_{Y_s\neq 0}|Z_s|^2ds\\
&\qquad+\int_{t\wedge\tau}^{\tau}p\left(\beta-\frac{2}{(2p-1)\wedge 1}\right)a_s^2e^{p\beta A_s}|Y_s|^{2p}ds\\
&\quad\leq X-2p\int_{t\wedge\tau}^{\tau}e^{p\beta A_s}|Y_s|^{2p-1}\langle\hat{Y}_s,Z_sdW_s\rangle.
\end{aligned} \tag{3.29}$$

The BDG inequality implies that $\{M_t = \int_0^{t\wedge\tau} e^{p\beta A_s}|Y_s|^{2p-1}\langle \widehat{Y}_s, Z_s dW_s\rangle\}_{t\geq 0}$ is a uniformly integrable martingale. Indeed, we have by Young's inequality

$$E[\langle M,M\rangle_\tau^{1/2}] \leq E\bigg[\sup_{0\leq t\leq\tau} e^{((2p-1)/2)\beta A_t}|Y_t|^{2p-1}\bigg(\int_0^\tau e^{\beta A_s}|Z_s|^2 ds\bigg)^{1/2}\bigg]$$
$$\leq \frac{2p-1}{2p}E\bigg[\sup_{0\leq t\leq\tau} e^{p\beta A_t}|Y_t|^{2p}\bigg] + \frac{1}{2p}E\bigg[\bigg(\int_0^\tau e^{\beta A_s}|Z_s|^2 ds\bigg)^p\bigg] < \infty, \tag{3.30}$$

the last inequality follows from Lemma 3.3.

Thus, we have

$$\frac{c(p)}{2}E\bigg[\int_{t\wedge\tau}^\tau e^{p\beta A_s}|Y_s|^{2p-2}\mathbf{1}_{Y_s\neq 0}|Z_s|^2 ds\bigg] \leq E[X] \tag{3.31}$$

and by BDG inequality, we get that

$$E\bigg[\sup_{0\leq t\leq\tau} e^{p\beta A_t}|Y_t|^{2p} + \int_0^\tau p\bigg(\beta - \frac{2}{(2p-1)\wedge 1}\bigg)a_s^2 e^{p\beta A_s}|Y_s|^{2p}ds\bigg]$$
$$\leq 2E[X] + k_p E[\langle M,M\rangle_\tau^{1/2}]. \tag{3.32}$$

On the other hand, we have also

$$k_p E[\langle M,M\rangle_\tau^{1/2}] \leq k_p E\bigg[\sup_{0\leq t\leq\tau} e^{(p/2)\beta A_t}|Y_t|^p\bigg(\int_0^\tau e^{p\beta A_s}|Y_s|^{2p-2}\mathbf{1}_{Y_s\neq 0}|Z_s|^2 ds\bigg)^{1/2}\bigg]$$
$$\leq \frac{1}{2}E\bigg[\sup_{0\leq t\leq\tau} e^{p\beta A_t}|Y_t|^{2p}\bigg] + \frac{k_p^2}{2}E\bigg[\int_0^\tau e^{p\beta A_s}|Y_s|^{2p-2}\mathbf{1}_{Y_s\neq 0}|Z_s|^2 ds\bigg]. \tag{3.33}$$

Thus, we obtain

$$E\bigg[\sup_{0\leq t\leq\tau} e^{p\beta A_t}|Y_t|^{2p} + d(p,\beta)\int_0^\tau a_s^2 e^{p\beta A_s}|Y_s|^{2p}ds\bigg] \leq k_p E[X], \tag{3.34}$$

where $d(p,\beta) = p(\beta - (2/(2p-1)\wedge 1))$.

For $p > 1$, let us estimate $E[X]$, then $d(p,\beta) = p(\beta - 2)$ and we have

$$k_p[X] = k_p e^{p\beta A_\tau}|\xi|^{2p} + K_p\int_0^\tau e^{p\beta A_s}|Y_s|^{2p-1}|f(s,0,0)|ds, \tag{3.35}$$

now we estimate the second term of the right-hand side,

$$
\begin{aligned}
K_p & \int_0^\tau e^{p\beta A_s} |Y_s|^{2p-1} |f(s,0,0)| ds \\
& = K_p \int_0^\tau (a_s e^{(p/2)\beta A_s} |Y_s|^p)(e^{((p-1)/2)\beta A_s} |Y_s|^{p-1}) \left(e^{(1/2)\beta A_s} \left| \frac{f(s,0,0)}{a_s} \right| \right) ds \\
& \le p \int_0^\tau a_s^2 e^{p\beta A_s} |Y_s|^{2p} ds + K_p \left(\sup_{0\le t\le\tau} e^{(p-1)\beta A_t} |Y_t|^{2p-2} \right) \left(\int_0^\tau e^{\beta A_s} \left| \frac{f(s,0,0)}{a_s} \right|^2 ds \right) \\
& \le p \int_0^\tau a_s^2 e^{p\beta A_s} |Y_s|^{2p} ds + \frac{1}{2} \sup_{0\le t\le\tau} e^{p\beta A_t} |Y_t|^{2p} + K_p \left(\int_0^\tau e^{\beta A_s} \left| \frac{f(s,0,0)}{a_s} \right|^2 ds \right)^p,
\end{aligned} \tag{3.36}
$$

where we use the notation K_p for a constant depending on p and whose value could be changing from line to line.

Coming back to estimate (3.34), since $\beta > 3$, we get that

$$
\begin{aligned}
& E\left[\sup_{0\le t\le\tau} e^{p\beta A_t} |Y_t|^{2p} + p(\beta-3) \int_0^\tau a_s^2 e^{p\beta A_s} |Y_s|^{2p} ds \right] \\
& \qquad \le K_p E\left[e^{p\beta A_\tau} |\xi|^{2p} + \left(\int_0^\tau e^{\beta A_s} \left| \frac{f(s,0,0)}{a_s} \right|^2 ds \right)^p \right].
\end{aligned} \tag{3.37}
$$

The first result follows easily.

Now we study the case that $p \in (1/2,1)$. Noting that the estimate (3.34) also holds for $p \in (1/2,1)$, we have

$$
E\left[\sup_{0\le t\le\tau} e^{p\beta A_t} |Y_t|^{2p} + d(p,\beta) \int_0^\tau a_s^2 e^{p\beta A_s} |Y_s|^{2p} ds \right] \le k_p E[X], \tag{3.38}
$$

where $d(p,\beta) = p(\beta - 2/(2p-1))$ and $X = e^{p\beta A_\tau} |\xi|^{2p} + 2p \int_0^\tau e^{p\beta A_s} |Y_s|^{2p-1} |f(s,0,0)| ds$.

Just like the proof of the first result, we estimate

$$
K_p \int_0^\tau e^{p\beta A_s} |Y_s|^{2p-1} |f(s,0,0)| ds. \tag{3.39}
$$

Since $p \in (1/2,1)$, we have, $\mathbb{P}$-a.s.,

$$
\begin{aligned}
K_p & \int_0^\tau e^{p\beta A_s} |Y_s|^{2p-1} |f(s,0,0)| ds \\
& = K_p \int_0^\tau \left(a_s^{(2p-1)/p} e^{(2p-1)/2\beta A_s} |Y_s|^{2p-1} \right) \left(a_s^{(1-p)/p} \right) \left(e^{(1/2)\beta A_s} \left| \frac{f(s,0,0)}{a_s} \right| \right) ds \\
& \le \frac{d(p,\beta)}{2} \int_0^\tau a_s^2 e^{p\beta A_s} |Y_s|^{2p} ds + K_{p,\beta} \int_0^\tau (a_s^{2(1-p)}) \left(e^{p\beta A_s} \left| \frac{f(s,0,0)}{a_s} \right|^{2p} \right) ds \\
& \le \frac{d(p,\beta)}{2} \int_0^\tau a_s^2 e^{p\beta A_s} |Y_s|^{2p} ds + K_{p,\beta} \left(\int_0^\tau a_s^2 ds \right)^{1-p} \left(\int_0^\tau e^{\beta A_s} \left| \frac{f(s,0,0)}{a_s} \right|^2 ds \right)^p \\
& \le \frac{d(p,\beta)}{2} \int_0^\tau a_s^2 e^{p\beta A_s} |Y_s|^{2p} ds + K_{p,\beta,L} \left(\int_0^\tau e^{\beta A_s} \left| \frac{f(s,0,0)}{a_s} \right|^2 ds \right)^p.
\end{aligned} \tag{3.40}
$$

Coming back to estimate (3.34), we get

$$E\Big[\sup_{0\le t\le\tau} e^{p\beta A_t}\,|\,Y_t\,|^{2p}+\frac{d(p,\beta)}{2}\int_0^\tau a_s^2 e^{p\beta A_s}\,|\,Y_s\,|^{2p}ds\Big] \\ \le K_{p,\beta,L}E\Big[e^{p\beta A_\tau}|\xi|^{2p}+\Big(\int_0^\tau e^{\beta A_s}\Big|\frac{f(s,0,0)}{a_s}\Big|^2 ds\Big)^p\Big]. \tag{3.41}$$

The second result follows easily. □

4. Existence and uniqueness of a solution

With the help of the above a priori estimates, now we can prove our existence and uniqueness result.

THEOREM 4.1. *For $p>1/2$, let (A1), (A2), (A3), (A4), (A5), and (A6) hold for a sufficient large β, the BSDE (2.1) has a unique solution in $\mathcal{B}^{2p}(\beta,a,\tau)$.*

Proof. Let us start by studying the uniqueness part.

Assuming that (Y,Z) and (Y',Z') are two solutions of BSDE (2.1) in $\mathcal{B}^{2p}(\beta,a,\tau)$, we denote by $(\widetilde{Y},\widetilde{Z})$ the process $(Y-Y',Z-Z')$. It is obvious that $(\widetilde{Y},\widetilde{Z})$ is a solution in $\mathcal{B}^{2p}(\beta,a,\tau)$ to the following BSDE:

$$\widetilde{Y}_{t\wedge\tau}=\int_{t\wedge\tau}^{\tau} h(s,\widetilde{Y}_s,\widetilde{Z}_s)ds-\int_{t\wedge\tau}^{\tau}\widetilde{Z}_s dW_s, \tag{4.1}$$

where h stands for the random function

$$h(t,y,z)=f(t,y+Y_t',z+Z_t')-f(t,Y_t',Z_t'). \tag{4.2}$$

It is easy to verify that BSDE (4.1) satisfies the assumption (A1), (A2), (A3), (A4), (A5), and (A6). Noting that $h(t,0,0)=0$, by Proposition 3.6, we get immediately that $(\widetilde{Y},\widetilde{Z})=0$.

Let us turn to the existence part.

For each $n\ge 1$, let us define $\xi_n=e^{-(1/2)\beta A_\tau}q_n(e^{(1/2)\beta A_\tau}\xi)$ and

$$f_n(t,y,z)=f(t,y,z)-f(t,0,0)+a_te^{-(1/2)\beta A_t}q_n\Big(e^{(1/2)\beta A_t}\Big|\frac{f(t,0,0)}{a_t}\Big|\Big), \tag{4.3}$$

where $q_n(x)=x(n/|x|\vee n)$.

It is easy to show that each pair (ξ_n,f_n) satisfies the condition demanded by Theorem 3.1, then for each $n\ge 1$, the BSDE

$$Y_t^{(n)}=\xi_n+\int_{t\wedge\tau}^{\tau} f_n(s,Y_s^{(n)},Z_s^{(n)})ds-\int_{t\wedge\tau}^{\tau}Z_s^{(n)}dW_s \tag{4.4}$$

has a unique solution in $\mathcal{B}^2(\beta,a,\tau)$. Moreover, according to Lemma 3.2,

$$Y^{(n)}\in\mathcal{N}^{2p}(\beta,a,\tau,\mathbb{R}^m)\cap\mathcal{N}^{2p,a}(\beta,a,\tau,\mathbb{R}^m). \tag{4.5}$$

By Proposition 3.6, for each $(n,k) \in \mathbb{N} \times \mathbb{N}$,

$$||Y^{(n+k)} - Y^{(n)}||_{\mathcal{N}^{2p}}^{2p} + ||Y^{(n+k)} - Y^{(n)}||_{\mathcal{N}^{2p,a}}^{2p} + ||Z^{(n+k)} - Z^{(n)}||_{\mathcal{M}^{2p}}^{2p}$$
$$\leq C_{p,\beta,L} E\Bigg[\left| q_{n+k}(e^{(1/2)\beta A_\tau}\xi) - q_n(e^{(1/2)\beta A_\tau}\xi) \right|^{2p}$$
$$+ \left(\int_0^\tau \left| q_{n+k}\left(e^{(1/2)\beta A_t} \left| \frac{f(t,0,0)}{a_t} \right| \right) - q_n\left(e^{(1/2)\beta A_t} \left| \frac{f(t,0,0)}{a_t} \right| \right) \right|^2 ds \right)^p \Bigg]. \tag{4.6}$$

Since (A4) and (A5) hold, by dominated convergence theorem, we obtain that the right-hand side of the last inequality clearly tends to 0, as $n \to \infty$, uniformly in k, so $(Y^{(n)}, Z^{(n)})$ is a Cauchy sequence in $\mathcal{B}^{2p}(\beta, a, \tau)$. It is easy to pass to the limit in the approximating equation, yielding a solution of the BSDE (2.1) in $\mathcal{B}^{2p}(\beta, a, \tau)$. □

References

[1] É. Pardoux and S. G. Peng, "Adapted solution of a backward stochastic differential equation," *Systems & Control Letters*, vol. 14, no. 1, pp. 55–61, 1990.

[2] N. El Karoui, S. G. Peng, and M. C. Quenez, "Backward stochastic differential equations in finance," *Mathematical Finance*, vol. 7, no. 1, pp. 1–71, 1997.

[3] Ph. Briand and R. Carmona, "BSDEs with polynomial growth generators," *Journal of Applied Mathematics and Stochastic Analysis*, vol. 13, no. 3, pp. 207–238, 2000.

[4] Ph. Briand, B. Delyon, Y. Hu, E. Pardoux, and L. Stoica, "L^p solutions of backward stochastic differential equations," *Stochastic Processes and Their Applications*, vol. 108, no. 1, pp. 109–129, 2003.

[5] N. El Karoui and S.-J. Huang, "A general result of existence and uniqueness of backward stochastic differential equations," in *Backward Stochastic Differential Equations (Paris, 1995-1996)*, vol. 364 of *Pitman Research Notes in Mathematics Series*, pp. 27–36, Longman, Harlow, London, UK, 1997.

[6] C. Bender and M. Kohlmann, "BSDEs with stochastic Lipschitz condition," http://cofe.uni-konstanz.de/Papers/dp00_08.pdf preprint.

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