Journal of Applied Mathematics and Stochastic Analysis 4, Number 2, Summer 1991, 95-110

OSCILLATIONS OF A POLARIZABLE VACUUM[1]

JAMES G. GILSON
School of Mathematical Sciences
Queen Mary College, Mile End Road
London E1 4NS, United Kingdom

ABSTRACT

A classical basis for one-dimensional Schrödinger quantum theory is constructed from simple vacuum polarization harmonic oscillators within standard stochastic theory. The model is constructed on a two-dimensional phase configuration surface with phase velocity vectors that have a speed of light zitterbewegung behaviour character. The system supplies a natural Hermitian scalar product describing probability density which is derived from angular momentum considerations. The generality of the model which is extensive is discussed.

Key words: Schrödinger, quantum, classical, phase, stochastic.

AMS Subject Classification: 81C05, 76C05.

1. INTRODUCTION

In previous papers [1,2,3,4,5,19,20], the author has shown that a mathematical model for a "physical" structure that involves negative mass particles in addition to the usual positive mass particles free to move in an extended configuration space can be used as a basis for the construction of a "classical" theory that fully accounts for the form taken by Schrödinger quantum theory. With this model, the Schrödinger equation becomes deducible from more fundamental and familiar classical assumptions. One of the motivations for this line of investigation has been the conviction that the probabilistic aspects of Schrödinger theory should arise naturally from built in stochastical [22] features of any accounting underlying alternative theory, rather than in the add hoc way in which probability appears in the orthodox theory. In this paper, it will be shown that a correct and convincing stochastic basis can be introduced and woven about the structure that the alternative theory presents. This is achieved by making a

[1]Received: October 1990, Revised: December 1990.

substantial generalization of work [27] done by the author some years ago in an attempt to construct theoretically a massive particle from "contained" random photon motion. It is then shown that this generalization can be used as a more basic and completely sound "probabilistic" foundation for the alternative theory. The particular topic of importance in this context, an erratic zig-zag motion of a photon or light particle probably first mentioned by Dirac [28], and often referred to as zitterbewegung. It shows up in some aspects of the theory of quantized fields [29]. It has appeared in the past to be a process likely to be amenable to some sort of "classical" stochastic explanation similar to Brownian motion and some authors [17,18] have made use of the process in their work on quantum foundations. The work to be described in this article has in a special sense the character of zitterbewegung on the two dimensional surface which is the configuration space of the alternative theory. Related work or work with related motivations can be found in references [8-16,23,25,26].

2. OSCILLATORS

Attention will be confined to one-dimensional quantum theory such as would normally be described by the one-dimensional Schrödinger equation. The main basis for the alternative theory are positive and negative mass or energy monopolar "particle" fields such as normal electrons with their positive rest mass m_0 and negative charge - $|e|$ and more controversially truly "negative" rest mass - m_0 and "positively" charged $|e|$, "negative mass positrons" in constrained motion on a "two" dimensional surface. Thus in the case of electronic polar monopoles, the additional anti-polar particles are not positrons. However, in this theory, these two types of monopole always appear locked together in a balanced dipolar condition of zero rest mass and zero charge so that the theory does not necessarily imply that actual negative mass anti-polar type particles should be detectable as free particles nor indeed does it preclude the existence of such free particles. The actual "nonzero" energy carrying capacity of these "dipoles" depends on kinematic differences of the constituent monopoles such as differences in their velocities. The positive mass monopoles are only free to move in $\pm e$ directions that is parallel with the x-axis. The negative mass monopoles are only free to move in the $\pm e'$ directions that is parallel with the y-axis. The $y = 0$ cross section of the extended configuration space is to be regarded as the normal space of common experience and "measurement" and it is on this cross section that the normal one dimensional Schrödinger equation is recovered from the theory. This scheme gives a means of describing subtle mass polarization processes that can transform the "empty" two dimensional surface into complex patterns of flowing and rotating dipolar vortices instantaneously formed from the basic monopoles. The connection of this polarization picture with the Schrödinger structure has been described in detail in earlier publications [1,2,3,4]. However, the

account of the theory given in this article is self contained and more fundamentally based. Here we wish to concentrate on the instantaneous structure of single dipoles, their oscillator character, their variety of "internal" states and also most importantly on the interaction between dipoles in different internal states. This effectively rebounds the theory at a more fundamental level where only oscillatory processes can take place and, as will be seen, in a special sense only with the velocity of light. The higher level velocity fields in which the theory was originally formulate can then be seen as statistical averages with the alternative theory retaining its original structure. We view the basic two dimensional surface as being embedded in a "three-space". The three-space was originally introduced so that physical properties such as vorticity of the polarization fluid motion and magnetic fields orthogonal to the xy-plane could be "visualized" as vectors normal to the configuration plane. The full value of using the three-space arises from what will be called the primary polarization field as this shares with the vorticity the property of being a vector perpendicular to the xy-plane, the plane of monopolar motions. From this quantity which has the versatile character of being able to represent mechanical, electrical and magnetic characteristics simultaneously, all of the alternative theory is deducible. In this article, the third direction will be more fully utilized in a way which enhances the "explanatory" nature of the theory in its role as a foundation for Schrödinger quantum mechanics. The two-dimensional vacuum oscillators which are here being introduced as a new basis add an internal structure to the dipoles or vacuum polarization units of the original theory. They are formed from a "pair" of oppositely signed "energy" oscillators constructed from the positive and negative mass monopoles which are able to oscillate on the configuration plane in the orthogonal direction e and e'. Thus at the new "lower" level of analysis only small separations of the positive and negative monopoles are important and large displacements or free translations do not contribute to the process. Translatory velocity fields will be recovered later as averages over the oscillatory "fundamental" process. As was the case with earlier work, the now "oscillating" positive mass monopoles are constrained to movement in the $\pm e$ directions whilst the now "oscillating" negative monopoles are constrained to movement in the $\pm e'$ directions. The oscillations that occur are of the "dipole moments" associated with the "planar" polarization in the first instance. Using the electronic rest energy m_0c^2 and a fixed length l_0 which is taken to be the "Compton" wave length divided by 4π we introduce the magnitude of a basic electronic "energy" dipole moment $|\mathcal{M}_{ngy}|$ associated with the mass monopoles and their "planar" freedom of movement as,

$$|\mathcal{M}_{ngy}| = 2m_0c^2l_0, \tag{2.1}$$

where l_0 is taken to have the value $l_0 = \kappa/2m_0c$. Attention will be confined to simulation of the quantum behaviour of systems that have a finite number of discrete non-negative energy levels

denoted by $E_j > 0$, say, where the subscript j ranges over the integer values 1 to n. However, as all the processes associated with this model are a consequence of vacuum polarization this will also imply the parallel existence of a related set of anti-polar energy levels $-E_i$ being involved in the overall system structure. The system being simulated can be thought of as one "pole" of an enlarged embedding construction. even in the orthodox theory there is an embryonic version of such a structure. This takes the form of the complex conjugate ϕ_1^* of an eigenfunction ϕ_i of the time evolution operator with energy E_i being "effectively" an eigenfunction of the time evolution operator with negative energy $-E_i$. More general systems which have continuous as well a discrete eigenvalues are only excluded because of technical complications involved in their description but could be covered by the theory with some elaboration. Thus for each energy level E_j, we associate a configuration vector $q+j$ for e direction polarization and a configuration vector $q-j$ for e' direction polarization under oscillatory conditions which are solutions to the simple harmonic equation $\ddot{q}_{\pm j} = -(E_j/\kappa)^2 q_{\pm j}$ according to the formulae,

$$E_j q_{+j} = 2m_0 c^2 l_0 cos(\phi_j/2)e \tag{2.2}$$

$$E_j q_{-j} = 2m_0 c^2 l_0 sin(\phi_j/2)e', \tag{2.3}$$

the two oscillatory processes involve being taken to be $\pi/2$ radians out of phase and where at the very least the angle ϕ_j depends on abstract time t. With this prescription a "phase-point" vector defined by combining the positive and negative configuration vectors of the oscillators above in the form,

$$\overline{q}_j = (2m_0 c^2 l_0/E_j)(cos(\phi_j/2)e - sin(\phi_j/2)e') \tag{2.4}$$

will execute a circular motion with radius $2m_0 c^2 l_0/E_j$, with angular position given $-\phi_j/2$ and angular velocity $-\partial(\phi_j/2)/\partial t$. A "phase-point" vector defined by,

$$q_j = (2m_0 c^2 l_0/E_j)(cos(\phi_j/2)e + sin(\phi_j/2)e') \tag{2.5}$$

will execute a circular motion with the same radius but with angular position $\phi_j/2$ and with angular velocity $\partial(\phi_j/2)/\partial t$. The two phase-point vectors defined above act like hands of a clock with angular separation ϕ_j and moving contrary wise according to how $\phi_i/2$ depends on abstract time t with their motion representing a fundamental "time-keeping" mechanism associated with the basic monopolar oscillations. The vector $\overline{q}$ always points along the direction of the line of centers of the constituent monopolar pair and so has a definite associated physical image related to the local process. The angular velocities $\omega_j = \partial(\phi_j/2)/\partial t$ will be identified with the state energies by the usual quantum formula,

$$E_j = \kappa\omega_j. \tag{2.6}$$

Thus, in this context, the usual quantum "angular" frequencies have a visual image in terms of the rotation rates of the phase vectors q_j. The velocities associated with the polarization configuration vectors are obtained by partially differentiating them with respect to time t to obtain,

$$\overline{\beta}_j = \partial\overline{q}_j/\partial t = -c(sin(\phi_j/2)e + c\phi(\phi_j/2)e') \tag{2.7}$$

and

$$\beta_j = \partial q_j/\partial t = -c(sin(\phi_j/2)e - cos(\phi_j/2)e'), \tag{2.8}$$

$$= \beta_{+j} + \beta_{-j}, \text{ say.} \tag{2.9}$$

Thus the circular motion described by the "phase" velocities (2.7) and (2.8) takes place at the speed of light and this is true for all the n eigenstate. If the phase motion of the system is measured, the phase points will always be found to have the velocity of light and to be moving in any one of the 2π possible directions on the configuration plane. This feature taken with the statistics associated with the involvement of the various eigenstates constitutes the "zitterbewegung" character of the basic oscillations. The statistical uncertainties associated with the eigenstates will be discussed later. It is intended that the quantum energy states of the system should be seen to be "constructed" out of the polarizability of the hypervacuum of "extra" energy the "vacuum" can hold as a consequence. The polarized condition locally is the result of the movement and generation of the fundamental amount of "mass dipole moment $m_0 l_0$" in various configurations or oscillatory conditions. In general any specific quantum state will have an energy that is not necessarily an integral number of electron rest energies. Thus the monopolar "particles" associated with the quantum states will have monopolar "mass" values given by the formulae

$$\pm M_i = \pm E_i/c^2 = \pm N_{ei} m_0. \tag{2.10}$$

The $\pm$ *sign* of (2.10) depending on the polarity of the state and the N_{ei} or the number of electronic rest masses in the mass of the state i are usually not integers. Thus the quantum states cannot be thought of as just a collection of non-interacting electronic rest energies. We shall assume that given the simultaneous existence of a collection of quantum monopolar mass state $\pm M_i$ for some range of values of i, there will also exist an attraction between any such mass monopole and every other mass monopole of opposite sign in the collection. It is not necessary to specify the precise form that this attractive force takes. However, it is possible to calculate various of its consequences. That it should exist at all is highly plausible given that oppositely

signed simple electronic monopoles are of opposite electronic charge. The dipoles are in turn constructed form the positive and negative mass monopoles and the general pattern of behaviour of the whole system arises from the attraction between any mass monopole and every mass monopole of opposite sign. It is the attraction between these opposite sign fields that leads to the possibility of oscillatory motions of the system at the basic level. The movement of the quantum mass monopoles can be envisaged as a square dance on the configuration plane in which members of the oppositely signed monopolar fields oscillate in pairwise relationships in a motion which can be interpreted and analyzed as rotational in terms of the polarization configuration vectors (2.4) and (2.5). The mutual rotational interaction between the positive and negative monopolar fields generates angular momenta perpendicular to the xy-plane for each of the dancing pairs of eigenstates. The angular momentum induced by this motion will be examined in the next section. In the earlier work, the assumption was made that the vacuum polarization process enabling this model has the symmetry property connection electron mass and electronic charge built into the mass-charge values of the assumed underlying basic electronic monopoles. Here, this same symmetry will be assumed to apply generally to "monopoles" that may consist of more than one or a non-integral number of the basic monopoles. Thus a monopole of mass M will also be considered to carry a charge of magnitude $Q = eM/m_0$. This leads to the important conclusion that each one of the basic 2-dimension oscillators generates a specific amount of magnetic moment orthogonal to the two-space. This can be seen by considering the polarization vector $\overline{q}_j$ from (2.4) which besides pointing in the direction of separation of the two monopolar constituents has a magnitude equal to their separation distance. As the system performs one complete cycle the positive monopole of the pair perform one circular motion around and relative to the negative monopole of the pair at the radial distance $|\overline{q}_j|$. It is to be emphasized that this "circular" motion is relative motion and is possible in spite of the restriction of the monopolar motion on the configuration plane to straight lines parallel with the e, e' directions. Thus we analyze the consequences of this rotatory activity by viewing the motion of positive mass particles as taking place relative to the negative mass particles. If the positive mass monopole M_i carries the charge Q the relative circular motion of the charge Q at the radius $|\overline{q}_j|$ will generate a magnetic dipole moment $\mathcal{M}_{mag}$ given by the usual formula from electrodynamics involving the area of the loop and the current flow in it,

$$\mathcal{M}_{mag} = (\pi q^2 Q/T)k, \tag{2.11}$$

where T is the periodic time for one cycle. q has the value $2m_0c^2l_0/E_i$ using (2.5). The conventional current is anti-clockwise and according to the mass-charge symmetry rule Q has the value $-|e|E_i/m_0c^2$. If the periodic time is given the value $T = 2\pi/\omega = 2\pi\kappa/E_i$, then the

magnetic dipole moment of the loop (2.11) assumes the value,

$$\mathcal{M}_{mag} = l_0 c \mid e \mid k, \tag{2.12}$$

which conveniently is just one Bohr magneton. This is the important contribution to the "primary" magnetic field orthogonal to the configuration plane from the one oscillator or state j in self interaction and this clearly does not depend on j. Each of the n states will give a contribution with the same value as (2.12) arising from the interaction between its positive and negative aspects but there will be many other contribution from pairwise interactions between positive aspects of a state with negative aspects of a different state which take a slightly more complex form than (2.12). However, the argument leading to the value (2.12) for the magnetic moment contributed by one oscillator does supply the clue that it is necessary to examine the motion of the positive mass field "relative" to the negative mass field in order to find the general form for the induced magnetic moment. It seems this is most easily done in terms of a "relative" angular momentum of the positive and negative mass fields, the topic to be examined in the next section.

3. ANGULAR MOMENTUM

The angular momentum orthogonal to the configuration plane induced by the dancing eigenstate pairs can be examined in terms of the vector product combination,

$$a_{ij} = q_{-i}(-M_i) \wedge \beta_{+j} - q_{+j}(M_j) \wedge \beta_{-i}. \tag{3.1}$$

$$= (M_j q_{+j} - M_i q_{-i}) \wedge (\beta_{+j} - \beta_{-i}). \tag{3.2}$$

The "relative" nature of this quantity being evident from the second form (3.2) which involves the velocity of the phase point of the positive monopole from the state j relative to the velocity of the phase point of the negative monopole from state i and the contribution to the mass polarization of those monopoles. This quantity is one contribution to the rotational interaction angular momentum between states i and j. In this situation the mass values $-M_i, M_j$ which appear in the first factors of (3.1), the planar mass dipolar contributions, are not the mass values associated with the positions and velocities β_{+j}, β_{-i} respectively of the moving monopoles as is usually the case in an angular momentum vector product but never the less (3.1) is still an induced angular momentum. The other contribution to the angular momentum interaction between the two state, the angular momentum of the positive monopole from state i relative to the negative monopole from state j, is given by

$$a_{ji} = q_{-j}(-M_j) \wedge \beta_{+i} - q_{+i}(M_i) \wedge \beta_{-j}. \tag{3.3}$$

$$= (M_i q_{+i} - M_j q_{-j}) \wedge (\beta_{+i} - \beta_{-j}). \tag{3.4}$$

Thus there are two contributions to the angular momentum when the two interacting states are different ($i \neq j$) and only the one contribution,

$$a_{ii} = q_{-i}(-M_i) \wedge \beta_{+i} - q_{+i}(M_i) \wedge \beta_{-i}, \tag{3.5}$$

when the positive and negative monopoles sides of a state i are in self interaction. Evaluation of these vector products gives

$$a_{ij} = -2m_0 c l_0 cos((\phi_i - \phi_j)/2)k. \tag{3.6}$$

The self interacting term for the state i is obtained by putting $j = i$ in (3.6) to give,

$$a_{ii} = -2m_0 c l_0 k. \tag{3.7}$$

Comparing this with (2.12), it can be seen that the step from an angular "momentum" contribution to a similarly constituted magnetic moment contribution is just multiplication by $-|e|/2m_0$ which is the "classical" gyromagnetic ration. This observation enables us to infer any magnetic contribution from two interacting states from the corresponding angular momentum contribution or indeed to infer the full magnetic contribution from the full angular momentum contribution. If there is present an assembly of oscillators composed of n_i members in the state E_i for $i = 1$ to n, then the total angular momentum generated by all the n^2 interactions between the oppositely signed fields is

$$a = \Sigma_{ij} n_i n_j a_{ij}$$

$$= -2m_0 c l_0 (\Sigma_i n_i^2 + 2\Sigma_{i<j} n_i n_j cos((\phi_i - \phi_j)/2))k \tag{3.8}$$

$$= -2m_0 c l_0 (\Sigma_i n_i exp(\phi_i/2i))(\Sigma_i n_i exp(-\phi_i/2i))k. \tag{3.9}$$

Inspection of (3.9) suggests that the angular momentum a dividend by $-2c$ can be interpreted as the mass dipole moment orthogonal to the xy-plane or the mass polarization vector orthogonal to the plane and as has been shown is generated by the "rotary" phase or oscillatory polarizations on the plane. This interpretation is suggested in the first instance by the multiplicative dimensioned factors that appear in the expression (3.9). It is likely possible to give a more elaborate justification for such an interpretation in terms of Lorentz forces acting on the electric charges of the monopoles. However, here we shall be content with identifying the existence of the primary mass polarization condition as being indicated by the factors appearing in the formula (3.9). The dimensionless real quantity

$$\rho^{(0)} = (\Sigma_i n_i exp(\phi_i/2i))(\Sigma_i n_i exp(-\phi_i/2i)), \tag{3.10}$$

also appearing in (3.9) is the number of dipoles present each with the fundamental mass dipole moment $m_0 l_0 k$ and the quantity (3.9) divided by $-2c$ and in which it first appears will be called "the primary mass polarization field" in spite of the fact that, in this work, it has been shown to be generated form the motions of the planar polarizations. It was referred to as the primary polarization field in earlier work because from it all the usual quantum mechanical quantities can be derived by simple mathematical operations. Here we are working from a lower level of structure with the quantity regarded as primary earlier having been shown to arise from the basic "speed of light" underlying oscillatory polarization process. It should be noted that the step from the angular moment (3.9) to the primary mass dipole moment in the k direction involves the factor $-2c$ and that the 2 in this factor is a direct consequence of the classical gyromagnetic ratio being operative. The form (3.10) which is to be directly related to the Hermitian quadratic product of orthodox quantum theory arises naturally from the polarizing angular momentum (3.9) and a number of important quantum characteristics can be read off from its structure. It is a product of two quantities one being the complex conjugate of the other and each involves the linear superposition of the eigenfunctions ψ_i. It thus gives a strong indication of how general space dependence will have to be incorporated into the structure particularly when, as intended, the initial real xy-plane representation is replaced by a complex plane representation. Thus if the first factor of this product is taken to be a regular function of a complex variable $z = x + iy$ and consequently has real and imaginary parts satisfying the Cauchy-Riemann equations, the form (3.10) is such that $ln\rho^{(0)}$ will be a solution of the two dimensional Laplace equation. That is to say $ln\rho^{(0)}$ will be a scalar two-space potential while the first factor or mixed wave function will be a complex two-space potential. The quantity $cl_0 ln\rho^{(0)}$ turns out to be the stream function or imaginary part of the complex fluid potential that controls the "average" relative velocity of the basic monopoles. This complex potential is an important quantity exponentially nonlinearly related to the wave function.

In order to define a "hyper" density over the two-surface, it is necessary to introduce a fundamental comparison volume in which all the $\rho^{(0)}$ local dipoles given by (3.9) are considered to reside. If the comparison volume is denoted by v_0 then the dipolar three-density can be defined as,

$$\rho = \rho^{(0)}/v_0, \tag{3.11}$$

whilst the quantity (3.9) can be converted into an angular momentum density by dividing it by the comparison volume v_0. The comparison volume is a dimensioned constant but various other

quantities will have to depend on space and time in order that a Schrödinger field type of situation can be seen to arise from the local space polarization oscillators that have been introduced. In other words, we need to have oscillators of the type just discussed spread over the configuration space according to some density specification and this, of course, is also necessary to make sense of the density function for the oscillators (3.11). To install spacial variability in the structure the assembly numbers n_i will be made spatially dependent according to the following prescription. Each n_i will be factored into a constant part N_i not depending on position or time and a part $n_{ci}(x,y)$ that only depends on space,

$$n_i = N_i n_{ci}(x,y). \tag{3.12}$$

The functional dependence of the $n_{ci}(x,y)$ on the x and y coordinates is not arbitrary but is determined by choosing the wave functions ψ_i to have the special x,y form of dependence of a regular or analytic function $\psi(x+iy,t)$ of the complex variable $z = x+iy$ in those regions of the z-plane of interest. This has the consequence that the real and imaginary parts of each $\psi_i(z,t)$ satisfies the Cauchy-Riemann equations,

$$\partial\psi_{i1}/\partial x = \partial\psi_{i2}/\partial y, \quad \partial\psi_{i1}/\partial y = -\partial\psi_{i2}/\partial x. \tag{3.13}$$

Various physical arguments can be used to justify the introduction of the Cauchy-Riemann equations (3.13) at the level of the average fluid flow that is generated by the statistical basis and in terms of vorticity of velocity fields and dilatational effects. However, here we just recognize that these equations simply facilitate the passage from the real plane representation in which the system has been introduced to the complex plane representation in which the Schrödinger equation is to be recovered. The magnitude $|\psi_i|$ of an eigenstate wave function ψ_i will then be identified as

$$|\psi_i| = n_{ci}(x,y)/l_0. \tag{3.14}$$

Thus here, wave functions will be defined to be inverse lengths. Under this prescription, the formula (3.10) takes the form,

$$\rho^{(0)} = (\Sigma_i N_i n_{ci}(x,y) exp(\phi_i/2i))(\Sigma_i N_i n_{ci}(x,y) exp(-\phi_i/2i)), \tag{3.15}$$

or

$$\rho^{(0)} = l_0^2 \Sigma_i N_i \psi_i))(\Sigma_i N_i \psi_i{}^*), \tag{3.16}$$

or

$$\rho^{(0)} = l_0^2 \psi(x,y,t)\psi^*(x,y,t), \tag{3.17}$$

where ψ is a generalized "mixed" state wave function defined by,

$$\psi(x,y,t) = \Sigma_i N_i \psi_i(x,y,t). \tag{3.18}$$

The usual "one-dimensional" quantum wave eigenfunctions $\psi_i(x,t)$ are to be recovered from,

$$\psi_i(x,y,t) = l_0^{-1} n_{ci}(x,y) exp(\phi_i(x,y,t)/2i) = |\psi_i| exp(\phi_i/2i, \tag{3.19}$$

or

$$\psi_i(z,t) = |\psi_i(z,t)| exp(\phi_i(z,t)/2i \tag{3.20}$$

in terms of the complex variable, when y is set equal to zero. In (3.18), the absolute constants N_i that were factored out of the assembly numbers n_i are now to be seen as the usual linear superposition constants of orthodox quantum theory. The structure that has been set up using the basic set of frequencies n_i is a standard type of stochastic system [24] where quantities such as $\rho^{(0)}$ can be expressed as expectation values of random variables. Certainly two types of expectation value or other statistical measure can be formulated using the probabilistic basis. A linear one, by converting the original assembly numbers n_i into the probabilities $p_i = n_i/N$ or a "quadratic" one by converting the interaction products $n_i n_j$ numbers into the probabilities $p_{ij} = n_i n_j / N_f$ where $N_f = N^2$ and $N = \Sigma_i n_i$. The primary polarization field is formed from electronic mass dipoles with dipole moment $m_0 l_0 k$. Such a dipole can be thought of as a pair of electronic "mass" monopoles separated by a displacement of magnitude l_0 pointing orthogonally away from the xy-plane. Thus generally, if this polarization process is such that charge polarization is always involved with the mass polarization so that one electronic mass m_0 monopole always carries with it one electronic charge $-|e|$ as the symmetry principle implies then, the primary mass polarization will also involve a primary charge "polarization" density field given by,

$$-|e| l_0 (\Sigma_i n_i exp(\phi_i/2i))(\Sigma_i n_i exp(=\phi_i/2i))k/v_0 \tag{3.21}$$

as can be read off from (3.9) and using the density function introduced earlier it is more succinctly expressed as

$$g_{ele} = -|e| l_0 \rho(x,y,t)k, \tag{3.22}$$

with the dipolar mass polarization density vector being expressed as

$$g_{mas} = m_0 l_0 \rho(x,y,t)k. \tag{3.23}$$

The original formula for the induced angular momentum contains the velocity of light in the factor $-2c$. If the conceptual steps from (3.23) to (3.22) through (3.9) are made with the

velocity of light left in or account is taken of the magnetic inductive property of the phase loops currents discussed earlier, we are led to consider the co-existence and contribution of fields like

$$g_{mag} = \pm \mid e \mid cl_0\rho(x,y,t)k \tag{3.24}$$

which represents a density of magnetic dipoles with magnetic pole strength $\mid e \mid c$ or with magnitude of magnetic dipole moment $\mid e \mid cl_0$ which is exactly one Bohr magneton. Thus the system of "rotating" dipole phases in the plane together with their probabilistic weighting seem to generate this orthogonal field of dipoles which are composite in that they are simultaneously electric, magnetic and mass orientated. The various dipolar densities being given by the formulae (3.22), (3.23) and (3.24). The density of magnetic dipoles is the key to the next step as it generates the primary field of "magnetic induction" orthogonal to the xy-plane. The primary magnetic induction field B_0 can be taken to be,

$$B_0 = \mu_0 g_{mag}. \tag{3.25}$$

The quantity (3.9) from which the magnetic indication (3.25) is derived is essentially a statistical quantity depending as it does on the assembly numbers n_i and it can easily be expressed as an expectation value of a random variable by making us of the probabilities p_{ij}. Thus B_0 itself is essentially an average or an expectation value and so quantities derived from B_0 will also be averages. It is possible to derive the average or an expectation value and so quantities derived from B_0 will also be averages. It is possible to derive the average dipolar flow field from B_0 by introducing an "average" dipolar electric current via a Maxwell equation [6] which can be taken to be,

$$\mu_0 J = \nabla \wedge B_0. \tag{3.26}$$

Substituting B_0 into this equation with a restriction to the positive sign in (3.23) gives an "average" electric current density which can be written in the form,

$$J = - \mid e \mid \rho(x,y,t)(s_1(x,y,t)e + s_2(x,y,t)e'). \tag{3.27}$$

The current flow in the e' direction can only involve positively charged monopoles whereas the current flow in the e direction can only involve the negatively charged $- \mid e \mid$ monopoles. Consequently, the e direction component of "velocity" flow u is defined in terms of the e current component directly as $s_1(x,y,t)$ while the e' direction component of "velocity" flow v is defined as $-s_2(x,y,t)$ or of opposite sign to the corresponding term appearing in the current to take account of the e' velocity component carrying the positive charge $\mid e \mid$ [7]. That is to say, the "average" monopolar velocity field given by,

$$\beta = u(x,y,t)e + v(x,y,t)e' = \overline{s}\,, \tag{3.28}$$

where

$$u(x,y,t) = -cl_0 \partial ln\rho(x,y,t)/\partial y \tag{3.29}$$

and

$$v(x,y,t) = -cl_0 \partial ln\rho(x,y,t)/\partial x, \tag{3.30}$$

as follows from the Maxwell equation (3.26). All that is required now to demonstrate that this system is equivalent to one dimensional Schrödinger quantum mechanics is an equation of continuity for the movement of dipole or oscillator density over the configuration surface. This needs to have the form,

$$\partial\rho/\partial t + \nabla \cdot (\rho\beta^{(c)}) = \Gamma, \tag{3.31}$$

involving the source term Γ. The source term is necessary on the configuration plane to take account of external influences on the system behavior. Its form is determined by the external field function on the ordinary space cross section $y = 0$ and itself is generally zero on this cross section so giving the usual conservation of probability on the x axis. The velocity field $\beta^{(c)}$ is the dipolar "centroidal" velocity field,

$$\beta^{(c)} = \beta/2 = (u(x,y,t)e + v(x,y,t))e')/2. \tag{3.32}$$

This is a velocity as would be measured midway between points separated in the k direction and moving with the two component monopolar velocities. The two component velocities can here be thought of as projecting to the same (x,y) position on the plane but separated by the primary polarization length vector $l_0 k$ so that the velocity of the mid-point of the moving dipole is given by (3.32). If the expression for $\beta^{(c)}$ as given by (3.32) through (3.29) and (3.30) is substituted into the continuity equation (3.31), the equation of "centrifugal diffusion" [22],

$$\partial\rho/\partial t = cl_0 \partial^2\rho/\partial x\partial y + \Gamma, \tag{3.33}$$

is obtained. If the expression for ρ given by given (3.16) through (3.11) is substituted into (3.33), some rearrangements are made and $x + ly$ is denoted by z, the analytically [21] continued Schrödinger equation,

$$i\kappa\partial\psi/\partial t = -(\kappa^2/2m_0)\partial^2\psi/\partial z^2 + V(z)\psi, \tag{3.34}$$

is easily derived using the Cauchy-Riemann equations (3.13) and on the real axis when $z = x$, this reduces to the usual one-dimensional Schrödinger equation. The detailed steps involved in

deriving the Schrödinger equation have been given elsewhere [2,3] and so only the briefest outline has been given here.

4. CONCLUSIONS

The higher dimensional "classical" model for vacuum oscillations described in this article completely replicates the type of physical process that is usually described by the one-dimensional Schrödinger equation, its involvement of the imaginary unit, the failure of the orthodox theory to supply any physical picture for the "complex" wave function and of the wave function linearity superposition principle, it seems unlikely that any "classical" model could ever by produced in just the one configuration dimension. In fact, the orthodox view is that "any" classical model is impossible. It has been demonstrated here that a very simple fully satisfactory classical model can be produced in a higher dimensional space based on standard stochastic theory, in which the quantum probability on density Hermitian bilinear form arises naturally and the wave function can be given a clear physical image. The basic vector $\overline{q}$ indicates the direction of separation of the local pair of polarizing monopoles in the real plane and also has the character of the hand of a clock while the argument of $\overline{q}$ has the character of an angular analogue time measure as indicated by the clock hand. The two vectors q and $\overline{q}$ describe a basic zitterbewegung type of process in terms of their velocities of motion always with the speed of light and onto this probabilistic or stochastic activity the quantum simulation process is built. The unitary part $exp(\phi_i/2i)$ of the wave functions ψ_i are simple the complex plane representations of the $\overline{q}_i$ and so the wave functions acquire physical images by association. The implication of the existence of this model and the results derived from it is that the orthodox view of the philosophical significance of the Schrödinger equation may need to be modified. The successful construction of such a simple classical model on a sound stochastic basis suggests that quantum processes may well sound stochastic basis suggest that quantum processes may well indeed involve activities of a physical nature that actually do occur in a higher dimensional space in which the three space of common experience and experiment is imbedded. The model is of substantial generality and, as has been shown, it supplies a complete account of the characteristics of the linear Schrödinger equation with which this article has been concerned and which is essentially conditioned by the equation of continuity (3.31) as a classical precursor. The model is "intrinsically" nonlinear with an exponential type of nonlinearity connecting the argument of the wave function which can be shown to be a complex velocity potential to the space of wave function superpositions but this only shows up partially in the work here in the appearance of the "quadratic" form for the quantum probability density. The intrinsic nonlinearity has the important consequence that the same structure can be used to generate many nonlinear generalizations [19] of the Schrödinger

equation and to simultaneously add explanation for the underlying processes involved in terms of fluid vorticities and local states of compression. Such explanations can be given in terms of nonlinear feedback from the two-dimensional fluid potential into the space of quantum wave functions. These generalizations are simply achieved by replacing the simple equation of continuity (3.31) by other modified forms of continuity equation involving some appropriate form of feedback into the rate of flow or local production of density on the configuration plane with the main structure of the model remaining unaltered.

ACKNOWLEDGEMENTS

I am very grateful to Professor M.S. Bartlett for the remarks that many years ago set me off on the search for a sound stochastic basis for quantum theory and to Professor Sir Michael Atiyah who more recently drew my attention to the central importance of the nonlinear relation between the quantum fluid complex potential and the space of wave function linear superposition.

REFERENCES

[1] Gilson, J.G., "A classical basis for quantum mechanics", *Ann. Inst. Henri Poincaré,* XXXII n^0 4, pp. 319-325, (1980).

[2] Gilson, J.G., "The nature of a quantum state", *Acta Physica Hungarica,* **60** (3-4), pp. 145-160, (1986).

[3] Gilson, J.G., "An alternative view of the Schrödinger Process", *Speculations in Science and Technology,* Vol. **9**, (1986).

[4] Gilson, J.G., "Six-dimensional flow and the three-dimensional Schrödinger equation", *Journal of Mathematical and Physical Sciences,* Vol. **20**, No. 1, pp. 55-63, Feb. (1986).

[5] Gilson, J.G., "A real plane representation of the electrodynamics of the complex Schrödinger configuration plane", *Annals de la Fondation Louis de Broglie,* Vol. **6**, No. 1, (1981).

[6] Cowing, T.G., "*Magnetohydrodynamics*", Adam Hilger, (1976).

[7] Milne-Thomson, "*Theoretical Hydrodynamics*", Macmillan and Co. Ltd., New York, (1949). John Wiley and Sons, Ltd., (1954).

[8] Madelung, E., *Zeits. Für. Phys.* **40**, 332, (1926).

[9] Jánossy, L., *Foundations of Physics* **6**, 341, (1976).

[10] Takabayasi, T., *Progress of Theor. Phys.,* **9**, 187, (1953).

[11] De Broglie, L., "*La Thermodynamique de la Particle Isoleé*", Gauthier-Villars, Paris, (1964).

[12] Jammer, M., "*The Philosophy of Quantum Mechanics*", John Wiley and Sons, New York, (1974).

[13] Peres, A., *Am. J. Phys.*, **54** (8), August, (1986).

[14] Bohm, D., Hiley, B.J., Kaloyeru, P.N., "An Ontological Basis for the Quantum Theory", *Physics Reports*, **144**, No. 6, pp. 323-349, North-Holland, Amsterdam (1987).

[15] Barut, A.D., "Class", *Quantum Gravity* **4**, p. 141, (1987).

[16] Barut, A.D., *International Centre for Theor. Phys.*, Preprint **157**, Triest, (1987).

[17] Cavalleri, G., "*Lettere al Nuovo Cimento*", Vol. **43**, No. 6, 16, Luglio (1985).

[18] Cavalleri, G., Spavieri G., "*Il Nuovo Cimento*", Vol. **95**, B., No. 2, 11, Ottobre (1986).

[19] Gilson, J.G., "Classical Fluid Aspects of Nonlinear Schrödinger Equations and Solutions", *Journal of Applied Mathematics and Simulation,* Vol. 1, Issue 2, (1987).

[20] Gilson, J.G., "Classical Variational Basis for Schrödinger Quantum Theory", *Journal of Applied Mathematics and Simulation,* Vol. 1, Number 4, pp. 287-303, (1988).

[21] Beardon, A.F., "*Complex Analysis*", John Wiley & Sons, Ltd. , (1979).

[22] Gilson, J.G., "Centrifugal Diffusion Negative Mass and the Schrödinger Equation", *Math. Scientist,* **5**, pp. 79-89, (1980).

[23] Gilson, J.G., "On Stochastic Theories of Quantum Mechanics", *Proc. Camb. Phil. Soc.* **64**, pp. 1061-1070, (1968).

[24] Bartlett, M.S., "*An Introduction to Stochastic Processes*", Cambridge University Press, (1966).

[25] Nelson, E., *Phys. Rev.*, **150**, 1079, (1966).

[26] Kershaw, D., *Phys. Rev.*, **136**, 1850, (1964).

[27] Gilson, J.G., "Statistically Relativistic Particle", *J. Appl. Prob.* **4**, pp. 389-396, (1967).

[28] Dirac, P.M., "*The Principles of Quantum Mechanics*", Oxford University Press, (1947).

[29] Thirring, W.E., "*Principles of Quantum Electrodynamics*", Academic Press, Inc., (1958).